JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Design of Machine Elements-II {6ME4-04}

UNIT-IV

Co 3 :- To design the different types of gears due to gear forces.

Contents :- Design of gear teeth: Lewis and Buckingham equations, wear and dynamic load considerations.

Design and force analysis of spur, helical, bevel and worm gears, Bearing reactions due to gear tooth forces.

Unit-4 (i).

Spur Gears

→ There are two criterions to design the tooth of spur gears viz. beam strength by lewis equation and wear strength by Buckingham's equation.
→ lewis equation is based on bending failure, while Buckingham's equation is based on pitting failure. The effective load between two meshing teeth consists of two factors viz. tangential force due to transmitted torque and dynamic load.
→ There are two methods to account for dynamic load viz approximate estimation by velocity factor in the preliminary stages of gear design and precise calculation by Buckingham's equation in the final stages of gear design.

Advantages of Gear drives over chain or belt drives :-

Gear drives offer the following advantages compared with chain or belt drives :-

(i) It is a positive drive and the velocity ratio remains constant.
(ii) The centre distance between the shafts is relatively small, which results in compact construction.
(iii) It can transmit very large power, which is beyond the range of belt or chain drives.
(iv) It can transmit motion at very low velocity, which is not possible with the belt drives.
(v) The efficiency of gear drives is very high even up to 99 percent in case of spur gears.
(vi) A provision can be made in the gear box for gear shifting, thus changing the velocity ratio over a wide range.

Classification of gears :-

Gears are broadly classified into four groups :-

(i) Spur gears :-

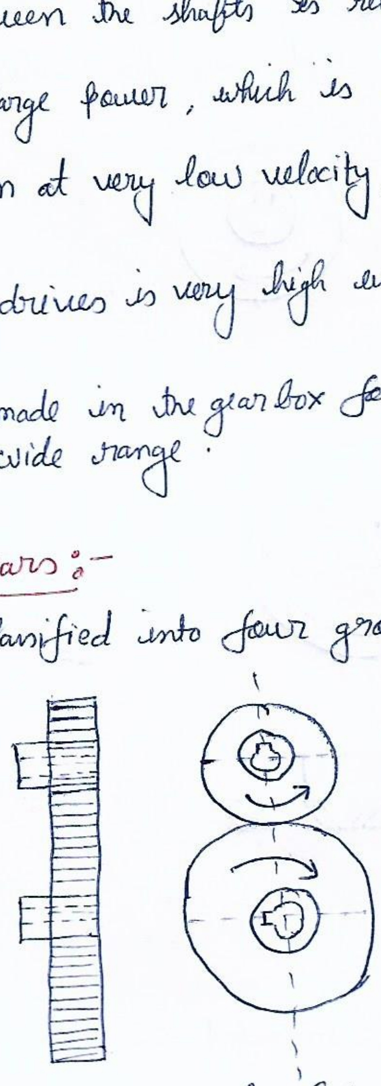
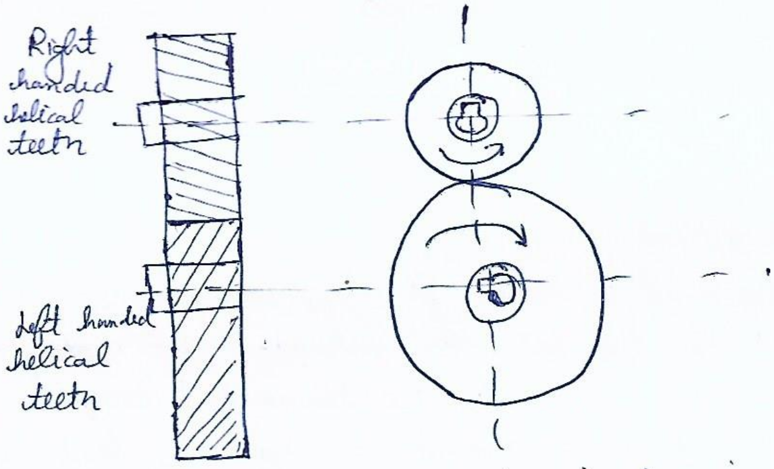


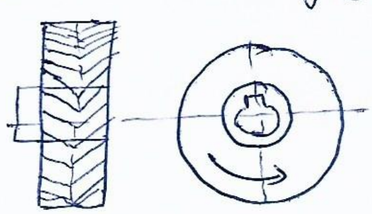
fig. Spur gears.

