Method and Philosophy of Statistical Process Control

6.1 : Basic SPC Tools

SPC can be applied to any process. Its seven major tools are

1. Histogram or stem-and –leaf plot
2. Check sheet
3. Pareto chart
4. Cause-and-effect diagram
5. Defect concentration diagram
6. Scatter diagram
7. Control chart

- In any production process, regardless of how well designed or carefully maintained it is, a certain amount of *inherent or natural variability* will always exist. This *natural variability* or "*background noise*" is the cumulative effect of many *small, essential unavoidable causes*.
- A process that is operating with only chance cause of variation present is said to be in statistical control.
- A process that is operating in the presence of assignable cause is said to be out of control.

  e.g. improperly adjusted or controlled *machines*, *operator* errors, or defective *raw material*.

6.2 : Chance and Assignable causes of Quality Variation

- A process is operating with only **chance causes of variation** present is said to be **in statistical control**.
- A process that is operating in the presence of **assignable causes** is said to be **out of control**.

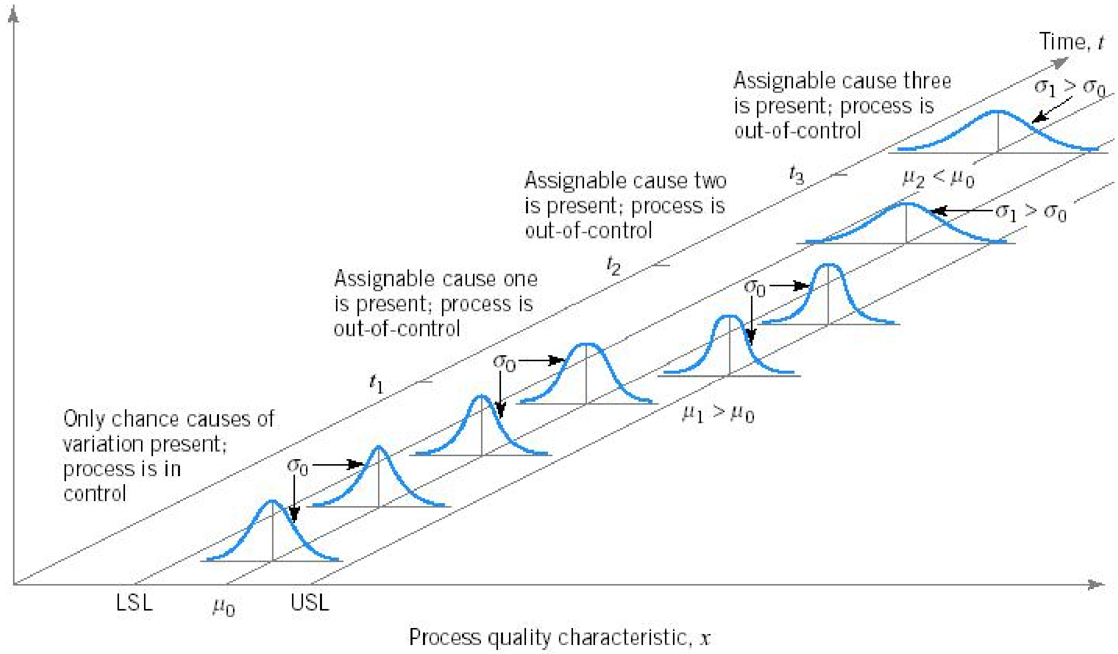

Figure 4-1 Chance and assignable causes of variation.

6.3 : Statistical Basis of the Control Chart

- A control chart contains
  - A **center line**
  - An **upper control limit**
  - A **lower control limit**
- A point that plots within the control limits indicates the process is in control
  - No action is necessary
- A point that plots outside the control limits is evidence that the process is out of control
  - Investigation and corrective action are required to find and eliminate assignable cause(s)
- There is a close connection between control charts and **hypothesis testing**

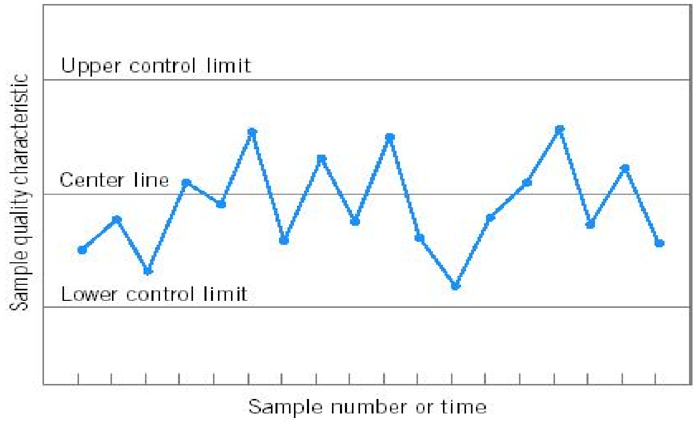

Figure 4-2 A typical control chart.

Photolithography Example

- Important quality characteristic in hard bake is resist flow width
- Process is monitored by average flow width
  - Sample of 5 wafers
  - Process mean is 1.5 microns
  - Process standard deviation is 0.15 microns
- Note that all plotted points fall inside the control limits
  - Process is considered to be in statistical control

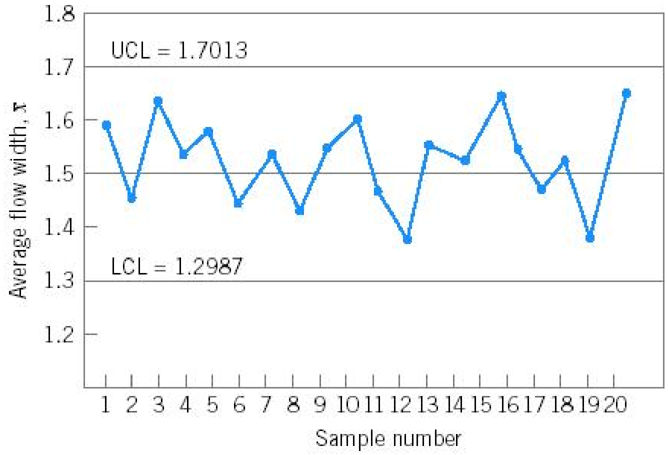

Figure 4-3 $\bar{x}$ control chart for flow width.

The process mean is 1.5 microns, and the process standard deviation is σ = 0.15 microns. Now if sample of size n = 5 are taken, the standard deviation of the sample average $\bar{x}$ is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.15}{\sqrt{5}} = 0.0671$$

Therefore, if the process is in control with a mean flow width of 1.5 microns, then by using the central limit theorem to assume that x is approximately normally distributed, we would expect 100 (1-α)% of the sample mean x to fall between $1.5 + Z_{\alpha/2}(0.0671)$ and $1.5 - Z_{\alpha/2}(0.0671)$. We will arbitrarily choose the constant $Z_{\alpha/2}$ to be 3, so that the upper and lower control limits become

$$UCL = 1.5 + 3(0.0671) = 1.7013$$

and

$$LCL = 1.5 - 3(0.0671) = 1.2987$$

As shown on the control chart. These are typically called "three-signma"[2] control limits. Shewhart Control Chart Model. We may give a general model for a control chart. Let w be a sample statistic that measures some quality characteristic of interest, and suppose that the mean of w is μ_w and the standard deviation of w is σ_w. Then the centre line, the upper control limit, and the lower control limit become

$$UCL = \mu_w + L\sigma_w$$

$$\text{Centre line} = \mu_w$$

$$LCL = \mu_w - L\sigma_w$$

Where L is the "distance" of the control limits from the centre line, expressed in standard deviation units. This general theory of control charts was first proposed by Walter A. Shewhart, and control charts developed according to these principles are often called Shewhart control charts.

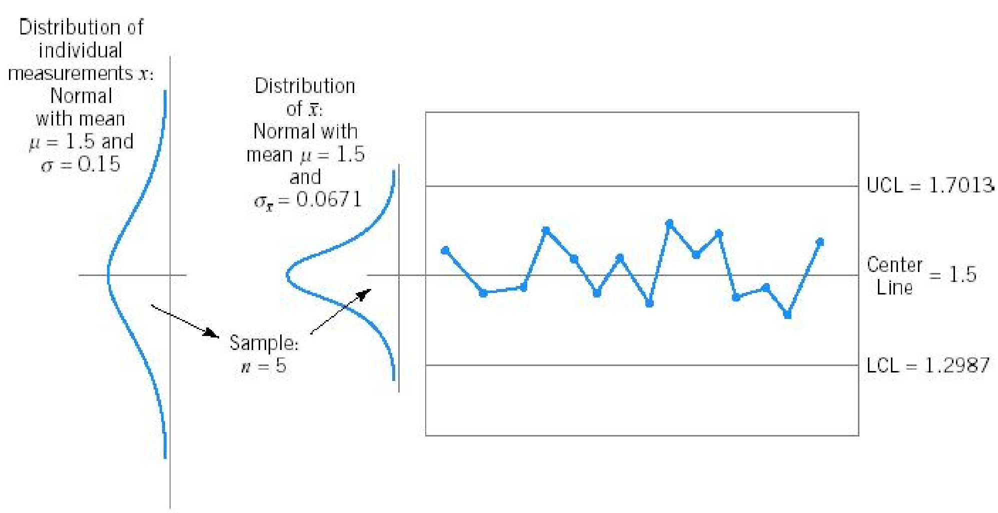

Figure 4-4 How the control chart works.

The most important use of a control chart is to improve the process. We have found that, generally,

1. Most processes do not operate in a state of statistical control.

2. Consequently, the routine and attentive use of control charts will identify assignable causes. If these causes can be eliminated from the process, variability will be reduced and the process will be improved.

This process improvement activity using the control chart is illustrated

3. The control chart will only detect assignable causes. Management, operator and engineering action will usually be necessary to eliminate the assignable causes.

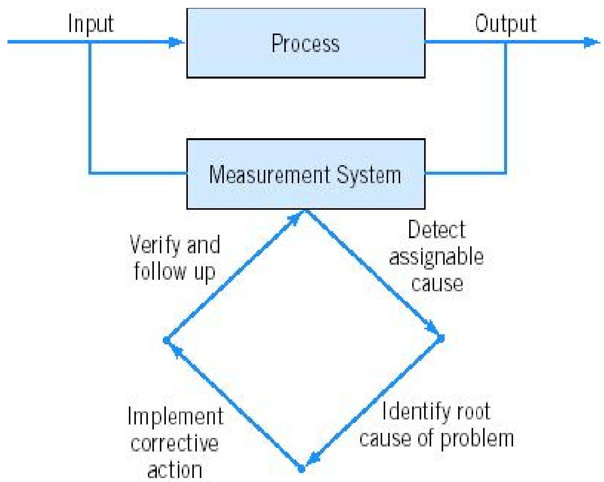

Figure 4-5 Process improvement using the control chart.

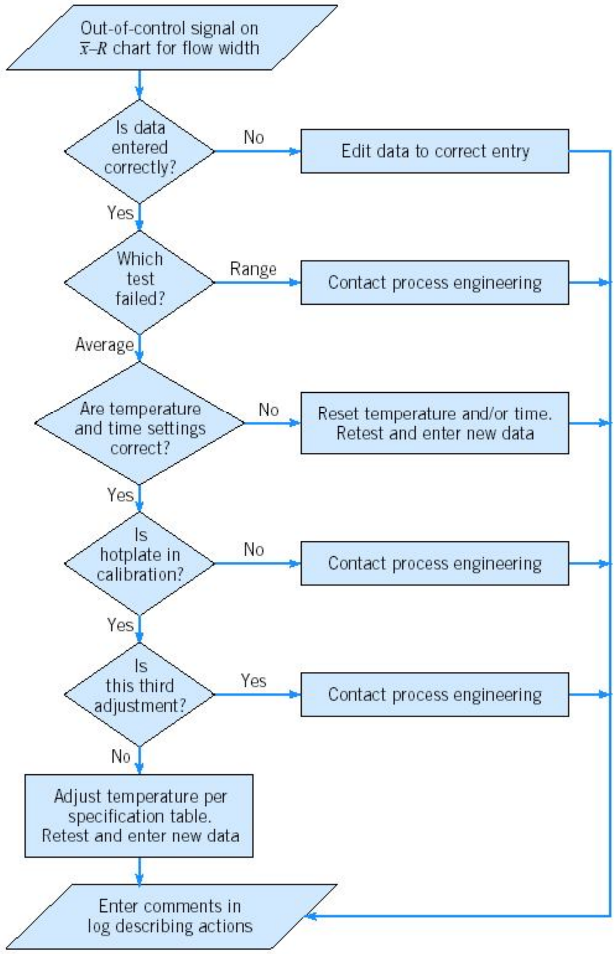

Figure 4-6 The out-of-control-action plan (OCAP) for the hard-bake process.

More Basic Principles

- Charts may be used to estimate process parameters, which are used to determine **capability**
- Two general types of control charts
  - Variables (Chapter 5)
    - Continuous scale of measurement
    - Quality characteristic described by central tendency and a measure of variability
  - Attributes (Chapter 6)
    - Conforming/nonconforming
    - Counts
- **Control chart design** encompasses selection of sample size, control limits, and sampling frequency

6.3 : Typical control chart

- Points plot within the control limits & no nonrandom pattern : process is in control, no action is necessary.
- A point plots outside of the control limits or random pattern exists : process is out of control, investigation and correction action are required to eliminate the assignable cause.
- *Type I error* of the control chart : the process is out of control when it is really in control.
- *Type II error* of the control chart : the process is in control when it is really out of control.

