# Introduction to Six Sigma Concepts

**What is six sigma**

Six sigma is several things. First, it is a statistical measurement. It tells us how good our products, services and processes really are. The Six sigma method allows us to draw comparisons to other similar or dissimilar products, services and processes. In this manner, we can see how far ahead or behind we are. Most importantly, we see where we need to go and what we must do to get there. In other words, Six sigma helps us to establish, our course and gauge our pace in the race for total customer satisfaction

For example, when we say a process is 6 sigma, we are saying it is Best-in-Class. Such a level of capability will only yield about 3 instances of nonconformance out of every million opportunities for nonconformance. On the other hand, when we say that some other process is 4 Sigma, we are saying it is average. This translates to about 6200 nonconformities per million opportunities for nonconformance. In this sense, the sigma scale of measure provides us with, a "goodness micrometer" for gauging the adequacy of our products, services and processes. Six Sigma as a business strategy can greatly help us to gain competitive edge. The reason for this is very simple – as you improve the sigma rating of a process, the product quality improves and costs go down. Naturally, the customer becomes more satisfied as a result. Let us remember there is no economics of quality. It is always cheaper to do "Right Things, "**Right First Time".**

**WHAT DOES "METRICS" STAND FOR ?**

| | |
|---|---|
| M | Measure |
| E | Everything |
| T | That |
| R | Results |
| I | In |
| C | Customer |
| S | Satisfaction |

**Applicability's of six sigma**

The first step toward improving the sigma capability of a process is defining what the 'customer' expectations are. Next, you "map" the process by which you get the work done to meet those expectations. This means that you create a 'box diagram' of the process flow; I.e.; identifying the steps within the process. With this done, you can now affix success criteria to each of the steps. Next, you would, want to record the number of times each of the given success criteria is not met and calculate the total defects-per-opportunity (TDPO). Following this, the TDPO information is converted to defects-per-opportunity (DPO) which in turn, is translated into, a sigma value (o). Now, you are ready to make direct comparisons – even apples and Oranges if you want.

**Three Sigma vs Six Sigma**

Three Sigma would be equivalent to one misspelled word per 15 pages of text . Six sigma would be equivalent to one misspelled over 300000 pages, quite a difference indeed. Now, let's put this in real world terms. Some corporations are already running Six Sigma. It is self-evident that they're going to perform better over the long haul. For example, several prestigious Japanese Companies(which are doing so well in the World market place) are currently running at or near the 6 sigma Level.

**SIGMA -** (o)

Sigma is a letter in Greek alphabet.
The term "sigma" is used to designate the distribution or spread about the me an (average) of any process or procedure.
The Sigma rating indicates how often defects are likely to occur. The higher the sigma rating, the less likely a process will produce defects. As sigma rating increases, costs go down, cycle time goes down and customer satisfaction goes up.

**QUALITY IMPROVEMENT**
**=**
**PRODUCTIVITY IMPROVEMENT**
**=**
**COST REDUCTION**

**RIGHT FIRST TIME AND EVERY TIME**

**What is a defect**

A defect is any variation of a required characteristic of the product (or its parts) or services which is far enough from its target value to prevent the product from fulfilling the physical and functional requirements of the customer, as viewed through the eyes of your customer.
A defect is also anything that causes the processor or the customer to make adjustments.

## Anything That Dissatisfies Your Customer

**The Common Metric: Defects per Unit (DPU)**

DPU is the best measure of the overall quality of the process.

- DPU is the independent variable.
- Process yields are dependent upon DPU

**Example:**

We checked 500 Purchase Orders (PO) and PO had 10 defects then,

d.p.u = d/u = 10/500 = 0.02 In a P.O. we check for the following :

a) Supplier address/approval.
b) Quantity as per the indent
c) Specifications as per the indent.
d) Delivery requirements
e) Commercial requirements.

Then there are 5 opportunities for the defects to occur. Then, the total no. opportunities =m u = 5x500 = 2500.
Defects per opportunity, d.p.o = d/(m u) = 10/2500 = 0.004
If expressed in terms of d.p.m.o. (defects per million opportunities) it becomes. d.p.m.o. = d.p.o x 106 = 4000 PPM
From d.p.o., we go to the normal distribution tables and calculate ZLT and corrected to ZST by adjusting for shift (1.5 o) then.

ZLT = 2.65; and
ZST = 2.65 + 1.5 = 4.15

No. of opportunities = No. of points checked.

If you don't check some points then it becomes a passive opportunity. We should take only active opportunities into our calculation of d.p.o., and Sigma level.

**Cost / Quality**

Six Sigma has shown that the Highest Quality Producer Is the Lowest Cost Producer

**Process capability process potential index (Cp)**

The greater the design margin, the lower the DPU.
Design Margin is measured by Capability Index (Cp),
Where :

The numerator is controlled by Design Engineer

$$Cp = \frac{\text{Maximum allowable Range of Characteristic}}{\text{Normal Variation of Process}}$$

The denominator is controlled by process Engineering.

**If Ford says,**

Cp should be more than 1.33 for regular production**.**

Cp should be more than or equal to 1.67 for new jobs.

Motorola says, Cp should be more than 2.0 for all jobs.

That implies, (U – L) / 6o = 2.0 or (U – L) = 12o i.e; (U – L) = ± 6o

Hence the name **Six Sigma.**

± 3o Process capability means 0.27%. I.e., 2700 PPM shall be out of specification.

± 6o Process capability shall mean 2.5 Parts per Billion, shall be outside the specification limits.

The six Sigma Methodology is a five phase improvement cycle that are employed in a project oriented fashion through the

1. Define
2. Measure
3. Analyze
4. Improve
5. Control

**Step 1 : Define :**

Define The Customer, Critical to quality (CTQ) issues, And the Core business Process involved. Define who customers are, what their requirements are and what their expectations are. Define Project boundaries – the start and stop of the process. Define the process to be improved by mapping the process flow.

**Step 2 : Measure :**

Develop a data collection plan for the process. Collect data, to determine types of defects and metrics.
Measure the current performance of the core business process involved.

**Step 3 : Analyze :**

Analyze the data collected to determine the root causes of defects and opportunities for improvement. Identify the gaps between current performance and goal performance. Prioritize opportunities to improve. Identify sources of variation.

**Step 4 : Improve :**

Improve target solutions by designing creative solutions to fix and prevent problems. Create innovative solutions using technology and discipline. Develop and deploy implementation plan.

**Step 5 : Control :**

Control the improvements to keep the process on the new course. Prevent reverting back to the "old way" Control the development, documentation and Implementation of an ongoing monitoring plan.

**Step 1 : Define:**

The Problem definition has five major elements. The Business Case.
Identifying the Customers of the project, their needs & requirements.
The problem statement. Project Scope. Goals & Objectives.

**Step 2 : Measure**

**Calculating Sigma Value for discrete data**

The data being collected for this project is discrete, to calculate sigma using the discrete method, there are three items being measured. They are :

**1. Unit** : The item produced.
**2. Defect** : Any event that does not meet customer's requirement.
**3. Opportunity** : A chance for a defect to occur.

**Step 2 : Measure**

The Formula to Calculate DPO.

$$\textbf{DPO} = \frac{\text{Number of Defects}}{\text{Number of opportunities x Number of units produced}}$$

**DPMO** = DPO x 1000000
This calculation is called defects per Million Opportunities.

**Step 2 : Measure**

Performance Measures

**For the Month of December** :

| | | |
|---|---|---|
| Total Number of rings produced | = | 86,702 |
| Number of Defective Rings | = | 47 |
| Number of Opportunities | = | 4 opportunities |
| Defects per Opportunity | = | 47/ 86,702 * 4 |
| | = | 1.355 * 10 – 4 |
| Defects per Opportunity | = | 47/ 86,702 * 4 |
| | = | 1.355 * 10 - 4 |
| Defects / Million Opportunities | = | 1.355 * 10 – 4 * 106 |
| | = | 135.55 DPMO |
| Converting DPMO to "o" value | = | 5.1 o |

**Performance Measure**

| Month | Number of Rings produced | Number of Defective Rings | DPO | DPMO | Sigma Value |
|---|---|---|---|---|---|
| December 2000 | 86,702 | 47 | 0.0001355 | 135.55 | 5.1 |
| January 2001 | 1,13,345 | 100 | 0.0002145 | 214.5 | 5.0 |
| February 2001 | 1,14,368 | 123 | 0.0002688 | 268.88 | 4.9 |
| March 2001 | 1,14,404 | 451 | 0.0009855 | 984.54 | 4.5 |

**The Practical Meaning of**

- 99% Good (3.8σ)
- 20,000 lost articles of mail/hr
- Unsafe drinking water 15 minutes / day.
- 5000 incorrect surgical procedure / week.
- 2 short or long landing at airport / day.
- 200,000 wrong drug prescriptions / year.
- No electricity for almost 7 hr/month

- 99.99966% Good (6σ)
- 7 articles lost / hr
- Unsafe drinking water 1 minutes / seven months.
- 1.7 incorrect surgical procedure / week.
- 1 short or long landing airport /
- 5 years
- 68 wrong drug prescriptions / year.
- One hour with out electricity / 34 years

**SIX SIGMA** o **AS A GOAL (Distribution shifted to ± 1,5**o )

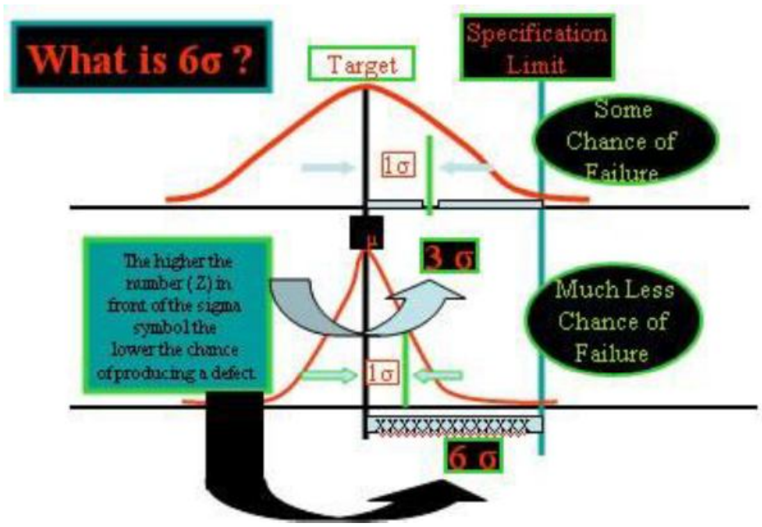
| Sigma Level | Defects in PPM | Yield in % |
|---|---|---|
| 2o | 308,538 | 69.1462 |
| 3o | 66,807 | 98.3198 |
| 4o | 6210 | 99.3790 |
| 5o | 233 | 99.9767 |
| 6o | 3.4 | 99.99966 |

