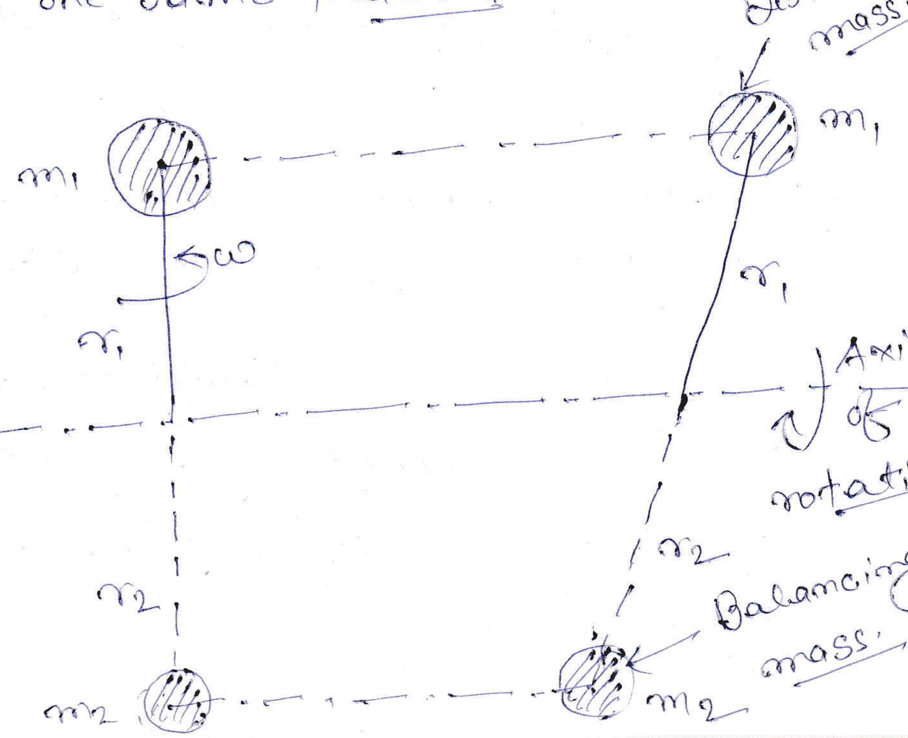
⇒ UNIT - V

BALANCING OF ROTATING MASSES:-

⇒ When A Certain mass is attached to a rotating shaft, it exerts some Centrifugal force, whose effect is to bend the shaft and to produce vibration in it. In order to prevent the effect of Centrifugal force force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the Centrifugal force of the first mass. This is done in such a way that the Centrifugal force of both the masses are made to be equal and opposite. The process of providing the second mass in order to Counteract the effect of the centrifugal force of the first mass, is called balancing of rotating masses.

1. Balancing of a Single Rotating Mass by a Single mass Rotating in the Same plane:-



The Centrifugal force exerted by the mass m_1 on the shaft:

$$\boxed{F_{c_1} = m_1 \cdot \omega^2 \cdot r_1} \longrightarrow (i)$$

where, r_1 = Radius of rotation of mass m_1

r_2 = Radius of rotation of mass m_2.

$\therefore$ Centrifugal force due to mass m_2

$$\boxed{F_{c_2} = m_2 \cdot \omega^2 \cdot r_2} \longrightarrow (ii)$$

Equating eqn (i) & (ii), we get.

$$F_{c_1} = F_{c_2}$$

$$\Rightarrow m_1 \cancel{\omega^2} r_1 = m_2 r_2 \cancel{\omega^2}$$

$$\therefore \boxed{m_1 r_1 = m_2 r_2}$$

2. Balancing of a Single Rotating mass by Two masses Rotating in Different planes:-

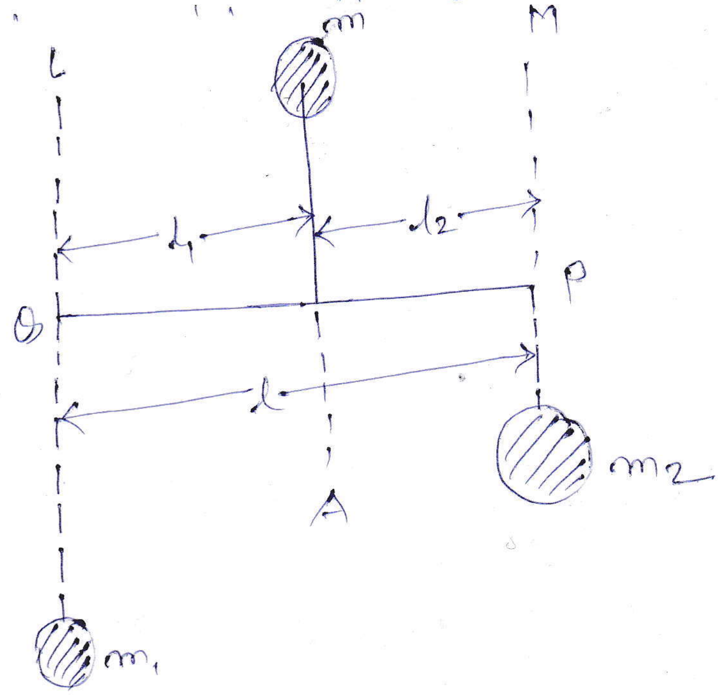
(a). When the plane of the disturbing mass lies in between the planes of the two balancing masses:-