→ In case of spur gears, the teeth are cut parallel to the axis of the shaft. spur gears are used only when the shafts are parallel. The profile of the gear teeth is in the shape of an involute curve and it remains identical along the entire width of the gear wheel. Spur gears impose radial loads on the shafts.

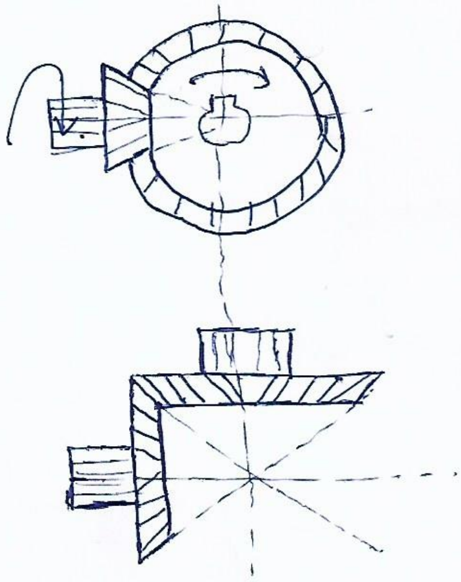
(ii) Helical gears :-



→ The teeth of these gears are cut at an angle with the axis of the shaft. Helical gears have an involute profile similar to that of spur gears. However, this involute profile is in a plane, which is perpendicular to the teeth element. The magnitude of the helix angle of pinion and gear is same. however, the hand of helix is opposite. A right hand pinion meshes with a left hand gear and vice-versa. Helical gears impose radial and thrust loads on shafts. There is a special type of helical gear, consisting of two helical gears with opposite hand of helix, it is called Herringbone gear. The construction results in equal and opposite thrust reactions, balancing each-other and imposing no thrust load on the shaft. Herringbone gears are used only for parallel shafts.

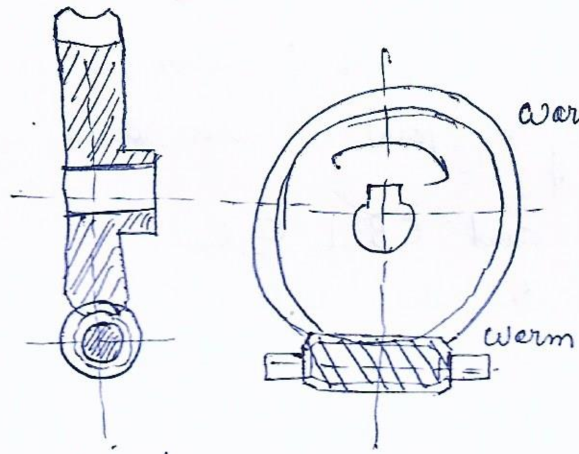


(iii) Bevel gears :-



→ Bevel gears have the shape of a truncated cone. The size of the gear teeth, including the thickness and height decreases towards the apex of the cone. Bevel gears are normally used for shafts which are at right angle to each-other. The teeth of

(iv) Worm gears:-



→ It consists of a worm and a worm wheel. The worm is in the form of a threaded screw, which meshes with the matching wheel. The threads on the worm can be single or multithread and usually have a small lead. Worm gear drives are used for shafts, the axes of which do not intersect and are perpendicular to each-other. The worm imposes high thrust load, while the worm wheel imposes high radial load on the shafts. Worm gear drives are characterized by high-speed reduction ratio.

Law of Gearing:- The fundamental law of gearing states —" The Common normal to the tooth ~~surface~~ profile at the point of contact should always pass through a fixed point, called the pitch point, in order to obtain a constant velocity ratio".