## Legends

| | |
|---|---|
| "m" | : Number of Opportunities. |
| "N" | : Number of parts. |
| "d" | : Number of defects. |
| "dpu" | : Defects per unit. |
| "dpo" | : Defects per opportunity. |
| "Yft" | : First time yield. |
| "Yrt" | : Rolled thru put yield. |
| "dpmo" | : Defects per million opportunities. |
| "TDPU" | : Total defects per unit. |
| "Zlt" | : Long term sigma level |
| "Zst" | : Short term sigma level |

## Formulae

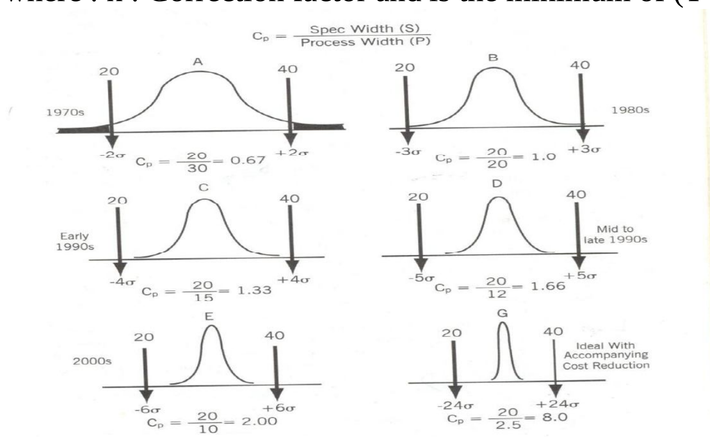
"dpu" = d / N
"dpo" = dpu/m
"Yft" = e-dpu
"dpmo" = dpo X 106
"TDPU" = sum of dpu
"Yrt" = e-TDPU
"YPO" = Yrt 1/m = e-dpo
"dpo" of the over all process = (1-Ypo)
Cpk = Zlt / 3
Z = (USL – X bar)/o
Cp = Zst/3



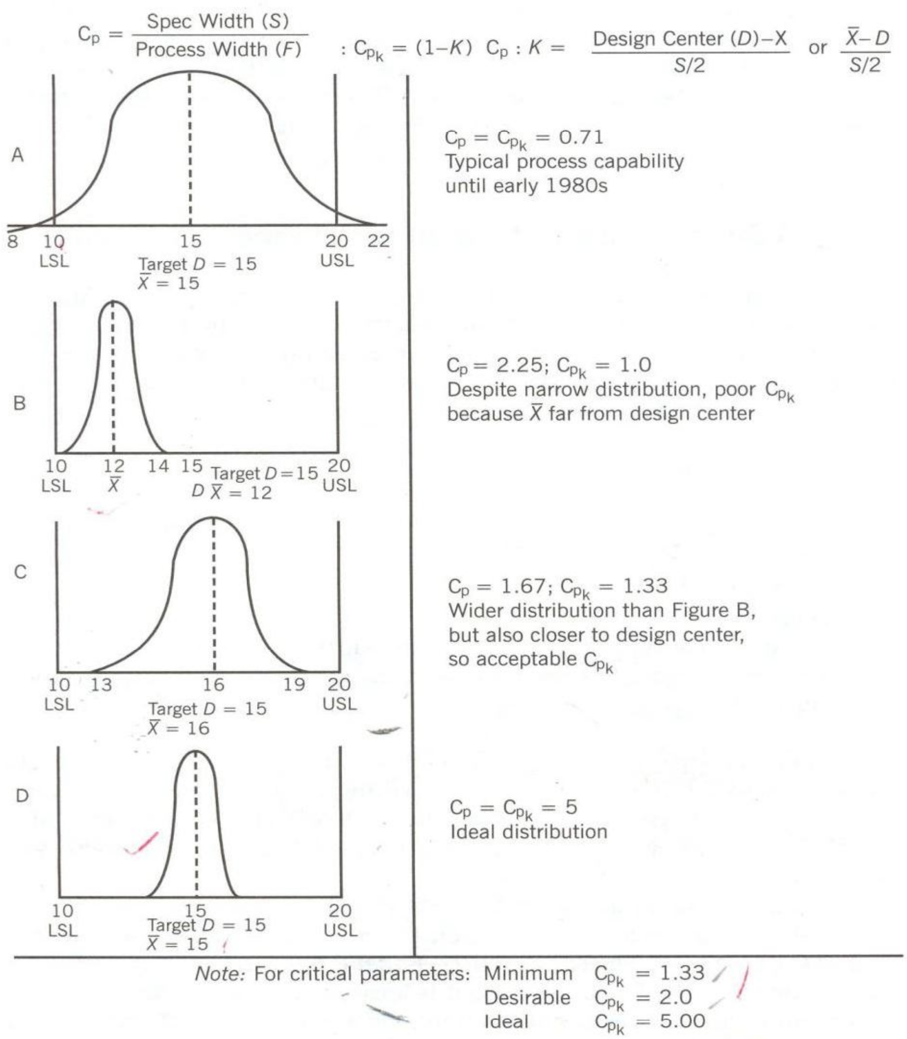

$u_i - u$

**Process Capability**
It is a measure of the inherent uniformity of the process. Before examining the sources and causes of variation and their reduction we must measure the variation.
YARD-STICKS OF PROCESS CONTROL:
Cp - measuring capability of a process
Cpk - Capability of process, but corrected for non-centering

**Process Capability Indices:**
Cp is a measure of spread.
Cp = Specification (S) / Process width (P)
Cpk is a measure of centering the process and its spread.
Cpk is minimum of Cpu= (USL - μ) / 3σ and Cpl = (μ - LSL) / 3σ
The relationship between Cp and Cpk is Cpk = (1 – k) Cp
where : k : Correction factor and is the minimum of (T -μ ) / S / 2 or (μ - T ) / S / 2

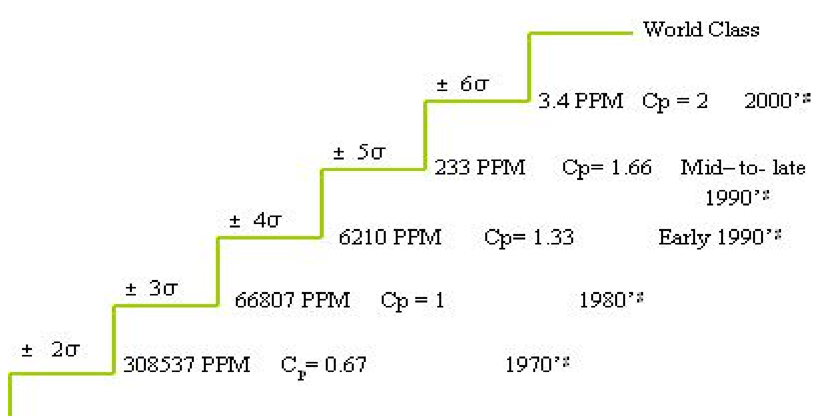


**Cp - Measure of variation**




**Process Capability**

**Sigma Levels and the Associated CP & Defects**

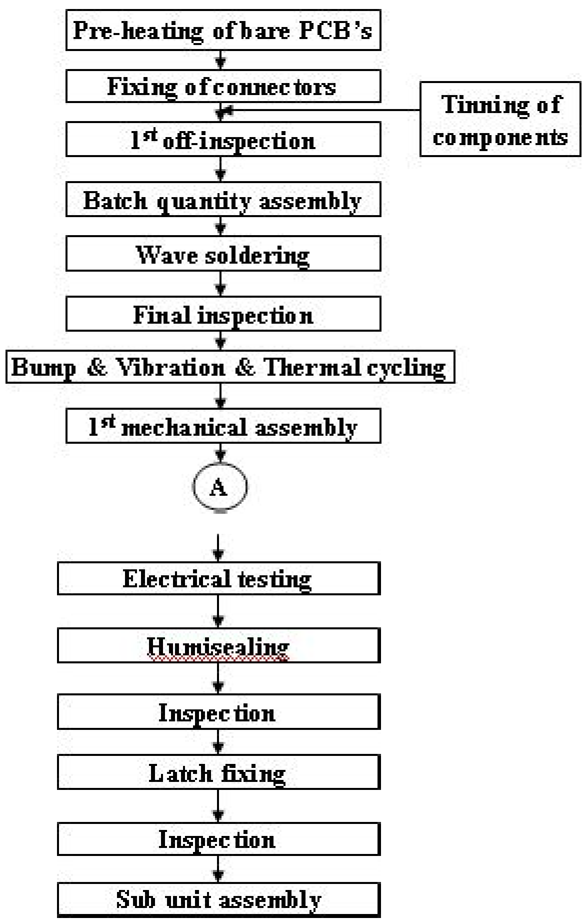


# CASE STUDY TO A REAL LIFE PROBLEM

In a company manufacturing and assembling of Printed Circuit Boards(PCB's),the rejection rate was found to be very high. Upon study it was noticed that there are 16 stages in the assembly process of Printed Wiring Boards(PWB's),out of which the rejection rate was more during the wave soldering process compared to the other stages.

This wave soldering process stage is a critical stage of assembly.

Figure: The assembly process



Hence Wave Soldering Process Stage was selected for the study, in order to reduce the process variability and to minimize the rejection rate. The PCB's are classified as single layered, bi-layered and multi-layered boards. In the assembly section of this company, two types of PCB's are being assembled.

On-line inspection data for the wave soldering process were collected and the attribute control charts (p and c) were plotted which showed that the process was not in a state of statistical control.

The fraction rejection was found to be 0.2 (i.e., 20%) and the average defects per unit were 1.67 for multi-layered boards and 0.5 for bi-layered boards respectively.

The on-line inspection data for wave soldering process with the existing process parameter values was collected and sigma($\sigma$) was calculated.

For bi-layered boards the sigma level was 3.39 and for multi-layered boards the sigma was 3.33,which are given below.

**The Wave Soldering Process**

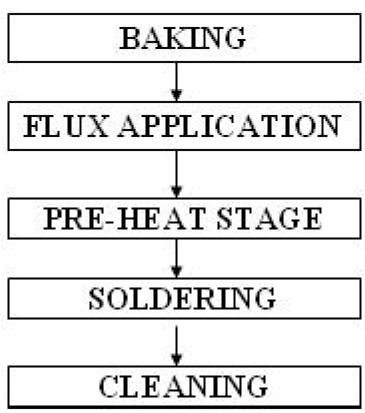



**Table 1. Product Type: Bi-layered Boards**

| Standard Process Parameters | |
|---|---|
| Baking Temperature | **75˚C** |
| Preheat Temperature | **300˚C** |
| Hot-air Temperature | **320˚C** |
| Solder Temperature | **245˚C** |
| Solder wave height | **11mm** |

On-line Data for Bi-layered Boards- calculation

Number of defects = 71

Total no. of soldering points = 233287

Defects per opportunity =

$$\frac{\text{Total no. of defects}}{\text{Total no. of soldering points}} = \frac{71}{233287} \quad \ldots(1)$$

= 0.000304

From the Normal Tables ,the value of sigma is 3.39

**Table 2. Product Type: Multi-layered Boards**

| Standard Process Parameters | |
|---|---|
| Baking Temperature | 75˚C |
| Preheat Temperature | 320˚C |
| Hot-air Temperature | 340˚C |
| Solder Temperature | 255˚C |
| Solder wave height | 12.5mm |

On-line Data for Multi-layered Boards- calculation

Number of defects = 38

Total no. of soldering points = 106828

Defects per opportunity =

$$\frac{\text{Total no. of defects}}{\text{Total no. of soldering points}} = \frac{38}{106828} \quad \ldots(2)$$

= 0.00036

From the Normal Tables, the value of sigma is 3.33

After conducting the Brain storming session with the operators, foreman and the manager, the causes for the rejection of the PWA's were traced out. During the inspection of the wave soldered PWA's, it was found that the rejections were due to the following causes.