Consider a disturbing mass m lying in a plane A to be balanced by two rotating masses m_1 & m_2 lying in two different planes L and M. Let r, r_1 & r_2 be the radii of rotation of the masses in planes A, L and M respectively.

Let, l_1 = Distance between the planes A & L

l_2 = " " " " A & M

l = " " " " L & M



Centrifugal force exerted by the mass m in the plane A,

$$\boxed{F_c = m \cdot \omega^2 \cdot r}$$

Similarly

$$\boxed{F_{c_1} = m_1 \omega^2 r_1}$$

and

$$\boxed{F_{c_2} = m_2 \omega^2 r_2}$$

Since, The net force acting on the shaft must be equal to zero. Therefore, the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses,

$$\therefore \quad F_c = F_{c_1} + F_{c_2}$$

$$\Rightarrow m\omega^2 r = m_1 \omega^2 r_1 + m_2 \omega^2 r_2$$

$$\therefore \quad \boxed{m r = m_1 r_1 + m_2 r_2} \quad \text{———} (i).$$

Take moment about point P,

$$F_{C_1} \times l = F_C \times l_2$$

$$\Rightarrow m_1 \cancel{\omega^2} r_1 \times l = m \cancel{\omega^2} r . l_2$$

$$\Rightarrow m_1 r_1 l = m r l_2$$

$$\therefore \boxed{m_1 r_1 = m.r.\frac{l_2}{l}} \longrightarrow (ii)$$

Taking moment about point Q, we get,

$$F_{C_2} \times l = F_C \times l_1$$

$$\Rightarrow m_2 \cancel{\omega^2} r_2 \times l = m \cancel{\omega^2} r \times l_1$$

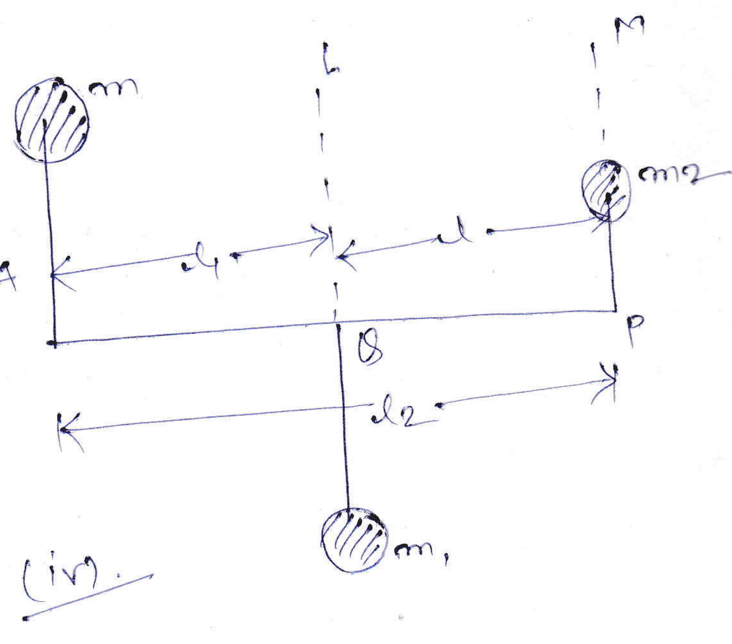
$$\Rightarrow \boxed{m_2 r_2 = m.r.\frac{l_1}{l}} \longrightarrow (iii)$$

(b): When the plane of the disturbing mass lies on one end of the planes of the balancing masses.

$$\Rightarrow F_C + F_{C_2} = F_{C_1}$$

$$\Rightarrow m\omega^2 r + m_2 \omega^2 r_2 = m_1 \omega^2 r_1$$

$$\Rightarrow \boxed{mr + m_2 r_2 = m_1 r_1} \longrightarrow (iv)$$



Taking moment about point P, we get,

$$F_{C_1} \times l = F_C \times l_2$$

$$\Rightarrow m_1 \cancel{\omega^2} r_1 \times l = m \cancel{\omega^2} r \times l_2$$

$$\Rightarrow \boxed{m_1 r_1 = m \cdot r \cdot \frac{l_2}{l}} \quad \text{—— (v)}$$

Now Taking moment about point B, we get,

$$F_{C_2} \times l = F_C \times l_1$$

$$\Rightarrow m_2 \cancel{\omega^2} r_2 \times l = m \cancel{\omega^2} r \times l_1$$

