5ME4-04 : Design of Machine Elements-I.

Unit-4

CO2 : To differentiate the basic analytical design under different loading conditions.

CO3 : To estimate the stresses and strains induced in different m/c element subjected to torsion and bending.

Content :- Design of Members in Torsion

Shaft and Keys :- Design for strength, rigidity, solid and hollow shafts. Shafts under combined loading. Sunk keys.

Couplings :- Design of muff coupling, flanged coupling: rigid and flexible.

UNIT-4

Shafts, Keys and Couplings

SHAFTS:-

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears, etc are mounted on it. These members along with the forces exerted upon them cause the shaft to bending. In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.

Properties of Materials used for shafts :-

The material used for shafts should have the following properties :-

1) It should have high strength.
2) It should have good machinability.
3) It should have low notch sensitivity factor.
4) It should have good heat treatment properties.
5) It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40C8, 45C8, 50C4 and 50C12.

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

Manufacturing of shafts :- Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses.

Stresses in shafts :- The following stresses are induced in the shafts :-

1) Shear stresses due to the transmission of torque (i.e due to torsion load)
2) Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc as well as due to the weight of the shaft itself.
3) Stresses due to combined torsional and bending loads.

Design of shafts :-

The shafts may be designed on the basis of :-

(a) Strength and (b) Rigidity and Stiffness

In designing shafts on the basis of strength, the following cases are considered.

1. Shafts subjected to twisting moment or torque only.
2. Shafts subjected to bending moment only.
3. Shafts subjected to combined twisting and bending moments.
4. Shafts subjected to axial loads in addition to combined torsional and bending loads.

~~Shaft~~ Shafts subjected to Twisting Moment only :-

When the shaft is subjected to a twisting moment (torque) only, then the diameter of the shaft is obtained by using the torsion equation.

$$\boxed{\frac{T}{J} = \frac{\tau}{r}} \quad \text{—(i)}$$

T = Twisting moment acting on the shaft ;
J = Polar moment of inertia of the shaft about the axis of rotation ;
τ = Torsional shear stress ; r = distance from neutral axis to the outermost fibre

for round solid shaft, $\boxed{J = \frac{\pi}{32} d^4}$

→ substituting in eqn (i)

$$\frac{T}{\frac{\pi}{32} d^4} = \frac{\tau}{d/2} \Rightarrow \boxed{T = \frac{\pi}{16} \tau d^3} \quad \text{—(ii).}$$

→ from this d can be calculated.

for hollow shaft :- $J = \frac{\pi}{32}[d_o^4 - d_i^4]$, $r = \frac{d_o}{2}$ — Substituting these values in eqn (i).

$$\frac{T}{\frac{\pi}{32}(d_o^4 - d_i^4)} = \frac{\tau}{\frac{d_o}{2}} = \boxed{T = \frac{\pi}{16} \times \tau \left[\frac{d_o^4 - d_i^4}{d_o}\right]} \quad \text{—(iii)}$$

Let $k = \frac{d_i}{d_o}$.

$$T = \frac{\pi}{16} \times \tau \times \frac{d_o^4}{d_o}\left(1 - \frac{d_i^4}{d_o^4}\right)$$

$$\boxed{T = \frac{\pi}{16} \tau d_o^3 (1 - k^4)} \quad \text{—(iii)}$$

Note :- ① The power transmitted (in watts) by the shaft :-

$$P = \frac{2\pi N T}{60} \Rightarrow \boxed{T = \frac{60P}{2\pi N}}$$

{N = speed of the shaft in rpm
T = Twisting moment in N-m}

② In case of belt drives :-

$$\boxed{T = (T_1 - T_2)R}$$

{R = radius of pulley
T_1 = Tension in the tight side of the belt
T_2 = Tension in the slack side of the belt}

Shafts subjected to Bending Moment only :- When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation.