Measles
Blow holes
Solder bridge
Solder splash and
Icicles

The necessary calculations are made for both Bi-layered and Multi-layered boards and analysis was carried out for the collected data with the help of Pareto Diagram which showed that blow holes and solder bridges constituted for majority of rejection.

After discussions with the operators, foreman and manager, the causes for the blow holes and solder bridges were identified.

The **cause and effect** diagrams were drawn and critical process parameters (control factors) that influence the wave soldering process were identified as:

Baking Temperature
Pre-heat Temperature
Hot-air Temperature
Solder Temperature
Solder wave height

Two noise factors viz., ambient temperature and humidity, each with two levels are considered for the experimentation. In order to optimize the above identified wave soldering process parameters, the Orthogonal Array Approach of DOE was applied. Three levels were fixed for each of the above five critical factors which are shown in Tables 3 and 4 for both bi-layered and multi-layered boards respectively. With the application of the Linear Graphs the number of experiments to be conducted are 27 for the factors. OA Table and physical layout for the Bi-layered and Multi-layered boards are prepared. 27 experiments were carried out for both Bi-layered and Multi-layered PCB's separately with a sample size of two each.

**Table 3.Factors and Levels for Bi-layered Board**

| FACTORS | LEVEL 1 | LEVEL 2 | LEVEL 3 |
|---|---|---|---|
| Banking Temperature | 75˚C | 80˚C | 85˚C |
| Preheat Temperature | 300˚C | 305˚C | 310˚C |
| Hot air Temperature | 320˚C | 325˚C | 330˚C |
| Solder Temperature | 240˚C | 245˚C | 250˚C |
| Solder wave height | 10mm | 11mm | 12mm |

**Table 3. Factors and Levels for Bi-layered Board**

| FACTORS | LEVEL 1 | LEVEL 2 | LEVEL 3 |
|---|---|---|---|
| Banking Temperature | 75˚C | 80˚C | 85˚C |
| Preheat Temperature | 320˚C | 325˚C | 330˚C |
| Hot air Temperature | 340˚C | 345˚C | 350˚C |
| Solder Temperature | 250˚C | 255˚C | 260˚C |
| Solder wave height | 11.5mm | 12.5mm | 13.5mm |

## Analysis of Data and Results

The experimental results were analyzed to establish the optimum process parameter values for Baking Temperature, Pre-heat Temperature, Hot-air Temperature, Solder Temperature and Solder wave height. The responses were calculated for each of the experiments for both bi-layered and multi-layered PCB's. From the response matrices, the SIGNAL-TO-NOISE (S/N) ratios were calculated using the formula :

$y = 10 \log ((1/p) - 1)$ ……(3)

where $p = 1 - \%$ good …….(4)

For example ,

$p = 1 - (25/100) = 75$

$y = 10 \log ((1/0.3) - 1) = -4.7712$

The S/N ratios for each of the experiments were calculated for bi-layered and multi-layered boards. The analysis of variance (ANOVA) was carried out for bi-layered and multi-layered boards. The optimal levels of parameters were established based on the highest value of S/N ratios. Optimized Factor Level for the wave soldering process for both bi-layered and multi-layered PCB's are given below

| FACTORS | Bi-layered | Multi-layered |
|---|---|---|
| Banking Temperature | Level-3 85˚C | Level-3 85˚C |
| Preheat Temperature | Level-3 310˚C | Level-3 330˚C |
| Hot air Temperature | Level-3 330˚C | Level-3 260˚C |
| Solder Temperature | Level-3 250˚C | Level-3 260˚C |
| Solder wave height | Level-2 1.5mm | Level-3 2.5mm |

## Confirmation Run

Further experiments were carried out with the optimized levels of the above parameters for both bi-layered and multi-layered PCB's taking a sample size of 8 each to check the validity of the levels of the optimized parameters.

The sigma levels were calculated again for the data collected and were found to be 4.1 for bi-layered and 4.125 for multi-layered PCB's which shows that the process variability is decreased and process capability(cp and cpk) is increased. The percentage of rejections was again calculated which is reduced to 0.2% from 20%.

**Conclusions**

In this case study, both on-line quality control techniques like control charts, pareto diagram and cause and effect diagram as well as off-line quality control techniques are applied before the manufacture of the product to control the process. The sigma level for bi-layered PCB was improved from 3.39sigma to 4.1sigma and the multi-layered PCB from 3.33sigma to 4.125sigma.

Since the sigma levels were increased considerably using the Orthogonal approach of DOE, it is evident that the application of the DOE technique (during the early stages itself) is very effective in improving the quality of any process or product by optimizing the parameters in order to yield a product which can be produced with minimum cost and with minimum variation. The optimal levels for the factors obtained using OA approach and levels of sigma for both bi-layered and multi-layered PCB's are summarized below:

**Table 5: Comparison of the levels of five parameters for bi-layered PWS and multi-layered PWS**

| FACTORS | Bi-layered PWA's | | Multi-layered PWA's | |
|---|---|---|---|---|
| | Present | Optimum | Present | Optimum |
| Baking Temperature | 75˚C | 85˚C | 75˚C | 85˚C |
| Pre-heat Temperature | 300˚C | 310˚C | 320˚C | 330˚C |
| Hot-air Temperature | 320˚C | 330˚C | 340˚C | 345˚C |
| Solder Temperature | 245˚C | 250˚C | 255˚C | 260˚C |
| Solder Wave height | 11mm | 11mm | 12.5mm | 13.5mm |

**Table 6. Comparison of the Sigma Levels**

| Type of Printed Wiring Assembly | Present Level of Sigma | Improved Level of Sigma |
|---|---|---|
| Bi-layered | 3.39 | 4.1 |
| Multi-layered | 3.33 | 4.125 |

With lesser number of experiments in Orthogonal Array approach of DOE, it is possible to achieve the same effective results as compared to other techniques of DOE like Full Factorial, Fractional Factorial, Randomized Block Design etc.

**Orthogonal Array Table**

| Experiment Number | Baking Tempt. | Preheat Tempt. | Hot-air Tempt. | Solder Tempt. | Solder Wave Height |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 1 | 2 |
| 3 | 1 | 1 | 1 | 1 | 3 |
| 4 | 1 | 2 | 2 | 2 | 1 |
| 5 | 1 | 2 | 2 | 2 | 2 |
| 6 | 1 | 2 | 2 | 2 | 3 |
| 7 | 1 | 3 | 3 | 3 | 1 |
| 8 | 1 | 3 | 3 | 3 | 2 |
| 9 | 1 | 3 | 3 | 3 | 3 |
| 10 | 2 | 1 | 2 | 3 | 1 |
| 11 | 2 | 1 | 2 | 3 | 2 |
| 12 | 2 | 1 | 2 | 3 | 3 |
| 13 | 2 | 2 | 3 | 1 | 1 |
| 14 | 2 | 2 | 3 | 1 | 2 |
| 15 | 2 | 2 | 3 | 1 | 3 |
| 16 | 2 | 3 | 1 | 2 | 1 |
| 17 | 2 | 3 | 1 | 2 | 2 |
| 18 | 2 | 3 | 1 | 2 | 3 |
| 19 | 3 | 1 | 3 | 2 | 1 |
| 20 | 3 | 1 | 3 | 2 | 2 |
| 21 | 3 | 1 | 3 | 2 | 3 |
| 22 | 3 | 2 | 1 | 1 | 1 |
| 23 | 3 | 2 | 1 | 1 | 2 |
| 24 | 3 | 2 | 1 | 1 | 3 |
| 25 | 3 | 3 | 2 | 3 | 1 |
| 26 | 3 | 3 | 2 | 3 | 2 |
| 27 | 3 | 3 | 2 | 3 | 3 |

**Physical Layout For Bi-layered Boards**

| Experiment Number | Baking Tempt. | Preheat Tempt. | Hot-air Tempt. | Solder Tempt. | Solder Wave Height |
|---|---|---|---|---|---|
| 1 | 75˚C | 300˚C | 320˚C | 240˚C | 10mm |
| 2 | 75˚C | 300˚C | 320˚C | 240˚C | 11mm |
| 3 | 75˚C | 300˚C | 320˚C | 240˚C | 12mm |
| 4 | 75˚C | 305˚C | 325˚C | 245˚C | 10mm |
| 5 | 75˚C | 305˚C | 325˚C | 245˚C | 11mm |
| 6 | 75˚C | 305˚C | 325˚C | 245˚C | 12mm |
| 7 | 75˚C | 310˚C | 330˚C | 250˚C | 10mm |
| 8 | 75˚C | 310˚C | 330˚C | 250˚C | 11mm |
| 9 | 75˚C | 310˚C | 330˚C | 250˚C | 12mm |
| 10 | 80˚C | 300˚C | 325˚C | 250˚C | 10mm |
| 11 | 80˚C | 300˚C | 325˚C | 250˚C | 11mm |
| 12 | 80˚C | 300˚C | 325˚C | 250˚C | 12mm |
| 13 | 80˚C | 305˚C | 330˚C | 240˚C | 10mm |
| 14 | 80˚C | 305˚C | 330˚C | 240˚C | 11mm |
| 15 | 80˚C | 305˚C | 330˚C | 245˚C | 12mm |
| 16 | 80˚C | 310˚C | 320˚C | 245˚C | 10mm |
| 17 | 80˚C | 310˚C | 320˚C | 245˚C | 11mm |
| 18 | 85˚C | 310˚C | 320˚C | 245˚C | 12mm |
| 19 | 85˚C | 300˚C | 330˚C | 245˚C | 10mm |
| 20 | 85˚C | 300˚C | 330˚C | 245˚C | 11mm |
| 21 | 85˚C | 300˚C | 330˚C | 245˚C | 12mm |
| 22 | 85˚C | 305˚C | 320˚C | 250˚C | 10mm |
| 23 | 85˚C | 305˚C | 320˚C | 250˚C | 11mm |
| 24 | 85˚C | 305˚C | 320˚C | 250˚C | 12mm |
| 25 | 85˚C | 310˚C | 325˚C | 240˚C | 10mm |
| 26 | 85˚C | 310˚C | 325˚C | 240˚C | 11mm |
| 27 | 85˚C | 310˚C | 325˚C | 240˚C | 12mm |

**Conclusion**

In most of the Indian industries, the acceptance criterion is only on the basis of specification limits specified by the designer. If any characteristic of a product / process falls between the specified limits, it is taken for granted that the product is uniformly good. But as per Taguchi's QLF, as the functional characteristic of a product deviates from the target value, it causes loss to the society.