6.4 : Types of Process Variability

- **Stationary** and **uncorrelated** – data vary around a fixed mean in a stable or predictable manner
- **Stationary** and **autocorrelated** – successive observations are dependent with tendency to move in long runs on either side of mean
- **Nonstationary** – process drifts without any sense of a stable or fixed mean

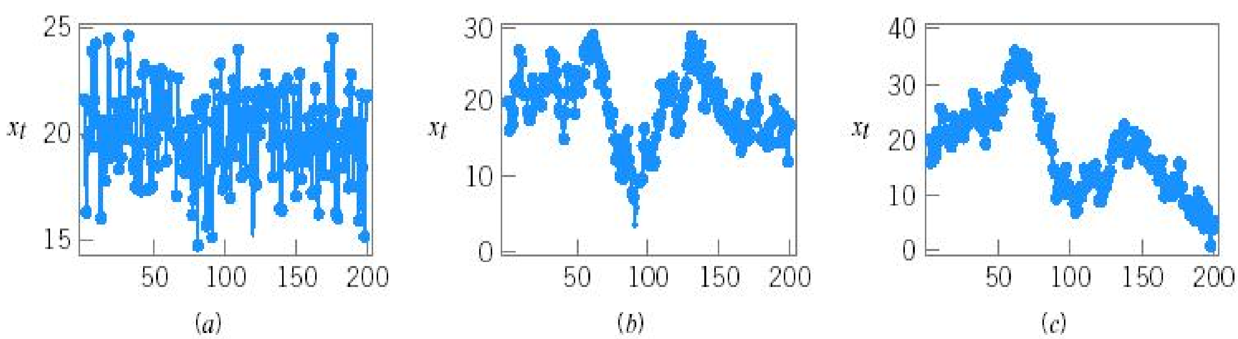

Figure 4-7 Data from three different processes. (*a*) Stationary and uncorrelated (white noise). (*b*) Stationary and autocorrelated. (*c*) Nonstationary.

6.5 : Reasons for Popularity of Control Charts

1. Control charts are a proven technique for improving productivity.
2. Control charts are effective in defect prevention.
3. Control charts prevent unnecessary process adjustment.
4. Control charts provide diagnostic information.
5. Control charts provide information about process capability.

Control Chart Theory

Control Chart Theory :

7.1 : Basic Principles

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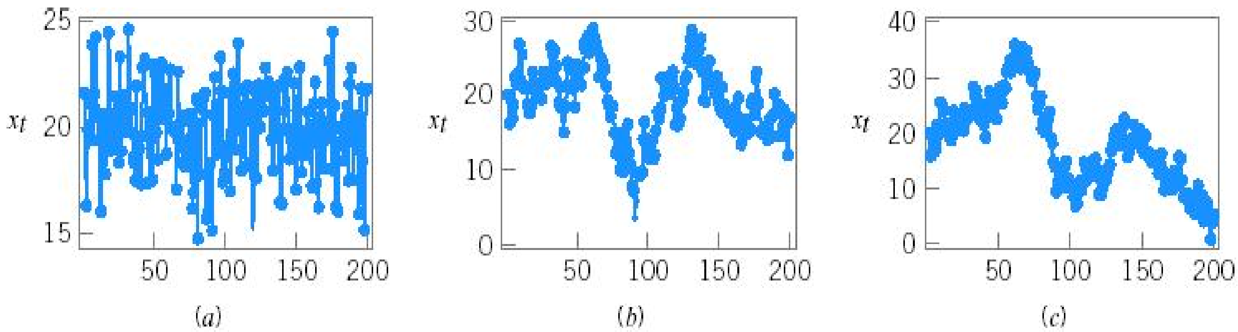

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7.5 : Choice of Control Limits

- 3-Sigma Control Limits
  - Probability of type I error is 0.0027
- Probability Limits
  - Type I error probability is chosen directly
  - For example, 0.001 gives 3.09-sigma control limits
- Warning Limits
  - Typically selected as 2-sigma limits

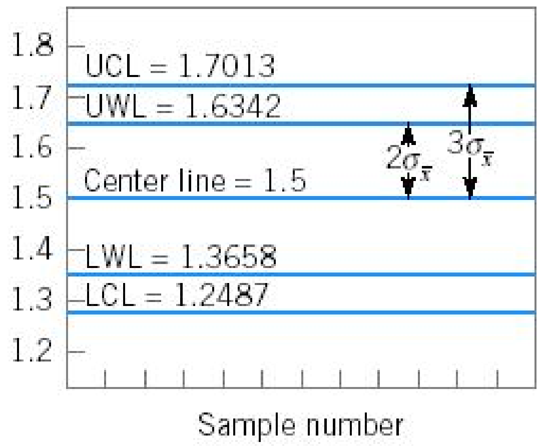

Figure 4-8 An $\bar{x}$ chart with two-sigma warning limits.

Another way to evaluate the decisions regarding sample size and sampling frequency is through the average run length (ARL) of the control chart. Essentially, the ARL is the average number of points that must be plotted before a point indicates an out-of-control condition. If the process observations are uncorrelated, then for any Shewhart control chart, the ARL can be calculated easily from

$$ARL = \frac{1}{P}$$

Where p is the probability that any point exceeds the control limits. This equation can be used to evaluate the performance of the control chart.

To illustrate, for the x chart with three-sigma limits, p = 0.0027 is the probability that a single point falls outside the limits when the process is in control. Therefore, the average run length of the x chart when the process is in control (called ARL_0) is

$$ARL_0 = \frac{1}{P} = \frac{1}{0.0027} = 370$$

That is, even if the process remains in control, an out-of-control signal will be generated every 370 samples, on the average.

The use of average run lengths to describe the performance of control charts has been subjected to criticism in recent years. The reasons for this arise because the distribution of run length for a Shewhart control chart is geometric distribution. Consequently, there are two concerns with ARL: (1) the standard deviation of the run length is very large, and (2) the geometric distribution is very skewed, so the mean of the distribution (the ARL) is not necessarily a very "typical" value of the run length.

For example, consider the Shewhart x control chart with three-sigma limits. When the process is in control, we have noted the p = 0.0027 and the in-control ARL_0 is ARL_0 = 1/p = 1/0.0027 = 370. This is the mean of the geometric distribution. Now the standard deviation of the geometric distribution is

$$\sqrt{(1-p)}\,p = \sqrt{(1-0.0027)}0.0027 \cong 370$$

That, is the standard deviation of the geometric distribution in this case is approximately equal to its mean. As a result, the actual ARL_0 observed in practice for the Shewhart x control chart will likely vary considerably. Furthermore, for the geometric distribution with p = 0.0027, the 10^{th} and 50^{th} percentiles of the distribution are 38 and 256, respectively. This mean that approximately 10% of the time the in-control run length will be less than or equal to 38 samples and 50% of the time it will be less than or equal to 256 samples. This occurs because the geometric distribution with p = 0.0027 is quite skewed to the right.

It is also occasionally convenient to express the performance of the control chart in terms of its average time to signal (ATS). If samples are taken at fixed intervals of time that are h hours apart, then

$$ATS = ARL.h$$

Consider the piston-ring process discussed earlier, and suppose we are sampling every hour. Equation 4-3 indicates that we will have a false alarm about every 370 hours on the average. .

Now consider how the control chart performs in detecting shifts in the mean. Suppose we are using a sample size of n = 5 and that when the process goes out of control the mean shifts to 74.015 mm. From the operating characteristic curve, we find that if the process mean is 74.015 mm, the probability of x falling between the control limits, is approximately 0.50. 'Therefore, p in equation 4-2 is 0.50, and the out-control ARL (called ARL_1) is

$$ARL_1 = \frac{1}{P} = \frac{1}{0.5} = 2$$

This is, the control chart will require two samples to detect the process shift, on the average, and since the time interval between samples is h = 1 hour, the average time required to detect this shift is

$$ATS = ARL_1 h = 2(1) = 2 \text{ hours}$$

Suppose that this is unacceptable, because production of piston rings with mean flow width of 1.725 microns results in excessive scrap costs and can result in further upstream manufacturing problems. How can we reduce the time needed to detect the out-of-control condition? One method is to sample more frequently. For example, if we sample every half hour, then the average time to signal for this scheme is $ATS = ARL_1\, h = 2(½) = 1$; that is, only one will elapse (on the average) between the shift and its detection. The second possibility is to increase the sample size. For example, if we use $n = 10$, then Fig. shows that the probability of x falling between the control limits when the process mean is 1.725 microns is approximately 0.1, so that $p = 0.9$, and from equation 4-2 the out-of-control ARL or ARL_1 is

$$ARL_1 = \frac{1}{P} = \frac{1}{0.9} = 1.11$$

and, if we sample every hour, the average time to signal is

$$\text{ATS} = \text{ARL}_1\, h = 1.11(1) = 1.11 \text{ hours}$$

Thus, the larger sample size would allow the shift to be detected about twice as quickly as the old one. If it became important to detect the shift in the (approximately) first hour after it occurred, two control chart designs would work:

Design 1	**Design 2**
Sample Size: n = 5	Sample Size: n = 10
Sampling Frequency: every half hour	Sampling Frequency: every hour

7.6 : Rational Subgroups

- The **rational subgroup** concept means that subgroups or samples should be selected so that if assignable causes are present, chance for differences *between* subgroups will be maximized, while chance for difference due to assignable causes *within* a subgroup will be minimized.
- Two general approaches for constructing rational subgroups:
    1. Sample consists of units produced at the same time − **consecutive** units
        - Primary purpose is to detect process shifts
    2. Sample consists of units that are representative of all units produced since last sample − **random sample of all process output over sampling interval**
        - Often used to make decisions about acceptance of product
        - Effective at detecting shifts to out-of-control state and back into in-control state *between* samples
        - Care must be taken because **we can often make any process appear to be in statistical control just by stretching out the interval between observations in the sample**.

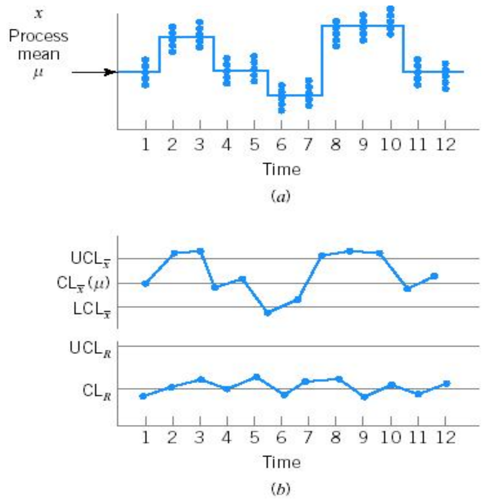

Figure 4-10 The "snapshot" approach to rational subgroups. (*a*) Behavior of the process mean. (*b*) Corresponding $\bar{x}$ and R control charts.

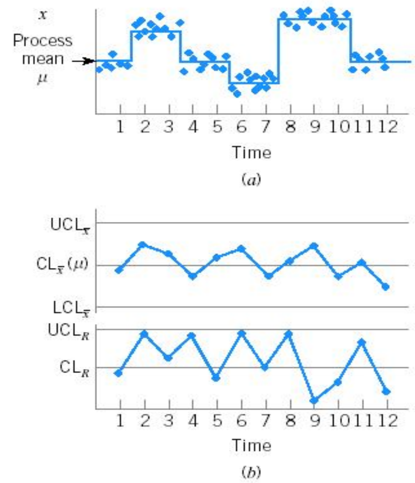

Figure 4-11 The random sample approach to rational subgroups. (*a*) Behavior of the process mean. (*b*) Corresponding $\bar{x}$ and R control charts.

Interpretation of Control Charts

Interpretation of Control Charts

8.1 : Analysis of Patterns on Control Charts

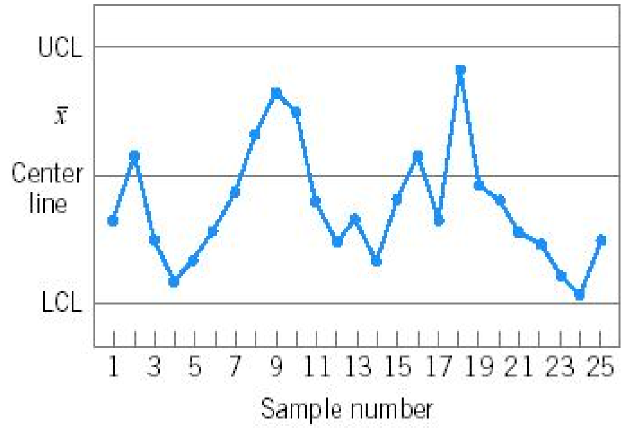

Figure 4-12 An $\bar{x}$ control chart.

- Pattern is very nonrandom in appearance
- 19 of 25 points plot below the center line, while only 6 plot above
- Following 4th point, 5 points in a row increase in magnitude, a *run up*
- There is also an unusually long *run down* beginning with 18th point

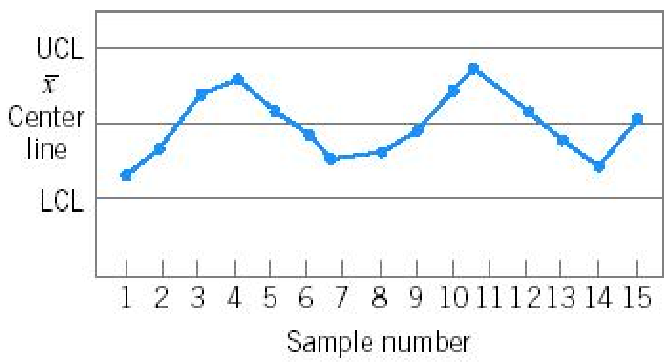

Figure 4-13 An $\bar{x}$ chart with a cyclic pattern.

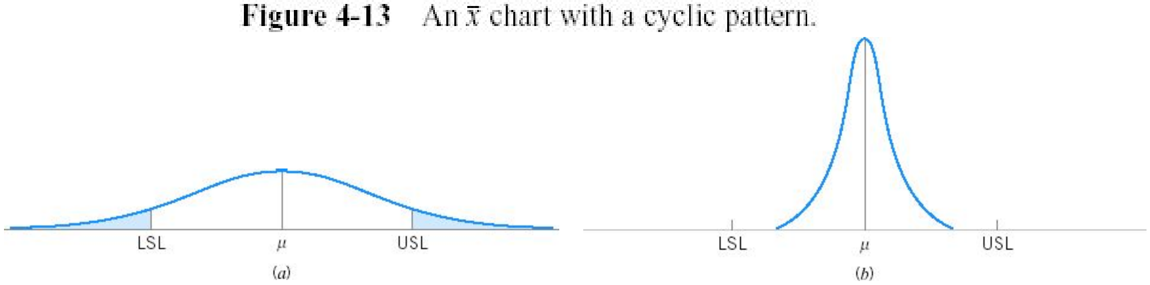

Figure 4-14 (*a*) Variability with the cyclic pattern. (*b*) Variability with the cyclic pattern eliminated.