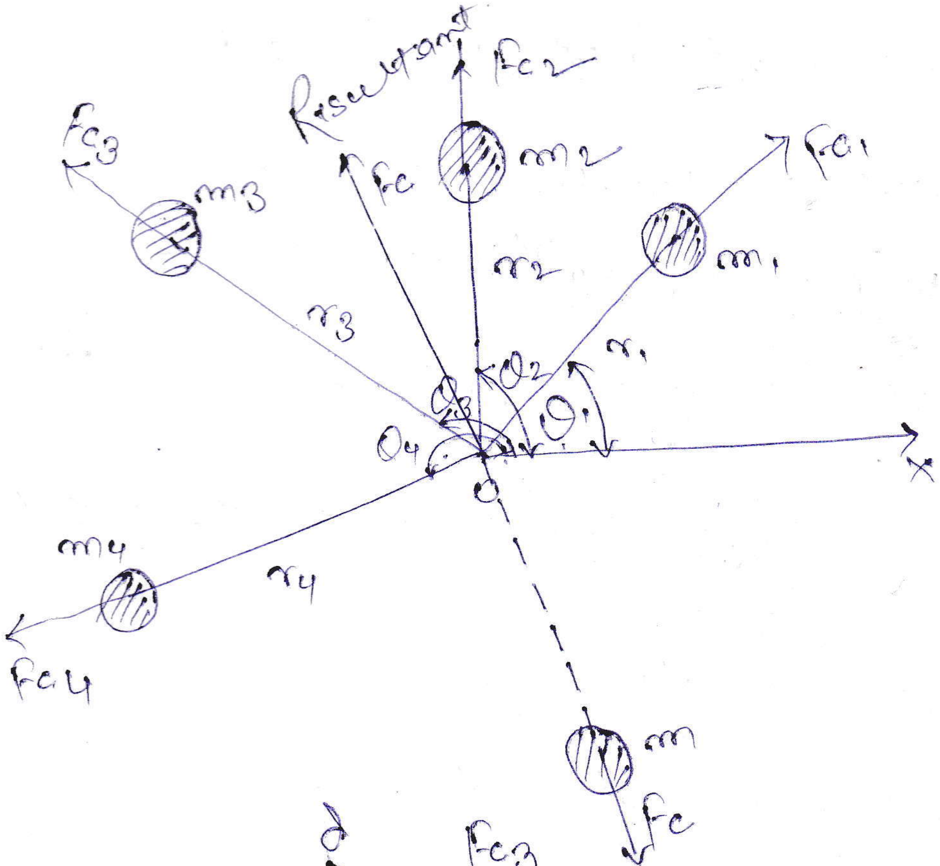
$$\therefore \boxed{m_2 r_2 = m r \frac{l_1}{l}} \quad \text{—— (vi)}$$

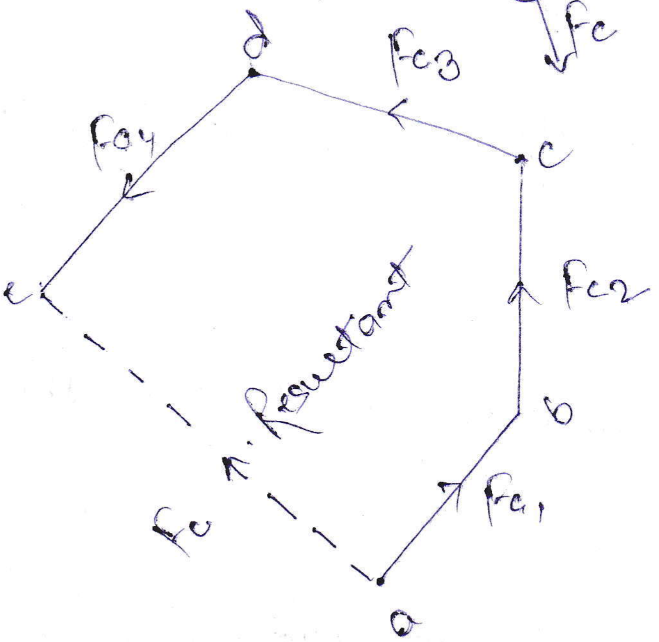
⇒ Balancing of several masses Rotating in the same plane:-

Let, Consider four masses of magnitude m_1, m_2, m_3 & m_4 at distance of r_1, r_2, r_3 & r_4 from the axis of rotating shaft.

Let θ_1, θ_2, θ_3 and θ_4 be the angles of these masses w.r.t. the horizontal line OX, which is given below.

Let these masses rotate about an axis through O and perpendicular to the plane of paper, with constant angular velocity ω rad/s.





These are two method to find Resultant of Several masses which rotate in the Same Plane.

(a) Analytical method.

(b) Graphical method.