The more the deviation, the more is the loss, even if it is within the specified limits. Robust engineering methods are recommended at the early stages of product design to achieve the higher sigma levels. Robust engineering also reduces the time to market with the help of two step optimization.

The results obtained from small scale laboratory experiments can be repeated under the large scale manufacturing conditions if the output characteristics are selected appropriately using S / N ratio.

## Introduction to six sigma

**Problem 1**

A press brake is set up to produce a formed part to a dimension of 3" ± 0.005". A process study reveals that the process limits are at 3.002" ± 0.006", i.e., at a minimum of 2.996" and a maximum of 3.008". After corrective action, the process limits are brought under control to 3.001" ± 002".

**Question:1**

Question 1. Calculate the Cp and Cpk of the old process.
Question 2. Calculate the Cp and Cpk of the corrected process

**Answers:**
Question 1.

specification with (s) = 0.010"; process width (p) = 0.012"

So Cp= S/P = 0.10/0.012 = 0.833
= 3.002"; design center (D) = 3.000"

$$SoK = \frac{\overline{X} - D}{S/2} = \frac{3.002 - 3.000}{0.005} = \frac{0.002}{0.005} = 0.4$$

**Question 2.**

Specification width (s) = 0.010"; process width (p) = 0.004".

Therefore Cpk = (1-0.4)0.833 = 0.5

So Cp= S/P = 0.10/0.004 = 2.5
= 3.001"; design center (D) = 3.000"

$$SoK = \frac{\overline{X} - D}{S/2} = \frac{3.001 - 3.000}{0.005} = \frac{0.001}{0.005} = 0.2$$

Therefore Cpk = (1-k) Cp = (1-0.2)2.5 = 2.0
Using the simpler and alternate formula for Cpk:

$$In\ question\ 1: C_{pk} = \frac{3.005 - 3.002}{0.006} = \frac{0.003}{0.006} = 0.5$$

$$In\ question\ 2: C_{pk} = \frac{3.005 - 3.001}{0.002} = \frac{0.004}{0.002} = 2.0$$

# <u>Control charts for attributes</u>

**Types of control chart**

There are two types of control charts that we deal with.

- Variables Control Charts

  These charts are applied to data that follow a continuous distribution.

- Attributes Control Charts

  These charts are applied to data that follow a discrete distribution.

**Attributes Data**

Data that can be classified into one of several categories or classifications is known as attribute data.

Classifications such as <u>conforming</u> and <u>nonconforming</u> are commonly used in quality control.

Another example of attributes data is the count of defects

**Types of attributes control chart:**

***p* chart**
This chart shows the fraction of nonconforming or defective product produced by a manufacturing process.
It is also called the control chart for fraction nonconforming.

***np* chart**
This chart shows the number of nonconforming. Almost the same as the *p* chart.

***c* chart**
This shows the number of defects or nonconformities produced by a manufacturing process.

***u* charts**
This chart shows the nonconformities per unit produced by a manufacturing process.

## Control Charts for Fraction Nonconforming

Fraction nonconforming is the ratio of the number of nonconforming items in a population to the total number of items in that population.

Control charts for fraction nonconforming are based on the binomial distribution

A quality characteristic follows a binomial distribution if:

- All trials are independent.
- Each outcome is either a "success" or "failure"
- The probability of success on any trial is given as p. The probability of a failure is 1-p
- The probability of a success is constant

The binomial distribution with parameters $n \neq 0$ and $0 < p < 1$, is given by

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

The mean and variance of the binomial distribution are

$$\mu = np \qquad \sigma^2 = np(1-p)$$

The sample fraction nonconforming is given as $\hat{p} = \frac{D}{n}$

where $\hat{p}$ is a random variable with mean and variance $\mu = p \qquad \sigma^2 = \frac{p(1-p)}{n}$

Let W be a simple statistic that measures some simple quality characteristic of interest, and suppose that mean of $W$ is $\mu_w$ and the standard deviation is $\sigma_w$. Then the Centre Line (CL), the upper control limit (UCL) and the lower control limit (LCL) become

$$\mathbf{UCL} = \boldsymbol{\mu_w} + \boldsymbol{L}\sigma_w$$

$$\mathbf{CL} = \boldsymbol{\mu_w}$$

$$\mathbf{LCL} = \boldsymbol{\mu w} - \boldsymbol{L}\sigma w$$

where $L$ is the "distance" of the of the control limits from the centre line, expressed in standard deviation units.

If a standard value of p is given, then the control limits for the fraction nonconforming are

$$UCL = p + 3\sqrt{\frac{p(1-p)}{n}}$$

$$CL = p$$

$$LCL = p - 3\sqrt{\frac{p(1-p)}{n}}$$

If no standard value of p is given, then the control limits for the fraction nonconforming are

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$
$$CL = p$$
$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

Where

$$\overline{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m}$$

**p-C hart construction for constant subgroup size**

1. Select the quality characteristics.
2. Determine the subgroup size and method
3. Collect the data.
4. Calculate the trial central line and control limits.

Establish the revised central line and control limits.
Quality characteristic may be

- A single quality characteristic
- A group of quality characteristics
- A part
- An entire product, or
- A number of products.

Subgroup size and method:
The size of subgroup is a function of the proportion nonconforming.
If $p$ = 0.001, and $n$ = 1000, then the average number nc, $np$ = 1. *Not good, since a large number of values would be zero.*
If $p$ = 0.15, and $n$ = 50, then $np$ = 7.5, would make a good chart.
Therefore, the selection of subgroup size requires some preliminary observations to obtain a rough idea of the proportion nonconforming.

Collection of the data:

The quality technician will need to collect sufficient data for at least 25 subgroups.
The data can be plotted as a run chart.

Since the run chart does not have limits, its is not a control chart.

**Trial and Revised Control Limits**

- Control limits that are based on a preliminary set of data can often be referred to as <u>trial control limits</u>.
- The quality characteristic is plotted against the trial limits, if any points plot out of control, assignable causes should be investigated and points removed.

With removal of the points, the limits are then recalculated.

**Example 1:**

A process that produces bearing housings is investigated. Ten samples of size 100 are selected.

| Sample # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| # Nonconf | 5 | 2 | 3 | 8 | 4 | 1 | 2 | 6 | 3 | 4 |

Is this process operating in statistical control?

n = 100, m = 10

| Sample # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| # Nonconf | 5 | 2 | 3 | 8 | 4 | 1 | 2 | 6 | 3 | 4 |
| Fraction Nonconf | 0.05 | 0.02 | 0.03 | 0.08 | 0.04 | 0.01 | 0.02 | 0.06 | 0.03 | 0.04 |

$$\bar{p} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m} = 0.038$$

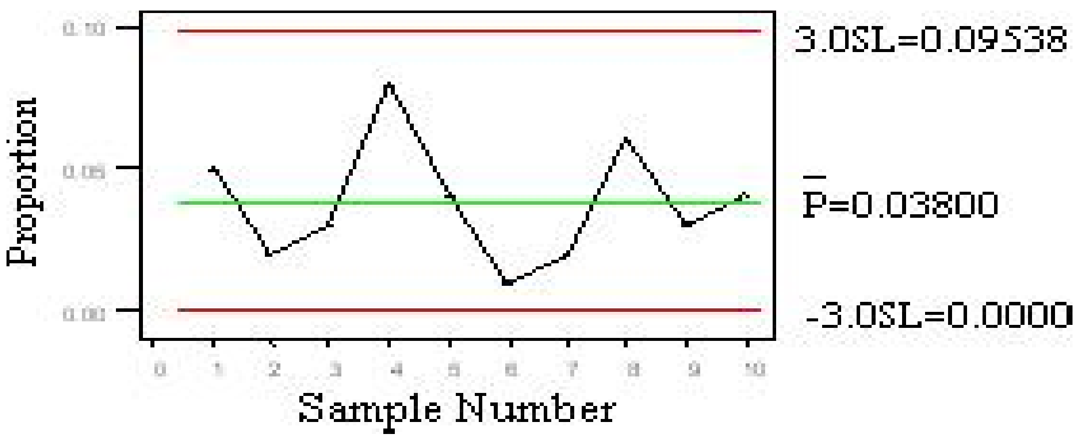
Control Limits are:

$$UCL = 0.038 + 3\sqrt{\frac{0.038(1-0.038)}{100}} = 0.095$$

$$CL = 0.038$$

$$LCL = 0.038 + 3\sqrt{\frac{0.038(1-0.038)}{100}} = -0.02 \rightarrow 0$$

The corresponding control chart is as follows



**Interpretation of Points on the Control Chart for Fraction Nonconforming**

- Care must be exercised in interpreting points that plot *below* the lower control limit.
- They often do not indicate a real improvement in process quality.

They are frequently caused by errors in the inspection process or improperly calibrated test and inspection equipment.

**Example 2:**

Frozen orange juice concentrate is packed in 6-oz cardboard cans.These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. Such a nonconforming can has an improper seal on either the side seam or the bottom panel.

| Sample Number | Number of Nonconforming cans, $D_i$ | Sample Fraction Nonconforming, $\hat{p}_i$ | Sample Number | Number of Nonconforming cans, $D_i$ | Sample Fraction Nonconforming, $\hat{p}_i$ |
|---|---|---|---|---|---|
| 1 | 12 | 0.24 | 17 | 10 | 0.20 |
| 2 | 15 | 0.30 | 18 | 5 | 0.10 |
| 3 | 8 | 0.16 | 19 | 13 | 0.26 |
| 4 | 10 | 0.20 | 20 | 11 | 0.22 |
| 5 | 4 | 0.08 | 21 | 20 | 0.40 |
| 6 | 7 | 0.14 | 22 | 18 | 0.36 |
| 7 | 16 | 0.32 | 23 | 24 | 0.48 |
| 8 | 9 | 0.18 | 24 | 15 | 0.30 |
| 9 | 14 | 0.28 | 25 | 9 | 0.18 |
| 10 | 10 | 0.20 | 26 | 12 | 0.24 |
| 11 | 5 | 0.10 | 27 | 7 | 0.14 |
| 12 | 6 | 0.12 | 28 | 13 | 0.26 |
| 13 | 17 | 0.34 | 29 | 9 | 0.18 |
| 14 | 12 | 0.24 | 30 | 6 | 0.12 |
| 15 | 22 | 0.44 | | 347 | $\bar{p}$ =0.2313 |
| 16 | 8 | 0.16 | | | |