$$\boxed{\frac{M}{I} = \frac{\sigma_b}{y}} \quad \text{(i)}$$

{M = Bending moment ; σ_b = Bending stress
I = Moment of Inertia of cross-sectional area of the shaft about the axis of rotation
y = Distance from neutral axis to the outer most fibre $= \frac{d}{2}$}

→ For a round solid shaft, MOI :-

$$I = \frac{\pi}{64} d^4 \; ; \; y = \frac{d}{2}$$ } Substituting these values in eqn(i)

$$\frac{M}{\frac{\pi}{64} d^4} = \frac{\sigma_b}{\frac{d}{2}} \Rightarrow \boxed{M = \frac{\pi}{32} \times \sigma_b \times d^3} \quad \text{—(ii)}$$

↳ from this d can be calculated.

→ For hollow shaft :-

$$I = \frac{\pi}{64}[d_o^4 - d_i^4] = \frac{\pi}{64}(d_o)^4(1-k^4)$$

$$y = \frac{d_o}{2}$$

$$\boxed{M = \frac{\pi}{32} \times \sigma_b \times (d_o)^3 (1-k^4)}$$

Shafts subjected to Combined Twisting Moment and Bending Moment :-

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been subjected to account for the elastic failure of the materials when they are subjected to various types of combined stresses. Following Two theories are important :-

① Maximum of shear stress theory or Guest's theory → used for ductile materials (mild steel)
② Maximum normal stress theory or Rankine's theory → used for brittle material (Cast Iron)

Let τ = Shear stress induced due to twisting moment

σ_b = Bending stress induced due to bending moment

According to maximum shear stress theory, maximum shear stress in the shaft :-

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \quad \text{—(i)}$$

Substituting $\sigma_b = \frac{32M}{\pi d^3}$ and $\tau = \frac{16T}{\pi d^3}$ in eqn (i)

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\sqrt{M^2+T^2}$$

or

$$\boxed{\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2+T^2}} \quad \text{—(ii)}$$

$$T_e = \text{Equivalent twisting moment} = \sqrt{M^2+T^2}$$

Equivalent twisting moment is defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the eqn (ii) may be written as:-

$$\boxed{T_e = \sqrt{M^2+T^2} = \frac{\pi}{16} \times \tau \times d^3} \quad \text{—(iii)}$$

→ d can be calculated.

According to the maximum normal stress theory, the maximum normal stress in the shaft :-

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$\sigma_{b(max)} = \frac{32}{\pi d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2+T^2}\right)\right]$$

or $$\frac{\pi}{32} \times \sigma_{b(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2+T^2}\right] \quad \text{—(iv)}.$$

$$M_e = \text{equivalent bending moment} = \frac{1}{2}\left[\left(M + \sqrt{M^2+T^2}\right)\right]$$

The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.

$$\boxed{M_e = \frac{1}{2}\left[M + \sqrt{M^2+T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3} \quad \text{—(V)}$$

→ d can be calculated. — iv

Note :- It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Shafts subjected to fluctuating loads

Till now we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (T) and bending moment (M).

Thus for a shaft subjected to combined bending and torsion :-

the equivalent twisting moment :-

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

and equivalent bending moment :-

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where :-

K_m = Combined shock and fatigue factor for bending

K_t = Combined shock and fatigue factor for torsion.

Recommended values for K_m and K_t

Nature of load.	K_m	K_t
1. Stationary Shafts		
(a) Gradually applied load	1	1
(b) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
2. Rotating Shafts		
a. Gradually applied or steady load	1.5	1.0
b. Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
c. Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

Unit-4 -b (Keys and Couplings)

Keys:- A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys:-

1. Sunk Keys
2. Saddle Keys
3. Tangent Keys
4. Round Keys
5. Splines

SUNK KEYS:- The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types:-

1. **Rectangular sunk Key:-** width of key $(w) = \frac{d}{4}$; $t = \frac{d}{6}$ { d = diameter of the shaft or diameter of the hole in the hub }

