VIBRATIONS

* GALILIO → Italian mathematician
  → Natural frequency
* Newton → force theory [English mathematician]
* DANIEL BERNOULLI → vibrations of Beams.
* L.B. EULER → Binding of Rod (vibrations)
* J.B.J Fourier → (mathematician)
  → Series, integral.
* LAGRAGE → Italian mathematician → Lagrange eq^n
* Rayleigh → English physicist
* S. P. TIMOSHENKO → BEAM THEORY → Elastic vibration
  (Russian engineer)
* Frahm → Torsional vibrations.
  ↓
  Frahm Read Techometer

Free or Natural vibrations: ⇒ The vibration in which there is no friction and there is no external force after the initial release of the system are known as natural vibrations.

Damped vibrations: → A vibration in which any kind of friction is present are known as damped vibrations. Because of friction there is a continuous energy loss in the form of heating therefore amplitude continuously decreases and ultimately it will be equal to zero after a sufficient long time.

Forced vibrations :→ The vibrations in which a repeated force continuously acts on the system are known as forced vibration.

Period :→ It's a time taken to repeat it's motion. It's a time taken in a one cycle.

Frequency :→ It's a number of cycle completed per second.

T = Time Period

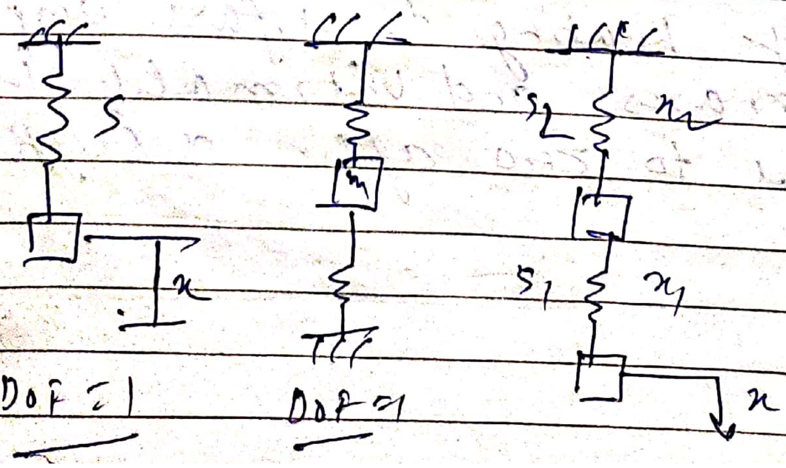
$f = \frac{1}{T}$ ⇒ frequency

Angular frequency

$$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$$

Resonance :→ It's a condition when the external forces frequency is equal to the natural frequency of the oscillation then it's is said as resonance. In this case the amplitude of the vibration will become very-very large and ultimately system will fail.

Degree of freedom of the vibrational system :→



DOF = 1   DOF = 1 $\boxed{x = x_1 + x_2}$

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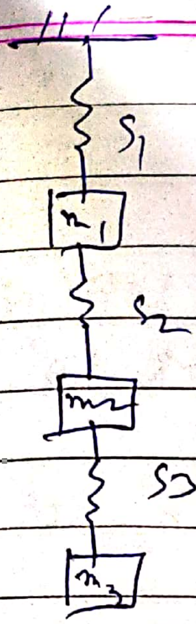
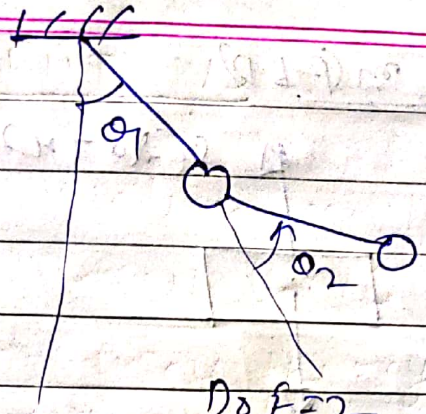
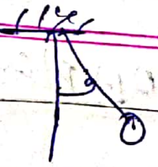


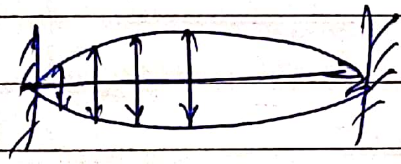
DOF = 3



DOF = 2



DOF = 1

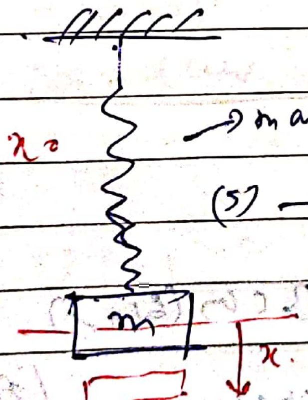


to find amplitude at every point in different

$\boxed{DOF = \infty}$

1 Ques in gate

Natural Frequency →



→ mass less

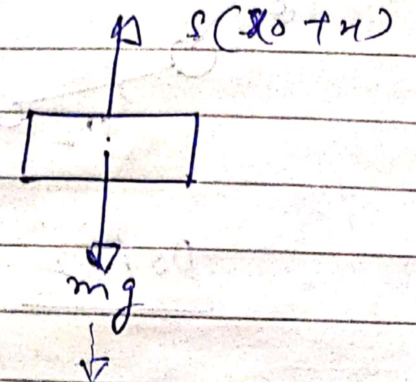
(s) → stiffness

eq^m (mean position)

m (eq^m)

$\Rightarrow \boxed{mg = sx_0}$

Displaced m (FBD)



$$mg - S(x_0 + x) = ma$$

$$mg - Sx_0 - Sx = m\ddot{x}$$

$$m\ddot{x} = -Sx$$

$$\ddot{x} = -\underbrace{\left(\frac{S}{m}\right)}_{\omega_n^2} x$$

$$\ddot{x} = -\omega_n^2 x$$

$$\boxed{\ddot{x} + (\omega_n^2) x = 0}$$

where $\omega_n^2 = \frac{S}{m}$

$$\boxed{\omega_n = \sqrt{\frac{S}{m}}}$$

Eqn of motion:-

$$\ddot{x} + (\omega_n^2) x = 0$$

Soln of this eqn :-

$$x = A \sin(\omega_n t) + B \cos(\omega_n t)$$

$\left.\begin{matrix} A \\ B \end{matrix}\right]$ unknown [A, B const]

$\left.\begin{matrix} A = R\cos\phi \\ B = R\sin\phi \end{matrix}\right]$ $R = \sqrt{A^2 + B^2}$, $\phi = \tan^{-1}(B/A)$

$$x = R\sin\omega_n t\cos\phi + R\cos\omega_n t\sin\phi$$

$$\boxed{x = R\sin(\omega_n t + \phi)}$$

$\left.\begin{matrix} R \\ \phi \end{matrix}\right]$

$x = A\sin(\omega_n t) + B\cos(\omega_n t)$] A, B

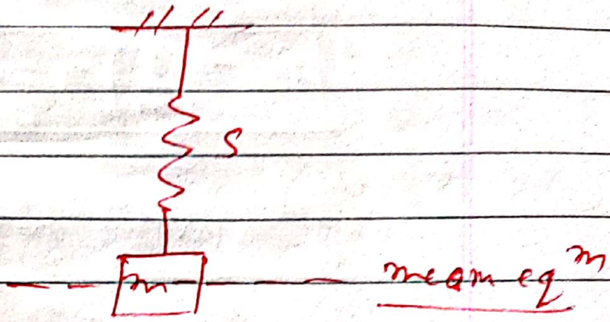
$x = R\sin(\omega_n t + \phi)$] R, ϕ

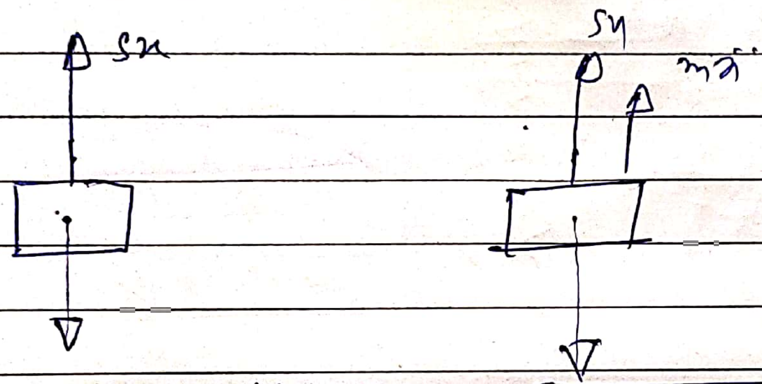
Initial conditions :-

(1) At $t=0$ < $x=0$, $\dot{x} = v_0$]