The Western Electric Handbook (1956) suggests a set of decision rules for detecting nonrandom patterns on control charts. Specifically, it suggests concluding that the process is out of control if either

1. One point plots outside the three-sigma control limits;
2. Two out of three consecutive points plot beyond the two-sigma warning limits;
3. Four out of five consecutive points plot at a distance of one-sigma or beyond from the center line;
4. Eight consecutive points plot on one side of the centre line.

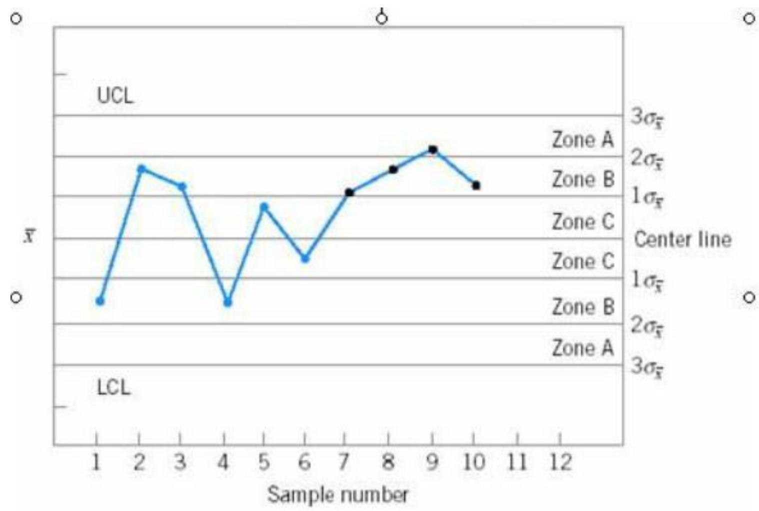

Figure 4-15 The Western Electric or zone rules, with the last four points showing a violation of the rule 3.

8.2 : Discussion of Sensitizing Rules for Control Charts

Some Sensitizing Rules for Shewhart Control Charts

Standard Action Signal

1. One or more points outside of the control limits.
2. Two of three consecutive points outside the two-sigma warning limits but still inside the control limits.
3. Four of five consecutive points beyond the one-sigma limits.
4. A run of eight consecutive points on one side of the center line.

Western Electric Rules

5. Six points in a row steadily increasing or decreasing.
6. Fifteen points in a row in zone C (both above and below the center line).
7. Fourteen points in a row alternating up and down.
8. Eight points in a row on both sides of the center line with none in zone C.
9. An unusual or nonrandom pattern in the data.

10. One or more points near a warning or control limit.

In general, care should be exercised when using several decision rules simultaneously. Suppose that the analyst uses *k* decision rules and that criterion *i* has type I error probability α_i. Then the overall type I error or false-alarm probability for the decision based on all *k* tests is

$$\alpha = 1 - \prod_{i=1}^{k}(1 - \alpha_i)$$

provided that all k decision rules are independent. However, the independence assumption is not valid with the usual sensitizing rules. Furthermore, the value of a; is not always clearly defined for the sensitizing rules, because these rules involve several observations.

Champ and Woodall (1987) investigated the average run length performance for the Shewhart control chart with various sensitizing rules. They found that the use of these rules does improve the ability of the control chart to detect smaller shifts, but the in control average run length can be substantially degraded. For example, assuming independent process data and using a Shew-bart control chart with the Western Electric rules results in an in-control ARL of 91.25, in contrast to 370 for the Shewhart control chart alone.

Some of the individual Western Electric rules are particularly troublesome. An illustration is the rule of several (usually seven or eight) consecutive points which either increase or decrease. This rule is very ineffective in detecting a trend, the situation for which it was designed. It done, however, greatly increase the false-alarm rate. See Davis and Woodall (1988) for more details.

8.2 : Phase I and Phase II of Control Chart Application

- Phase I is a **retrospective analysis** of process data to construct **trial control limits**
  - Charts are effective at detecting large, sustained shifts in process parameters, outliers, measurement errors, data entry errors, etc.
  - Facilitates identification and removal of assignable causes
- In phase II, the control chart is used to **monitor** the process
  - Process is assumed to be reasonably stable
  - Emphasis is on **process monitoring**, not on bringing an unruly process into control

Choice of Control Limits

- 3-Sigma Control Limits
  - Probability of type I error is 0.0027
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– For example, 0.001 gives 3.09-sigma control limits

- Warning Limits
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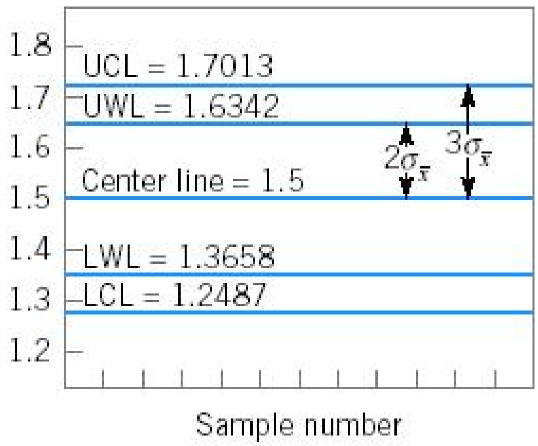

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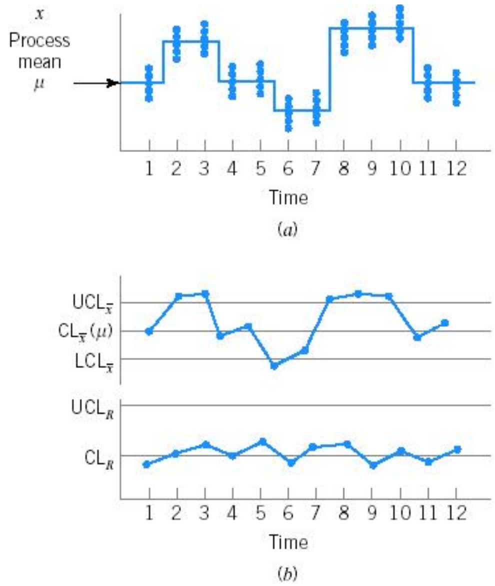


Figure 4-10 The "snapshot" approach to rational subgroups. (*a*) Behavior of the process mean. (*b*) Corresponding $\bar{x}$ and R control charts.

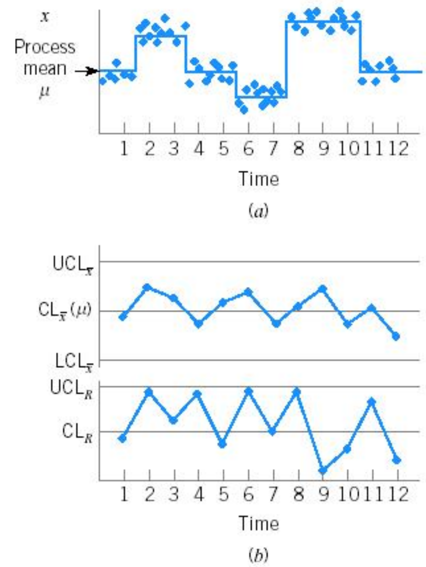


Figure 4-11 The random sample approach to rational subgroups. (*a*) Behavior of the process mean. (*b*) Corresponding $\bar{x}$ and R control charts.

SEVEN QUALITY CONTROL TOOLS

SEVEN QUALITY CONTROL TOOLS :

The Seven Quality Control tools as proposed by Dr.Kaoru Ishikawa, Professor at Tokyo University & Father of QC in Japan are:

1. Histogram or stem-and-leaf plot
2. Check sheet
3. Pareto chart
4. Cause-and-effect diagram
5. Defect concentration diagram
6. Scatter diagram
7. Control chart

Dr.Kaoru Ishikawa, Professor at Tokyo University & Father of QC in Japan further specified the following approach to Quality Problem solving.

1. Cause Analysis Tools are Cause and Effect diagram, Pareto analysis & Scatter diagram.
2. Evaluation and decision making tools are decision matrix and multivoting
3. Data Collection and analysis tools are check sheet, control charts, DOE, scatter diagram, stratification, histogram, survey.
4. IDEA CREATION TOOLS are Brainstorming, Benchmarking, Affinity diagram, Normal group technique.
5. Project Planning and Implementation tools are Gantt Chart and PDCA cycle.

Cause and effect diagram (also called Ishikawa or fishbone chart)

Description : The fishbone diagram identifies many possible causes for an effect or problem. It can be used to structure a brainstorming session. It immediately sorts ideas into useful categories.

When to Use : When identifying possible causes for a problem.

Especisally when a teams thinking tends to fall into ruts

Procedure for constructing Cause and Effect diagram :

Materials required : Flipchart (or) White Board, Marking Pens

Agree on a problem statement (effect). Write it at the center right of the flipchart or whiteboard. Draw a box around it and draw a horizontal arrow running to it.

Brainstorm the major categories of causes of the problem. If this is difficult use generic headings :
Methods
Machines (equipment)
People (manpower)
Materials
Measurement
Environment

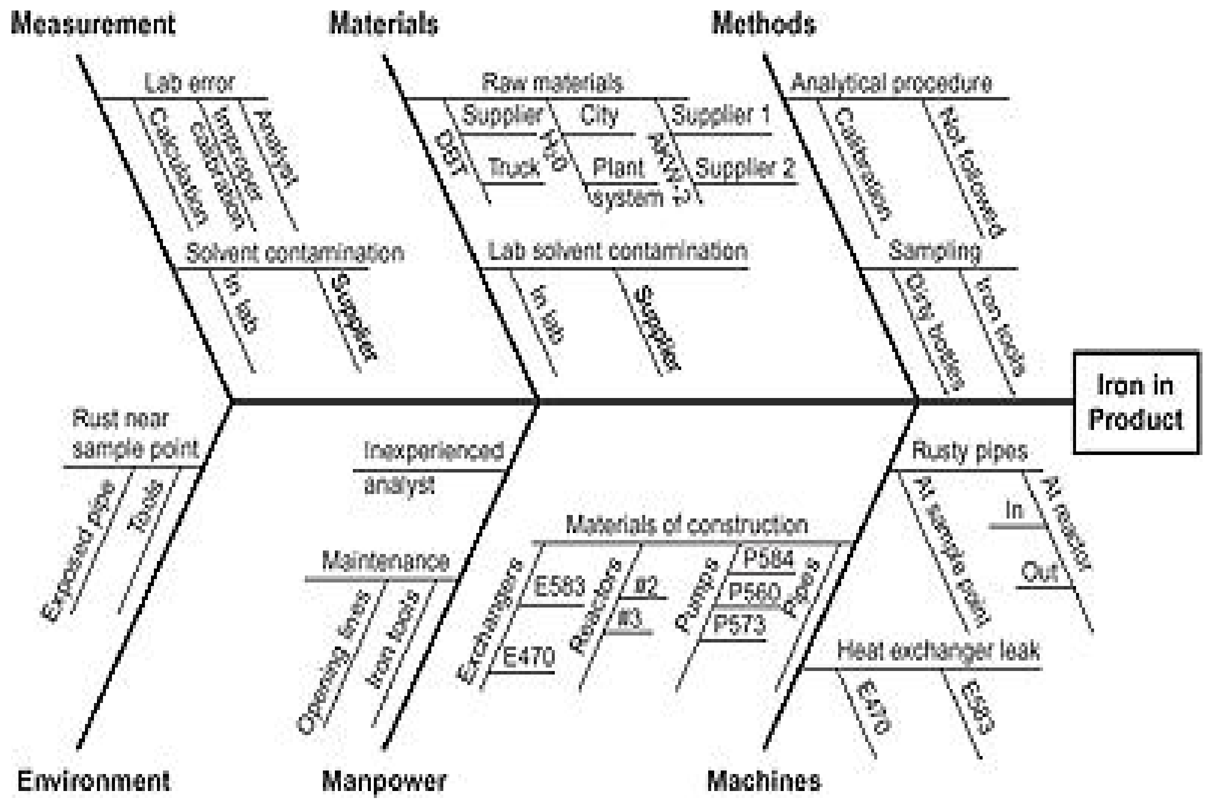
Write the categories of causes as branches from the main arrow.

Brainstorming all the possible causes of the problem. Ask: “Why does this happen?” As each idea is given, the facilitator writes it as a branch from the appropriate category. Causes can be written in several places if they relate to several categories.

Again ask “Why does this happen?” About each cause. Write sub-causes branching off the causes. Continue to ask “Why?” And generate deeper levels of causes. Layers of branches indicate causal relationships.

When the group runs out of ideas, focus attention to places on the chart where ideas are few.

Example : This fishbone diagram was drawn by a manufacturing team to try to understand the source of periodic iron contamination. The team used the six generic headings to prompt ideas. Layers of branches show thorough thinking about the causes of the problem.



For example, under the heading "Machines," the idea "materials of construction" shows four kinds of equipment and then several specific machine numbers.

Note that some ideas appear in two different places. "Calibration" shows up under "Methods" as a factor in the analytical procedure, and also under "Measurement" as a cause of lab error. "Iron tools" can be considered a "Methods" problem when taking samples or a "Manpower" problem with maintenance personnel.

Check Sheet (or) Defect Concentration Diagram :

Description : A check sheet is a structured, prepared form for collecting and analyzing data. This is a generic tool that can be adapted for a wide variety of purposes.

When to Use :

When data can be observed and collected repeatedly by the same person or at the same location.

When collecting data on the frequency or patterns of events, problems, defects, defect location, defect causes etc.

When collecting data from a production process.

Procedure :

Decide what event or problem will be observed. Develop operational definitions.