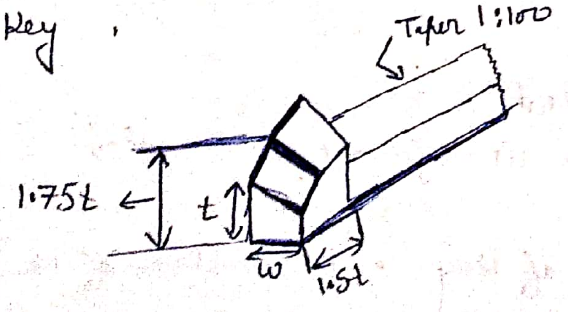
   ↳ The key has taper 1 in 100 on the top side only.

   $t = \frac{2w}{3}$

2. **Square sunk key:-** $w = t = \frac{d}{4}$

3. **Parallel sunk key:-** The parallel sunk keys may be of rectangular or square section, uniform in width and thickness throughout. It is taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. **Gib-head Key:-** It is a rectangular sunk key with a head at one end known as gib head. It is usually provided to facilitate the removal of key.



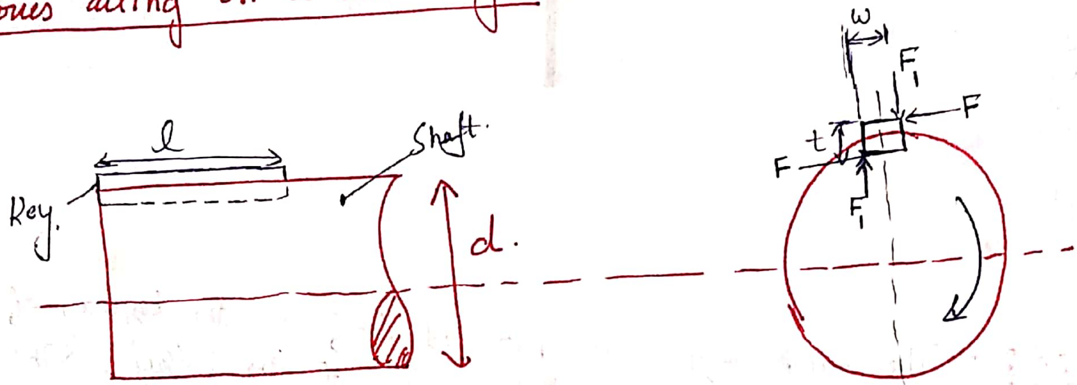
$w = \frac{d}{4}$

$t = \frac{d}{6} = \frac{2w}{3}$

5. **Feather Key :-** A key attached to one member of a pair and which permits relative axial movement is known as feather key. It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the keyway of the moving piece.

6. **Woodruff Key :-** easily adjustable key; largely used in machine tool and automobile construction

Forces acting on a Sunk key :-



When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :-

1. Forces (F_1) due to fit of the key in its keyway. These forces produce compressive stresses in the key which are difficult to determine in magnitude.
2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

Strength of a Sunk Key :-

Let T = Torque transmitted by the shaft

F = Tangential force acting at the circumference of the shaft

d = diameter of shaft

l = length of key ; w = width of key ; t = thickness of key

τ and σ_c = Shear & crushing stresses for the material of key.

Key may fail due to shearing or crushing due to the power transmitted by the shaft.

→ Considering shearing of the key

$F =$ Area resisting shearing × shear stress $= l * w * \tau$

∴ Torque transmitted by the shaft ⇒ $T = F \times \frac{d}{2} = l * w * \tau * \frac{d}{2}$ —(i).