(2) At $t=0$ < $x = x_0$, $\dot{x} = 0$]

(3) At $t=0$ < $x = x_0$, $\dot{x} = v_0$]

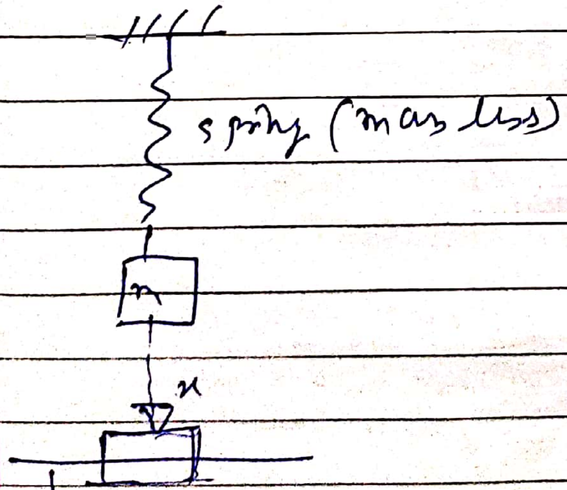




$0 - Sx = m\ddot{x}$

$\boxed{m\ddot{x} + Sx = 0}$ $\boxed{Sx + m\ddot{x} = 0}$

Newton's Law



$E = \frac{1}{2}mv^2 + \frac{1}{2}Sx^2$

Energy conserve) $\frac{dE}{dt} = 0 = \left(\frac{1}{2}m \cdot 2v\frac{dv}{dt} + \frac{1}{2}S \cdot 2x\frac{dx}{dt}\right)$

Energy method

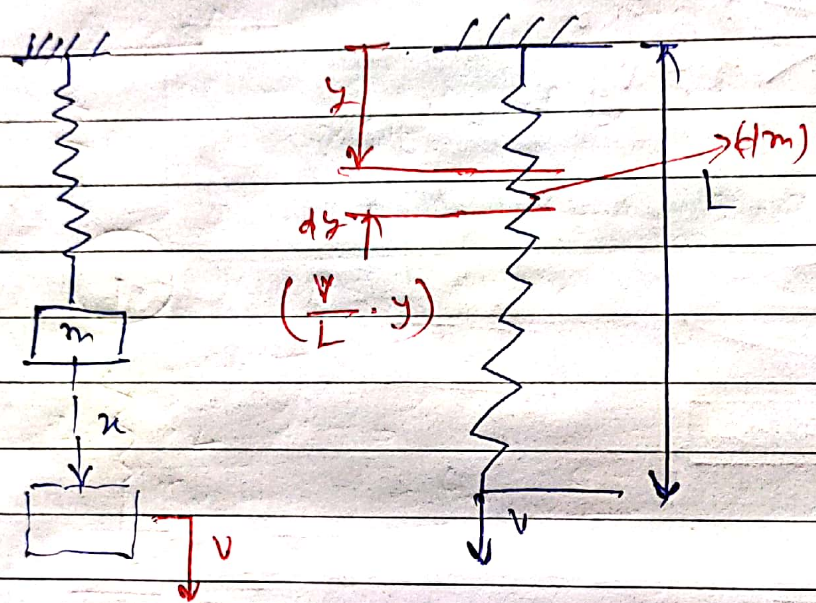
$$0 = \frac{1}{2}\{2m\dot{x}\ddot{x} + 2s x \dot{x}\} = 0$$

$$m\ddot{x} + s x = 0$$

$$\boxed{\ddot{x} + \left(\frac{s}{m}\right)x = 0} \rightarrow \omega_n^2 = \frac{s}{m}$$

$$\omega_n = \sqrt{\frac{s}{m}}$$

Spring having mass:→



$$dKE_{spring} = \frac{1}{2}(dm)\left(\frac{v}{L}\cdot y\right)^2$$

$$= \frac{1}{2} \times \left(\frac{m_s}{L}\right)\cdot dy \left(\frac{v^2}{L^2}\right)\cdot y^2$$

$$KE_{spring} = \frac{1}{2}\left(\frac{m_s}{L^3}\right)(v^2)\int_0^L y^2 dy$$

$$= \frac{1}{2}\left(\frac{m_s}{L^3}\right) v^2 \times \frac{L^3}{3}$$

$$KE_{spring} = \frac{1}{3}\left(\frac{1}{2}m_s v^2\right)$$

$$E = \frac{1}{2}mv^2 + \frac{1}{3}\frac{1}{2}(m_s v^2) + \frac{1}{2}s x^2$$

$$E = \frac{1}{2}v^2\left(m + \frac{m_s}{3}\right) + \frac{1}{2}s x^2$$

$$\frac{dE}{dt} = 0 = \frac{1}{2}\cdot\cancel{2}\dot{x}\ddot{x}\,(m + m_s$$

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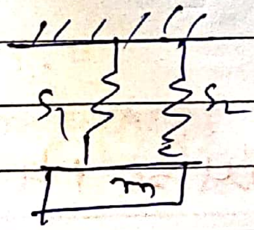
$$\left(m + \frac{m_s}{3}\right) \ddot{x} + sx = 0$$

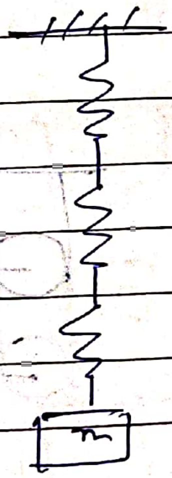
$$\ddot{x} + \frac{s}{\left(m + \frac{m_s}{3}\right)} \cdot x = 0$$

$$\boxed{\omega_n = \sqrt{\frac{s}{m + \frac{m_s}{3}}}}$$



$$S_{eq} = S_1 + S_2$$

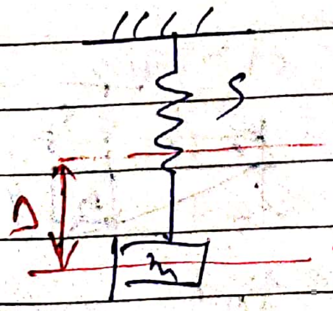
$$\boxed{\omega_n = \sqrt{\frac{S_{eq}}{m}}}$$



$$\boxed{\frac{1}{S_{eq}} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} + \cdots}$$

$$\boxed{\omega_n = \sqrt{\frac{S_{eq}}{m}}}$$

* Method of Static Deflection of mass :→



$$\omega_n = \sqrt{S/m}$$

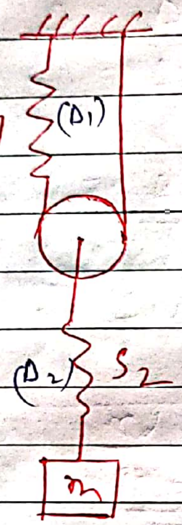
$$\omega_n = \sqrt{\frac{Sg}{mg}} \Rightarrow \omega_n = \sqrt{\frac{g}{\left(\frac{mg}{S}\right)}}$$

$$\omega_n = \sqrt{\frac{g}{\Delta}}$$

$$= \sqrt{\frac{g}{(mg/S)}}$$

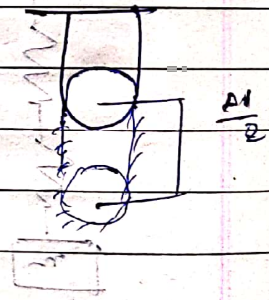
$$= \sqrt{\frac{S}{m}} = \omega_n$$

$$\boxed{\omega_n = \sqrt{\frac{S_{eq}}{m}} = \sqrt{\frac{g}{\Delta}}}$$

Q. 