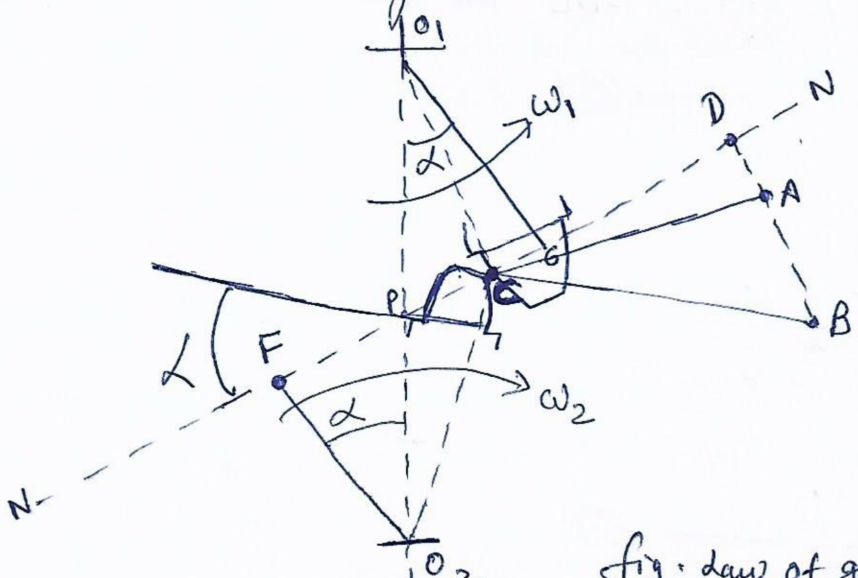


fig. Law of gearing

In above figure, O_1 and O_2 are centers of the two gears rotating with angular velocities ω_1 and ω_2 respectively. C is the point of contact between the teeth of the two gears and NN is the common normal at the point of contact.

$\vec{CA}$ is the velocity of point C, when it is considered on gear 1, $\vec{CB}$ is the velocity of point C, when it is considered on gear 2. Also,

$$CA \perp O_1C \quad \text{and} \quad CB \perp O_2C$$

The projections of the two vectors $\vec{CA}$ & $\vec{CB}$ i.e $\vec{CD}$ along the common normal NN must be equal, otherwise the teeth will not remain in contact and there will be a slip.

$$CA = \omega_1 \times O_1C$$
$$CB = \omega_2 \times O_2C$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2C}{O_1C} \times \frac{CA}{CB} \quad \text{— (a)}$$

Since $\triangle O_1CG$ and $\triangle CAD$ are similar, hence :-

$$\frac{O_1C}{CA} = \frac{O_1G}{CD} \quad \text{— (b)}$$

Similarly, $\triangle O_2FC$ and $\triangle CDB$ are similar, and thus,

$$\frac{O_2C}{CB} = \frac{O_2F}{CD} \quad \text{— (c)}$$

from (b) & (c)

$$\frac{CA}{CB} = \frac{O_1C}{O_2C} \times \frac{O_2F}{O_1G} \quad \text{— (d)}$$

from a and d,

$$\frac{\omega_1}{\omega_2} = \frac{O_2F}{O_1G} \quad \text{— (e)}$$

Similarly, $\triangle O_2FP$ and $\triangle O_1GP$ are similar, therefore,

$$\frac{O_2F}{O_1G} = \frac{O_2P}{O_1P} \quad \text{— (f)}$$

from (e) and (f),

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$

also $O_1P + O_2P = O_1O_2$ = constant. (h).

Therefore, for a constant velocity ratio (ω_1/ω_2), P should be a fixed point. This point P is called the pitch point.

→ It has been found that only involute and cycloidal curves satisfy the fundamental law of gearing.

(i) An Involute is a curve traced by a point on a line as the line rolls without slipping on a circle.

(ii) A cycloid is a curve traced by a point on the circumference of a generating circle as it rolls without slipping along the inside and outside of another circle. The cycloid profile consists of two curves namely, epicycloid and hypocycloid. An epicycloid is a curve traced by a point on the circumference of a generating circle as it rolls without slipping on the outside of pitch circle. A hypocycloid is a curve traced by a point on the circumference of a generating circle as it rolls without slipping on the inside of pitch circle.