(a) ~~Gr~~ Analytical method:-

- First of all to find Centrifugal force (which is the Product of mass and its radius of rotation ~~eet~~ exerted by each mass on the rotating Shaft.

→ Resolve Centrifugal force horizontaly & vertically.

$$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4 + \cdots$$

$$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4 + \cdots$$

→ find Resultant Centrifugal force magnitude of ΣH & ΣV

Resultant $F_c = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

→ Also find direction of Resultant Centrifugal force

$$\tan\theta = \left(\frac{\Sigma V}{\Sigma H}\right)$$

$$\therefore \theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

– The balancing force is equal to the resultant force but in opposite direction.

→ Then find out the magnitude of the balancing force

$$F_c = m \cdot r$$

where, m = Balancing mass

r = Radius of rotation of the balancing mass.

Q.No-1. Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles betn successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.

Soln :→

$m_1 = 200$ kg	$r_1 = 0.2$ m	$\theta_1 = 0°$
$m_2 = 300$ kg	$r_2 = 0.15$ m	$\theta_2 = 45°$
$m_3 = 240$ kg	$r_3 = 0.25$ m	$\theta_3 = 45° + 75° = 120°$
$m_4 = 260$ kg	$r_4 = 0.3$ m	$\theta_4 = 45° + 75° + 135° = 255°$

and $r = 0.2$ m.

Let m = Balancing mass

θ = The angles which the balancing mass makes with m_1.

Centrifugal force magnitude on each mass is given below.

$F_{c_1} = m_1 r_1 = 200 \times 0.2 = 40$ kg-m

$F_{c_2} = m_2 r_2 = 300 \times 0.15 = 45$ kg-m

$F_{c_3} = m_3 r_3 = 240 \times 0.25 = 60$ kg-m

$F_{c_4} = m_4 r_4 = 260 \times 0.3 = 78$ kg-m

By Analytical method:-

$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 +$

$m_4 r_4 \cos\theta_4$

$= 40 \cos 0^\circ + 45 \cos 45^\circ + 60 \cos 120^\circ + 78 \cos 2$

$= 40 + 31.8 - 30 - 20.2 = \underline{21.6 \text{ kg-m}}$

$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 +$

$m_4 r_4 \sin\theta_4$

$= 40 \sin 0^\circ + 45 \sin 45^\circ + 60 \sin 120^\circ + 78 \sin 255$

$= 0 + 31.8 + 52 - 75.3 = \underline{8.5 \text{ kg-m}}$

Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2}$

$= 23.2 \text{ kg.m}$

we already know that,

$\Rightarrow$ P_c = Balancing force

$\Rightarrow 23.2 = m \cdot r$

$\therefore m = \frac{23.2}{r} = \frac{23.2}{0.2} = \underline{116 \text{ kg}}$ Ans.

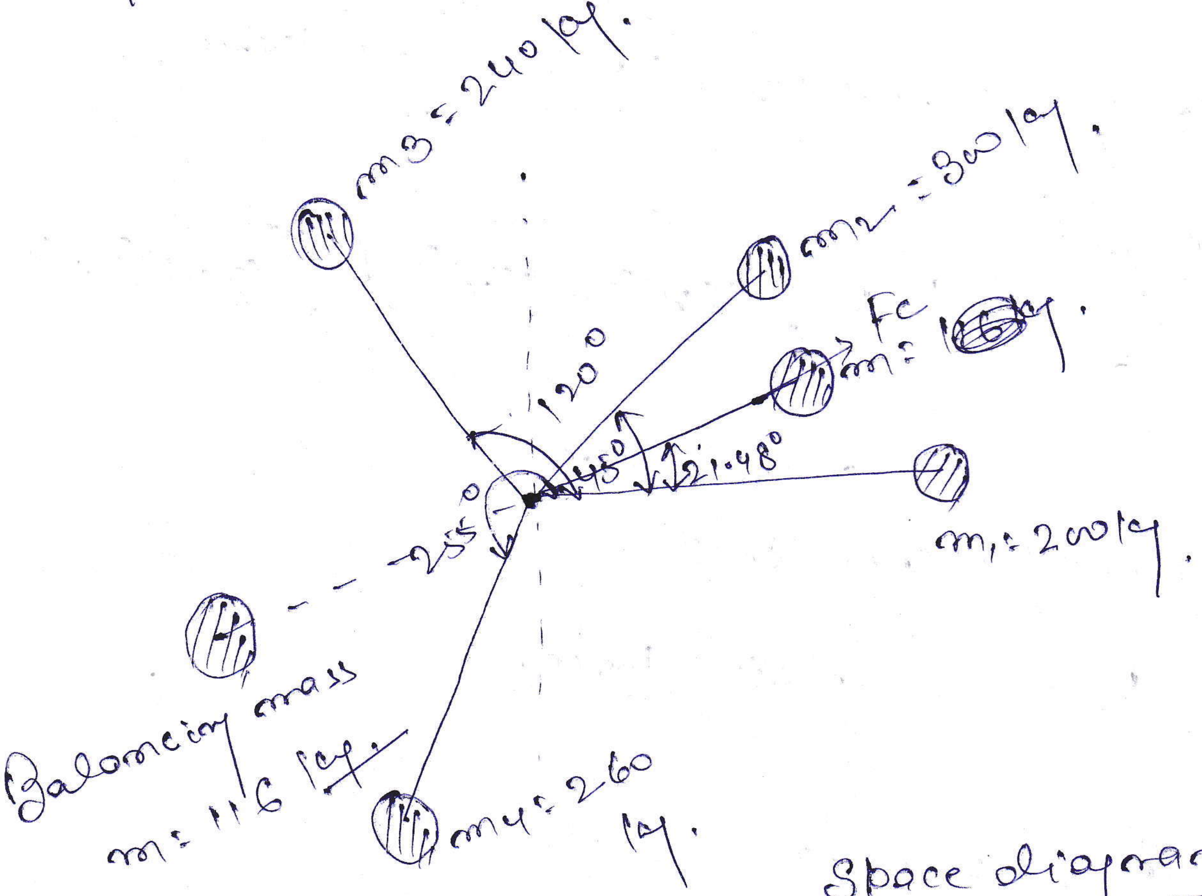
$\tan\theta = \left(\frac{\Sigma V}{\Sigma H}\right) = \left(\frac{8.5}{21.6}\right) \quad \therefore \theta = \tan^{-1}\left(\frac{8.5}{21.6}\right)$