What will be the natural oscillation of m

i.e. $\{\omega_n = ?\}$

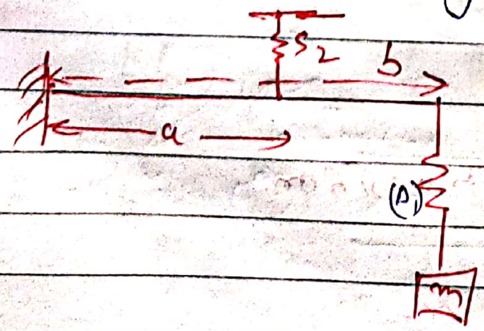


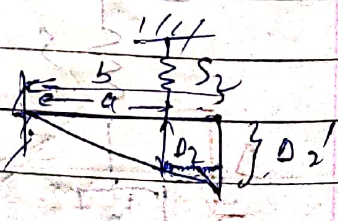
$$\Delta = \Delta_2 + \frac{\Delta_1}{2}$$

$$= \frac{mg}{S_2} + \frac{1}{2}\left[\frac{mg/2}{S_1}\right]$$

$$\Delta = \left(\frac{mg}{S_2}\right) + \left(\frac{mg}{4S_1}\right)$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{S_{eq}}{m}} = ?$$

Q. 

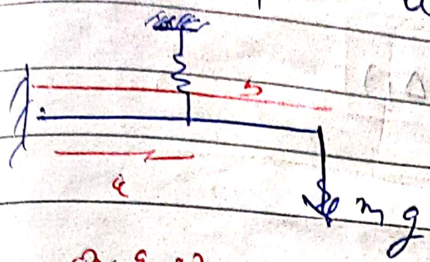


$$\frac{\Delta_2'}{b} = \frac{\Delta_2}{a}$$

$$\Delta_2' = (\Delta_2)\frac{b}{a}$$

$$\Delta = \Delta_1 + (\Delta_2)\frac{b}{a}$$

$$= \frac{mg}{s_1} + \left(\frac{b}{a}\right)\left\{\frac{mg(b/a)}{s_2}\right\}$$

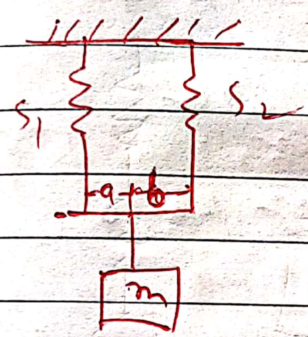


$$(mg \cdot b) = F' \cdot a$$

$$F' = mg(b/a)$$

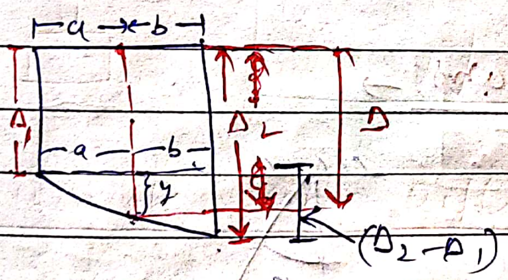
$$\boxed{\Delta = \frac{mg}{s_1} + \left(\frac{b}{a}\right)^2 \frac{mg}{s_2}}$$

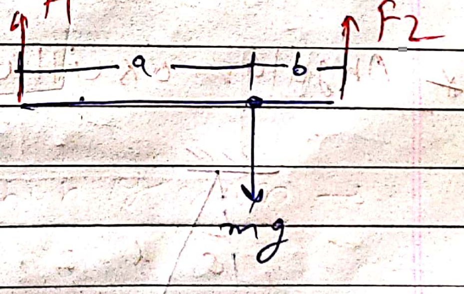
Q



$w_n = ?$

Q2





$$F_1 + F_2 = mg$$

$$F_1 \cdot a = F_2 \cdot b$$

$$F_2 = F_1\left(\frac{a}{b}\right)$$

$$F_1 + F_1\left(\frac{a}{b}\right) = mg$$

$$F_1\left(\frac{a+b}{b}\right) = mg$$

$$F_1 = mg\left(\frac{b}{a+b}\right), \quad \Delta_1 = F_1/s_1 = \frac{mg}{s_1}\left(\frac{b}{a+b}\right)$$

$$F_2 = mg\left(\frac{a}{a+b}\right) \longrightarrow \Delta_2 = \frac{F_2}{S_2} = \frac{mg}{S_2}\left(\frac{a}{a+b}\right)$$

$$\frac{\Delta_2 - \Delta_1}{a+b} = \frac{y}{a}$$

$$y = \frac{a}{a+b}(\Delta_2 - \Delta_1)$$

$$\boxed{\Delta = \Delta_1 + y}$$

$$\Delta = \Delta_1 + y$$

$$= \Delta_1 + \frac{a}{a+b}(\Delta_2 - \Delta_1)$$

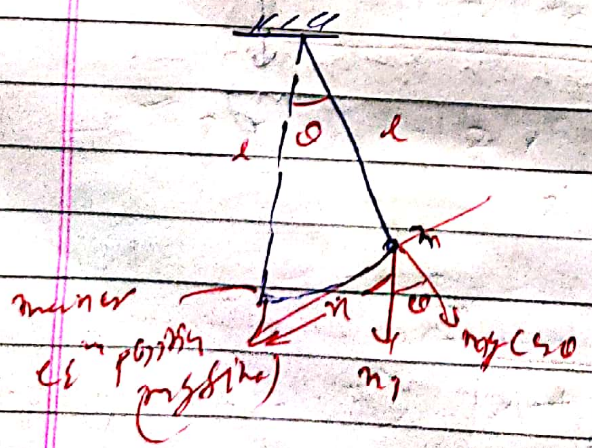
$$= \frac{\cancel{\Delta_1 a} + \Delta_1 b + \Delta_2 a - \cancel{\Delta_1 a}}{a+b}$$

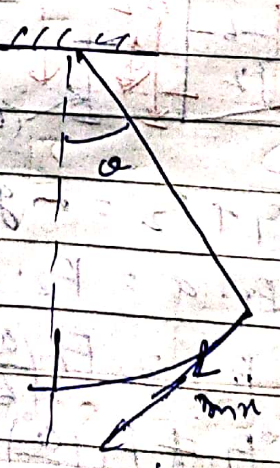
$$\Delta = \frac{\Delta_1 b + \Delta_2 a}{a+b}$$

$$\Delta = \frac{mg}{(a+b)^2}\left[\frac{b^2}{S_1} + \frac{a^2}{S_2}\right]$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{S_{eq}}{m}} = \ldots$$

Angular oscillations: →





$mg \sin\theta$

$$m\ddot{x} + mg\sin\theta = 0$$

$$\ddot{x} + g\sin\theta = 0$$

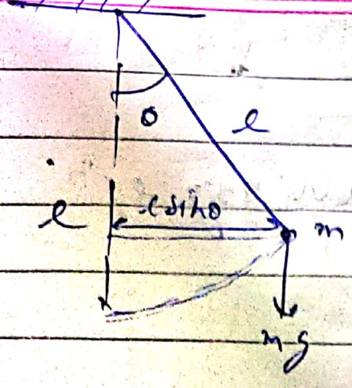
If θ is very small

$$\ddot{x} + g\theta = 0$$

$$\boxed{\ddot{x} + g\left(\frac{x}{l}\right) = 0}$$

$\theta = x/l$

$\omega_n^2 = \frac{g}{l}$



$$0 - mgl\sin\theta = I\cdot\alpha$$

$$I\alpha + mgl\sin\theta = 0$$

$$I\ddot{\theta} + mgl\sin\theta = 0$$

$$(ml^2)\ddot{\theta} + mgl\cdot\theta = 0$$

$$\boxed{\ddot{\theta} + \left(\frac{g}{l}\right)\theta = 0}$$

[$\theta \rightarrow$ very small
$\sin\theta \simeq \theta$]

$$\omega_n^2 = g/l$$

$$\boxed{\omega_n = \sqrt{g/l}}$$

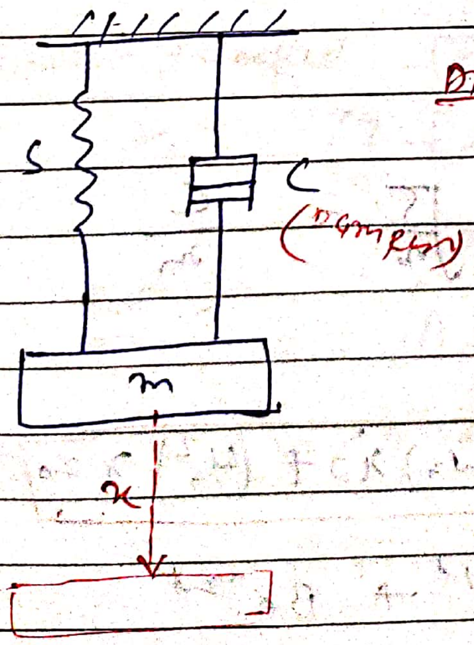
2 mark in gate

DAMPED VIBRATIONS :- (Reality)