→ Considering crushing of the key

$F =$ Area resisting crushing × crushing stress $= l * \frac{t}{2} * \sigma_c$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l * \frac{t}{2} * \sigma_c * \frac{d}{2} \quad \text{—(ii)}.$$

The key is equally strong in shearing & crushing, if

$$l w \tau \frac{d}{2} = l * \frac{t}{2} * \sigma_c * \frac{d}{2}$$

$$\boxed{\frac{w}{t} = \frac{\sigma_c}{2\tau}} \quad \text{—(iii)}$$

The permissible crushing stress for the usual key material is atleast twice the permissible shearing stress.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

⇓

Shearing strength of Key = Torsional shear strength of the shaft

$$l * w * \tau * \frac{d}{2} = \frac{\pi * \tau_1 * d^3}{16}$$

τ — shear stress for key material

τ_1 — shear stress for shaft material

Shaft Couplings :- Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following :-

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility
3. To reduce the transmission of shock loads from one shaft to another.
4. To Introduce protection against overloads.

Difference between Couplings and clutch :-

A coupling is termed as a device used to make permanent or semi-permanent connection where as a clutch permits rapid connection or disconnection at the will of the operator.

Types of Shaft Couplings :-

1. **Rigid Coupling :-** It is used to connect two shafts which are perfectly aligned. Following types of rigid couplings are mostly used :-
   - (a) Sleeve or muff coupling
   - (b) Clamp or split-muff coupling or compression coupling
   - (c) Flange Coupling

2. **Flexible Coupling :-** It is used to connect two shafts having both lateral and angular misalignment. Following types of coupling are mostly used :-
   - (a) Bushed pin type coupling
   - (b) Universal Coupling
   - (c) Oldham Coupling.

ii. Sleeve or Muff Coupling :- It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve.

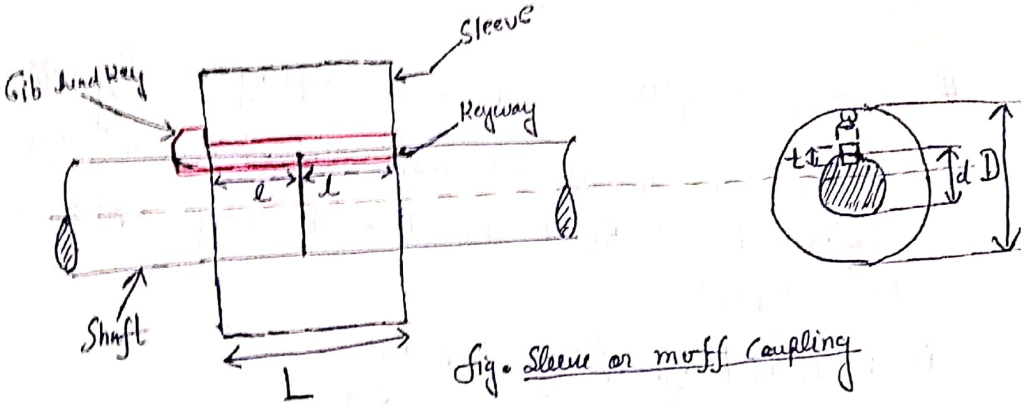


fig. Sleeve or muff coupling

Let :- d = diameter of the shaft ; D = outer diameter of the sleeve

L = length of the sleeve ; T = Torque transmitted by coupling

l = length of the key on each shaft ; w = width of key ; t = thickness of key

τ = torsional shear stress developed in the shaft ; τ_c = Induced shear stress for the sleeve material.

① Design of shaft :- $P = \frac{2\pi N T}{60}$ $\Rightarrow$ $\boxed{T = \frac{P \times 60}{2\pi N}}$

$= \boxed{T = \frac{\pi}{16} \tau d^3}$ → from this d can be calculated.

② Design for sleeve :- The sleeve is designed by considering it as a hollow shaft.

$\boxed{D = (2d + 13) \text{ mm}}$

$\boxed{L = 3.5\, d}$

Empirical relations.

$\boxed{T = \frac{\pi}{16} \tau_c \left(\frac{D^4 - d^4}{D} \right)}$ → from this τ_c can be calculated.

③ Design for key :-

→ w and t of the key can be obtained from the standard value from any design data book corresponding to the diameter of the shaft.