$\theta = 21.48^\circ$ = This is the angle of Resultant from horizontal mass m_1.

But we have to find angle of balancing mass is given below from mass m_1 is.

$$\theta' = 180^\circ + \theta$$
$$= 180^\circ + 21.48^\circ = 201.48^\circ$$ Ans.

(ii) A Graphical method :-



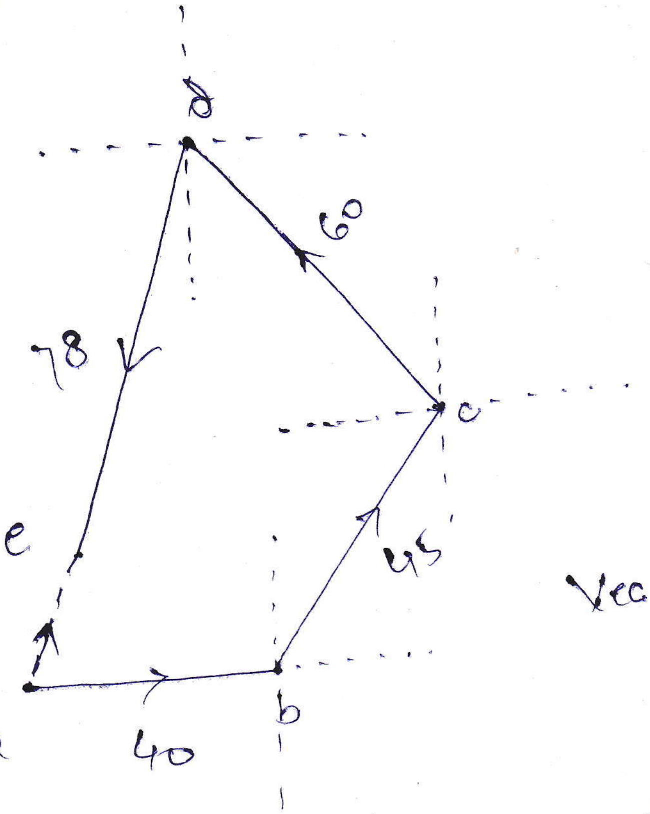
Space diagram.

$m_1 r_1 = 40$ kg.m

$m_2 r_2 = 45$ kg.m

$m_3 r_3 = 60$ kg.m

$m_4 r_4 = 78$ kg.m



Vector diagram

Vector ae is a Resultant force and vector ea i[s]
balancing force.

Now,

vector ea $=$ m.r

$\Rightarrow 23 = m \times 0.2$

$\therefore m = \frac{23}{0.2} = 115$ kg. Ans

By measuring $\theta = 21^{\circ}$.

