JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

DESIGN OF MACHINE ELEMENTS – II {6ME4-04}

UNIT – III

<u>CO1</u> :- To determine the finite and Infinite life of Mechanical Components due to fluctuating loads.

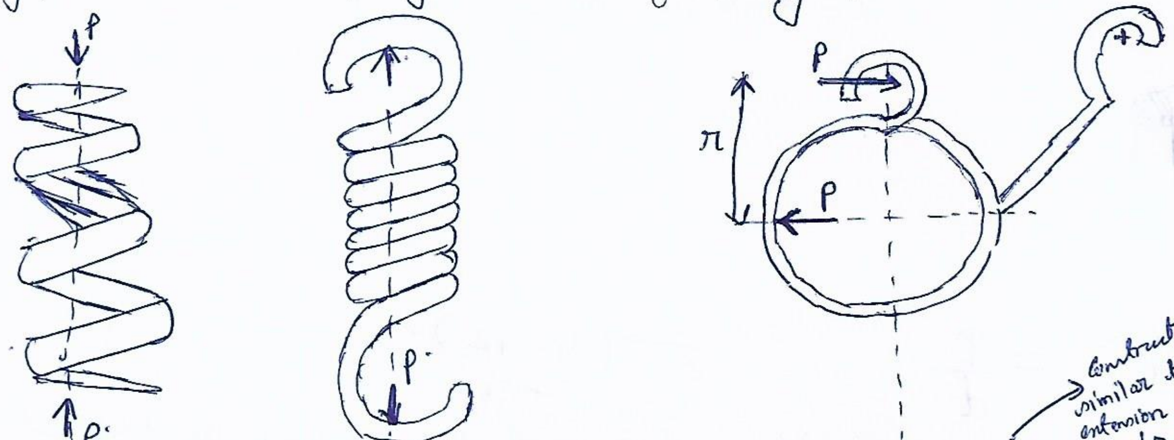
<u>Contents</u> :- Design of helical compression, tension, torsional springs, springs under variable stresses.

Design of belt, rope and pulley drive system.

→ Springs are flexible machine elements used primarily to deflect under load with the ability to return to its original shape when unloaded. Springs are designed to provide a push, a pull or a torque. They are also designed to store energy, measure a force, or absorb shocks and vibrations

→ There are three criterions in design of springs:-
ⓐ Sufficient strength to withstand external load
ⓑ desired load deflection characteristics.
ⓒ Sufficient buckling strength.

Types of springs:- Springs are classified according to their shape. The shape can be a helical coil of a wire, a piece of stamping, or a flat wound-up strip..
The popular types of mechanical springs are helical compression springs, helical extension spring, helical torsion spring and multileaf spring.



(a) Helical Compression Spring

(b) Helical Extension Spring.

(c) Helical torsion spring.

used in door-hinges, automobile starters and door locks

Construction of this spring is similar to that of compression or extension spring except that the ends are formed in such a way that the spring is loaded by a torque about the axis of the coils.

→ The helical spring is made from a wire, usually of circular cross-section which is bent in the form of a helix.

helical compression spring:- the external force tends to shorten the spring i.e spring is compressed.
helical torsion spring:- the external force tends to lengthen the spring i.e spring is elongated.
In both the cases, the external force acts along the axis of the spring and induces torsional shear stress in the spring wire.

Closely-coiled helical spring.	open coiled helical spring.
A helical spring is said to be closely-coiled spring when the spring wire is coiled so close that the plane containing each coil is almost at right angle to the axis of the helix - i.e the helix angle is very small (usually less than 10°).	A helical spring is said to be open-coiled spring, when the spring wire is coiled in such a way, that there is large gap between adjacent coils. i.e the helix angle is large