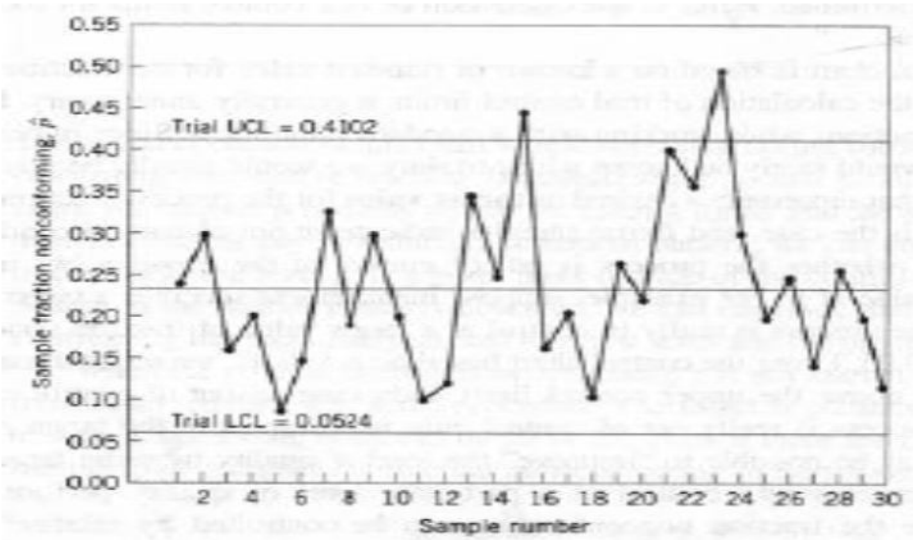
$$\bar{p} = \frac{347}{(30)(50)} = 0.2313$$

$$UCL = 0.2313 + 3\sqrt{\frac{0.2313(1-0.2313)}{50}} = 0.4102$$

$$CL = 0.2313$$

$$LCL = 0.2313 - 3\sqrt{\frac{0.2313(1-0.2313)}{50}} = 0.0524$$

The control chart is shown below.




**Observations from the Control Chart:**

Two points, those from samples 15 and 23, plot above the upper control limit, so the process is not in control

These points must be investigated to see whether an assignment cause can be determined

Analysis of the data from sample 15 indicates that a new batch of cardboard stock was put into production during that half-hour period.

Furthermore, during the half-hour period in which sample 23 was obtained, a relatively inexperienced operator had been temporarily assigned to the machine

Samples 15 and 23 are eliminated, and the new center line and revised control limits are calculated as

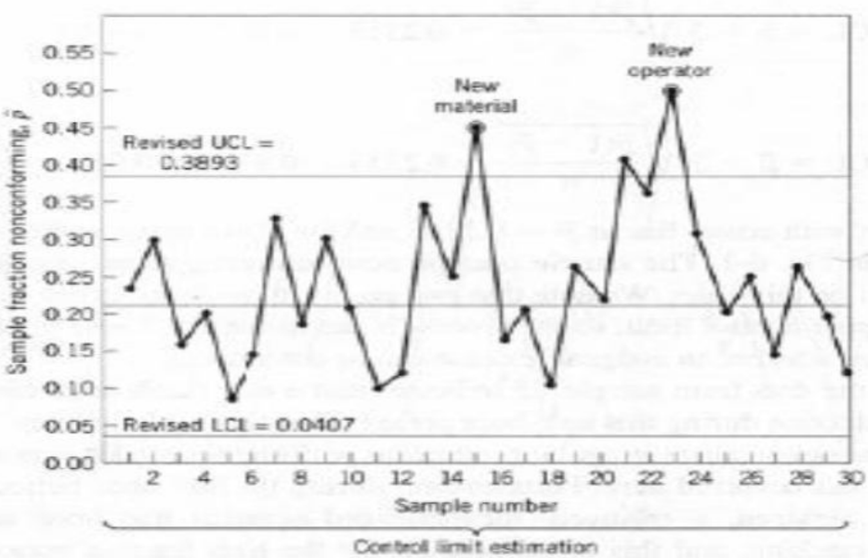
$$\bar{p} = \frac{301}{(28)(50)} = 0.2150$$

$$UCL = 0.2150 + 3\sqrt{\frac{0.2150(1-0.2150)}{50}} = 0.3893$$

$$CL = 0.2150$$

$$LCL = 0.2150 - 3\sqrt{\frac{0.2150(1-0.2150)}{50}} = 0.0407$$

The revised control chart is as follows.


0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Sample fraction nonconforming, $\hat{p}$
Revised UCL = 0.3893
Revised LCL = 0.0407
New material
New operator
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
Sample number
Control limit estimation

**Guidelines for the Design of the Control Chart**

If the rate of production is high, then more frequent sampling is better.

If false alarms or type I errors are very expensive to investigate, then it may be best to use wider control limits than three-sigma.

If the process is such that out-of-control signals are quickly and easily investigated with a minimum of lost time and cost, then narrower control limits are appropriate.

**More on sample size of fraction nonconforming control chart**

If p is very small, we should choose n sufficiently large (usually larger than 100) so that we have a high probability of finding at least one nonconforming unit in the sample.

Otherwise, we might find that the control limits are such that the presence of only one nonconforming unit in the sample would indicate an out-of control condition.

If P=0.01, n=8, then UCL=0.1155

If there is one nonconforming unit in the sample, then =1/8=0.1250, and we can conclude that the process is out of control.

Therefore, a sample size of 100 would have to be taken to expect to include even one defective item.

To avoid this pitfall, we can choose the sample size n so that the probability of finding at least one nonconforming unit per sample is at least y

For example, p=0.01, y=0.95, then we would like to find n such that .

Using Poisson distribution Z=np must exceed 3.0. Consequently p=0.01 implies n=300.

Duncan has suggested that the sample size can be determined so that a shift of some specified amount, $\delta$ can be *detected* with a stated level of probability (50% chance of detection). If $\delta$ is the magnitude of a process shift and L is the distance of control limits from centerline in standard deviation units, then *n* must satisfy:

$$\delta = L\sqrt{\frac{p(1-p)}{n}}$$

Therefore,

$$n = \left(\frac{L}{\delta}\right)^2 p(1-p)$$

If p=0.01, and we wish to detect a shift to p=0.05 with a probability of 0.5, then $\delta = 0.04$. and n must satisfy

$$\delta = L\sqrt{\frac{p(1-p)}{n}}$$

If three-sigma limits are used

$$n = \left(\frac{3}{0.04}\right)^2 0.01(1-0.99) = 56$$

**Positive Lower Control Limit**

The sample size *n*, can be chosen so that the lower control limit would be nonzero:

$$LCL = p - L\sqrt{\frac{p(1-p)}{n}} > 0$$

and

$$n > \frac{(1-p)}{p} L^2$$

If p=0.05 and L=3 then the sample size must be

$$n > \frac{(1-0.05)}{0.05} 3^2$$

i.e. $n > 171$

## Control Charts for Fraction Nonconforming –np chart

The actual number of nonconforming can also be charted. Let n = sample size, p = proportion of nonconforming. The control limits are:

$$UCL = np + 3\sqrt{np(1-p)}$$
$$CL = np$$
$$LCL = np - 3\sqrt{np(1-p)}$$

(if a standard p is not given, use $\bar{p}$)

**Example 3 - np Chart**

A process that produces bearing housings is investigated. Ten samples of size 100 are selected.

| Sample # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| # Nonconf | 5 | 2 | 3 | 8 | 4 | 1 | 2 | 6 | 3 | 4 |

Is this process operating in statistical control?

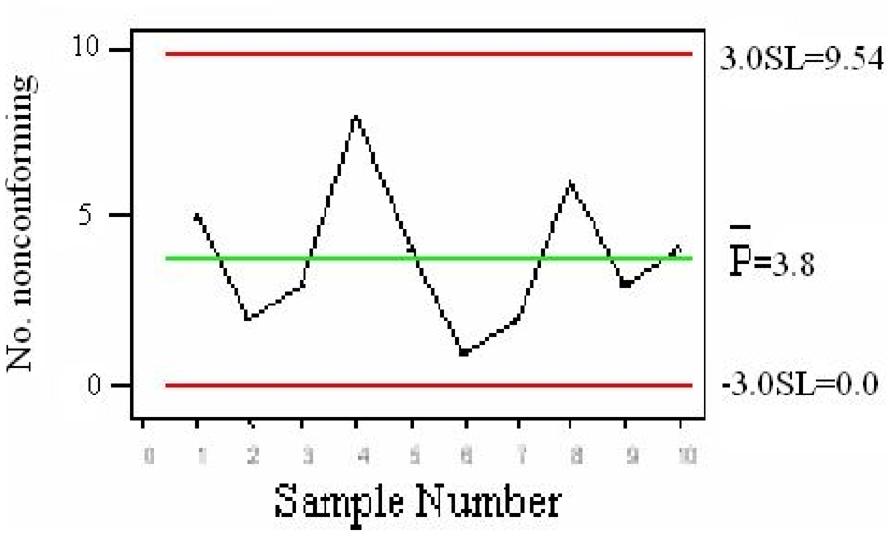
We have n = 100, m = 10

| Sample # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| # Nonconf | 5 | 2 | 3 | 8 | 4 | 1 | 2 | 6 | 3 | 4 |
| Fraction Nonconf | 0.05 | 0.02 | 0.03 | 0.08 | 0.04 | 0.01 | 0.02 | 0.06 | 0.03 | 0.04 |

$$\bar{p} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m} = 0.038$$

$$CL = 100\ (0.038\ ) = 3.8$$
$$UCL\ = 100\ (0.038\ ) + 3\sqrt{100\ (0.038\ )(1 - 0.038\ )} = 9.54$$
$$LCL\ = 100\ (0.038\ ) - 3\sqrt{100\ (0.038\ )(1 - 0.038\ )} = -2 = 0$$

The corresponding np chart is




## Control Charts for Nonconformities (Defects) – c Chart

The number of nonconformities in a given area can be modeled by the Poisson distribution. Let c be the parameter for a Poisson distribution, then the mean and variance of the Poisson distribution are equal to the value c.

The probability of obtaining x nonconformities on a single inspection unit, when the average number of nonconformities is some constant, c, is found using:

$$p(x) = \frac{e^{-c}c^{x}}{x!}$$

Standard Given:

$$UCL = c + 3\sqrt{c}$$
$$CL = c$$
$$LCL = c - 3\sqrt{c}$$

No Standard Given:

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$
$$CL = \bar{c}$$
$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

**Example 4: (c Chart)**

The number of weekly **customer complaints** are monitored in a large hotel using a c-chart. Develop **three sigma control limits** using the data table below.