Decide when data will be collected and for how long.

Design the form. Set it up so that data can be recorded simply by making check marks or Xs or similar symbols and so that data do not have to be recopied for analysis.

Label all spaces on the form.

Test the check sheet for a short trial period to be sure it collects the appropriate data and is easy to use.

Each time the targeted event or problem occurs, record data on the check sheet.

Example : The figure below shows a check sheet used to collect data on telephone interruptions. The tick marks were added as data was collected over several weeks.

Telephone Interruptions

Reason	Day					
	Mon	Tues	Wed	Thurs	Fri	Total
Wrong number	𝍸	||	|	𝍸	𝍸||	20
Info request	||	||	||	||	||	10
Boss	𝍸	||	𝍸||	|	||||	19
Total	12	6	10	8	13	49

Histogram : The most commonly used graph for showing frequency distributions, or how often each different value in a set of data occurs. The data are numerical values.

To see the shape of the data's distribution, especially when determining whether the output of a process is distributed approximately normally.

Analyzing whether a process can meet the customers requirements.

Analyzing whether a process can meet the customer's requirements

Analyzing what the output from a supplier's process looks like. Whether a process change has occurred from one time period to another.

To determine whether the outputs of two or more processes are different.

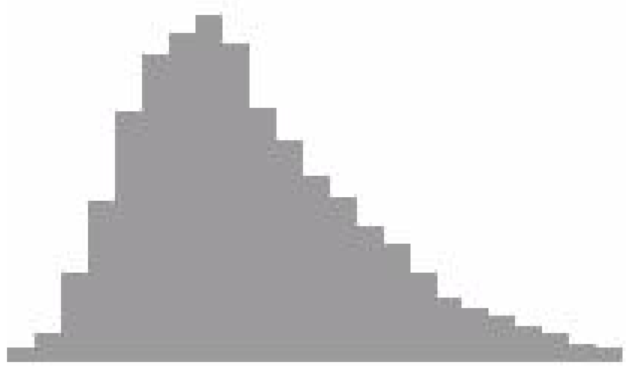
To communicate the distribution of data quickly and easily to others.

Histogram Shapes and Meaning

Normal : A common pattern is the bell shaped curve known as the "normal distribution" In a normal distribution, points are as likely to occur on one side of the average as on the other.

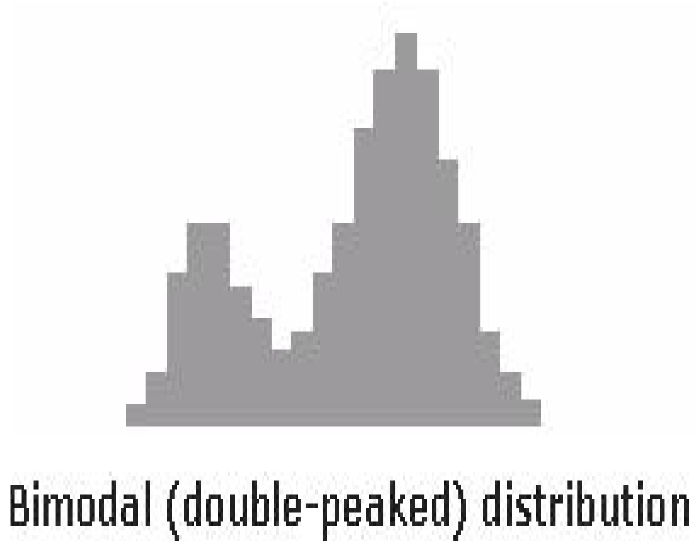
Normal distribution

Skewed : The skewed distribution is asymmetrical because a natural limit prevents outcomes on one side. The distribution's peak is off center toward the limit and a tail stretches away from it.



Right-skewed distribution

Double Peaked or bimodal : The bimodal distribution looks like the back of a two humped camel. The outcomes of two processes with different distributions are combined in one set of data. A two shift operation might be bimodal.



Plateau : The plateau might be called a "multimodal distribution". Several processes with normal distributions are combined. Because there are many peaks close together, the top of the distribution resembles a plateau.

Dog food : The dog food distribution is missing something – results near the average. If a customer receives this kind of distribution, someone else is receiving a heart cut, and the customer is left with the "dog food", the odds and ends left over after the masters meal.

CHECK SHEET **DEFECT DATA FOR 2002–2003 YTD**																		
Part No.: TAX-41 Location: Bellevue Study Date: 6/5/03 Analyst: TCB																		
	2002												2003					
Defect	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	Total
Parts damaged		1		3	1	2		1		10	3		2	2	7	2		34
Machining problems			3	3				1	8		3		8	3				29
Supplied parts rusted			1	1		2	9											13
Masking insufficient		3	6	4	3	1												17
Misaligned weld	2																	2
Processing out of order	2															2		4
Wrong part issued		1						2										3
Unfinished fairing			3															3
Adhesive failure				1							1		2			1	1	6
Powdery alodine					1													1
Paint out of limits						1								1				2
Paint damaged by etching			1															1
Film on parts						3		1	1									5
Primer cans damaged								1										1
Voids in casting									1	1								2
Delaminated composite										2								2
Incorrect dimensions											13	7	13	1		1	1	36
Improper test procedure										1								1
Salt-spray failure													4			2		4
TOTAL	4	5	14	12	5	9	9	6	10	14	20	7	29	7	7	6	2	166

Figure 4-16 A check sheet to record defects on a tank used in an aerospace application.

Pareto Chart

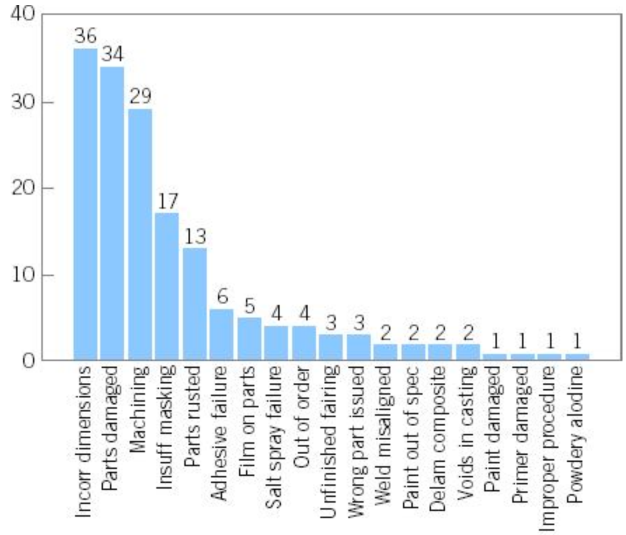


Figure 4-17 Pareto chart of the tank defect data.

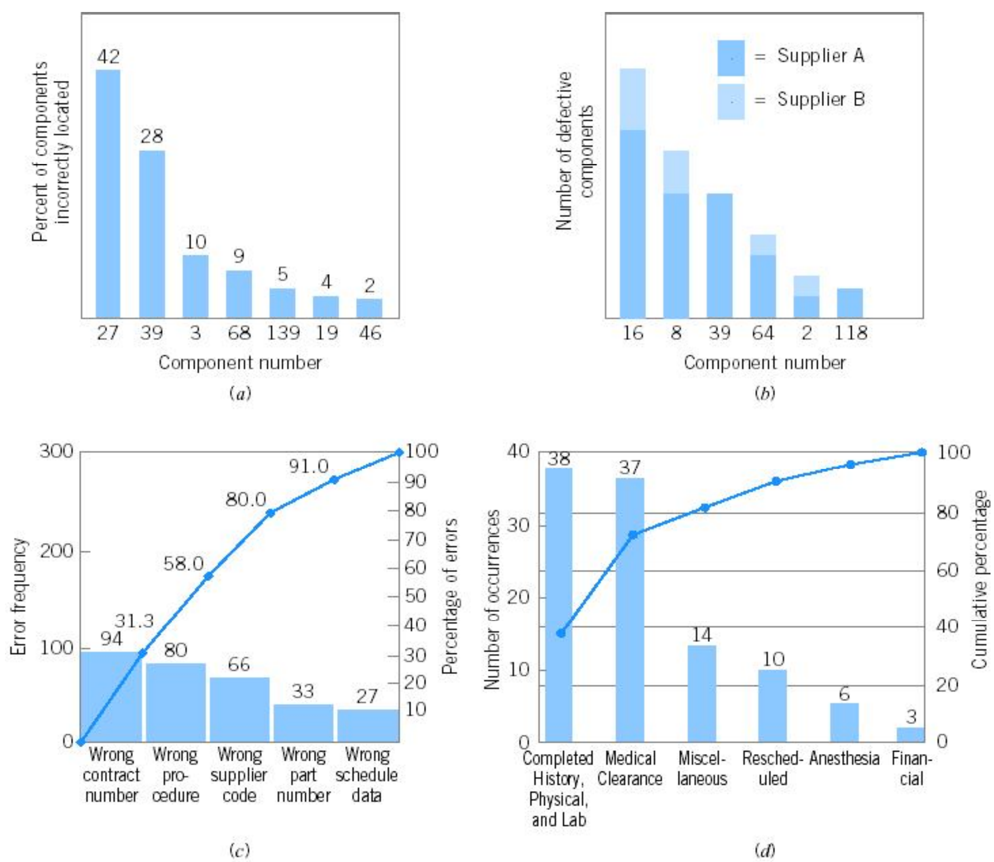


Figure 4-18 Various examples of Pareto charts.

Cause-and-Effect Diagram

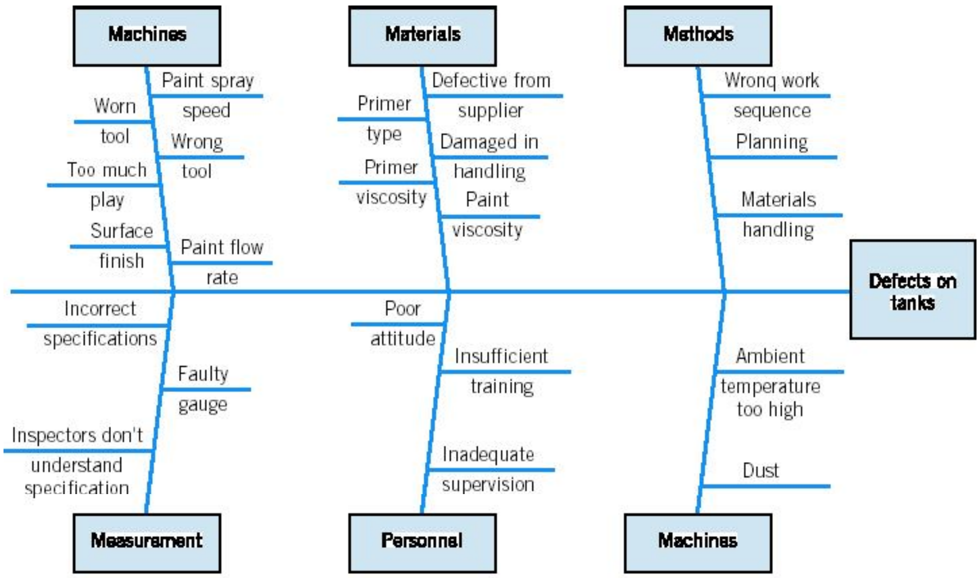


Figure 4-19 Cause-and-effect diagram for the tank defect problem.

How to Construct a Cause-and-Effect Diagram

1. Define the problem or effect to be analyzed.
2. Form the team to perform the analysis. Often the team will uncover potential causes through brainstorming.
3. Draw the effect box and the center line.
4. Specify the major potential cause categories and join them as boxes connected to the center line.
5. Identify the possible causes and classify them into the categories in step 4. Create new categories, if necessary.
6. Rank order the causes to identify those that seem most likely to impact the problem.
7. Take corrective action.

Defect Concentration Diagram

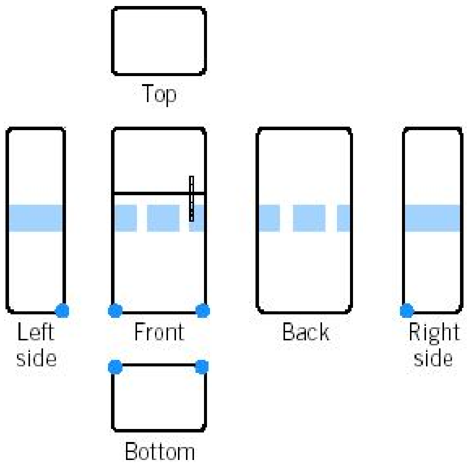


Figure 4-20 Surface-finish defects on a refrigerator.

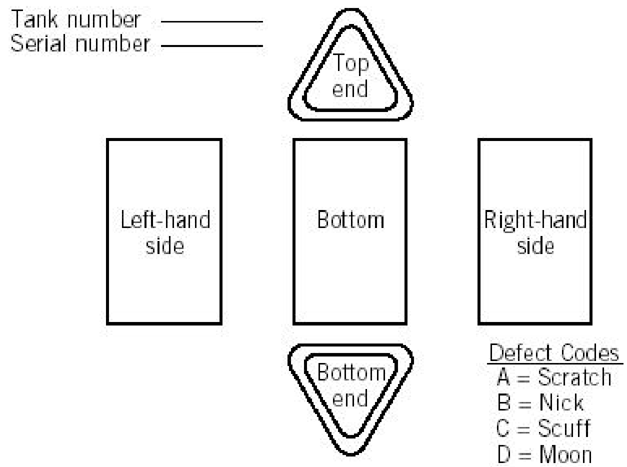


Figure 4-21 Defect concentration diagram for the tank.