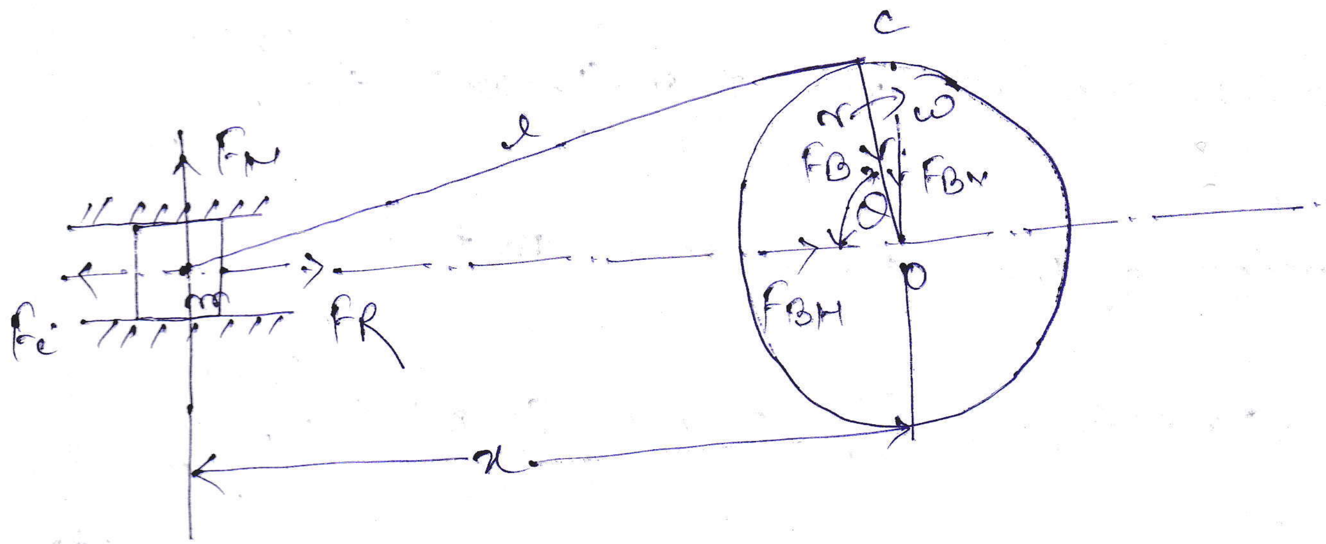
Resultant force make an angle with horizontal (a[e])
is calculated by measuring, so the angle of ba[l]ancing mass with horizontal is calculated
with addition of 180° to resultant angle.

$\boxed{\theta' = 180^{\circ} + \theta}$ $\theta' = 180^{\circ} + 21^{\circ} = 201^{\circ}$ An[s]

⇒ Balancing of Reciprocating masses :—

The various forces acting on the reciprocating parts of an engine. The resultant of all forces acting on the body of the engine due to inertia forces only is known as unbalanced force or shaking force.

If the resultant of all forces due to inertia effect is zero, then there will be no unbalance force, but unbalanced couple or shaking couple will be present.



Let F_R = Force required to accelerate the Piston (reciprocating parts)

F_i = Inertia force due to reciprocating parts

F_N = Force exerted on the side of the Cylinder wall or Normal force acting on the Cross-head of the guides.

F_B = force acting on the crankshaft bearing or main bearing.

F_R & F_Q same in magnitude but opposite in direction. & They balanced each other.

F_{BH} & F_Q is also same but opposite to each other.

This F_{BH} is unbalanced or shaking force which is denoted by F_U and required to be properly balanced.

The displacement of reciprocating parts is given below.

$$\text{Displacement } (x) = r\left[(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right]$$

velocity of the reciprocating parts is

$$\text{velocity} = \frac{dx}{d\theta} = \omega r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$$

Similarly acceleration of the reciprocating parts

$$\text{Acceleration} = \frac{d}{d\theta}\left(\frac{dx}{d\theta}\right) = \omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right].$$

Shaking Couple due to vertical Component of $F_{\ldots}$

is $\underline{F_{Bv} \times x}$

and Couple due to F_{Nv} is $\underline{F_{Nv} \times x}$ both

are equal & in opposite direction.

⇒ Primary and Secondary unbalanced forces of reciprocating masses.

Let,
m = mass of reciprocating parts
l = Length of connecting rod
r = Radius of the crank.
θ = Angle of inclination with the line of stroke.

n = l/r
ω = Angular Speed of the crank.

Acceleration of the reciprocation parts is

$$a_R = \omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

Force required to accelerate the reciprocating parts = Inertia force due to reciprocating parts =

$$F_I = F_R = m \times a_R = m\omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$F_R = F_Q = F_{BH} = F_U = m\omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$$\boxed{F_U = m\omega^2 r\cos\theta + m\omega^2 r\frac{\cos 2\theta}{n}}$$

$$\boxed{F_U = F_P + F_S}$$

Primary unbalanced force $(F_P) = m\omega^2 r\cos\theta$

Secondary " " $(F_S) = m\omega^2 r\frac{\cos 2\theta}{n}$

$F_{P(maximum)} = m\omega^2 r$ at $\theta = 0^\circ$ & 180°

Primary force is max^m twice in one revolution of the crank.

F_S (maximum) $= m\omega^2 \frac{r}{n}$ at $\theta = 0^\circ, 90^\circ, 180^\circ, 360^\circ$

Secondary force is max^m four times in one revolution of the crank.