→ length of the key on each shaft $= l = \frac{L}{2} = \frac{3.5d}{2}$;

→ Shearing stress τ induced in the key ⇒ $\boxed{T = l \times w \times \tau \times \frac{d}{2}}$ → τ for key can be calculated

→ Crushing stress σ_c in the key ⇒ $\boxed{T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}}$ → σ_c for key can be calculated.

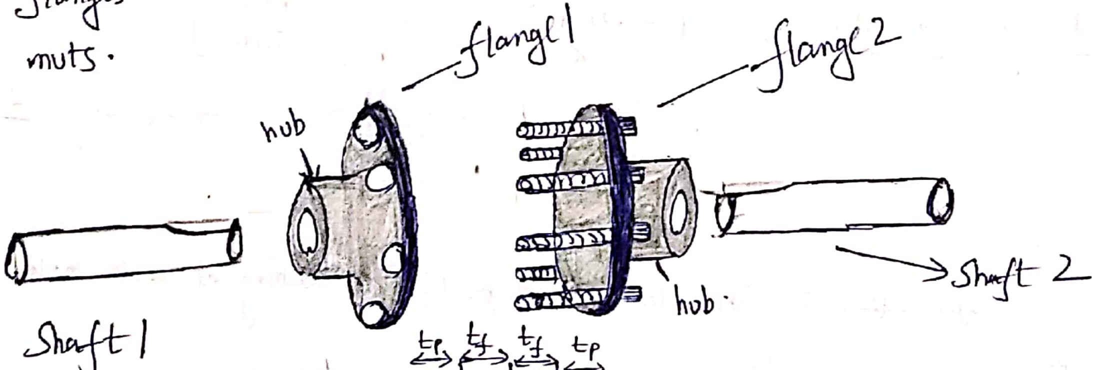
Flange Coupling :-

A flange coupling usually applies to a coupling having two seperate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. The flange coupling are of the following three types :-

1. Protected type flange coupling
2. Unprotected type flange coupling
3. Marine type flange coupling.

Design of protected type and unprotected type flange coupling :-

The two flanges mounted on the shaft are coupled together by means of bolts and nuts.



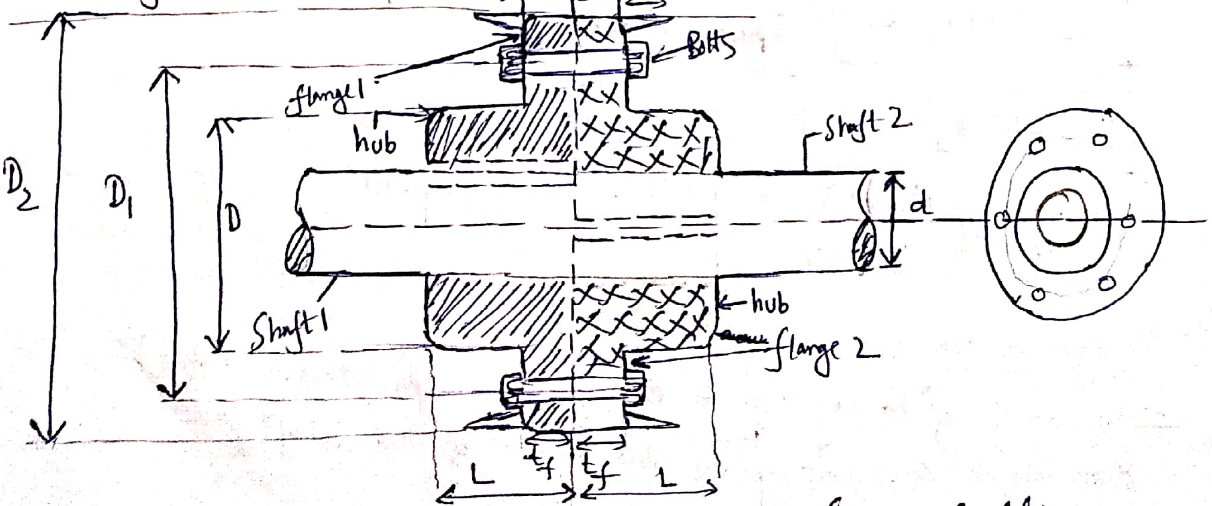


fig - protected type flange coupling.