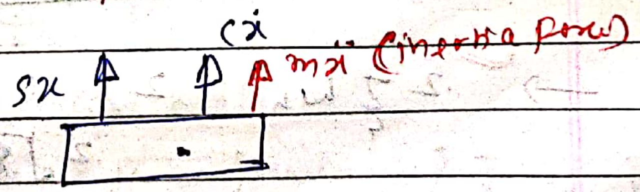
1. Potential energy storage device
2. K.E. device
3. Energy Dissipation (Loss) device

→ Real vibrating system

Friction → Dry friction b/w surface (Coulomb damping)
→ Drag force (friction by fluid on the body)
↳ viscous friction
→ friction inside the material



Displaced position :-



$$Sx + c\dot{x} + m\ddot{x} = 0$$

$$m\ddot{x} + c\dot{x} + Sx = 0$$

$$\boxed{\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \left(\frac{S}{m}\right)x = 0}$$

The Solⁿ of this eqⁿ is :-

$x = Ae^{\alpha_1 t} + B \cdot e^{\alpha_2 t}$

$\alpha_1, \alpha_2 \Rightarrow$ Roots of Auxilliary eqⁿ

$$\boxed{\alpha^2 + \left(\frac{c}{m}\right)\alpha + \left(\frac{s}{m}\right) = 0}$$

$$\alpha_{1,2} = \frac{-\left(\frac{c}{m}\right) \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{s}{m}\right)}}{2}$$

$$\alpha_{1,2} = \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{s}{m}\right)}$$

$$\frac{\left(\frac{c}{2m}\right)^2}{\left(\frac{s}{m}\right)} \rightarrow \text{Degree of Dampness}$$

$$\sqrt{\frac{\left(\frac{c}{2m}\right)^2}{(s/m)}} = \text{Damping factor or Damping Ratio } (\xi)$$

$$\sqrt{\frac{c^2}{4m^2} \times \frac{m}{s}} = \xi$$

Imp

Zeta $\boxed{\xi = \frac{c}{2\sqrt{sm}}}$ **

$$\rightarrow 2\xi\omega_n = 2 \times \frac{c}{2\sqrt{sm}} \times \frac{\sqrt{s}}{\sqrt{m}} = \frac{c}{m}$$

$$\omega_n^2 = \frac{s}{m}$$

Eqⁿ of motion $\Rightarrow$ $\boxed{\ddot{x} + (2\xi\omega_n)\dot{x} + (\omega_n^2)x = 0}$

Solⁿ $x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$

$\alpha_1, \alpha_2 \Rightarrow \alpha_{1,2} = -\xi\omega_n \pm \sqrt{(\xi\omega_n)^2 - \omega_n^2}$

$$\boxed{\alpha_{1,2} = \left\{-\xi \pm \sqrt{\xi^2 - 1}\right\}\omega_n}$$

If $\xi > 1 \Rightarrow$ over damped systems (No vibration)

$\xi = 1 \Rightarrow$ critically damped systems (No vibration)

$\xi < 1 \Rightarrow$ under damped systems. (vibration)

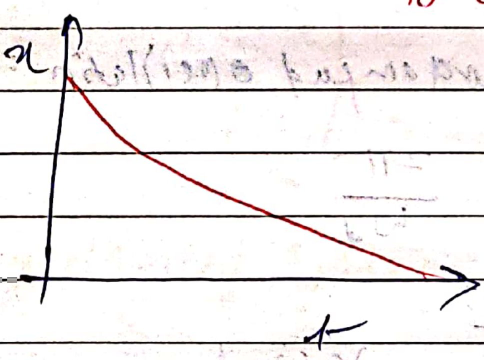
1. Overdamped System. ($\xi > 1$)

$\alpha_{1,2} = \left\{-\xi \pm \sqrt{\xi^2 - 1}\right\}\omega_n$ → Real roots

$x = Ae^{\alpha_1 t} + B_2 e^{\alpha_2 t}$

$$\boxed{x = Ae^{\{-\xi + \sqrt{\xi^2-1}\}\omega_n t} + Be^{\{-\xi - \sqrt{\xi^2-1}\}\omega_n t}}$$

$\left.\begin{matrix}A\\B\end{matrix}\right\}$ Initial conditions



2. Under damped : ($\xi < 1$)

$\alpha_{1,2} = (\xi + i\sqrt{1-\xi^2})\omega_n$ → Imaginary Roots

Soln:-

$x = \left\{Ae^{\{-\xi + i\sqrt{1-\xi^2}\}\omega_n t} + Be^{\{-\xi - i\sqrt{1-\xi^2}\}\omega_n t}\right\}$

$x = e^{-\xi\omega_n t}\left\{A^{i\sqrt{1-\xi^2}\,\omega_n t} + B^{-i\sqrt{1-\xi^2}\,\omega_n t}\right\}$

$\boxed{\left\{\sqrt{1-\xi^2}\,.\,\omega_n\right\} = \omega_d}$ → damped frequency

$$= e^{-\xi \omega_n t}\left\{A e^{i\omega_d t} + B e^{-i\omega_d t}\right\}$$

$$x = e^{-\xi \omega_n t}\left\{A(\cos\omega_d t + i\sin\omega_d t) + B(\cos\omega_d t - i\sin\omega_d t)\right\}$$

$$= e^{-\xi \omega_n t}\left\{\underline{(A+B)}\cos\omega_d t + \underline{i(A-B)}\sin\omega_d t\right\}$$

$A+B = X\sin\phi$

$i(A-B) = X\cos\phi$

$$= e^{-\xi \omega_n t}\left[X\cos\omega_d t\sin\phi + X\sin\omega_d t\cos\phi\right]$$

$$\boxed{x = X e^{-\xi \omega_n t}\sin(\omega_d t + \phi)}$$

$$\boxed{\omega_d = \sqrt{1-\xi^2}\,\omega_n = \text{const}}$$

$\left.\begin{matrix} X \\ \phi \end{matrix}\right\}$ initial condition

Freq^n of underdamped oscillation $= \omega_d = \sqrt{1-\xi^2}\,\omega_n$

$$T_d = \frac{2\pi}{\omega_d}$$

At $\underline{t = 0}$:-

$$x_0 = X\sin\phi$$

$\underline{\text{At} \Rightarrow t = T_d}$:-

$$x_1 = X e^{-\xi\omega_n T_d}\sin\left(\omega_d \cdot \frac{2\pi}{\omega_d} + \phi\right)$$

$$x_1 = X e^{-\xi\omega_n T_d}\sin\phi.$$

$\underline{\text{At } t = 2T_d}$:-

$$x_2 = X e^{-\xi\omega_n(2T_d)}\sin\left(\omega_d \cdot \frac{4\pi}{\omega_d} + \phi\right)$$

$$x_2 = X e^{-\xi\omega_n(2T_d)}\sin\phi$$

⋮

Date
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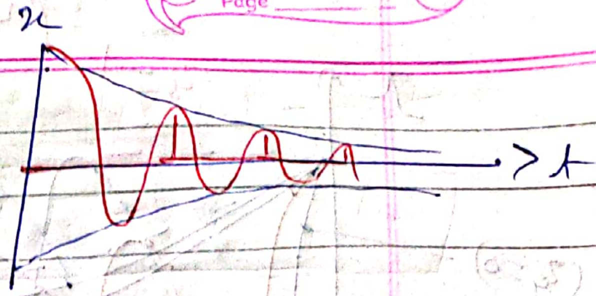


$$\frac{x_0}{x_1} = e^{\xi \omega_n T_d}$$

$$\frac{x_1}{x_2} = e^{\xi \omega_n T_d}$$

$$\boxed{\frac{x_0}{x_1} = \frac{x_1}{x_2} = \frac{x_2}{x_3} = \ldots \frac{x_{n-1}}{x_n} = \frac{x_n}{x_{n+1}} = e^{\xi \omega_n T_d}}$$