Helical springs, compression as well as extension have following advantages :-

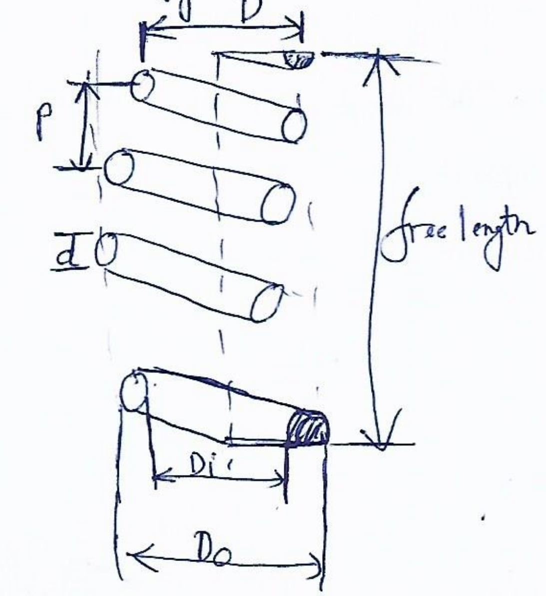
(i) They are easy to manufacture.

(ii) They are cheaper than other types of spring.

(iii) Their reliability is high.

(iv) The deflection of spring is linearly proportional to the force acting on the spring.

Terminology :-



d = wire diameter of spring (mm)

D_i = Inside diameter of spring coil (mm)

D_o = outside diameter of spring coil (mm).

D = mean coil diameter (mm)

Spring Index $(C) = \frac{D}{d}$ $\rightarrow (4-12)$.
$6-9$.

The spring index indicates the relative sharpness of the curvature of the coil. A low spring index means high sharpness of curvature. When the spring index is low ($C<3$) the actual stresses in the wire are excessive due to curvature effect. When the spring index is high ($C>15$), it results in large variation in coil diameter. Such a spring is prone to buckling and also tangles easily during handling.

Solid length $\Rightarrow N_t d$. (N_t → total no of coils)

total axial gap $\Rightarrow (N_t - 1) \times$ gap between adjacent coils.

free length $\Rightarrow$ Compressed length $+ \delta$.

$\Rightarrow$ solid length + total axial gap $+ \delta$.

pitch $\Rightarrow$ the pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state of spring.

$p =$ free length

Stiffness (K) :- stiffness of the spring (K) is defined as the force required to produce unit deflection. $K = \frac{P}{\delta}$.

→ rate of spring or gradient of spring or scale of spring or simply spring constant.

The stiffness of the spring represents the slope of load-deflection line.

→ Active coils are the coils in the spring, which contribute to spring action, support the external force and deflect under the action of force.

→ Inactive coils :- A portion of the coil at each end, which is in contact with the seat, does not contribute to spring action and called inactive coil. These coils do not support the load and do not deflect under the action of external force.

inactive coils = $N_t - N$.

$$\tau = K\left(\frac{8PD}{\pi d^3}\right)$$

K = stress factor or Wahl factor.

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}.$$

$$\delta = \frac{8PD^3N}{Gd^4}.$$

Series :- $\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$

Parallel :-

$$K = K_1 + K_2.$$

$S_{ut} = \frac{A}{\ }$

Surge in spring :- When the natural frequency of vibrations of the springs coincides with the frequency of external periodic force, which acts on it, resonance occurs. In this state, the spring is subjected to a wave of successive compressions of coils that travels from one end to the other and back. This type of vibratory motion is called 'surge' of spring. Surge is found in valve springs, which are subjected to periodic force.

The natural frequency of helical compression springs held between two parallel plates is given by :-

$$\omega = \frac{1}{2}\sqrt{\frac{K}{m}}$$

The natural frequency of helical compression springs with one end on the flat plate and the other end free, supporting the external force is given by :-

$$\omega = \frac{1}{4}\sqrt{\frac{K}{m}}$$

The mass of the spring is given by, $m = Al\rho$

where A = Cross-sectional area of spring $= \left(\frac{\pi}{4} d^2\right)$
l = length of spring $= (\pi D N_t)$
ρ = mass density of spring material.

<u>Surge in springs is avoided by the following methods :-</u>

(i) The spring is designed in such a way that the natural frequency of the spring is 15 to 20 times the frequency of excitation of the external force. This prevents the resonance condition to occur.

(ii) The spring is provided with friction dampers on central coils. This prevents propagation of surge wave.