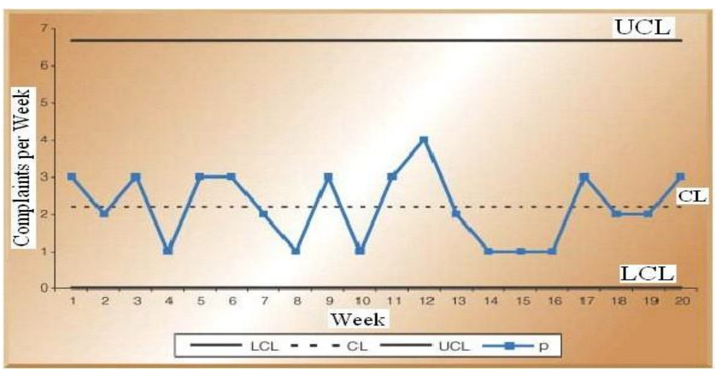
$$CL = \bar{c} = \frac{\#\text{ complaints}}{\#\text{ of samples}} = \frac{22}{10} = 2.2$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 2.2 + 3\sqrt{2.2} = 6.65$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 2.2 - 3\sqrt{2.2} = -2.25 = 0$$

The c-chart is

| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No of Complaints | 3 | 2 | 3 | 1 | 3 | 3 | 2 | 1 | 3 | 1 | 22 |




## Control Chart for Nonconformities per unit (u chart)

- If we find c, the total number of nonconformities in a sample of n inspection units, then the average number of nonconformities per inspection unit is c/n.

The control limits for the average number of nonconformities is

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

$$CL = \bar{u}$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

**Example 5: (u chart)**

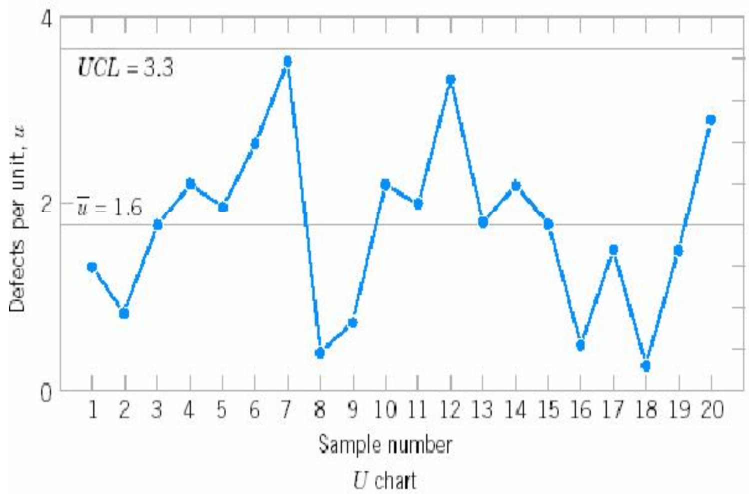
Number of Defects in Samples of Five Printed Circuit Boards

| Sample | Number of Defects $c_i$ | Sample | Number of Defects $c_i$ |
|---|---|---|---|
| 1 | 6 | 11 | 9 |
| 2 | 4 | 12 | 15 |
| 3 | 8 | 13 | 8 |
| 4 | 10 | 14 | 10 |
| 5 | 9 | 15 | 8 |
| 6 | 12 | 16 | 2 |
| 7 | 16 | 17 | 7 |
| 8 | 2 | 18 | 1 |
| 9 | 3 | 19 | 7 |
| 10 | 10 | 20 | 13 |

Number of Defects in Samples of Five Printed Circuit Boards

| Sample | Number of Defects $c_i$ | Defects per Unit $u_i$ | Sample | Number of Defects $c_i$ | Defects per Unit $u_i$ |
|---|---|---|---|---|---|
| 1 | 6 | 1.2 | 11 | 9 | 1.8 |
| 2 | 4 | 0.8 | 12 | 15 | 3.0 |
| 3 | 8 | 1.6 | 13 | 8 | 1.6 |
| 4 | 10 | 2.0 | 14 | 10 | 2.0 |
| 5 | 9 | 1.8 | 15 | 8 | 1.6 |
| 6 | 12 | 2.4 | 16 | 2 | 0.4 |
| 7 | 16 | 3.2 | 17 | 7 | 1.4 |
| 8 | 2 | 0.4 | 18 | 1 | 0.2 |
| 9 | 3 | 0.6 | 19 | 7 | 1.4 |
| 10 | 10 | 2.0 | 20 | 13 | 2.6 |

The u chart for the example is



*U* chart of defects per unit on printed circuit boards.

For the same example c- chart calculations are

$$CL = \bar{c} = \frac{160}{20} = 8$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 8 + 3\sqrt{8} = 16.485$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 8 - 3\sqrt{8} = -0.485 = 0$$

**Indications that special causes of variations are present**

One or more points outside the control limits
Seven or more consecutive points on one side of the centerline
Six points in a row steadily increasing or decreasing
Fourteen points alternating up and down
Two out of three consecutive points in the outer third of the control region
Fifteen points in a row within the center third of the control region
Eight points on both sides of the centerline with none in the center third of the control region
Two out of three consecutive points in the outer third of the control region
Fifteen points in a row within the center third of the control region
Eight points on both sides of the centerline with none in the center third of the control region

## <u>Control charts for variable sample size</u>

3 approaches to deal with variable sample size

1. **Variable-width control limits**: to determine control limits for each individual sample that are based on specific sample size
2. **Control limits based on average sample size:** to obtain an approximate set of control limits (constant control limits)
3. **The standardized control chart:**
   - The points are plotted in standard deviation units
   - Center line at zero
   - Upper and lower control limits € +3 and - 3

**Example 6:**

Consider the following example

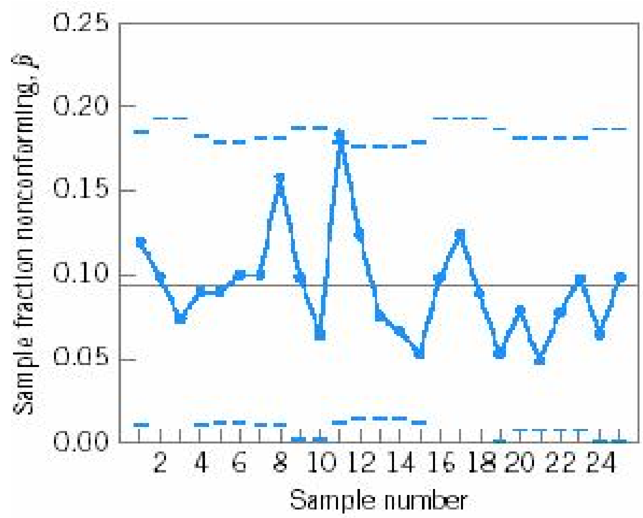
| Sample Number | Sample size, $n_i$ | Number of nonconfirming units, $D_i$ |
|---|---|---|
| 1 | 100 | 12 |
| 2 | 80 | 8 |
| 3 | 80 | 6 |
| 4 | 100 | 9 |
| --------- | --------- | --------- |
| 22 | 100 | 8 |
| 23 | 100 | 10 |
| 24 | 90 | 6 |
| 25 | 90 | 9 |
| | 2450 | 234 |

$$\bar{p} = \frac{\sum_{i=1}^{25} D_i}{\sum_{i=1}^{25} n_i} = \frac{234}{2450} = 0.096$$

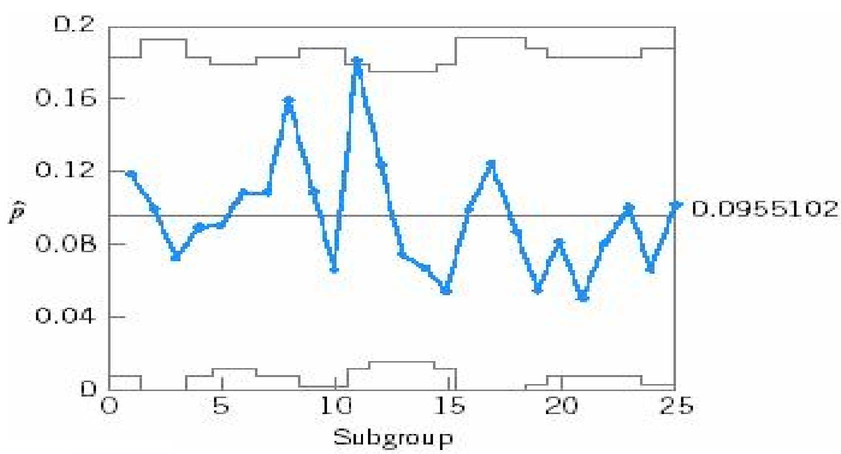
$$UCL = 0.096 + 3\sqrt{\frac{0.096(1-0.096)}{n_i}}$$

$$LCL = 0.096 - 3\sqrt{\frac{0.096(1-0.096)}{n_i}}$$

| Sample No | $n_i$ | $D_i$ | $\hat{p}_i$ | $\hat{\sigma}_{\hat{p}_i}$ | LCL | UCL |
|---|---|---|---|---|---|---|
| 1 | 100 | 12 | 0.120 | 0.029 | 0.009 | 0.183 |
| 2 | 80 | 8 | 0.100 | 0.033 | 0.000 | 0.195 |
| 3 | 80 | 6 | 0.075 | 0.033 | 0.000 | 0.195 |
| 4 | 100 | 9 | 0.090 | 0.029 | 0.009 | 0.183 |
| ------ | ------ | ------ | ------ | ------ | ------ | ------ |
| 22 | 100 | 8 | 0.080 | 0.029 | 0.009 | 0.183 |
| 23 | 100 | 10 | 0.100 | 0.029 | 0.009 | 0.183 |
| 24 | 90 | 6 | 0.067 | 0.031 | 0.003 | 0.189 |
| 25 | 90 | 9 | 0.100 | 0.031 | 0.003 | 0.189 |
| | 2450 | 234 | | | | |




Control chart for fraction nonconforming with variable sample size.



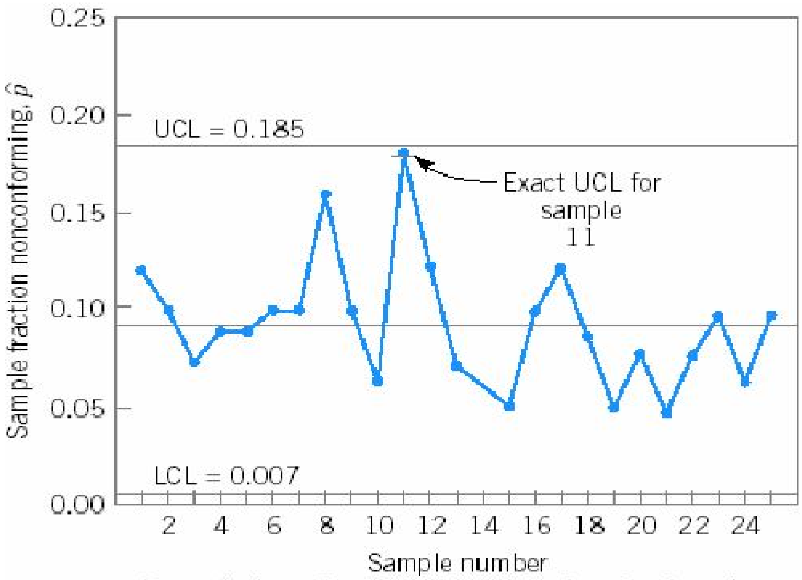

Control chart for fraction nonconforming with variable sample size using Minitab.