Logarithmic Decrement

$$\delta = \ln\left(e^{\xi \omega_n T_d}\right)$$

$$\delta = \xi \omega_n T_d$$

$$\delta = \xi \omega_n \times \frac{2\pi}{\omega_d}$$

$$= \frac{\xi \omega_n \times 2\pi}{\sqrt{1-\xi^2} \cdot \omega_n}$$

$$\boxed{\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

10/11/09

3. Critically damped system ($\xi = 1$)

$$\boxed{\ddot{x} + (2\xi\omega_n)\dot{x} + (\omega_n^2)x = 0}$$

$$\alpha_1 = \alpha_2 = \alpha = \left\{-\xi \pm \sqrt{\xi^2 - 1}\right\}\omega_n$$

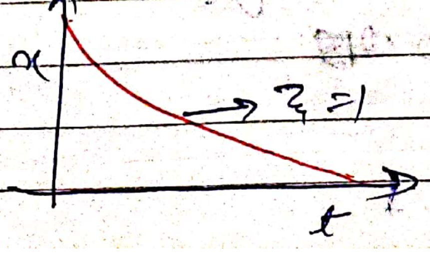
$$= -\omega_n$$

The solⁿ will be :-

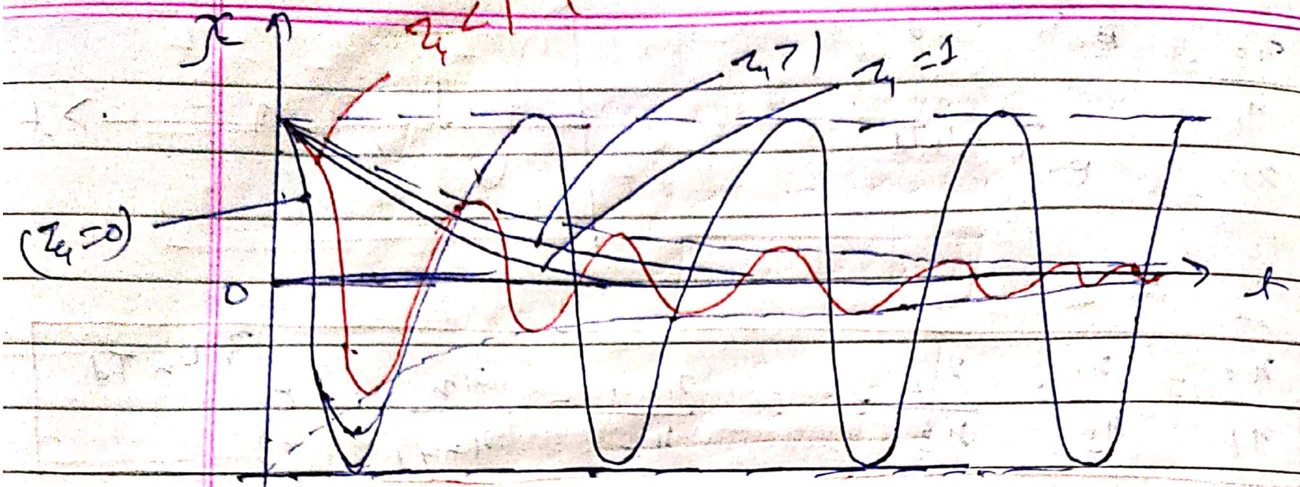
$$x = (A + Bt)e^{\alpha t}$$

$$\boxed{x = (A + Bt)e^{-\omega_n t}} \Rightarrow \text{No vibration}$$

$\left.\begin{matrix}A \\ B\end{matrix}\right\}$ can be obtained by initial condition



$(\omega_d = \sqrt{1-\xi^2}\,\omega_n)$ $(\omega_d = \cos\ldots)$



Q A machine weigh 18 kg and supported on springs and dashpots. The total stiffness of spring is 12 N/mm and damping is 0.2 N/mm/sec. The system is initially at rest and velocity of 120 mm/sec is imparted to the mass find

(i) The displacement and velocity of mass as a function of time.

(ii) The disp. and velocity after 0.4 seconds

Ans

$m = 18\,kg$

$t=0;\ x=0$

$t=0;\ V_0 = 0.12\ m/s$

$S = 12\ N/mm$

$S = 12{,}000\ N/m$

$c = 0.2\ N/mm/s$

$c = 200\ N/m/s$

$$x = Ye^{-\xi_y \omega_n t}\sin(\omega_d t + \phi)$$

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{12000}{18}} = 25.81\ rad/s.$$

$$\xi_y = \frac{c}{2\sqrt{Sm}} = \frac{200}{2\sqrt{12000 \times 18}} = 0.215$$

$$\xi_y = 0.215$$

$w_d = \sqrt{1-\xi^2}\, w_n$

$w_d = 26.21$ rad/sec.

$x = X e^{-5.55t} \sin(25.21t + \phi)$

At $t=0$, $x=0$

$0 = X \sin\phi$

$\sin\phi = 0$

$\phi = 0$

$x = X e^{-5.65t} \sin(25.21t)$

$\dot{x} = X\left[e^{-5.55t} \cos(25.21t) \times 25.21 + \sin(25.21t) \times e^{-5.55t}(-5.55)\right]$

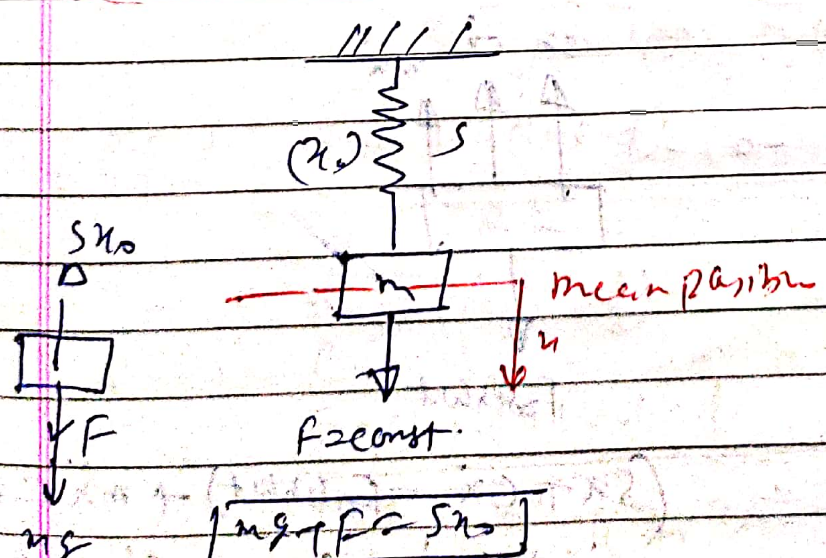
$t=0$, $\dot{x} = 0.12$ m/s

$0.12 = (25.21)X$

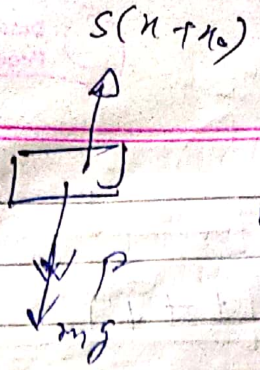
$X = (4.76 \times 10^{-3})$ m

$x = (4.76\times10^{-3}) e^{-5.55t} \sin(25.21t)$.