For a flange coupling :- d = diameter of shaft ; D = outer diameter of the hub
L = length of the hub or effective length of the key ; T = torque transmitted by the shaft
D_1 = pitch circle diameter of the bolt ; t_f = thickness of flange ; t_p = thickness of the protecting rim
D_2 = outside diameter of the flange ; n = number of bolt ;
d_1 = nominal diameter of bolt ; σ_{cb} and σ_{cR} = Allowable crushing stress for bolt & key respectively
τ_s, τ_b & τ_R = Allowable shear stress for shaft, bolt & key respectively.

Empirical relations for flange coupling :-

$D = 2d$; $L = 1.5\,d$; $D_1 = 3d$; $D_2 = 4d$; $t_f = 0.5\,d$; $t_p = .25\,d$

$n = 3$, for d upto 40 mm
$n = 4$, for d upto 100 mm
$n = 6$, for d upto 180 mm

Design Procedure :-

① Design for shaft :- $\boxed{T = \frac{60 * P}{2\pi N}}$; $\boxed{T = \frac{\pi}{16} * \tau_s\, d^3}$ → d can be calculated.

② Design for hub :- the hub is designed by considering it as a hollow shaft

$\boxed{D = 2d}$; $\boxed{T = \frac{\pi}{16} * \tau_c \left(\frac{D^4 - d^4}{D}\right)}$ → from this equation induced shearing stress in the hub can be checked.

$\boxed{L = 1.5\,d}$

③ Design for flange :- $\boxed{t_f = 0.5\,d}$; $\boxed{t_p = 0.25\,d}$; $\boxed{D_2 = 4\,d}$

Induced shear stress in the flange :- Flange at the junction of the hub is under shear while transmitting the torque.

$\therefore$ T = Circumference of hub × thickness of flange × Shear stress × Radius of hub

$$T = \pi D * t_f * \tau_c * \frac{D}{2}$$

$\boxed{T = \frac{\pi D^2}{2} * t_f * \tau_c}$ → from this τ_c for flange can be checked.

④ Design for Key :- → w & t for key can be taken from data book as per d.

→ $L = 1.5\,d$.

→ $\boxed{T = L * w * \tau_R * \frac{d}{2}}$ → τ_R for key can be checked.

→ $\boxed{T = L * \frac{t}{2} * \sigma_{cR} * \frac{d}{2}}$ → σ_{cR} for key can be checked

(5) Design for bolts :-

(a) Nominal diameter of the bolts :-

Load on each bolt $= \frac{\pi}{4}(d_1)^2 \tau_b$

Total load on all the bolts $= \frac{\pi}{4}(d_1)^2 \tau_b\, n$

∴ Torque transmitted ⇒ $\boxed{T = \frac{\pi}{4} d_1^2 \tau_b\, n * \frac{D_1}{2}}$ → from this d_1 can be calculated.