**Example 7: (Control limit based on average sample size)**

$$\bar{n} = \frac{\sum_{i=1}^{25} n_i}{25} = \frac{2450}{25} = 98$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.096 + 3\sqrt{\frac{0.096(1-0.096)}{98}} = 0.185$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.096 - 3\sqrt{\frac{0.096(1-0.096)}{98}} = 0.007$$



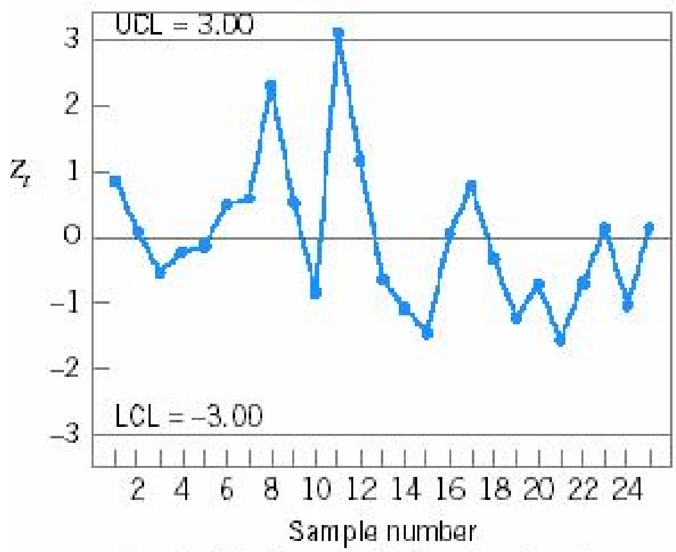

Control chart for fraction nonconforming based on average sample size.

**Example 8: (The standardized control Chart)**

$$Z_i = \frac{p_i - p}{\sqrt{\frac{p(1-p)}{n_i}}}$$

$$Z_i = \frac{\hat{p}_i - \bar{p}}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}} = \frac{\hat{p}_i - \bar{p}}{\sqrt{\frac{(0.096)(0.904}{n_i}}}$$

| Sample No | $n_i$ | $D_i$ | $\hat{p}_i$ | $\hat{\sigma}_{\hat{p}_i}$ | $z_i$ |
|---|---|---|---|---|---|
| 1 | 100 | 12 | 0.120 | 0.029 | 0.830 |
| 2 | 80 | 8 | 0.100 | 0.033 | 0.120 |
| 3 | 80 | 6 | 0.075 | 0.033 | -0.640 |
| 4 | 100 | 9 | 0.090 | 0.029 | -0.210 |
| ------ | ------ | ------ | ------ | ------ | ------ |
| 22 | 100 | 8 | 0.080 | 0.029 | -0.550 |
| 23 | 100 | 10 | 0.100 | 0.029 | 0.140 |
| 24 | 90 | 6 | 0.067 | 0.031 | -0.940 |
| 25 | 90 | 9 | 0.100 | 0.031 | 0.130 |




Standardized control chart for fraction nonconforming.

**Example 9 (u Chart – variable width control limit)**

In a textile finishing plant, dyed cloth inspected for the occurrence of defects per 50 $m^2$. The data on 10 rolls of cloth are shown in the table. Set up a control chart for nonconformities per unit.

| Roll Number | No. of $m^2$ | Total No. of Nonconformities | No. of inspection unit in roll, n | No. of Nonconformities per inspection unit |
|---|---|---|---|---|
| 1 | 500 | 14 | 10.0 | 1.40 |
| 2 | 400 | 12 | 8.0 | 1.50 |
| 3 | 650 | 20 | 13.0 | 1.54 |
| 4 | 500 | 11 | 10.0 | 1.10 |
| 5 | 475 | 7 | 9.5 | 0.74 |
| 6 | 500 | 10 | 10.0 | 1.00 |
| 7 | 600 | 21 | 12.0 | 1.75 |
| 8 | 525 | 16 | 10.5 | 1.52 |
| 9 | 600 | 19 | 12.0 | 1.58 |
| 10 | 625 | 23 | 12.5 | 1.84 |
| | | Σ=153 | Σ=107.5 | |

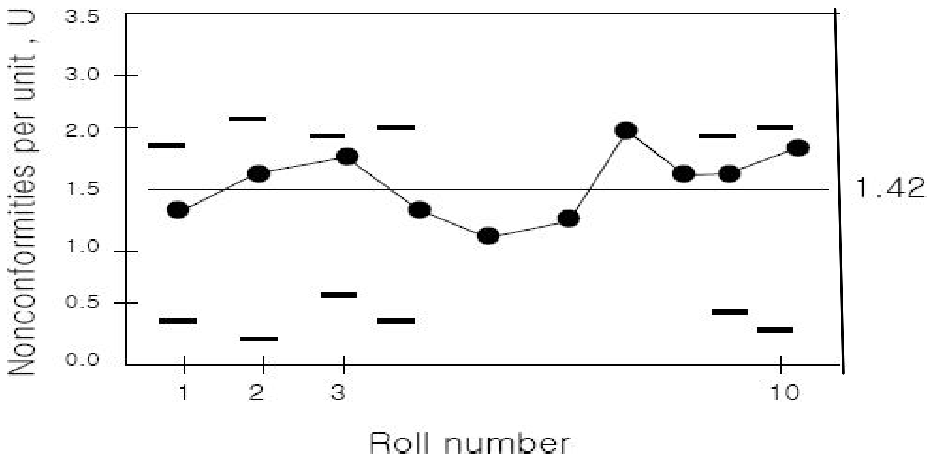
u Chart – variable sample size

$$\overline{u} = 153/107.5 = 1.42$$

$$UCL = \overline{u} + 3\sqrt{\frac{\overline{u}}{n_i}}$$

$$LCL = \overline{u} - 3\sqrt{\frac{\overline{u}}{n_i}}$$

| Roll No., i | $n_i$ | UCL | LCL |
|---|---|---|---|
| 1 | 10.0 | 2.55 | 0.29 |
| 2 | 8.0 | 2.68 | 0.16 |
| 3 | 13.0 | 2.41 | 0.43 |
| 4 | 10.0 | 2.55 | 0.29 |
| 5 | 9.5 | 2.58 | 0.26 |
| 6 | 10.0 | 2.55 | 0.29 |
| 7 | 12.0 | 2.45 | 0.39 |
| 8 | 10.5 | 2.52 | 0.32 |
| 9 | 12.0 | 2.45 | 0.39 |
| 10 | 12.5 | 2.43 | 0.41 |



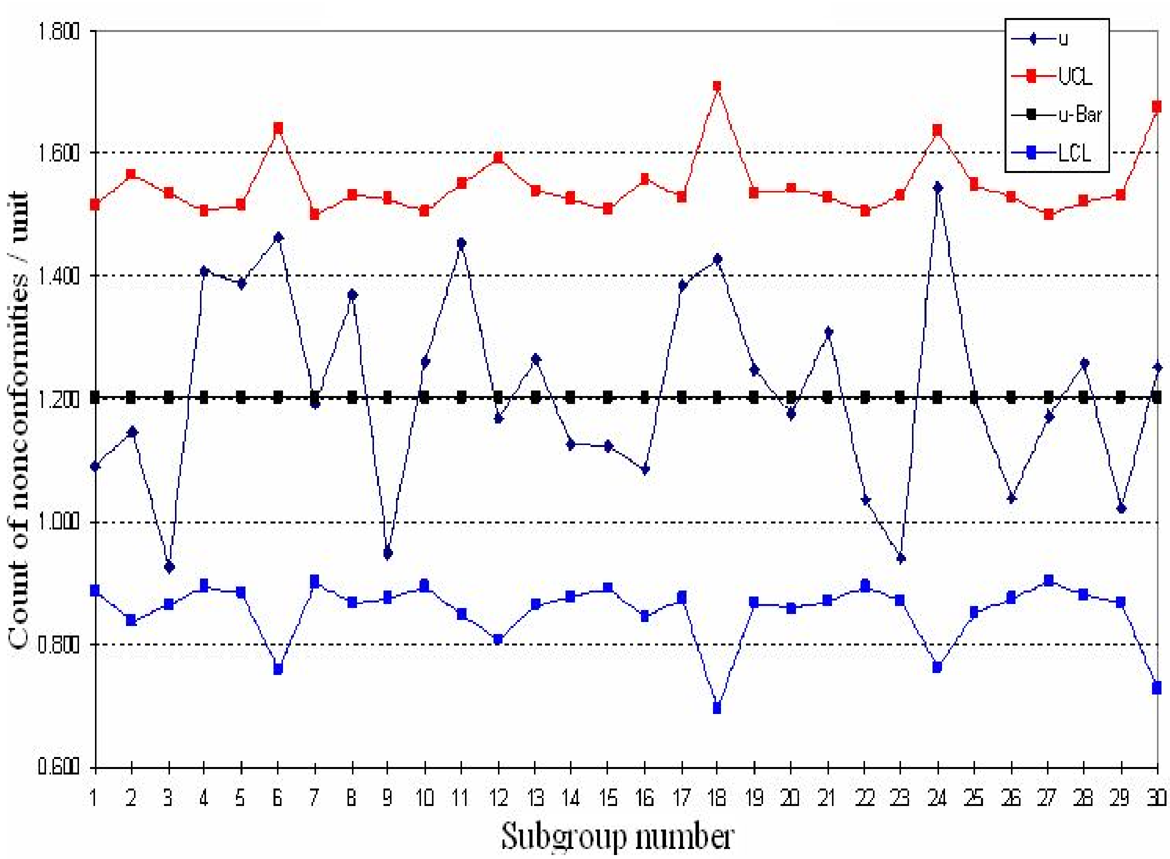

| ID Number | Subgroup | *n* | *c* | *u* | UCL | *u* -Bar | LCL |
|---|---|---|---|---|---|---|---|
| 30-Jan | 1 | 110 | 120 | 1.091 | 1.51 | 1.20 | 0.89 |
| 31-Jan | 2 | 82 | 94 | 1.146 | 1.56 | 1.20 | 0.84 |
| 1-Feb | 3 | 96 | 89 | 0.927 | 1.54 | 1.20 | 0.87 |
| 2-Feb | 4 | 115 | 162 | 1.409 | 1.51 | 1.20 | 0.89 |
| 3-Feb | 5 | 108 | 150 | 1.389 | 1.52 | 1.20 | 0.88 |
| 4-Feb | 6 | 56 | 82 | 1.464 | 1.64 | 1.20 | 0.76 |
| 28-Feb | 26 | 101 | 105 | 1.040 | 1.53 | 1.20 | 0.87 |
| 1-Mar | 27 | 122 | 143 | 1.172 | 1.50 | 1.20 | 0.90 |
| 2-Mar | 28 | 105 | 132 | 1.257 | 1.52 | 1.20 | 0.88 |
| 3-Mar | 29 | 98 | 100 | 1.020 | 1.53 | 1.20 | 0.87 |
| 4-Mar | 30 | 48 | 60 | 1.250 | 1.67 | 1.20 | 0.73 |

$$\bar{u} = \frac{\sum c}{\sum n} = \frac{3389}{2823} = 1.20$$

$$u_{Jan30} = \frac{c}{n} = \frac{120}{110} = 1.091$$

$$UCL_{Jan30} = 1.20 + 3\sqrt{\frac{1.20}{110}} = 1.51$$

$$LCL_{Jan30} = 1.20 - 3\sqrt{\frac{1.20}{110}} = 0.89$$




**Control Limits Based on an Average Sample Size**

- Control charts based on the average sample size results in an approximate set of control limits.
- The average sample size is given by

$$\bar{n} = \frac{\sum_{i=1}^{m} n_i}{m}$$

- The upper and lower control limits are

$$\bar{u} \pm 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

**Example `10: (The Standardized Control Chart)**

- The points plotted are in terms of standard deviation units. The standardized control chart has the follow properties:
  - Centerline at 0
  - UCL = 3  LCL = -3