Important terms related to gears:-

(i) Circular pitch ⇒ The circular pitch (p) is the distance measured along the pitch circle between two similar points on adjacent teeth.

$$p = \frac{\pi d'}{Z} \qquad \begin{Bmatrix} d' = \text{pitch circle diameter} \\ Z = \text{number of teeth} \end{Bmatrix}$$

(ii) Diametral pitch (P) ⇒ The diametral pitch (P) is the ratio of the number of teeth to the pitch circle diameter. Therefore,

$$P = \frac{Z}{d'}$$

(iii) module :– The module (m) is defined as the inverse of diametral pitch. Therefore,

$$m = \frac{1}{P} = \frac{d'}{Z} \; ; \quad \text{or} \quad d' = mZ$$

The centre to centre distance between two gears having Z_p and Z_g teeth is given by :-

$$a = \frac{1}{2}(d'_p + d'_g) = \frac{1}{2}(mz_p + mz_g)$$

$$= \frac{m(z_p + z_g)}{2}$$

(iv) gear ratio (i) :– The ratio of the number of teeth on gear to that on pinion.

$$i = \frac{Z_g}{Z_p} = \frac{n_p}{n_g}$$

Interference and Undercutting :-

→ Interference :– Gear tooth has involute profile only outside the base circle. The involute profile begins at the base circle. In some cases, the dedendum is so large that it extends below this base circle. In such situations, the portion of the tooth below the base circle is not involute. The tip of the tooth on the mating gear, which is involute, interferes with this non-involute portion of dedendum. This phenomenon of tooth profiles overlapping and cutting into each-other is called 'interference'. In this case, the tip of the tooth overlaps and digs into the root section of its mating gear. Interference is non conjugate action and results in excessive wear, vibrations and jamming.

→ Undercutting → When the gears are generated by involute rack cutters, interference is automatically eliminated because the cutting tool removes the interfering portion of the flank. This is called 'undercutting'. Undercutting solves the problem of interference. However, undercut tooth is considerably weaker. Undercutting not only weakens the tooth but also removes

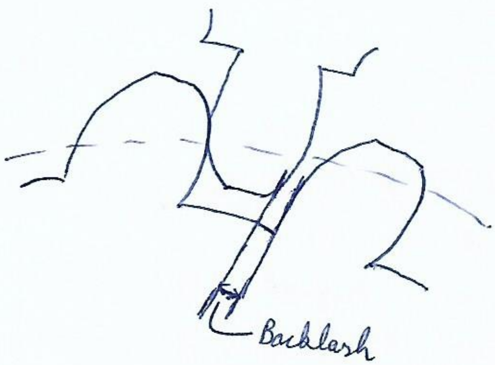
Methods to eliminate Interference :-

(i) Increase the number of teeth on the pinion.

(ii) Increase pressure angle.

(iii) Use long and short addendum gearing.

Backlash :- Backlash is defined as the amount by which the width of tooth space exceeds the thickness of the engaging tooth measured along the pitch circle.



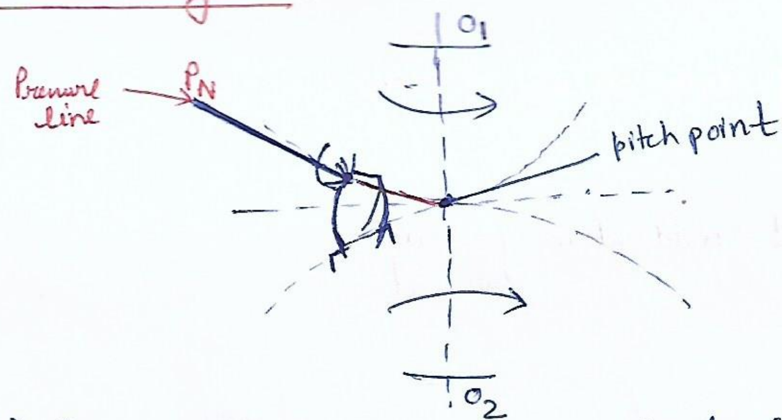
Objective of providing backlash :-

1. Backlash prevents the mating teeth from jamming together. The mating teeth do not make contact on both sides simultaneously. This makes the teeth roll together freely and smoothly.
2. Backlash compensates for machining errors.
3. Backlash compensates for thermal expansion of teeth.