(b) Checking crushing stress for bolts :-

Area resisting crushing of all the bolts $= n * d_1 * t_f$

~~Crushing strength of all the bolts~~ $= \cancel{(n*d_1*t_f)}$

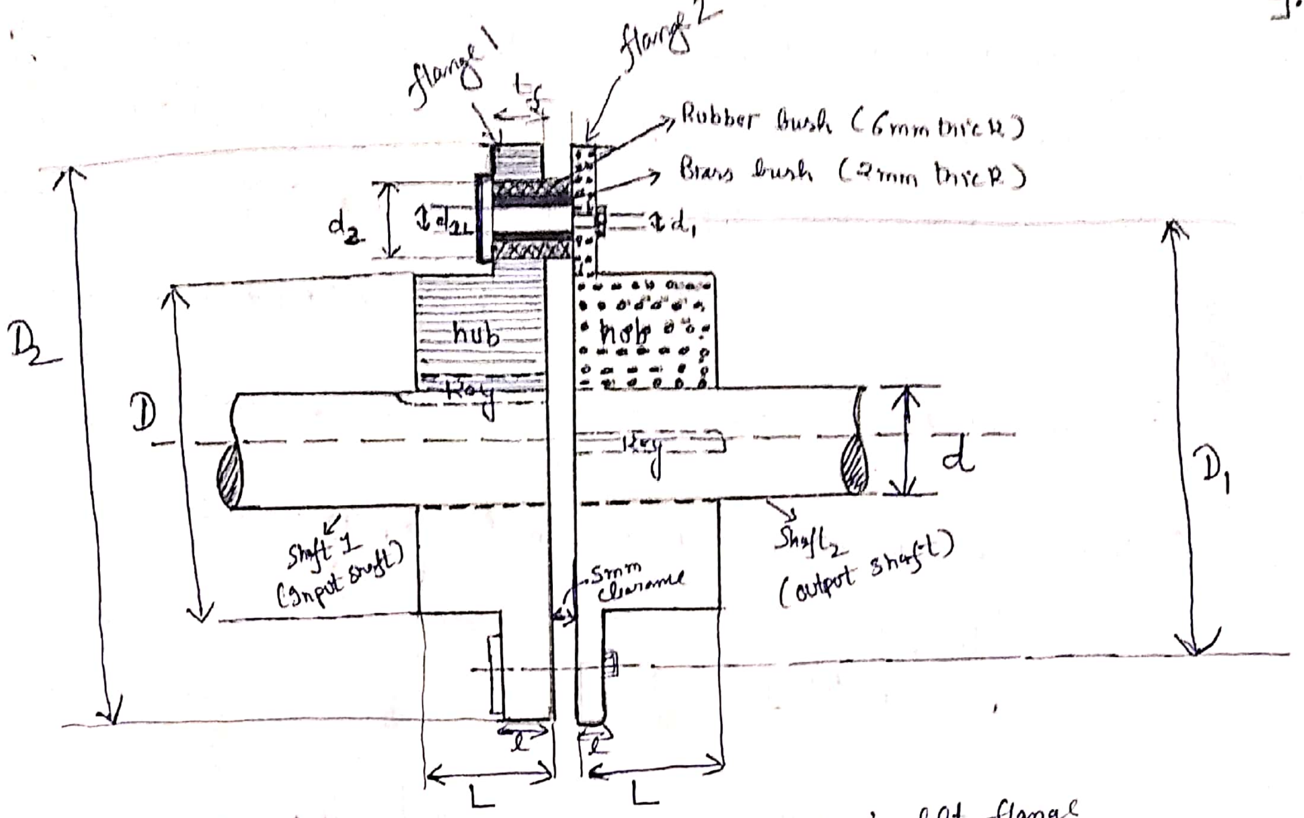
$\cancel{∴ \text{Torque} \Rightarrow T = n d_1 t_f \frac{D_1}{2}}$

∴ crushing strength of all the bolts $= (n * d_1 * t_f)\, \sigma_{cb}$

$\boxed{∴ \text{Torque} \Rightarrow T = (n*d_1*t_f)\,\sigma_{cb} * \frac{D_1}{2}}$ → from this σ_{cb} for bolts can be checked.