* forced oscillations: →



$\boxed{mg + F = sx_0}$



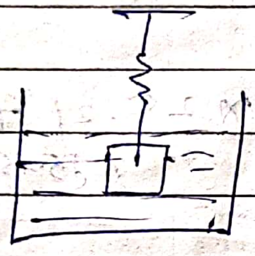
$$mg + f - s(x+x_0) = m\ddot{x}$$

$$mg + f - sx - sx_0 = m\ddot{x}$$

$$\boxed{m\ddot{x} + sx = 0}$$

$$\boxed{\ddot{x} + \left(\frac{s}{m}\right)x = 0}$$

$$\omega_n = \sqrt{\frac{s}{m}}$$



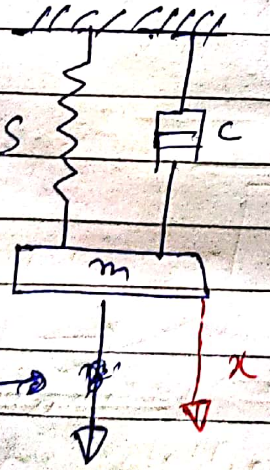
$$\left(\frac{mg - F_B}{m}\right) = \left(g - F_B/m\right)$$

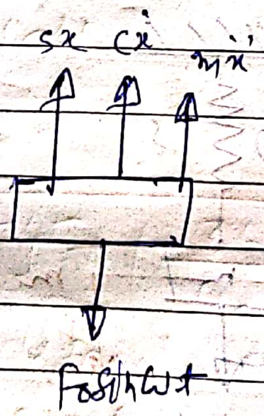
$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{g - \frac{F_B}{m}}{l}}$$

$$\boxed{\omega_n = \sqrt{\frac{g - \frac{F_B}{m}}{l}}}$$

Forced damped oscillations :-

harmonic force   underdamped





$$(sx + c\dot{x} - F_0 \sin\omega t) + m\ddot{x} = 0$$

$$m\ddot{x} + c\dot{x} + sx = F_0 \sin\omega t$$

$$\boxed{\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \left(\frac{s}{m}\right)x = \left(\frac{F_0}{m}\right)\sin\omega t}$$

The sol^n is:-

$$\boxed{x = CF + PI}$$

CF: $\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \left(\frac{s}{m}\right)x = 0$

$$CF = y e^{-\xi \omega_n t} \sin(\omega_d t + \phi_1) \quad \text{——} \; (1)$$

P.I. → $\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \left(\frac{s}{m}\right)x = \left(\frac{F_0}{m}\right)\sin\omega t$

$$\frac{c}{m} = a.$$

$$\frac{s}{m} = b$$

$$\frac{F_0}{m} = d.$$

$$\ddot{x} + a(\dot{x}) + (b)x = (d)\sin\omega t$$

$$P.I = \frac{d\sin\omega t}{\{(D^2 + aD + b)\}} \cdot \frac{\{(b-\omega^2) - aD\}}{\{(b-\omega^2) - aD\}} \quad [D^2 = -\omega^2]$$

$$= \frac{d\{(b-\omega^2)\sin\omega t - aD(\sin\omega t)\}}{(b-\omega^2)^2 + a^2\omega^2}$$

$$= \frac{d\{(b-\omega^2)\sin\omega t - (a\omega)\cos\omega t\}}{(b-\omega^2)^2 + a^2\omega^2}$$

R cos φ, R sin φ

$$R = \sqrt{(b-\omega^2)^2 + (a\omega)^2}$$

$$P.I = \frac{d \cdot R \sin(\omega t - \phi)}{R^2}$$

$$= \frac{d}{R}\sin(\omega t - \phi)$$

$$= \frac{(F_0/m)\sin(\omega t - \phi)}{\sqrt{\left(\frac{s}{m} - \omega^2\right)^2 + \left(\frac{c}{m}\omega\right)^2}}$$

$$= \frac{(F_0/m)\sin(\omega t - \phi)}{\omega_n^2 \sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}$$

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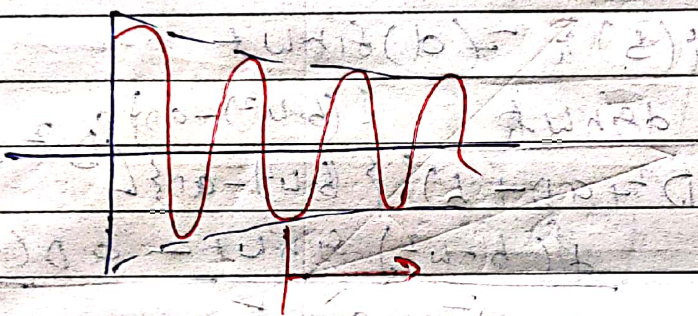
$$P.I = \frac{F_0/S}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}} \sin(\omega t - \phi)$$

The soln is:

$$x = CF + PI$$

$$x = \underbrace{Xe^{-\xi\omega_n t}}_{\text{Damped Free Response}} \sin(\omega_d t + \phi_1) + \underbrace{\frac{F_0/S}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}} \sin(\omega t - \phi)}_{\text{Steady state response}}$$



Vibrations will Never stop

Steady State Response (Never Ending): → Amplitude of Steady State Response

$$A = \frac{(F_0/S)}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}} = \text{const.}$$

Magnification factor)

M.F: → It is defined as a ratio of amplitude of steady state Response to the static deflection of spring under the constant force F_0

$$MF = \frac{A}{(F_0/S)} = \frac{1}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}$$

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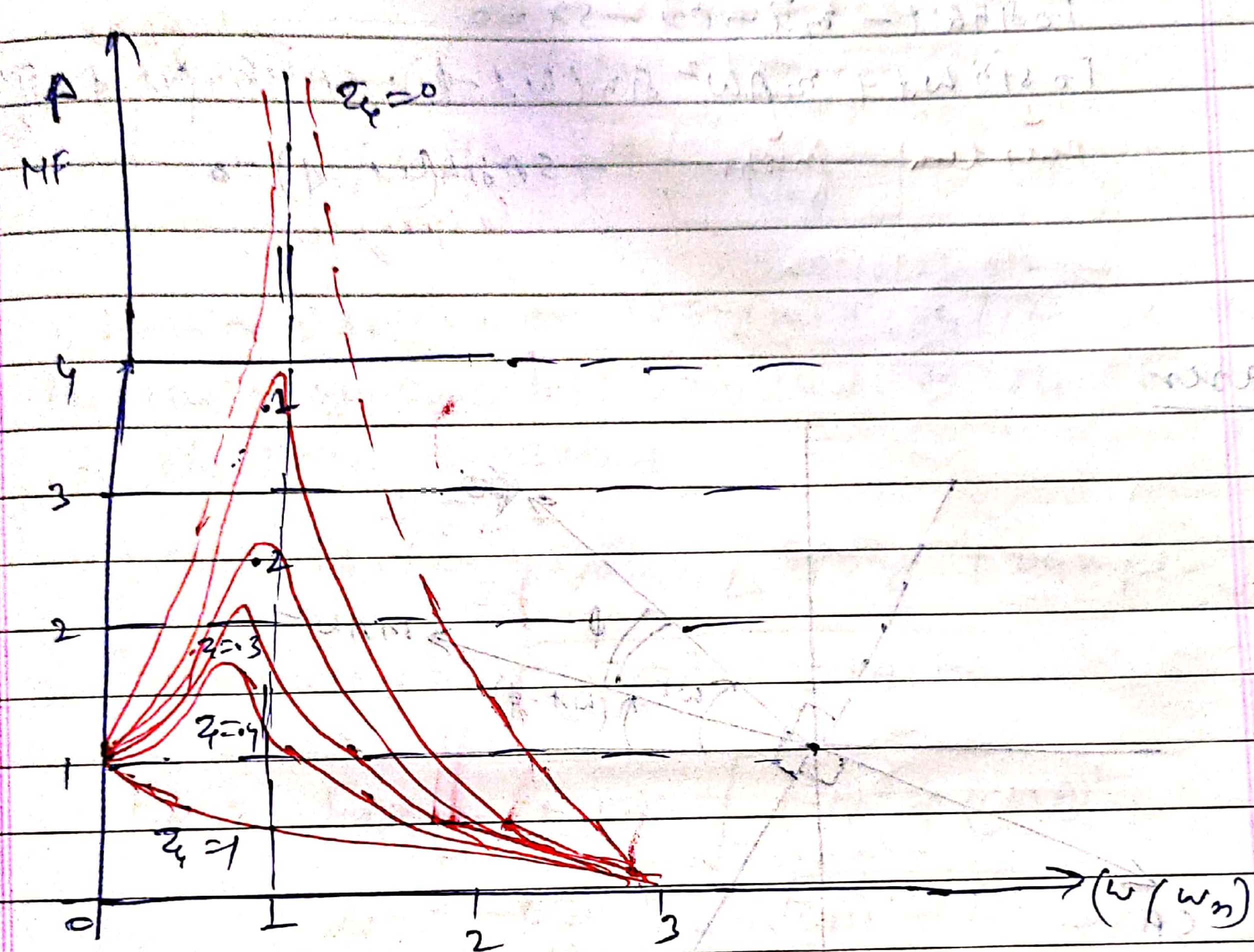
This magnification factor (MF) depends upon