Scatter Diagram

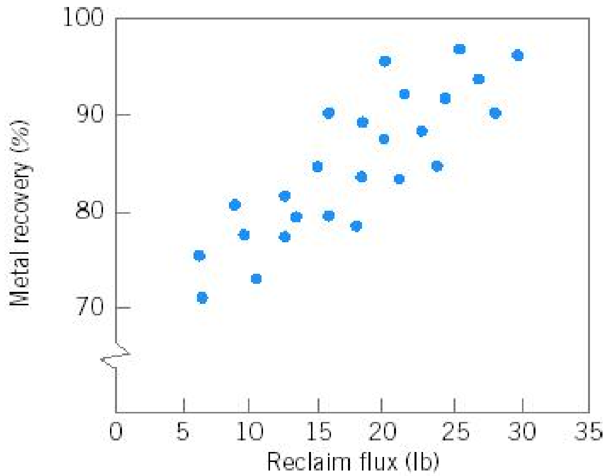


Figure 4-22 A scatter diagram.

Elements of a Successful SPC Prouam

1. Management leadership
2. A team approach
3. Education of employees at all levels
4. Emphasis on reducing variability
5. S. Measuring success in quantitative (economic) terms
6. A mechanism for communicating successful results throughout the organization

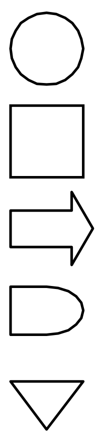
Nonmanufacturing Applications of Statistical Process Control

- Nonmanufacturing applications do not differ substantially from industrial applications, but sometimes require ingenuity
    1. Most nonmanufacturing operations do not have a natural measurement system
    2. The observability of the process may be fairly low
- **Flow charts** and **operation process charts** are particularly useful in developing process definition and process understanding. This is sometimes called **process mapping**.
    1. Used to identify **value-added** versus **nonvalue-added** activity

Ways to Eliminate Nonvalue-Add Activities

1. Rearranging the sequence of work steps
2. Rearranging the physical location of the operator in the system
3. Changing work methods
4. Changing the type of equipment used in the process
5. Redesigning forms and documents for more efficient use
6. Improving operator training
7. Improving supervision
8. Identifying more clearly the function of the process to all employees
9. Trying to eliminate unnecessary steps
10. Trying to consolidate process steps

Operation Process Chart Symbols



= Operation

= Inspection

= Movement or transportation

= Delay

= Storage

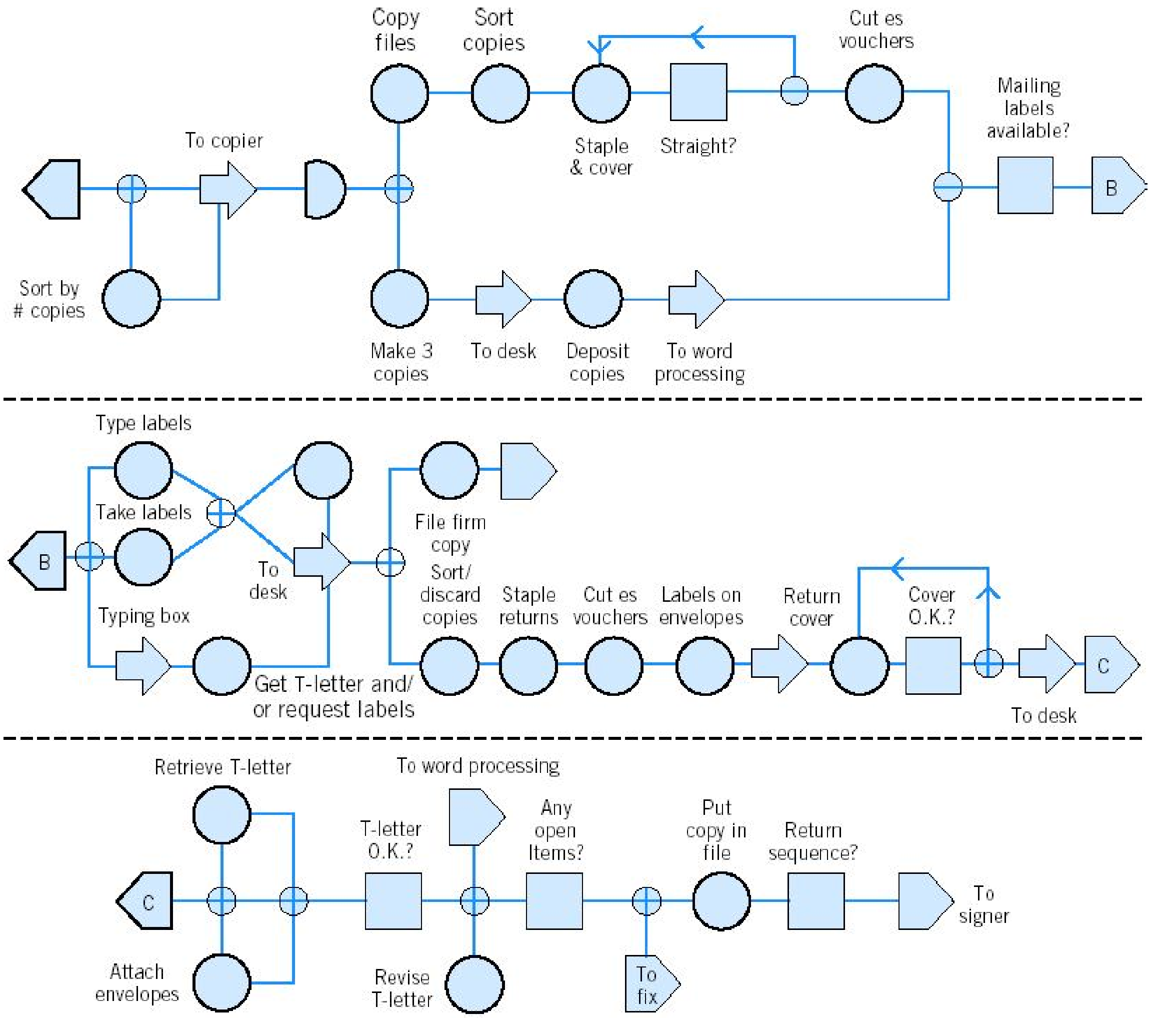


Figure 4-31 Flow chart of the assembly portion of the Form 1040 tax return process.

Important Terms and concepts

- Assignable causes of variation
- Average run length (ARL)
- Average time to signal
- Cause-and-effect diagram
- Chance causes of variation
- Control Chart
- Control limits
- Defect concentration diagram
- Designed experiments
- Flow charts and operations process charts
- Histogram

- In-control process
- “Magnificent Seven”
- Out-of-control-action plan (OCAP)
- Out-of-control process
- Pareto Chart
- Patterns no control charts
- Phase I and Phase II application of control charts
- Rational subgroups
- Sample size for control charts
- Sampling frequency for control charts
- Scatter diagram
- Sensitizing rules for control charts
- Shewhart control charts
- Statistical Control of a process
- Statistical process control (SPC)
- Steam-and-leaf plot
- Three sigma control limits
- Warning limits

Introduction to Control charts

Statistical process control

- Statistical process control is a collection of tools that when used together can result in process stability and variability reduction.
- A stable process is a process that exhibits only common variation, or variation resulting from inherent system limitations.
- A stable process is a basic requirement for process improvement efforts.

Advantage of a stable process

- Management knows the process capability and can predict performance, costs, and quality levels.
- Productivity will be at a maximum, and costs will be minimized.
- Management will be able to measure the effects of changes in the system with greater speed and reliability.
- If management wants to alter specification limits, it will have the data to back up its decision.

Categories of variation in piece part production

- Within-piece variation
- Piece-to-piece variation
- Time-to-time variation

Source of variation

Variation is present in every process due to a combination of the equipment, materials, environment, and operator.

The first source of variation is the equipment. This source includes tool wear, machine vibration, work holding-device positioning, and hydraulic and electrical fluctuations. When all these variations are put together, there is a certain capability or precision within which the equipment operates.

The second source of variation is the material. Since variation occurs in the finished product, it must also occur in the raw material (which was someone else's finished product). Such quality characteristics as tensile strength, ductility, thickness, porosity, and moisture content can be expected to contribute to the overall variation in the final product.

A third source of variation is the environment. Temperature, light, radiation, electrostatic discharge, particle size, pressure, and humidity can all contribute to variation in the product. In order to control this source, products are sometimes manufactured in white rooms. Experiments are conducted in outer space to learn more about the effect of the environment on product variation.

A fourth source is the operator This source of variation includes the method by which the operator performs the operation. The operator's physical and emotional well-being also contribute to the variation. A cut finger, a twisted ankle, a personal problem, or a headache can make an operator's quality performance vary. An operator's lack of understanding of equipment and material variations due to lack of training may lead to frequent machine adjustments, thereby compounding the variability.

The above four sources account for the true variation. There is also a reported variation, which is due to the inspection activity. Faulty inspection equipment, the incorrect application of a quality standard, or too heavy a pressure on a micrometer can be the cause of the incorrect reporting of variation. In general, variation due to inspection should be one-tenth of the four other sources of variations. It should be noted that three of these sources are present in the inspection activity-an inspector, inspection equipment, and the environment.

Chance and Assignable Causes of Quality Variation

As long as these sources of variation fluctuate in a natural or expected manner, a stable pattern of many chance causes (random causes) of variation develops. Chance causes of variation are inevitable. Because they are numerous and individually of relatively small importance, they are difficult to detect or identify.

When only chance causes are present in a process, the process is considered to be in a state of statistical control. It is stable and predictable. However, when an assignable cause of variation is also present, the variation will be excessive, and the process is classified as out of control or beyond the expected natural variation.

- A process that is operating with only chance causes of variation present is said to be in statistical control.
- A process that is operating in the presence of assignable causes is said to be out of control.
- The eventual goal of SPC is reduction or elimination of variability in the process by identification of assignable causes.

Control chart

- Control chart was developed to recognize constant patterns of variation.
- When observed variation fails to satisfy criteria for controlled patterns, the chart indicate this.
- Control chart allow us to distinguish between controlled and uncontrolled processes

Statistical Basis of the Control Chart

Basic Principles

A typical control chart has control limits set at values such that if the process is in control, nearly all points will lie between the upper control limit (UCL) and the lower control limit (LCL).

Definition :

A control chart is defined as a statistical tool used to detect the presence of assignable causes in any manufacturing systems and it will be influenced by the pure system of chance causes only

Control charts are of two types : Variable control charts and attribute control charts

Variable Control charts : A variable control chart is one by which it is possible to measure the quality characteristics of a product. The variable control charts are

(i) $\overline{x}$ - chart

(ii) R – chart

(iii) o – chart

Attribute Control chart : An attribute control chart is one in which iti is not possible to measure the quality characteristics of a product i.e., it is based on visual inspection only like good or bad success or failure, accepted or rejected. The attribute control charts are.

(i) p - chart
(ii) np – chart
(iii) c – chart
(iv) u - chart

Objectives of control charts

- Control charts are used as one source of information to help whether an item or items should be released to the customer.
- Control charts are used to decide when a normal pattern of variation occurs, the process should be left alone when an unstable pattern of variable occurs which indicates the presence of assignable causes it requires an action to eliminate it.
- Control charts can be used to establish the product specification.
- To provide a method of instructing to the operating and supervisory personnel (employees) in the technique of quality control.

Notations

x : Mean of the sample

$\sigma_{\bar{x}}^{1}$: Standard deviation of the sample

$\bar{x}^{1}$: Mean of the population or universe

σ^{1} : Standard deviation of the population

Central Limit Theorem

Irrespective of the shape of the distribution of the universe, the average value of a sample size 'n' (X bar1, X bar2, X bar3 ------------- n) drawn from the population will tend towards a normal distribution as n tends to infinity.

Relation between R bar and σ^1 - $u_i{}^1 = \frac{\bar{R}}{d_2}$

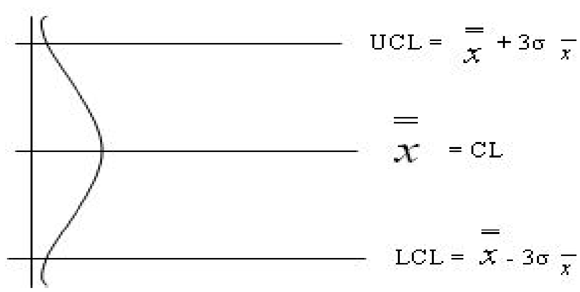
$\bar{R}$ = Mean Range

d2 = Depends upon sample size from the tables

Also $\sigma_{\bar{x}} = \frac{\sigma^1}{\sqrt{n}}$

Where n = sample size

Determination of control limits for $\bar{x}$ chart when the range is known.



$\bar{\bar{x}}$ = Mean of all the sample

$$UCL = \bar{\bar{x}} + 3\sigma_{\bar{x}}$$

$$= \bar{\bar{x}} + 3\frac{\sigma'}{\sqrt{n}}$$

$$= \bar{\bar{x}} + 3\frac{\bar{R}}{d_2\sqrt{n}}$$

$$UCL = \bar{\bar{x}} + A_2\bar{R}$$

Where $A_2 = \dfrac{3}{d_2\sqrt{n}}$

A_2 constant which is obtained from the tables

Similarly LCL $= \bar{\bar{x}} - A_2\bar{R}$

Control Limits for $\bar{\bar{x}}$ chart

$$UCL = \bar{\bar{x}} + A_2\bar{R}$$

$$LCL = \bar{\bar{x}} - A_2\bar{R}$$

$$CL = \bar{\bar{x}}$$

Control Limits for R chart

$$UCL = D_4\ R$$

$$LCL = D_3\ \bar{R}$$

$$CL = \bar{R}$$

Interpretation of Control Charts

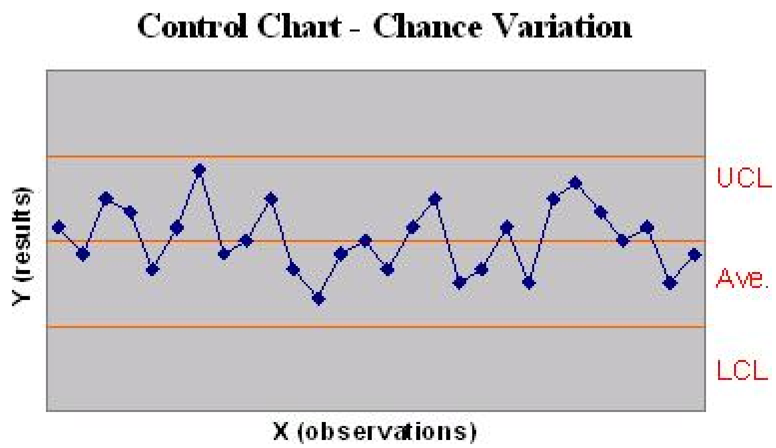
After plotting the points on the X bar - R charts, it shows two possible states of control. They are

1. State of statistical control and
2. State of lack of control.

State of Statistical Control

A manufacturing process is said to be in a state of statistical control whenever it is operated upon by a pure system of chance causes. The display of points in the X bar chart and R chart will be distributed evenly and randomly around the center line and all the points should fall between the UCL and LCL.