Methods of providing backlash :-

1. The teeth of the gear are cut slightly thinner. This is obtained by setting the cutting tool deeper into the blank resulting in thinner tooth and wider space.
2. The centre distance between mating gears is slightly increased.

Force Analysis :-



→ In gears, power is transmitted by means of a force exerted by the tooth of the driving gear on the meshing teeth of the driven gear.

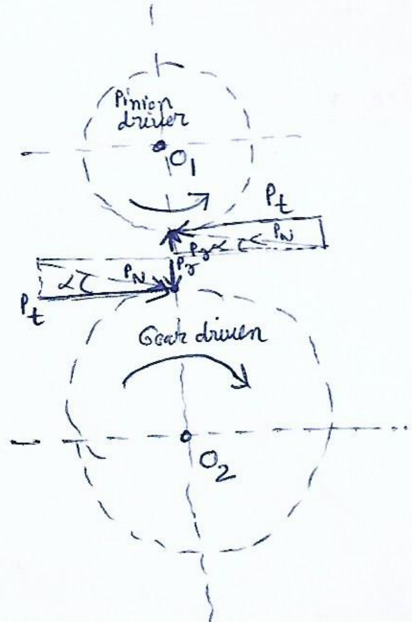
→ According to the fundamental law of gearing, this resultant force P_N always acts along the pressure line. The resultant force P_N can be resolved into two components — tangential component P_t and radial component P_r at the pitch point.

→ The tangential component P_t is a useful load because it determines the magnitude of the torque and consequently the power, which is transmitted. The radial component P_r is a separating force, which is always directed towards the centre of the gear.

The torque transmitted by the gears is given by :-

$$M_t = \frac{60 \times 10^6 (KW)}{2\pi n}$$

M_t = torque transmitted by gears (N-mm)
KW = power transmitted by gears (KW)
n = speed of rotation (rpm).



→ The tangential component P_t acts at the pitch circle radius. Therefore,

$$P_t\left(\frac{d'}{2}\right) = M_t$$

$$P_t = \frac{2M_t}{d'}$$

$$P_r = P_t \tan\alpha \qquad \tan\alpha = \frac{P_r}{P_t}$$

$$P_N = \frac{P_t}{\cos\alpha} \qquad \cos\alpha = \frac{P_t}{P_N}$$

Gear tooth failures :-

(i) Abrasive wear

(ii) Corrosive wear

(iii) Initial pitting

(iv) Destructive pitting

(v) Scoring

Number of teeth :- In the design of gears, it is required to decide the number of teeth on the pinion and gear. There is a limiting value of minimum number of teeth on pinion. As the number of teeth decreases a point is reached when there is interference and the standard tooth profile requires modification. The minimum number of teeth to avoid interference is given by,

$$Z_{min} = \frac{2}{\sin^2\alpha}$$

→ In practice, giving a slight radius to the tip of tooth can further reduce the value of Z_{min}.

Pressure angle (α)	14.5°	20°	25°
Z_{min} (Practical)	27	14	9
Z_{min} (theoretical)	32	17	11

→ for the 20° full depth involute tooth system, it is always safe to assume

Face width:- In the design of gears, it is required to express the face width in terms of module.

→ The optimum range of the face width is :-

$$(8m) < b < (12m)$$

→ In the preliminary stages of gear design, the face width is assumed as ten times of module.

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Beam Strength of Gear tooth — Lewis equation.

→ In the Lewis analysis, the gear teeth is treated as a cantilever beam.

→ The tangential component (P_t) causes the bending moment about the base of the tooth.

– The Lewis equation is based on the following assumptions:-

(i) The effect of the radial component (P_r), which induces compressive stress is neglected.

(ii) It is assumed that the tangential component (P_t) is uniformly distributed over the face width of the gear. This is possible when the gears are rigid and accurately machined.

(iii) The effect of stress concentration is neglected.

(iv) It is assumed that at any time only one pair of teeth is in contact and takes the total load.