(iii) A spring made of stranded wire reduces the surge.

Design against fluctuating load :-

P_{max} to P_{min}.

mean force $P_m = \frac{1}{2}(P_{max} + P_{min})$

force amplitude $P_a = \frac{1}{2}(P_{max} - P_{min})$

direct. Shear stress correction factor.

$$\tau_m = K_s\left(\frac{8 P_m D}{\pi d^3}\right) \quad \left\{K_s = \left(1 + \frac{0.5}{C}\right)\right\}$$

For torsional stress amplitude (τ_a) it is necessary to also consider the effect of stress concentration due to curvature in addition to direct shear stress.

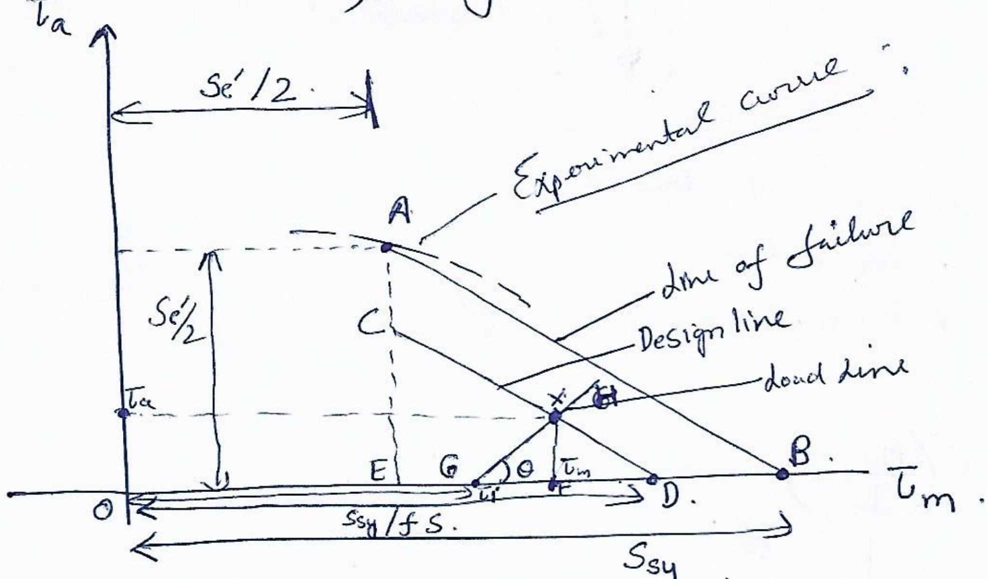
$$\tau_a = K_s K_c\left(\frac{8 P_a D}{\pi d^3}\right) \Rightarrow \tau_a = K\left(\frac{8 P_a D}{\pi d^3}\right).$$

For patented and cold drawn steel wires

$$S'_{se} = 0.21\, S_{ut} \; ; \; S_{sy} = 0.42\, S_{ut}.$$

For oil hardened and tempered steel wires (SW & VW)

$$S'_{se} = .22\, S_{ut} \; ; \; S_{sy} = 0.45\, S_{ut}.$$



Considering similar triangles XFD & AEB :-

$$\frac{\overline{XF}}{\overline{FD}} = \frac{\overline{AE}}{\overline{EB}} \Rightarrow \frac{\tau_a}{\frac{S_{sy}}{fs} - \tau_m} = \frac{\frac{1}{2}S'_{se}}{S_{sy} - \frac{1}{2}S'_{se}}$$