Control Charts - in Control VS Chance Variation



State of Lack of Control

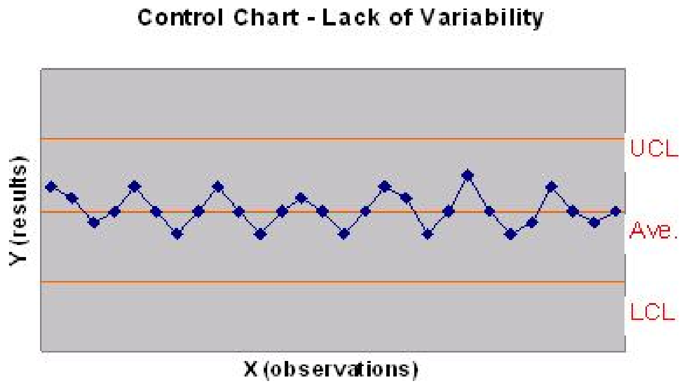
A process is said to be in a state of lack of control whenever the state of statistical control does not hold good. In such a state we interpret the presence of assignable causes, the reason for lack of control are

- Points violating the control limits
- Run
- Trend
- Clustering
- Cycle pattern

Control Charts Interpretation

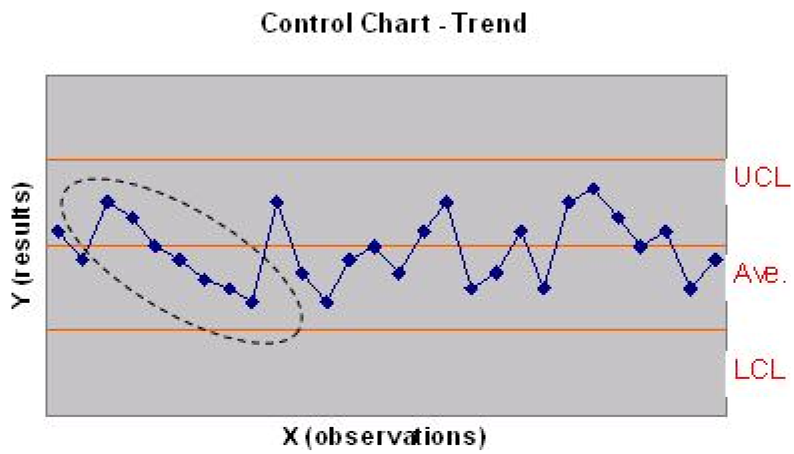
- Special: Any point above UCL or below LCL
- Run : > 7 consecutive points above or below centerline
- 1-in-20: more than 1 point in 20 consecutive points close to UCL or LCL
- Trend: 5-7 consecutive points in one direction (up or down)

Control Charts - Lack of Variability



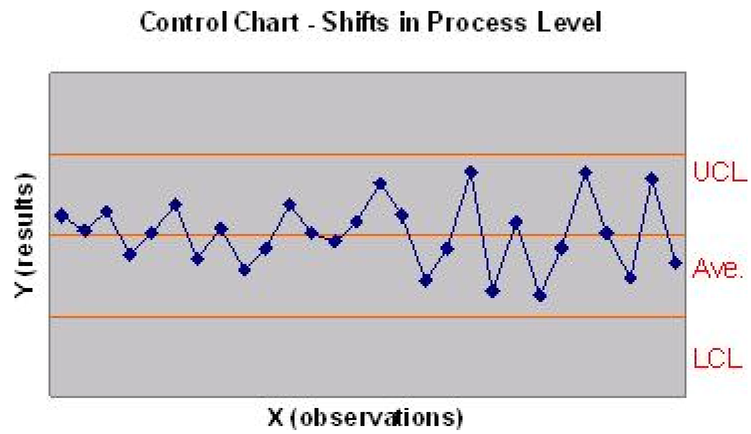
Control Charts – Lack of Variability

X- Bar Causes	R Causes
- Incorrect calculation of control limits - Significant process gains Operator not making checks or Checks wrong	- Significant process gains
Corrective Actions	
- Check control limits - Validate subgroup/lot sampling techniques - Verify checking procedures - Reevaluate operator training	



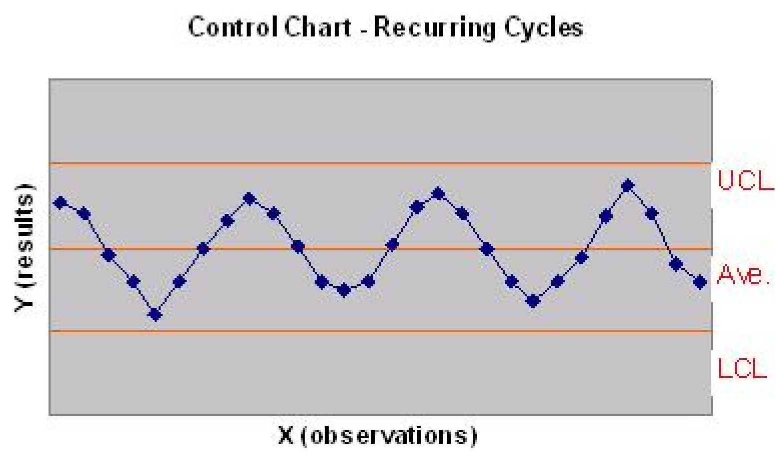
X- Bar Causes	R Causes
Deterioration of machine - Tired operator - Tool wear	- Improvement/deteriorate- TION OF OPERATOR skill - Tired operator - Change in incoming material quality
Corrective Actions	
- Repair or use alternate machine if available - Discuss operations with operator to find cause - Rotate operator to verify common/special cause - Change, Repair, or Sharpen tools - Investigate incoming material condition	

Control Charts shifts in Process Levels



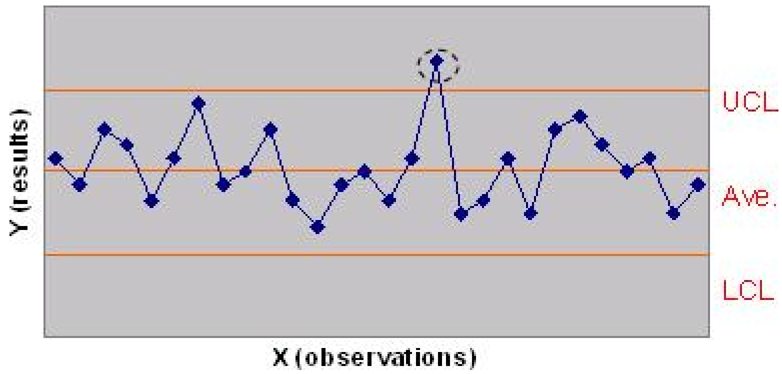
X- Bar Causes	R Causes
- Changes in materials pro- Portions - New operator or machine - Changes in process - Changes in inspection	- Change in material - Change in method - Change in operator - Change in inspection
Corrective Actions	
- More consistent material supply - Investigate source of material - CHECK machine - Check operator procedures/methods - Check test equipment calibration	

Control Charts Recurring Cycles



X- Bar Causes	R Causes
- Physical environment: Temperature or humidity - Tired operator - Scheduled rotation of machine and/or operator	- Scheduled maintenance - Tired operator - Tool wear
Corrective Actions	
- Adjust equipment more frequently and accurately - Service equipment - Rotate operators - Evaluate machine maintenance - Replace, Sharpen or Repair tool	

<u>**Control Charts points near or outside limits**</u>



X- Bar Causes	R Causes
- Over control - Large system deltas in material quality - Large system deltas in Test equipment/method	- Mixture of material of Distinctly different Quality
Corrective Actions	
- Check control limits - Investigate material variation - Evaluate Test procedure - Evaluate inspection frequency or method - Possible Over adjustment by operator	

Applications of X bar – R Chart with Real life data

Problem 3.
The following are the X bar - R values of 20 subgroup of 5 readings each

S.G No	**X bar**	**R**
1	34.0	4
2	31.6	2
3	30.8	3
4	33.8	5
5	31.6	2
6	33.0	5
7	**28.2**	**13**
8	33.8	**19**
9	**37.8**	6
10	35.8	4
11	**38.4**	4
12	34.0	**14**
13	35.0	4
14	33.8	7
15	31.6	5
16	33.0	7
17	32.6	3
18	31.8	9
19	35.6	6
20	33.0	4

$$\Sigma \bar{x} = 669.2$$

$$\Sigma R = 126.0$$

(a) Determine the control limits for X bar and R chart.
(b) Construct the and R chart and interpreter the result.
(c) What is process capability?
(d) Does it appear that the process is capable of meeting the specification limits.
(e) Determine the percentage age of rejection if any
The specification limits are =33±5.

$$\Sigma \bar{x} = 669.2$$

$$\Sigma R = 126.0$$

$$x = \frac{\Sigma \bar{x}}{k} = \frac{669.2}{20} = 33.46$$

$$\bar{R} = \frac{\sum R}{K} = \frac{126}{20} = 63$$

For a subgroup size of 5 from tables

A2 = 0.58
d2 = 2.326
D3 = 0.0
D4 = 2.11

Control limits for R- chart

UCL= D4 $\bar{R}$ = 2.11 x 6.3
LCL = D3 $\bar{R}$ = 0.0
CL = $\bar{R}$ = 6.3

It is seen from the data two subgroup are crossing the UCL which indicates the presence of assignable causes. So the homogenization is necessary.

$$\bar{R} = \frac{126.0 - 14 - 19}{20 - 2}$$

$$= \frac{93}{18} = 5.17$$

Again control limits for R-chart

UCL= $D_4 \bar{R}_1$ = 2.11 x 5.17

LCL= $D_3 \bar{R}_1$ = 0.0

CL = $\bar{R}_1$ = 5.17

Again one more subgroup is crossing the UCL

$$\bar{R}_2 = \frac{126 - 14 - 19 - 13}{20 - 3} = 4.7$$

Again control limits for R-chart

UCL= $D_4 \bar{R}_2$ = 2.11 x 4.17 = 9.917

LCL= $D_3 \bar{R}_2$ = 0 x 4.7 = 0.0

CL = $\bar{R}_2$ = 4.7

Now all the points are falling with the control limits. The final values are

UCL = 9.917
LCL = 0.0
CL = 4.7

Control limits for $\bar{\bar{X}}$ - chart

UCL = $\bar{\bar{X}} + A_2 \bar{R}_2$

$= 33.46 + 0.58 \times 4.7$
$= 36.186$

LCL $= \overline{\overline{X}} - A_2 \overline{R_2}$
$= 30.734$

CL $= \overline{\overline{X}} = 33.46$

It is seen from the data that three subgroup are crossing the control limits. Which indicates the presence of assignable causes. So homogenization is necessary

$$X_1 = \frac{669.2 = 37.8 - 38.4 - 28.2}{20 - 3}$$
$= 33.22$

Again control limits for X-bar -chart

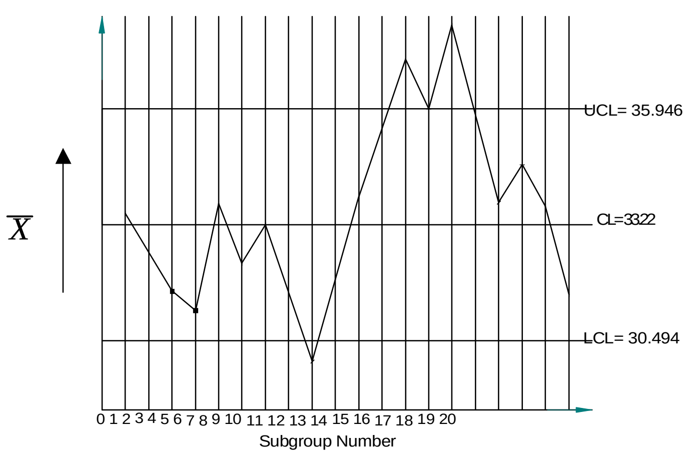
UCL $= \overline{\overline{X}}_1 + A_2 \overline{R_2}$
$= 33.22 + 0.58 \times 4.7$
$= 35.946$

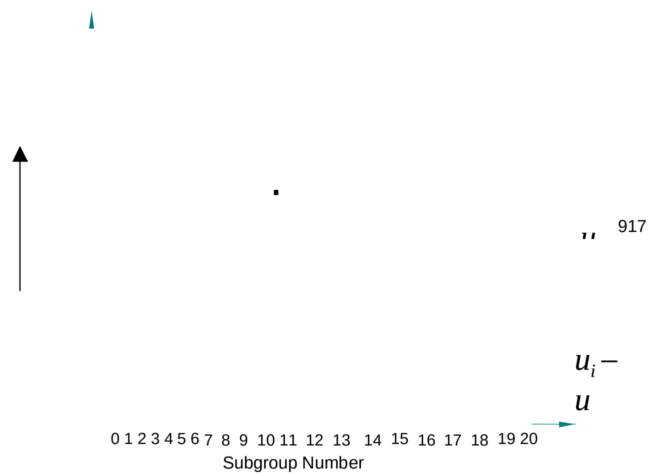
LCL $= \overline{\overline{X}}_1 - A_2 \overline{R_2}$
$= 33.22 - 0.58 \times 4.7$
$= 30.494$

CL $= \overline{\overline{X}}_1 = 33.22$

Now all the points are falling within the control limits. The final value are
UCL = 35.946
LCL = 30.494
CL = 33.22

The Charts are plotted for the final values





(b) Interpretation R-chart is not in control. Some points are crossing the UCL, - chart is not in control. Points are crossing the control limits. So process is not in a of statistical control

$$o1 = \frac{\bar{R}_2}{d_2} = \frac{4.7}{2.326} = 2.02$$

The process capability = 6o1

= 6 x 2.02

= 12.12

(d) UCL – LSL =10

Since 6o1 > (UCL – LCL), the process is not capable of meeting the specifications limits.