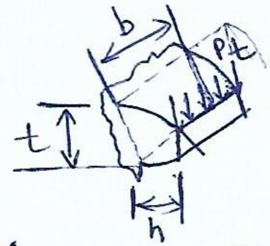

fig. Gear as Cantilever

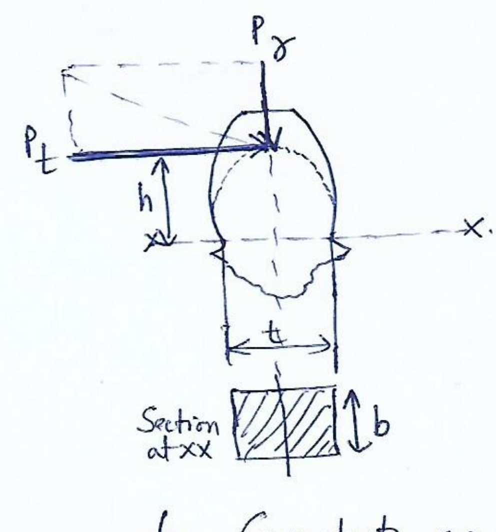

fig. Gear tooth as Parabolic beam.

It is observed that the cross-section of the tooth varies from free to ... within the tooth profile. The advantage

For this beam, the stress at any cross-section is uniform or same. The weakest section of the gear tooth is at section XX, where the parabola is tangent to the tooth profile.

At section XX, $M_b = P_t \times h$

$$I = \left(\frac{1}{12}\right) b t^3$$

$$y = \frac{t}{2}$$

$$\sigma_b = \frac{M_b y}{I} = \frac{(P_t \times h)\left(\frac{t}{2}\right)}{\left(\frac{1}{12}\right) b t^3}$$

$$P_t = b \sigma_b \left(\frac{t^2}{6h}\right)$$

Multiplying the numerator and denominator of the right hand side by m,

$$P_t = m b \sigma_b \left(\frac{t^2}{6hm}\right)$$

Defining a factor $Y = \frac{t^2}{6hm}$

$$\boxed{P_t = m b \sigma_b Y} \text{ ———— (a)}$$

When the tangential force is increased, the stress also increases. When the stress reaches the permissible magnitude of bending stresses, the corresponding force (P_t) is called the beam strength. Therefore, the beam strength (S_b) is the maximum value of the tangential force that the teeth can transmit without bending failure.

$$\boxed{S_b = m b \sigma_b Y} \rightarrow \text{Lewis Equation.}$$

→ In order to avoid the breakage of tooth of gear due to bending, the beam strength should be more than the effective force between the meshing teeth.

$$\boxed{S_b \geq P_{eff}}$$

Deciding the weaker pinion or gear

In the design of gears, it is required to decide the weaker between pinion and gear.

Writing the Lewis equation:-

$$S_b = m b \sigma_b Y$$

It is observed that m and b are same for pinion as well as for gear. When different materials are used the product ($\sigma_b \times Y$) decides the weaker between pinion and gear. The Lewis form factor Y is always less for a pinion compared with gear. When the same material is used for pinion and gear, the pinion is always weaker than the gear.

Permissible Bending stress :-

The teeth are subjected to fluctuating stresses, endurance limit stress (S_e) is the criterion of design. Therefore, the maximum bending stress is equal to the endurance limit stress of the gear teeth.

→ Earle Buckingham has suggested that endurance limit stress of gear tooth is approximately one-third of the ultimate tensile strength of the material

$$\boxed{\sigma_b = S_e = \left(\frac{1}{3}\right) S_{ut}}$$

→ In case of bronze gears, the endurance limit stress is taken as 40% of the ultimate tensile strength.

Effective load on gear tooth :-

Service factor (C_s) = $\dfrac{\text{maximum torque}}{\text{rated torque}}$

$$C_s = \frac{(M_t)_{max}}{M_t} = \frac{(P_t)_{max}}{P_t} \qquad \{(P_t)_{max} = C_s P_t\}$$

For electric motor :-

→ There are two methods to account for the dynamic load – approximate estimation by the velocity factor in the preliminary states of gear design and precise calculation by Buckingham's equation in the final stages of gear design.