Bushed-pin Flexible Coupling :-

A bushed-pin flexible coupling is a modification of the rigid type of flange coupling. The coupling bolts are known as pins. The rubber or leather bushes are used over the pins. The two halves of the coupling are dissimilar in construction. That is holes on left flange are larger in diameter than the holes on right hand flange. Due to this thickness of left hand flange is more than the thickness of right hand flange. A clearance of 5 mm is left between the face of the two halves of the coupling. There is no rigid connection between them and the drive takes place through the medium of the compressible rubber or leather bushes. In designing the bushed-pin flexible coupling, the proportions of the rigid type flange coupling are modified. The main modification is to reduce the bearing pressure on the rubber or leather bushes and it should not exceed 0.5 N/mm². In order to keep the low bearing pressure, the pitch circle diameter and pin size is increased.



d_1 = diameter of pin in right flange ; d_{1L} = diameter of pin in left flange
d_2 = diameter of rubber bush or diameter of hole to be made in left flange
D = outer diameter of hub ; D_2 = Outside diameter of flange ; D_1 = Diameter of pitch circle of the pins
l = length of bush in the flange ; P_b = bearing pressure on the bush or pin ;
n = number of pins ; L = length of hub or length of Key ; T = Torque transmitted by the shaft.

① <u>Design for Shaft</u> :- $\boxed{P = \frac{2\pi NT}{60}}$; $\boxed{T = \frac{\pi}{16} \tau d^3}$ → d can be calculated.

② <u>Design for pins and rubber bush</u> :-

ⓐ Diameter of pins, $d_1 = \frac{0.5 d}{\sqrt{n}}$ {n = no of pins}.

ⓑ $d_{1L} = (d_1 + 4)$ mm

ⓒ $d_2 = d_{1L} + 2 *$ thickness of brass bush + 2 x thickness of Rubber bush.

$d_2 = d_{1L} + 2\times2 + 2\times6$

$\boxed{d_2 = (d_{1L} + 16)\text{ mm}}$

ⓓ <u>Shear stress in the Pins</u> :- In Bushed Pin coupling analysis, we assume that power is transmitted by the shear resistance of the pins

Bearing load acting on each pin $= W = P_b * d_2 * l$ ——(i)

$\therefore$ Total bearing load on the bush or pins $= W * n = P_b * d_2 * l * n$

Torque transmitted by the coupling $= T = W * n * \left(\frac{D_1}{2}\right)$

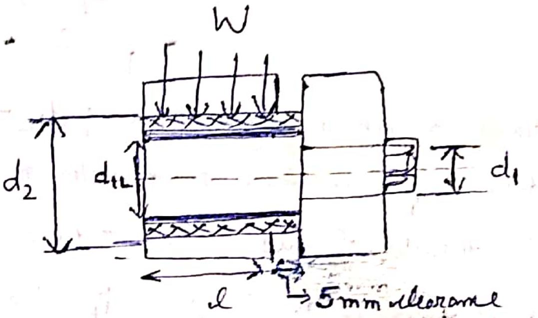
$$T = P_b * d_2 * l * n * \frac{D_1}{2} \quad \text{——(ii)}$$

from eqn (i) and (ii) $\Rightarrow$ l and W can be calculated.

$\therefore$ Direct stress due to pure torsion in the coupling halves:-

$$\boxed{\tau = \frac{W}{\frac{\pi}{4}(d_1)^2}}$$

e. Bending stress of the pin :-



$$M = W\left(\frac{l}{2} + 5\right)$$

$$\sigma_b = \frac{M}{Z} \quad \left\{Z = \frac{\pi}{32} d_1^3\right\}$$

Maximum principal stress $= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$

Maximum shear stress $= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\tau^2}\right]$

3. Design for hub :-

$\boxed{D = 2d}$; $\boxed{L = 1.5\,d}$; $\boxed{T = \frac{\pi}{16} \tau_c \left(\frac{D^4 - d^4}{D}\right)}$ $\rightarrow$ τ_c can be calculated for hub

(4) Design for Key :- Same procedure as in earlier cases.

(5) Design for flange :- $\boxed{t_f = 0.5d}$; $\boxed{D_2 = 4d}$

$\boxed{T = \frac{\pi D^2}{2} * \tau_c * t_f}$ $\rightarrow$ τ_c for flange can be calculated.