(e) UNTL = $\bar{X}^1$+3o1

= 33.22 + 3 x 2.02

= 39.28

LNTL = $\bar{X}^1$-3o1

= 33.22 - 3 x 2.02

= 27.16

CL = $\bar{\bar{X}}_1$ = 33.22

UCL = 38

LSL = 28

$$Below\ Z = \frac{28 - 33.22}{2.02} = - 2.58$$

Probability = 0.0052 = 0.52%

Above Z = $\frac{38 - 33.22}{2.02}$ – 2.36

Probability = 0.9909 = 99.09%

Therefore 100 – 99.09 = 0.91%

Total Rejection = .052 + 0.91 = 1.43%

Problem - 4

A Control charts has been used to monitor a certain characteristic. The process is sampled in a subgroup size of 4 at an interval of 2 hours. $\bar{X}$ - chart has 3σ control limits of 121 and 129 with the target value of $\bar{X}'$, = 125.

(a) If the product is sold to a user who has a specification of 127 ±8. What percentage of the product will not meet the specification assuming normally distributed output.

(b) If the target value of the process can be shifted without effect on the process standard deviation, what target value would minimise the amount of product being outside the specifications.

(c) At this new target value what percentage of the product will not meet the specification requirements.

Solution. Subgroup size 'n' = 4

UCL = 129

LCL = 121

CL = $\bar{\bar{X}} = \bar{X}' = 125.$

Specification limits = 127 ±8

USL = 135

LSL = 119

From tables, for a subgroup size of 4.

A2 = 0.73

d2 = 2.059

D3 = 0.0

D4 = 2.28

UCL = $\bar{\bar{X}}$ + A2 $\bar{R}$

$$\bar{R} = \frac{UCL - \bar{\bar{X}}}{A_2}$$

$$= \frac{129 - 125}{0.73} = 5.48$$

Process capability = 6 σ'

= 6 x 2.6

= 15.96

USL – LSL = 16

Science $6o^1$ < (USL – LSL). The process in capable of meeting the specification limits.

$$UNTL = \bar{X}^1 + 3\,o^1$$
$$= 125 + 3 \times 2.66$$
$$= 132.98$$

$$LNTL = \bar{X}^1 - 3\,o^1$$
$$= 125 - 3 \times 2.66$$
$$= 117.02$$

USL = 135

LSL = 119

Percentage of rejection

$$Z = \frac{119 - 125}{2.66} = \frac{LSL - CL}{\sigma^1}$$
$$= -2.25$$

(a) Probability from table 0.0122 = 1.22%

(b) In order to minimize the percentage of rejection change the process target from 125 to 127.

The percentage of rejection

$$Z = \frac{119 - 127}{2.66}$$
$$= -3.00$$

Probability from tables = 0.00135

Percentage of rejection = 0.135

(c) Since it is symmetric the total percentage of rejection = 0.135 x 2 = 0.27%.

Problem -5

For a certain characteristic of a product of sample size2 after 25 sub-groups. ΣR =

0.81 and Σ X bar = 27.635. The specification limits are 1.12 ± 0.087.

(a) In the process harmonized to the specifications.
(b) What are the rejections percentages if any?
(c) Is the process capable of meeting the specifications.
(d) Harmonise the process to the specifications and obtain the control limits for X bar-R chart after harmonizing the process to specification.

Solution. n = 2, K = 25, ΣR = 0.81, Σ X bar = 27.635.
Specification limits = 1.12 ± 0.087
USL = 1.207
LSL = 1.033
From tables for a subgroup size of 2.
d2 = 1.128
A2 = 1.88
D3 = 0.0
D4 = 3.27

$$X = \frac{\Sigma \overline{X}}{K} = \frac{27.635}{25} = 1.1054 = \overline{X}^1$$

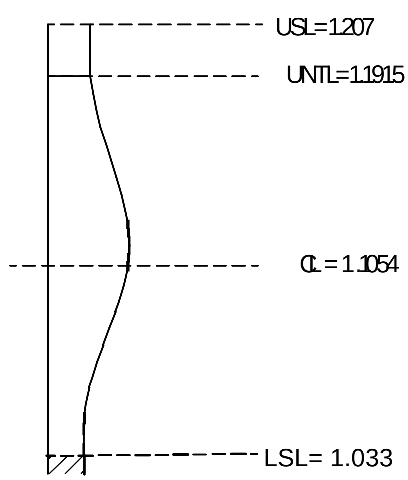
$$\overline{R} = \frac{\Sigma R}{K} = \frac{0.81}{25} = 0.0324$$

$$\sigma^1 = \frac{\overline{R}}{d_2} = \frac{0.0324}{1.128} = 0.0287$$

UNTL = $\overline{X}^1 + 3\sigma^1$
= 1.1054 + 3 x 0.0287
= 1.1915

LNTL = $\overline{X}^1 - 3\sigma^1$
= 1.1054 – 3 x 0.0287
= 1.0193

CL = $\overline{X}^1$ = 1.1054



(a) It is clear from the figure that the process is not harmonised with the specifications (LNTL is below LSL) (For a process to be harmonised, LNTL, UNTL must fall well With in the USL and LSL or must be just equal to them.)

(b) The percentage of rejections

$$Z = \frac{1.033 - 1.1054}{0.0287} = -2.5$$

Probability = 0.0059
Percentage of rejection = 0.59%

(c) USL – LSL = 1.207 – 1.033
= 0.174
6 o1 = 6 x 0.0287
= 0.1722
6 o1 (USL-LSL) i.e., 0.1722 < 0.174 the process is capable of meeting the specification limits.

(d) In order to harmonise the process to the specifications change the process centre to the specifications mean

i.e., $\overline{\overline{X}} = 1.12$
The control limits for X-bar-chart

$$\text{UCL} = \overline{\overline{X}} + A_2\overline{R}$$
$$= 1.12 + 1.88 \text{ x } 0.0324$$
$$= 1.1809$$

$$\text{LCL} = \overline{\overline{X}} - A_2\overline{R}$$
$$= 1.059$$

CL = X bar = 1.12

Control limits for R-chart

$$UCL = D_4\overline{R} = 3.27\ x\ 0.0324$$
$$= 0.1059$$

$$LCL = D_3\overline{R} = 0.0 \times 0.0324 = 0.0$$

$$CL = \overline{R} = 0.0324.$$

<u>Development and use of X bar – R Chart</u>

In order to establish a pair of control charts for the average (X bar) and the range (R), it is desirable to follow a set procedure. The steps in this procedure are as follows:

1. Select the quality characteristic.
2. Choose the rational subgroup.
3. Collect the data (20 to 25 samples).
4. Calculate the mean (X bar) and R for each sample.
5. Determine the trial control limits.
6. Establish the revised control limits.
7. Construction of X bar - R – Chart.
8. Interpretation of the Results.

Equations for computing 3-sigma limits on Shewhart control charts for variables

Method	$\overline{X}$-chart	R chart	S chart
μ and σ^1 known or assumed (X_o, σ_o)	$CL = \overline{X}_o = \mu$ $UCL_{\overline{X}} = \mu + A\sigma$ $LCL_{\overline{X}} = \mu - A\sigma$	$CL = R_o = d_2\sigma$ $UCL_R = D_2\sigma$ $LCL_R = D_1\sigma$	$CL = s_0 = C_4\sigma$ $UCL_s = B_6\sigma$ $LCL_s = B_5\sigma$

Method	-chart	R chart	S chart
μ and σ^1 estimated from $\overline{\overline{X}}$ and $\overline{R}$	$CL = \overline{X}$ $UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R}$ $LCL_{\overline{X}} = \overline{X} - A_2\overline{R}$	$CL = \overline{R}$ $UCL_R = D_4\overline{R}$ $LCL_R = D_3\overline{R}$	

Method	-chart	R chart	S chart
μ and σ^1 estimated from $\overline{\overline{X}}$ and $\overline{s}$	$CL = \overline{X}$ $UCL_{\overline{X}} = \overline{\overline{X}} + A_3\overline{s}$ $LCL_{\overline{X}} = \overline{X} - A_3\overline{s}$		$CL = \overline{s}$ $UCL_s = B_4\overline{s}$ $LCL_s = B_3\overline{s}$

Problem 1.

Control charts for X bar and R are maintained on a certain dimension of a manufactured part which is specified as 2.05 ± 0.02 cms. Subgroup size is 4. The values of X bar and R are computed for each subgroup. After 20 subgroups. $\Sigma \bar{X}$ = 41.283 and ΣR = 0.280. If the dimensions fall above USL, rework is required, if below LSL, the part must be scrapped. If the process is in statistical control and normally distributed.

(a) Determine the 3σ control limits for X bar and R chart.
(b) What is process capability
(c) What can you conclude regarding its ability to meet specifications
(d) Determine the percentage of scrap and rework
(e) What are your suggestions for improvement.

Solution. $\Sigma \bar{X}$ = 41.283
ΣR = 0.280
n = Sample size = 04
Number of subgroup (K) = 20

The specification limits are 2.05 ± 0.02
Upper specification limit USL = 2.07 cm
Lower specification limit LSL = 2.03 cm
From the tables, for a subgroup size 4

$A_2 = 0.73$ $d_2 = 2.059$
$D_3 = 0.0$ $D_4 = 2.28$

$$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{K} = \frac{41.283}{20} = 2.06415$$

$$\bar{R} = \frac{\Sigma R}{K} = \frac{0.28}{20} = 0.014$$

(a) Control limit for R – chart

$$UCL = D_4 \bar{R} = 2.28 \times 0.014 = 0.0319$$

$$LCL = D_3 \bar{R} = 0 \times 0.014 = 0.0$$

$$CL = \bar{R} = 0.014$$

Control limits for $\overline{X}$- chart

$UCL = \overline{\overline{X}} + A_2 \overline{R}$

$= 2.06415 + 0.73 \times 0.014$

$= 2.07437$

$LCL = \overline{\overline{X}} - A_2 \overline{R}$

$= 2.06415 - 0.73 \times 0.014$

$= 2.05393$

$CL = \overline{\overline{X}} = 2.06415$

(b) Process capability

Since the process is in a state of statistical control.

$CL = \overline{\overline{X}} = \overline{X}^1 = 2.06415$

$$\sigma^1 = \frac{\overline{R}}{d2} = \frac{0.014}{2.059}$$

$= 0.00679$

The process capability $= 6\sigma^1$

$= 6 \times 0.00679$

$= 0.04074$

(c) USL – LSL = 2.07 – 2.03 = 0.04

Since the 6 o1 is greater than USL – LSL , the process is not capable of meeting the specification limit i.e., 0.0407 > 0.04.

Note:

1. If 6 o1 is less than (USL – LSL). The process is capable of meeting the specification. There should not be any rejection. If rejection occurs we can conclude that, the process is not centered properly.
2. If 6o1 is equal to (USL – LSL), the process is exactly capably of meeting the specification limits. But tight tolerances are provided. We have to prefer a skilled operator for operating the machine.
3. If 6 o1 is greater than to (USL – LSL), the process is not capable of meeting the specifications limits. The rejections are inevitable.

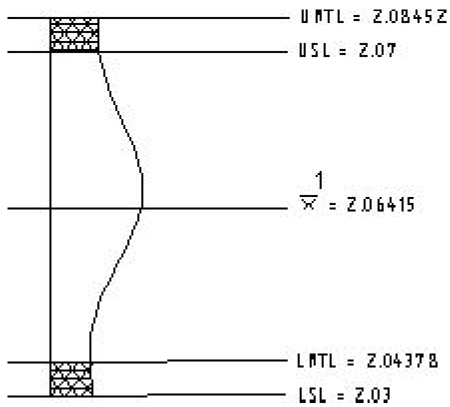
(d) UNTL (upper natural tolerance limit)

$$UNTL = \overline{X}^1 + 3\sigma^1$$
$$= 2.06415 + 3(0.00679) = 2.08452$$

LNTL (lower natural tolerance limit)

$$LNTL = \overline{X}^1 - 3\sigma^1 = 2.04378$$

USL = 2.07 LSL = 2.03



It is clear from the figure that the percentage of scrap is zero. The percentage of rework is

$$Z = \frac{USL - \overline{X}^1}{\sigma^1} = \frac{2.07 - 2.06415}{0.00679} = 0.86$$

The probability from the normal tables for Z = 0.86 is

0.8051 i.e. 80.51%

Therefore the rework is 100 – 80.51 = 19.49%

(e) Since the percentage of rework is 19.49%, to minimize this, the possible ways are

(i) Change the process centre to the specification mean i.e., from 2.06415 to 2.05. The calculations are shown below:

$$Z = \frac{2.07 - 2.05}{0.00679} = 2.94$$

Probability from Normal tables is 0.9984

That is 1 – 0.9984 = 0.0016 i.e. 0.16%

The percentage of rework is 0.16%
Since it is symmetric the percentage of scrap is also 0.16%.

(ii) Widening the specification limits, for this we have to consult the design engineer, whether the product performs its function satisfactorily or not.
(iii) Decrease the dispersion, for this we have to prefer a skilled operator and very good raw material and a new machine, practically which is difficult.
(iv) Leave the process alone and do the 100% Inspection.
(v) Calculate the cost of scrap and rework, whichever is costly make it zero, accordingly change the process centre.

Problem 2.