(i) $\frac{\omega}{\omega_n}$

(ii) ξ



Note:→ M.F. is max. for $\left(\frac{\omega}{\omega_n}\right)$ ~~as~~ slightly less than 1 not equal to $\left[\frac{\omega}{\omega_n}=1\right]$

* VIBRATION ISOLATION:→ (Transmissibility):→

$$x = \frac{(F_0/S)}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2+\left(\frac{2\xi\omega}{\omega_n}\right)^2}} \sin(\omega t-\phi)$$

→ A

$$\boxed{x = A\sin(\omega t-\phi)}$$

$m\ddot{x} + c\dot{x} + sx = F_0 \sin \omega t$

$x = A \sin(\omega t - \phi)$

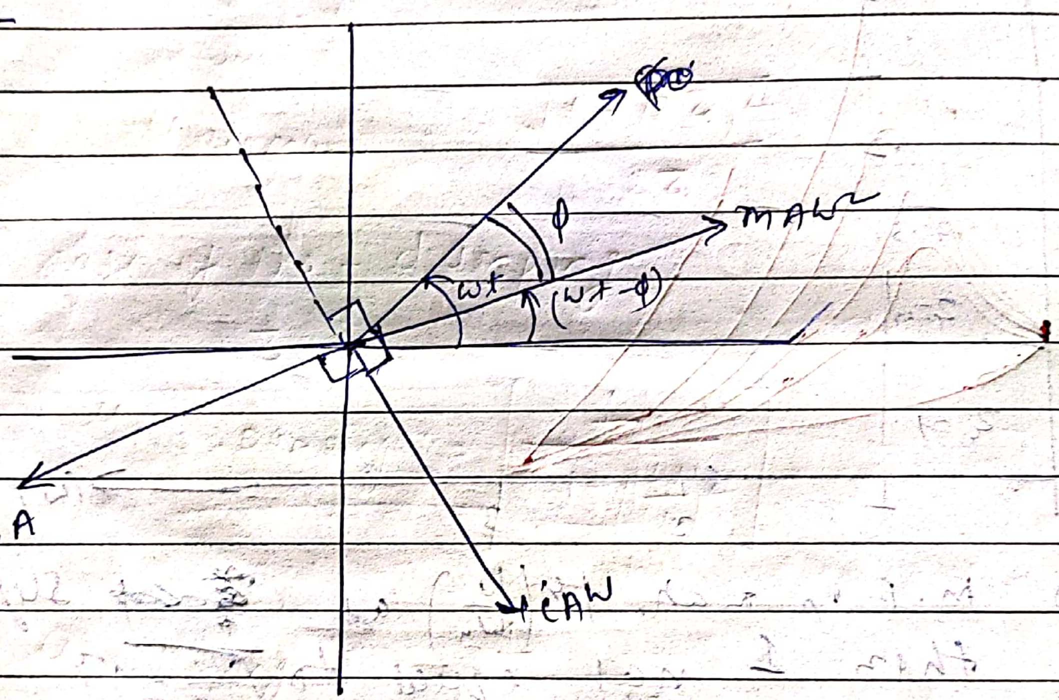
$\dot{x} = A\omega \cos(\omega t - \phi) = A\omega \sin(\omega t - \phi + \pi/2)$

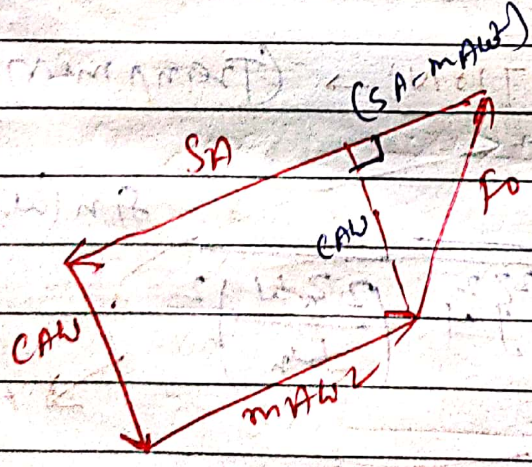
$\ddot{x} = -A\omega^2 \sin(\omega t - \phi)$

$F_0 \sin \omega t - m\ddot{x} - c\dot{x} - sx = 0$

$\underbrace{F_0 \sin \omega t}_{\text{Appl. term}} + \underbrace{mA\omega^2}_{\text{Inertia}} \sin(\omega t - \phi) - \underbrace{cA\omega \sin\left(\omega t - \phi + \frac{\pi}{2}\right)}_{\text{Damping}}$
$\underbrace{- sA \sin(\omega t - \phi)}_{\text{Spring term}} = 0$

Phaser





$F_0 = \sqrt{(SA - mA\omega^2)^2 + (CA\omega)^2}$

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$$F_0 = SA\sqrt{\left(1-\frac{w^2}{S/m}\right)^2+\left(\frac{C}{S}\omega\right)^2}$$

$$\left(\frac{F_0}{S}\right) = A\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2+\left(\frac{2\xi\omega}{\omega_n}\right)^2}$$

$$\boxed{A = \frac{(F_0/S)}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2+\left(\frac{2\xi\omega}{\omega_n}\right)^2}}}$$

→ vibration isolation or transmissibility is defined as the ratio of force transmitted to the foundation to the force applied

~~vibration isolation system [spring + Damper]~~

$$F_T = F_{transmitted} = \sqrt{(SA)^2+(CWA)^2}$$

$$F_T = F_{transmitted} = \sqrt{(SA)^2+(CWA)^2}$$

$$\epsilon = \frac{F_T}{F_0} = \frac{\sqrt{(SA)^2+(CWA)^2}}{F_0}$$

$$= \frac{SA\sqrt{1+\left(\frac{2\xi\omega}{\omega_n}\right)^2}}{F_0}$$

Transmissibility

$$\boxed{\epsilon = \frac{\sqrt{1+\left(\frac{2\xi\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(\frac{2\xi\omega}{\omega_n}\right)^2}}}$$

Transmissibility :→

$$\mu = \frac{F_t}{F_0}$$

$$\mu = \frac{\sqrt{1+\left(\frac{2\xi\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(\frac{2\xi\omega}{\omega_n}\right)^2}}$$

At Resonance!

$\omega = \omega_n$

$\frac{\omega}{\omega_n} = 1$

$$\mu_{resonance} = \frac{\sqrt{1+(2\xi)^2}}{2\xi}$$

In case of No-damping

$\xi = 0$

$$\mu_{No\ Damping} = \left[\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$\frac{F_t}{F_0} = \mu_{No\ damping} = \frac{1}{\pm\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}$$

(I) When :-

$\frac{\omega}{\omega_n} < \sqrt{2}$ ⇒ $\mu > 1$ → more damping is required in vibration isolation system

$\frac{F_t}{F_0} > 1$

$\boxed{F_t > F_0}$ ⟶ Danger.

(II) If $\frac{\omega}{\omega_n} = \sqrt{2}$ ⇒ $\mu = 1$

⇒ $\boxed{F_t = F_0}$ ⟶ useless

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(iii) If $\frac{\omega}{\omega_n} > \sqrt{2} \Rightarrow \mu < 1$