Spring Materials :-

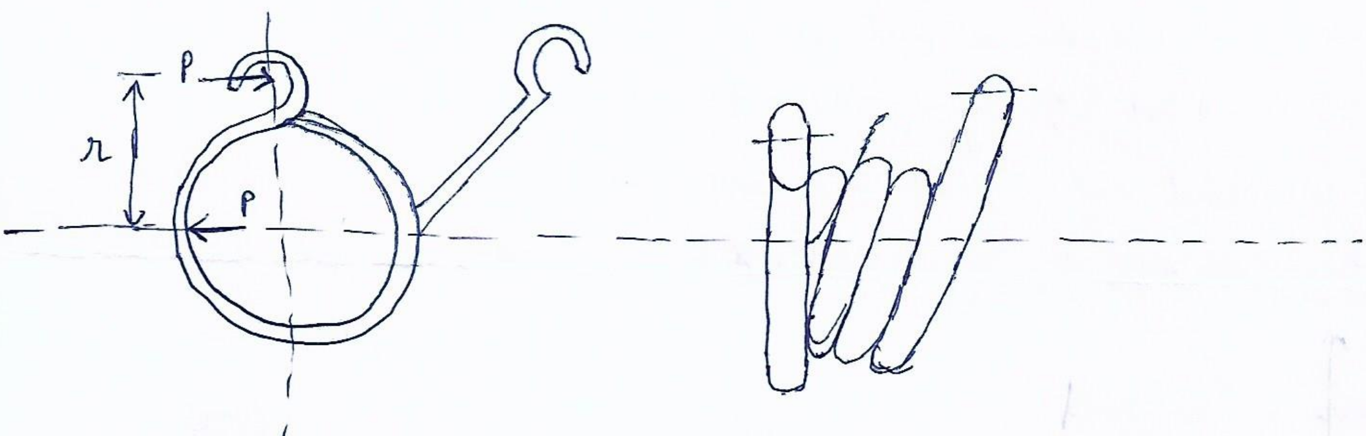
(i) Patented and Cold draw steel wires (unalloyed).

(ii) oil-hardened and tempered spring steel wires and value spring wires,

(iii) oil hardened and tempered steel wires (alloyed).

(iv) Stainless steel spring wires.

Helical torsion springs :- A helical torsion spring is a device used to transmit the torque to a particular component of a machine or mechanism. It is widely used in door hinges, brush holders, automobile starters and door locks.



The helical torsion springs resists the bending moment $(P \times r)$, which tends to wind up the spring. Each individual section of the torsion spring is, in effect a portion of a curved beam. Using the curved beam theory, the bending stresses are given by :-

$$\sigma_b = K\left(\frac{M_b y}{I}\right) \Rightarrow y = \frac{d}{2}, \quad I = \frac{\pi d^4}{64}.$$

$$\boxed{\sigma_b = K\left(\frac{32 M_b}{\pi d^3}\right)}$$

$$\boxed{K_i = \frac{4C^2 - C - 1}{4C(C-1)}}$$

$$K_o = \frac{4C^2 + C - 1}{4C(C+1)}$$

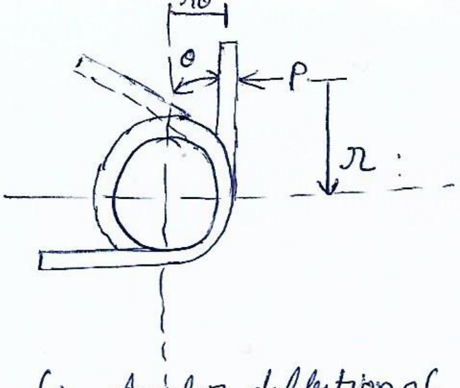


fig. Angular deflection of spring.

$$M_b = Pr$$

The strain energy stored in the spring is given by :-

$$U = \int \frac{(M_b)^2 dx}{2EI}$$

$$U = \int_0^{\pi DN} \frac{P^2 r^2 dx}{2EI} \Rightarrow U = \frac{P^2 r^2 (\pi DN)}{2EI}.$$

The deflection in the direction of force P is approximately $(r\theta)$.
Using Castigliano's theorem.

$$r\theta = \frac{\partial U}{\partial P} = \frac{Pr^2 (\pi DN)}{EI}$$

Substituting $I = \pi d^4 / 64$.

$$\boxed{\theta = \frac{64 P r DN}{E d^4}}$$

The stiffness of the helical torsion spring is defined as the bending moment required to produce unit angular displacement.

$$\boxed{K = \frac{Pr}{\theta}} \quad \text{or} \quad \boxed{K = \frac{E d^4}{64 DN}}.$$