→ It is difficult to calculate the exact magnitude of dynamic load in the preliminary stages of gear design. To overcome this difficulty, a velocity factor C_v developed by Barth is used.

The values of velocity factor are as follows :-

(i) For ordinary and commercially cut gears made with form cutters and with velocity ($v < 10\,m/s$).

$$C_v = \frac{3}{3+v}$$

(ii) For accurately hobbed and generated gears with ($v < 20\,m/s$)

$$C_v = \frac{6}{6+v}$$

(iii) For precision gears with shaving, grinding and lapping operations and with ($v > 20\,m/s$).

$$C_v = \frac{5.6}{5.6+\sqrt{v}}$$

→ The pitch line velocity is given by :- $v = \frac{\pi d' n}{60 \times 10^3}$

→ The effective load between two meshing teeth is given by :-

$$\boxed{P_{eff} = \frac{C_s P_t}{C_v}}$$

→ In the final stages of gear design when gear dimensions are known, errors specified and the quality of gears determined, the dynamic load is calculated by equations derived by Earle Buckingham.

The effective load is given by :- $P_{eff} = (C_s P_t + \ldots)$

$$P_d = \frac{21v\,(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}}$$

P_d = dynamic load or incremental dynamic load (N)
v = pitch line velocity (m/s)
C = deformation factor (N/mm^2)
e = Sum of errors between two meshing teeth (mm)
b = face width of tooth (mm)
P_t = tangential force due to rated torque (N)

$$C = \frac{K}{\left[\frac{1}{E_p} + \frac{1}{E_g}\right]}$$

K = 0.107 (for 14.5° full depth teeth)
K = 0.111 (for 20° full depth teeth)
K = 0.115 (for 20° stub teeth)

$$e = e_p + e_g$$

$$\phi = m + 0.25\sqrt{d'}$$

ϕ = tolerance factor
m = module (mm)
d' = pitch circle diameter (mm)

→ The errors depends upon the quality of the gear and the method of manufacture. There are twelve different grades from Gr 1 to Gr 12 in decreasing order of precision.

Grade		e (microns)
1	⟶	$0.80 + 0.06\phi$
2	⟶	$1.25 + 0.10\phi$
3	⟶	$2.00 + 0.16\phi$
4	⟶	$3.20 + 0.25\phi$
5	⟶	$5.00 + 0.40\phi$
6	⟶	$8.00 + 0.63\phi$
7	⟶	$11.00 + .90\phi$
8	⟶	$16.00 + 1.25\phi$
9	⟶	$22.00 + 1.80\phi$
10	⟶	$32.00 + 2.50\phi$
11	⟶	$45.00 + 3.55\phi$
12	⟶	$63.00 + 5.00\phi$

Estimation of module Based on Beam strength :-

$$m = \left[\frac{60 \times 10^6}{\pi} \left\{ \frac{(KW)\, C_s\, (fs)}{z n C_v \left(\frac{b}{m}\right)\left(\frac{S_{ut}}{3}\right) Y} \right\} \right]^{1/3}$$

↳ The above equation is used in the preliminary stages of gear design

Wear strength of Gear tooth :-

$$\boxed{P_t = b Q d_p' K}$$

$$\boxed{S_w = b Q d_p' K}$$ — Buckingham's equation for wear

A load-stress factor K is defined as :-

$$K = \frac{\sigma_c^2 \sin\alpha \cos\alpha \left(1/E_1 + 1/E_2\right)}{1.4}$$

Ratio factor (Q) for internal gears :-

$$Q = \frac{2 z_g}{z_g - z_p}$$

Note :- The expression for the load-stress factor K can be simplied when both the gears are made of steel with a 20° pressure angle.

$$\boxed{K = 0.16 \left(\frac{BHN}{100}\right)^2}$$

Estimation of module based on wear strength :-

$$m = \left[\frac{60 \times 10^6}{\pi} \left\{ \frac{(kW)\, C_s\, (fs)}{z_p^2\, n_p\, C_v \left(\frac{b}{m}\right) Q K} \right\} \right]^{1/3}$$