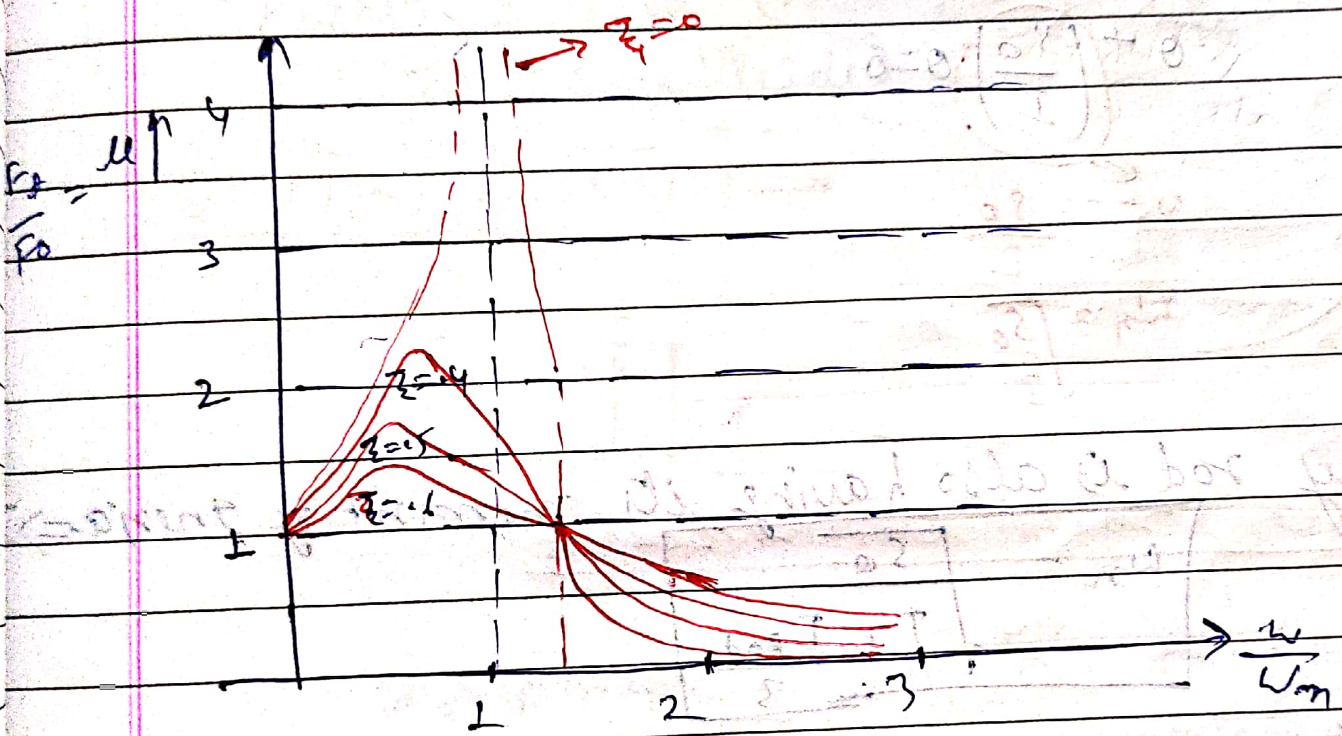
$\Rightarrow F_t/F_0 < 1$

$\boxed{F_t < F_0} \longrightarrow$ useful

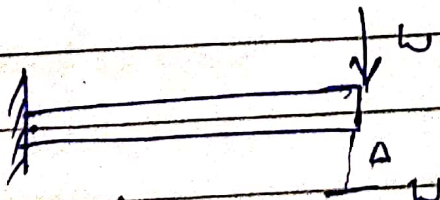
μ depends upon

(i) ω/ω_n

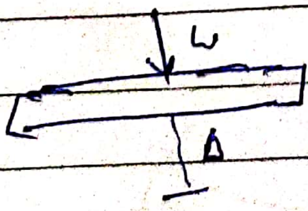
(ii) ξ



✗ Transverse vibrations :→



$\omega_n = \sqrt{\frac{g}{\Delta}}$



$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}}$

$T_n = 2\pi\sqrt{\frac{\Delta}{g}}$

Torsional Vibration! →

$$(S_o \cdot \theta) + I\alpha = 0$$

$$I\ddot{\theta} + S_o \cdot \theta = 0$$

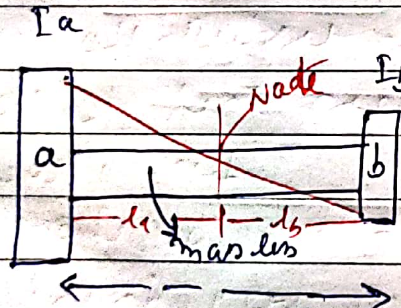
$$\ddot{\theta} + \left(\frac{S_o}{I}\right)\theta = 0$$

$$\omega_n^2 = \frac{S_o}{I}$$

$$\omega_n = \sqrt{\frac{S_o}{I}}$$

if rod is also having its moment of inertia ⇒ I_{rod}

$$\boxed{\omega_n = \sqrt{\frac{S_o}{I + \frac{I_{Rod}}{3}}}}$$



$$l_a + l_b = L \longrightarrow (1)$$

$$\omega_a = \sqrt{\frac{S_{\theta_1}}{I_a}}$$

$$\omega_b = \sqrt{\frac{S_{\theta_2}}{I_b}}$$

$$\omega_a = \omega_b$$

$$\sqrt{\frac{S_{\theta_1}}{I_a}} = \sqrt{\frac{S_{\theta_2}}{I_b}}$$

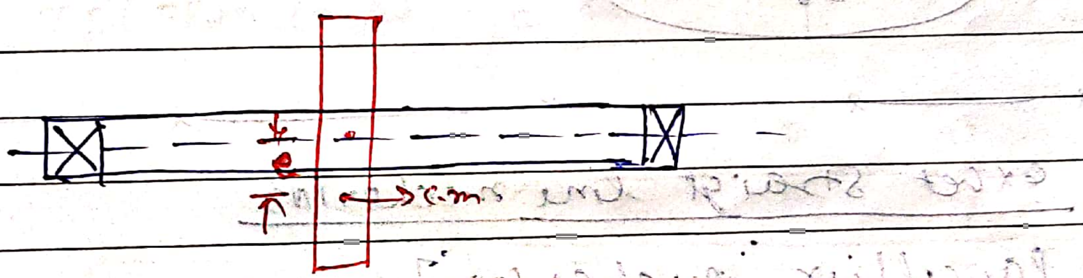
$$\frac{S_{\theta_1}}{I_a} = \frac{S_{\theta_2}}{I_b}$$

$$\frac{GJ}{l_a . I_a} = \frac{GJ}{l_b I_b}$$

$$\boxed{\frac{l_a}{l_b} = \frac{I_b}{I_a}} \quad \text{———} \quad (2)$$

→ if numbers of rotor are n number of node points will be (n-1) (amplitude of vibration is exact zero amplitude).

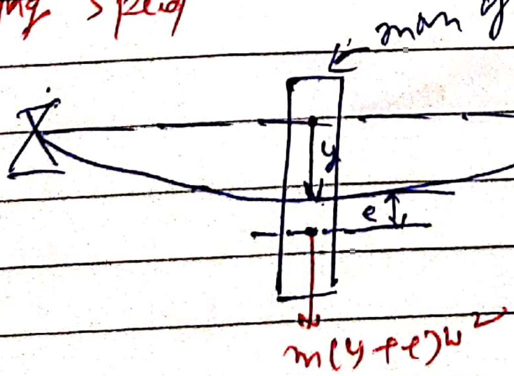
Whirling of Shaft! →



e ⇒ Distance of Rotor C.M. from the Centre of Rotor (eccentricity)

$$\omega = \omega_n = \sqrt{\frac{s}{m}} \Rightarrow \text{Resonance}$$

→ Whirling speed
→ Critical speed
→ Whipping speed



$$m(y+e)\omega^2 = s.y$$

$$my\omega^2 + me\omega^2 = sy$$

$$sy - my\omega^2 = me\omega^2$$

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$$y(s - m\omega^2) = m e \omega^2$$

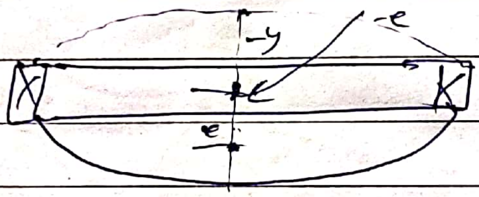
$$y = \frac{m e \omega^2}{(s - m\omega^2)}$$

$$= \frac{m e \omega^2}{m\omega^2\left(\frac{s}{m\omega^2} - 1\right)}$$

$$\boxed{y = \left\{\frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}\right\}}$$

if $\omega > \omega_n$ then at some position

$y = -e$ → vibration will stop



Exact straight line mechanism

(1) Paucellier mechanism
(2) Hart mech.] Turning pair only
(3) Scott-Russel mech.] one sliding pair

Approx st line mechanism

1) Watt's mech.
2) modified Scott-Russel mech
3) Grasshopper mech.
4) Tchebicheff mech.
5) Robert's mech