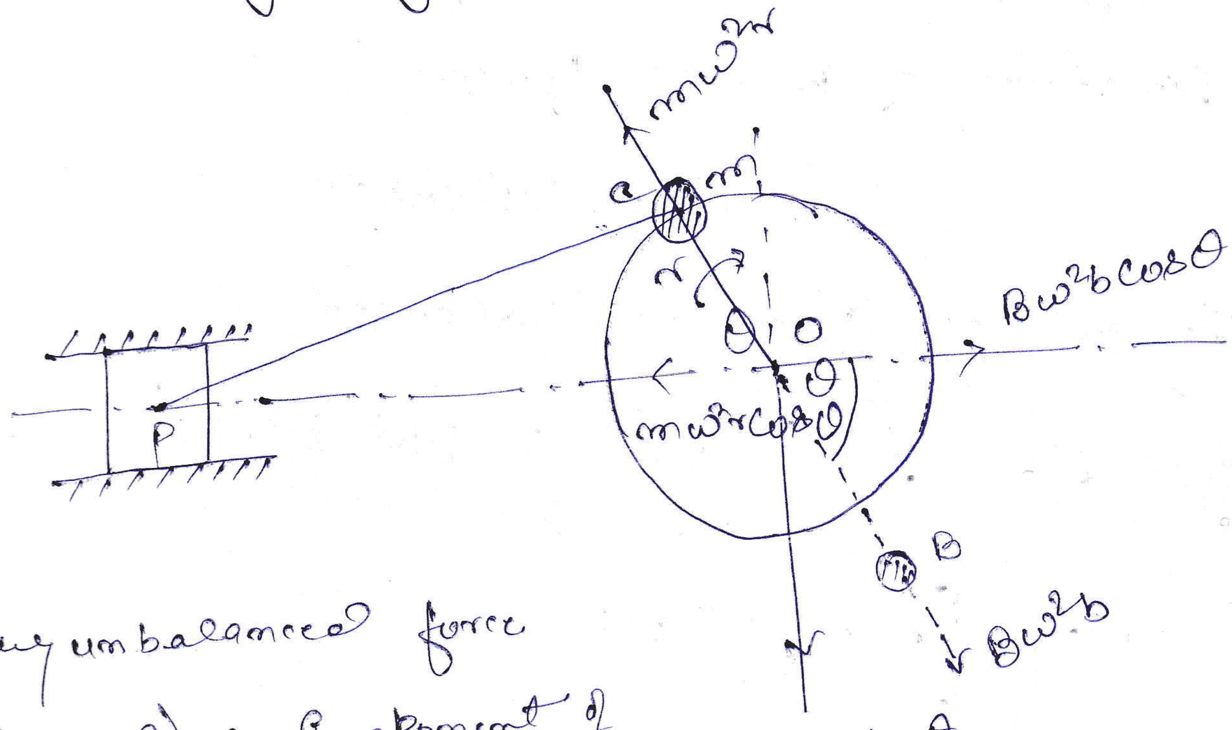
→ Max^m Secondary force is $1/n$ times the max^m Primary force.

→ In case of moderate speed secondary unbalanced force is so small that it may be neglected as

Compared to Primary unbalanced force.

→ The unbalanced force due to reciprocating masses varies in magnitude but constant in direction. But due to the Revolving masses, the unbalanced force is constant in magnitude but varies in direction.

⇒ Partial Balancing of unbalanced Primary force in Reciprocating Engine.



Primary unbalanced force ($m\omega^2 r\cos\theta$) : Component of the Centrifugal force exerted by a rotating mass m placed at the Crank radius r. which is given in fig.

This Primary force acts from O to P along the line stroke.

Balancing of Primary force is considered as equivalent to the balancing of mass m rotating at the crank radius r. This mass is balanced by a mass B at a radius b, placed diametrically opposite to the crank pin C.

F_c due to mass B $= B\omega^2 b$

Horizontal component of this force acting in opposite direction of Primary force.

$$= B\omega^2 b \cos\theta$$

The Primary force is balanced if

$$\Rightarrow m\cancel{\omega^2} r \cancel{\cos\theta} = B\cancel{\omega^2} b \cancel{\cos\theta}$$

$$\Rightarrow \boxed{mr = Bb}$$

The vertical component of this force of mass (B) is $\perp$r to the line of stroke

$$= B\omega^2 b \sin\theta$$

This force remains unbalanced. The maxm value of this force is $B\omega^2 b$ at 90° & 270° which is same for the maxm value of Primary force $m\omega^2 r$.

The Primary unbalanced force acts along the line of stroke. But in the second case the unbalanced force acts along the perpendicular to the line of stroke. The max^m value of the forces remains same in both the cases.

* The change of direction of the max^m unbalanced force from the line of stroke to the perpendicular to the line of stroke.