Subgroup of 5 item each are taken from a manufacturing process at regular intervals. A certain quality characteristic is measured and X bar , R values computed for each subgroup. After 25 subgroup
$\Sigma \bar{x} = 357.5$, $\Sigma R = 8.8$. Assume that all the points are within the control limits on both the charts. The specifications are 14.4 ± 0.4

(a) Compute the control limits for X bar and R chart
(b) What is the process capability
(c) Determine the percentage of rejections if any
(d) What can you conclude regarding its ability to meet the specifications.
(e) Suggest the possible scrap for improving the situation. (note: n=5 from tables A2=0.5, d2=2.236, D3 = 0, D4 = 2.11)

Development and use of X bar– S Chart With Real life data

Note :

Although X- bar and R charts are widely used, it is a occasionally desirable to estimate the process standard deviation directly instead of indirectly through the use of the range R. This leads to control charts for X-bas and S, where S is the sample standard deviation. Generally X-bar and s charts are preferable to their more familiar counter parts, X – bar and R charts when either

1. The sample size n is moderately large ---say n>10 or 12.
2. The sample size n is variable

Problem – 6

The following data presents the inside diameter measurements on the piston rings to illustrate the construction and the operation of X bar and S chart. The subgroup size is five.

Sample no	$\overline{X}_i$	Si
1	74.010	0.0148
2	74.001	0.0075
3	74.008	0.0147
4	74.003	0.0091
5	74.003	0.0122
6	73.996	0.0087
7	74.00	0.0055
8	73.997	0.0123
9	74.004	0.0055
10	73.998	0.0063
11	73.994	0.0029
12	74.001	0.0042
13	73.998	0.0105
14	73.990	0.0153
15	74.006	0.0073
16	73.997	0.0078
17	74.001	0.0106
18	74.007	0.0070

19	73.998	0.0085
20	74.009	0.0080
21	74.000	0.0122
22	74.002	0.0074
23	74.002	0.0119
24	74.005	0.0087
25	73.998	0.0162

We will illustrate the construction of $\overline{X}$ and S chart using the piston-ring inside diameter measurement in table above. The grand average and the average standard deviation are

$$\overline{\overline{X}} = \frac{1}{25}\sum_{i=1}^{25}\overline{X}_i = \frac{1}{25}(1850.028) = 74.001$$

and

$$\overline{S} = \frac{1}{25}\sum_{i=1}^{25}\overline{S}_i = \frac{1}{25}(0.2350) = 0.0094$$

Respectively. Consequently, The parameters for the $\overline{X}$ chart are

$$UCL = \overline{\overline{X}} + A_3\overline{S} = 74.001 + (1.427)(0.0094) = 74.014$$

$$CL = \overline{\overline{X}} = 74.001$$

$$LCL = \overline{\overline{X}} - A_3\overline{S} = 74.001 - (1.427)(0.0094) = 73.988$$

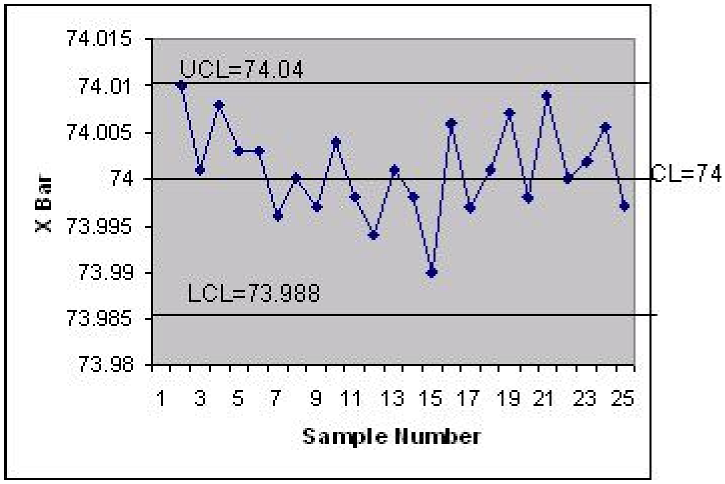
And for the S chart

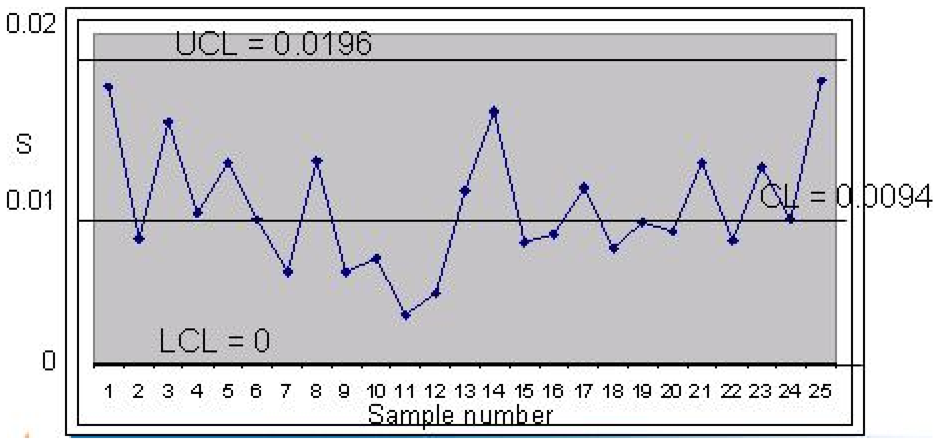
$$UCL = B_4\overline{S} = (2.089)(0.0094) = 0.0196$$

$$CL = \overline{S} = 0.0094$$

$$LCL = B_3\overline{S} = (0)(0.0094) = 0$$

The control chart are show in the fig below





Note:

The control limits for the x bar chart based on S bar are identical to the X bar chart control limits, where the limits were based on R bar They will not always be the same, and in general, the X bar chart control limits based on S bar will be slightly different than limits based on R bar.

We can estimate the process standard deviation using the fact that S/c4 is an unbiased estimate of o. Therefore, since c4 = 0.9400 for samples of size five, our estimate of the process standard deviation is

$$\hat{\sigma} = \frac{S}{c_4} = \frac{0.0094}{0.9400} = 0.01$$

This estimate is very similar to that of o obtained via the range method.

Problem -7

A certain product has a specification of 120 ±5. At present the estimated process average is120 and o1 = 1.5

(a) Compute the 3o1limits for X bar , R chart based on a subgroup size of 4

(b) If there is a shift in the process average by 2%, What percentage of product will fail to meet the specification.

(c) What is the probability of detecting the shift by X bar - chart

solution. Specification limits = 120 ±5

USL = 125

LSL = 115

$\overline{X}^1$ = 120, σ^1 = 1.5

$\overline{X}^1$ = $\overline{\overline{X}}$

n = 4

From tables for a subgroup size of 4

A_2 = 0.73

D_2 = 2.059

D_3 = 0.0

D_4 = 2.28

$$\sigma^1 = \frac{\overline{R}}{d_2}$$

$$\overline{R} = \sigma^1 \ x \ d_2 = 1.5x2.059$$

= 3.0885

Control limits for R-chart

UCL = $D_4 \ \overline{R}$

= 2.28 x 3.0885

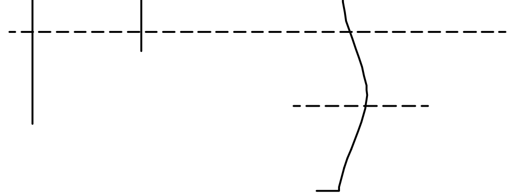
= 7.04178

$$LCL = D_3 \overline{R}$$
$$= 0.0$$
$$CL = \overline{R} = 3.0885$$

Control limits for $\overline{X}$ - chart

$$UCL = \overline{\overline{X}} + A_2 \overline{R}$$
$$= 120 + 0.73x3.0885$$
$$= 122.2546$$

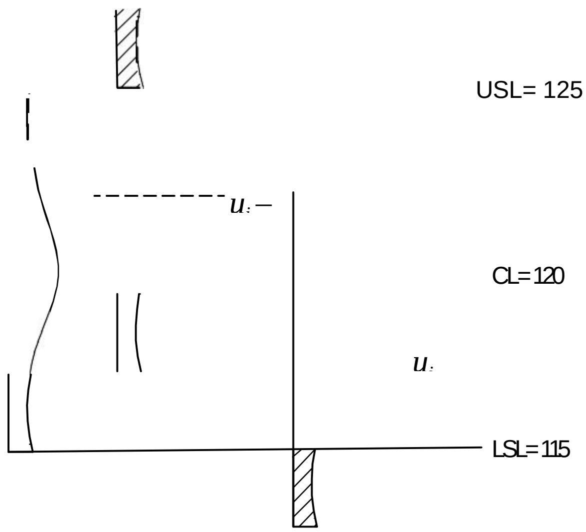
$$LCL = \overline{\overline{X}} - A_2 \overline{R}$$
$$= 120 - 0.73x3.0885$$
$$= 117.7454$$
$$CL = \overline{\overline{X}} = 120$$



(b) shift in the process average = ± 2%

$\overline{X^1}$ new = 120 x 1.02 (+2%)
= 122.4

$\overline{X^1}$ new = 120 x 0.98 (-2%)
= 117.6



Below Z = $\frac{115 - 117.6}{1.5}$

= -1.73

Probability = 0.0418
= 4.18%

Above: Z = $\frac{125 - 122.4}{1.5}$

= 1.73

Probability = 0.9582

= 95.82%

i.e. 100-95.82 = 4.18%

(c) With respect to X bar - chart

$$o_{\bar{x}} = \frac{\sigma^1}{\sqrt{n}} = \frac{1.5}{\sqrt{4}}$$

$o_n = 0.75$

$$\text{Below } Z = \frac{122.26 - 122.4}{0.75} = 0.1866$$

Probability = 0.5714

= 57.14%

$$\text{Above } Z = \frac{122.26 - 122.4}{0.75} = -0.1866$$

= 0.1866

Probability = 0.4287 = 42.86%

i.e. =100 – 42.86 = 57.14%

Problem - 8

Subgroup of 4 items each are taken from a manufacturing process at regular intervals. A certain quality characteristic is measured and X bar , R values are computed for each subgroup. After 25 subgroup. Σ X bar = 15350, Σ R = 411.1.

(a) Compute the control limits for X bar, R chart.

(b) Assume all the points are falling within the control limits on both the charts. The specification limits are 610 ± 15. If the quality characteristic is Normally distributed what percentage of product would fail to meet the specifications.

(c) Any product that falls below L will be scrapped and above U must be reworked. It is suggested that the process can be centered at a level so that not more than 0.1% of the product will be scrapped. What should be the aimed $\bar{X}$ value of to make the scrap exactly 0.1%.

(d) What percentage of rework can be expected with this centering.

Solution. $\sum \overline{X} = 15350$

$S\, R = 411.4$

$K = 25$

$n = 4$

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{K} = \frac{15350}{25} = 614$$

$$\overline{R} = \frac{\Sigma \overline{R}}{K} = \frac{411.4}{25} = 16.456$$

From tables, for a subgroup size of 4

A2 = 0.73

d2 = 2.059

D3 = 0.0

D4 = 2.28

Control limits for X bar - chart

$$UCL = \overline{\overline{X}} + A_2 \overline{R}$$
$$= 614 + 0.73 \times 16.456$$
$$= 626.012$$

$$LCL = \overline{\overline{X}} - A_2 \overline{R}$$
$$= 601.987$$

$$CL = \overline{X} = 614$$

Control limits for R –chart

$$UCL = D_4 \overline{R}$$
$$= 2.28 \times 16.456$$
$$= 37.5196$$

$$LCL = D_3 \overline{R}$$
$$= 0 \times 16.450$$
$$= 0.0$$

$$CL = \overline{R} = 16.456$$

(b) Specification limits are

610 ± 15

USL = 625

LCL = 595

$$\overline{X}^{1} = X = 614$$

$$\sigma^1 = \frac{\overline{R}}{d_2} = \frac{16.456}{2.059} = 7.99$$

$$UNTL = \overline{X}^1 + 3\sigma^1$$
$$= 614 + 3 \times 7.99$$
$$= 637.97$$
$$LNTL = \overline{X}^1 - 3\sigma^1$$
$$= 590.03$$

USL – LSL = 30

Process capability $= 6\sigma^1$
$$= 6 \times 7.99$$
$$= 47.94$$

$u_i - u$

$u\ n_i$

UNTL = 637.97

USL = 625

CL = 614

LSL = 595

LNTL = 590.03

Percentage of scrap

Science $6\sigma^1 > (USL - LSL)$, the process is not capable of meeting the specification limits. Rejections are inevitable.

Percentage of scrap

$$Z = \frac{LSL - \overline{X}^1}{\sigma^1}$$
$$= \frac{595 - 614}{7.99} = -2.37$$

Probability from tables = 0.0089 = 0.89%

Percentage of rework

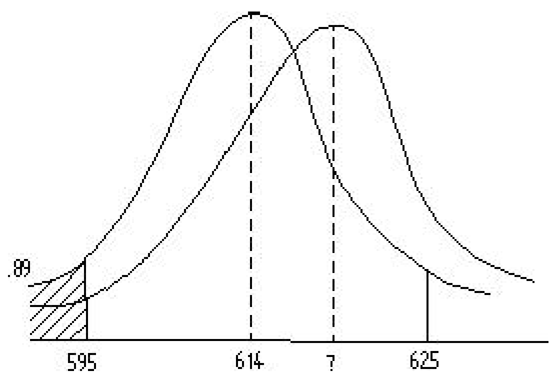
$$Z = \frac{USL - \overline{X}^{\cdot}}{\sigma^1}$$
$$= \frac{625 - 614}{7.99} = 1.37$$

Probability from tables = 0.947 i.e 91.47%

Rework = 100 – 91.47%

= 8.53%

For the probability 0.001 the Z value from the normal table is -3



$$-3 = \frac{595 - \overline{X}'\, new}{7.99}$$

The percentage of rework now is

$$\overline{X}^{1}\, new = 595 + 3 \times 7.99 = 618.97$$

$$Z = \frac{625 - 618.97}{7.99} = 0.75$$

For Z 0.75 the probability from normal table is 0.7734

i.e 77.34%

Percentage of rework = 100 - 77.34 = 22.66%.