The points plotted are given by:

$$z_i = \frac{u_i - \overline{u}}{\sqrt{\frac{\overline{u}}{n_i}}}$$

**Application of Control Charts**

- The control chart, though originally developed for quality control in manufacturing, is applicable to all sorts of repetitive activities in any kind of organization.
- They can be used for services as well as products, for people, machines, cost, and so on.

For example, we can plot errors on engineering drawings, errors on plans and documents, and errors in computer software as c or u charts.

- Sometimes, the quality control engineer has a choice between variable control charts and attribute control charts.

**Advantages of attribute control charts**

- Allowing for quick summaries, that is, the engineer may simply classify products as *acceptable* or *unacceptable*, based on various quality criteria.
- Thus, attribute charts sometimes bypass the need for expensive, precise devices and time-consuming measurement procedures.
  More easily understood by managers unfamiliar with quality control procedures.

**Advantages of variable control charts**

- More sensitive than attribute control charts.
- Therefore, variable control charts may alert us to quality problems before any actual "unacceptables" (as detected by the attribute chart) will occur.
- Montgomery (1985) calls the variable control charts *leading indicators* of trouble that will sound an alarm before the number of rejects (scrap) increases in the production process.

## Guidelines for Implementing Control Charts

1. Determine which process characteristics to control.
2. Determine where the charts should be implemented in the process.
3. Choose the proper type of control chart.
4. Take action to improve processes as the result of SPC/control chart analysis.
5. Select data-collection systems and computer software

   Determining Which Characteristics to Control and Where to put the control charts:

   At the start of a control chart program, it is usually difficult to determine which product or process characteristics should be controlled and at which points in the process to apply control charts . Some useful guidelines are given below.

   1. At the beginning of a control chart program, control charts should be applied to any product characteristics or manufacturing operations believed to be important. The charts will provide immediate feedback as to whether they are actually needed.
   2. The control charts found to be unnecessary should be removed, and others that engineering and operator judgment indicates may be required should be added. More control charts will usually be employed at the beginning than after the process has stabilized.
   3. Information on the number and types of control charts on the process should be kept current. It is best to keep separate records on the variables and attributes charts. In general, after the control charts are first installed, we often find that the number of control charts tends to increase rather steadily. After that it will usually decrease. When the process stabilizes, we typically find that it has the same number of charts from one year to the next. However, they are not necessarily the same charts.
   4. If control charts are being used effectively and if new knowledge is being gained about the key process variables, we should find that the number of x-bar and R charts increases and the number of attributes control charts decreases.
   5. At the beginning of a control chart program there will usually be more attributes control charts, applied to semifinished and finished units near the *end* of the manufacturing process. As we learn more about the process, these charts will be replaced with x -bar and r charts applied *earlier* in the process to the critical parameters and operations that result in nonconformities in the finished product. Generally, the earlier that process control can be established, the better. In a complex assembly process, this may imply that process controls need to be implemented at the vendor or supplier level.
   6. . Control charts are an on-line, process monitoring procedure. They should be implemented and maintained as close to the work center as possible, so that feedback will be rapid. Furthermore, the process operators and manufacturing engineering should have direct responsibility for collecting the process data, maintaining the

charts, and interpreting the results. The operators and the engineers have the detailed knowledge of the process required to correct process upsets and use the control charts as a device to improve process performance. Microcomputers can speed up the feedback and should be an integral part of any modern, on-line, process control procedure.

Choosing the proper type of Control Chart:

A. x-bar and R (or x-bar and s) charts. Consider using measurements control charts in these situations:

1. A new process is coming on stream, or a new product is being manufactured by an existing process.
2. The process has been in operation for some time, but it is chronically in trouble or unable to hold the specified tolerances.
3. The process is in trouble, and the control chart can be useful for diagnostic purposes (troubleshooting).
4. Destructive testing (or other expensive testing procedures) is required.
5. It is desirable to reduce acceptance-sampling or other downstream testing to a minimum when the process can be operated in control.
6. Attributes control charts have been used, but the process is either out of control or in control but the yield is unacceptable.
7. There are very tight situations, overlapping assembly tolerances, or other difficult manufacturing problems.
8. The operator must decide whether or not to adjust the process, or when a set-up must be evaluated.
9. A change in product specification is desired.
10. Process stability and capability must be continually demonstrated, such as in regulated industries.

B. Attributes Charts (p charts, c charts, and u charts). Consider using attributes control charts in these situations:

1. Operators control the assignable causes, and it is necessary to reduce process fallout.
2. The process is a complex assembly operation and product quality is measured in terms of the occurrence of the nonconformities, successful or unsuccessful product function, and so forth. (Examples include computers, office automation equipment, automobiles, and the major subsystems of these products.)
3. Process control is necessary, but measurement data cannot be obtained.
4. A historical summary of process performance is necessary. Attributes control charts, such as p charts, c charts and u charts, are very effective for summarizing information about the process for management review.
5. Remember that attributes charts are generally inferior to charts for measurements. Always use x-bar and R or x-bar and S charts whenever possible.

C. Control Charts for individuals. Consider using control chart for individuals in conjunction with a moving-range chart in these situations:

1. It is inconvenient or impossible to obtain more than one measurement per sample, or repeat measurements will differ by laboratory or analysis error. Examples often occur in chemical processes.

2. automated testing and inspection technology allow measurement of every unit produced. In these cases, also consider the cumulative sum control chart and the exponentially weighted moving average control chart.

3.The data become available very slowly, and waiting for a larger sample wil be impractical or make the control procedure too slow to react to problems. This often happens in nonproduct situations; for example, accounting data may become available only monthly.

4. Generally, individuals charts have poor performance in shift detection and can be very sensitive to departures from normality. Always use the EWMA and cusum charts instead of individuals charts whenever possible.

Actions taken to Improve the Process

Process improvement is the primary objective of SPC

If we take a less precise interpretation of capability as a qualitative assessment of whether or not the level of nonconforming units produced is low enough to warrant no immediate additional effort to further improve the process, then following two questions are relevant.

Is the process in control?

Is the process capable?

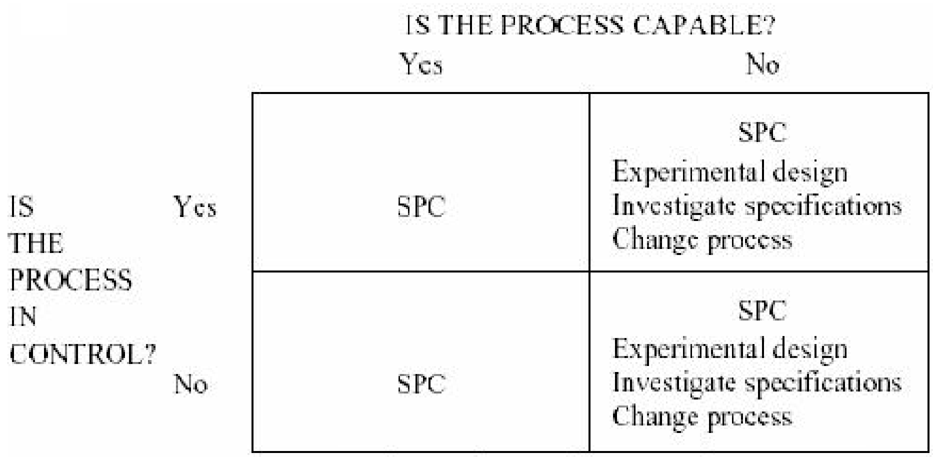
Process improvement is the primary objective of SPC

If we take a less precise interpretation of capability as a qualitative assessment of whether or not the level of nonconforming units produced is low enough to warrant no immediate additional effort to further improve the process, the following two questions are relevant.

Is the process in control?

Is the process capable?

The following figure is useful in answering these questions.




Actions taken to improve the process

1. NW cell - is in ideal state

- SPC methods are valuable for process monitoring and warning against assignable causes that could cause slippage in performance.

2. NE cell – statistical control bur poor capability.

-SPC methods may be useful primarily through recognition of patterns

-Control charts will not produce many out of control signals-Active intervention necessary to improve the process (Refer to suggestions indicated in the diagram)

3. SE cell – process out of control and not capable

- Actions recommended same as for NE cell
- SPC is expected to yield rapid results, control charts should lead to identification of assignable causes

4. SW cell – lack of statistical control

- Does not produce many defectives because the specifications are wide
- SPC methods should be used to establish control and reduce variability because

Customer may require both control and capability
Specifications can change without notice
Process experiences assignable causes. This may lead to poor capability in curse of time

Selection of Data-Collection Systems and Computer Software
Computer has a useful role in SPC
Computer is a great productivity improvement device.
SPC data can be made part of company-wide manufacturing data base – useful to management, engineering, marketing, and so on in addition to manufacturing and quality
Provides more information than manual systems
Permits many quality characteristics to be monitored and
Provides automatic signaling of assignable causes

Choosing a Computer Software:

1. Capable of stand alone operation as well as on multi-terminal LAN
2. Should be user friendly – customization for any application
3. Video display of control charts for at least 25 samples
4. Storage sufficient to accommodate reasonable amount of process history –easy editing - transfer of data to master manufacturing database
5. Simultaneous multiple file handling so that a number of quality characteristics can be examined
6. User should be able to calculate control limits from any subset of the data – capability to input center lines and control limits directly
7. Acceptance of a variety of inputs – manual data entry, RS-232 input from an electronic gage, input from another computer or instrument controller- real-time process monitoring or transfer of data from a real-time data acquisition system are becoming increasingly important
8. Should support other statistical applications, including as a minimum histograms and computation of process capability indices
9. Service and support from software supplier