Let a fraction 'c' of the reciprocating masses is balanced, such that

$$\boxed{c\,m \cdot r = B\,b}$$

$\therefore$ unbalanced force along the line of stroke

$$= m\omega^2 r\cos\theta - B\omega^2 b\cos\theta$$

$$= m\omega^2 r\cos\theta - c\,m\omega^2 r\cos\theta$$

$$= (1-c)\,m\omega^2 r\cos\theta \quad \text{—— (i)}$$

unbalanced force along the perpendicular to the line of stroke

$$= B\omega^2 b\sin\theta = c\,m\omega^2 r\sin\theta \quad \text{—— (ii)}$$

$\therefore$ Resultant unbalanced force at any instant

$$= \sqrt{\left[(1-c)\,m\omega^2 r\cos\theta\right]^2 + \left[c\,m\omega^2 r\sin\theta\right]^2}$$

$$= m\omega^2 r \sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

* If the balancing mass is required to balance revolving masses as well as reciprocating masses then

$$B b = m_1 r + c m \cdot r = r(m_1 + c m)$$

where, m_1 = Magnitude of the revolving mass

m = " " " Reciprocating mass

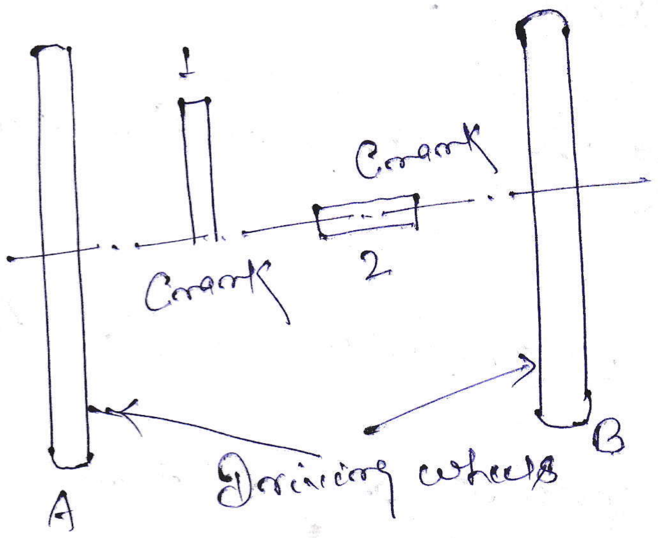
⇒ Partial balancing of Locomotives: →

(i) Inside cylinder locomotives.

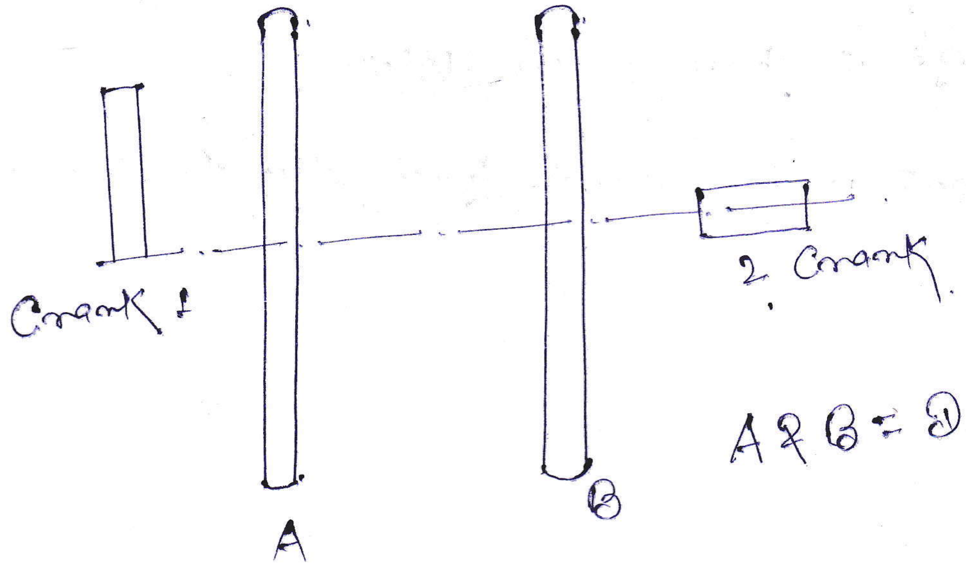
(ii) outside cylinder Locomotives.

(i) Two cylinders are placed in betn the plane of two driving wheels.

(ii)



(ii) The two cylinders are placed outside the driving wheels, one on each side of the driving wh



locomotives.
→ (i) Single or uncoupled locomoti
(ii) Coupled locomotives.

A single or uncoupled locomotives is one in which the effort is transmitted to one pair of the wheels only.
But in coupled locomotives the driving wheels are connected to the leading and trailing wheel b an outside coupling rod.

⇒ The effect of an unbalanced primary force along the line of stroke produce

(i) Variation in tractive force along the line of stroke
(ii) Swaying couple.

- The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as a **Hammer Blow**.

- The resultant unbalanced force due to the two cylinders