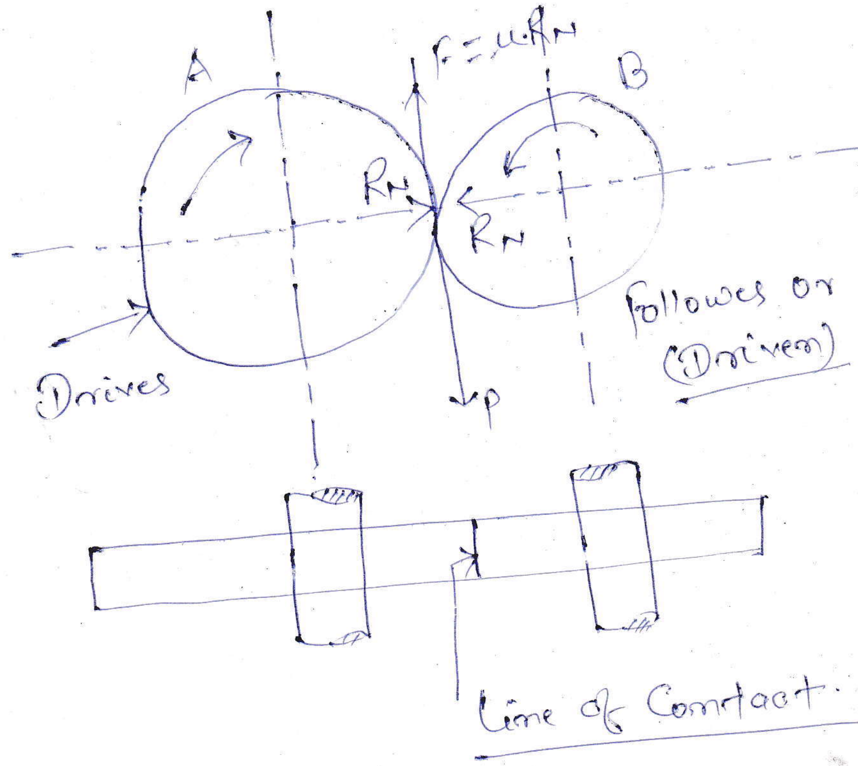
SEM.-V th ⇒ Toothed GEAR UNIT-03

(a) Friction wheel:-



⇒ Classification of Toothed gears:-

(i) According to the position of axes of the shaft

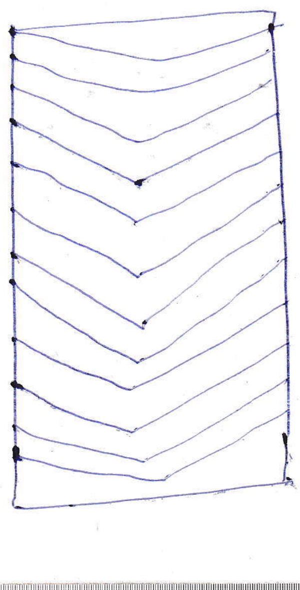
(a) Parallel

(b) Intersecting

(c) Non-intersecting and Non-Parallel.

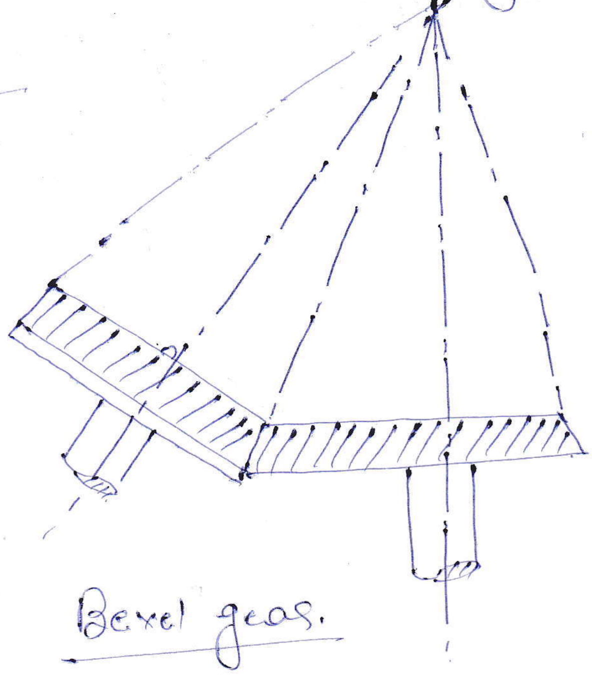
→ Single helical gears:-

→ Double ~~hical~~ helical gears:-



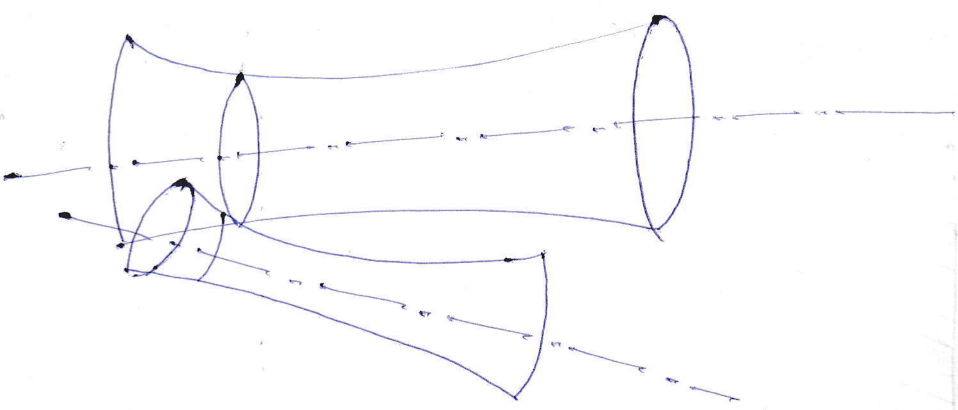
It is also known as herringbone gears.

⇒ The two non-Parallel or intersecting, but Coplanar shaft connected by gears is shown below. These gears are called bevel gears. And the arrangement is known is bevel gearing.



Bevel gear.

⇒ The two-non-intersecting and non-Parallel i.e non Co-planar shaft connected by gears is shown below. These gears are known as skew bevel gear or spiral gear. and the arrangement is known as skew bevel gearing or spiral gearing.



⇒. Spiral gear

⇒ According to peripheral velocity of the gears:-

(a) Low velocity → velocity < 3 m/sec.

(b) Medium velocity → $3 \leq V \leq 15$

(c) High velocity → $V \geq 15$ m/sec.

⇒ According to the type of gearing:-

(a) External gearing.

(b) Internal gearing → Large gear (Annular gear) & Small gear is known as Pinion.

(c) Rack and Pinion gearing.

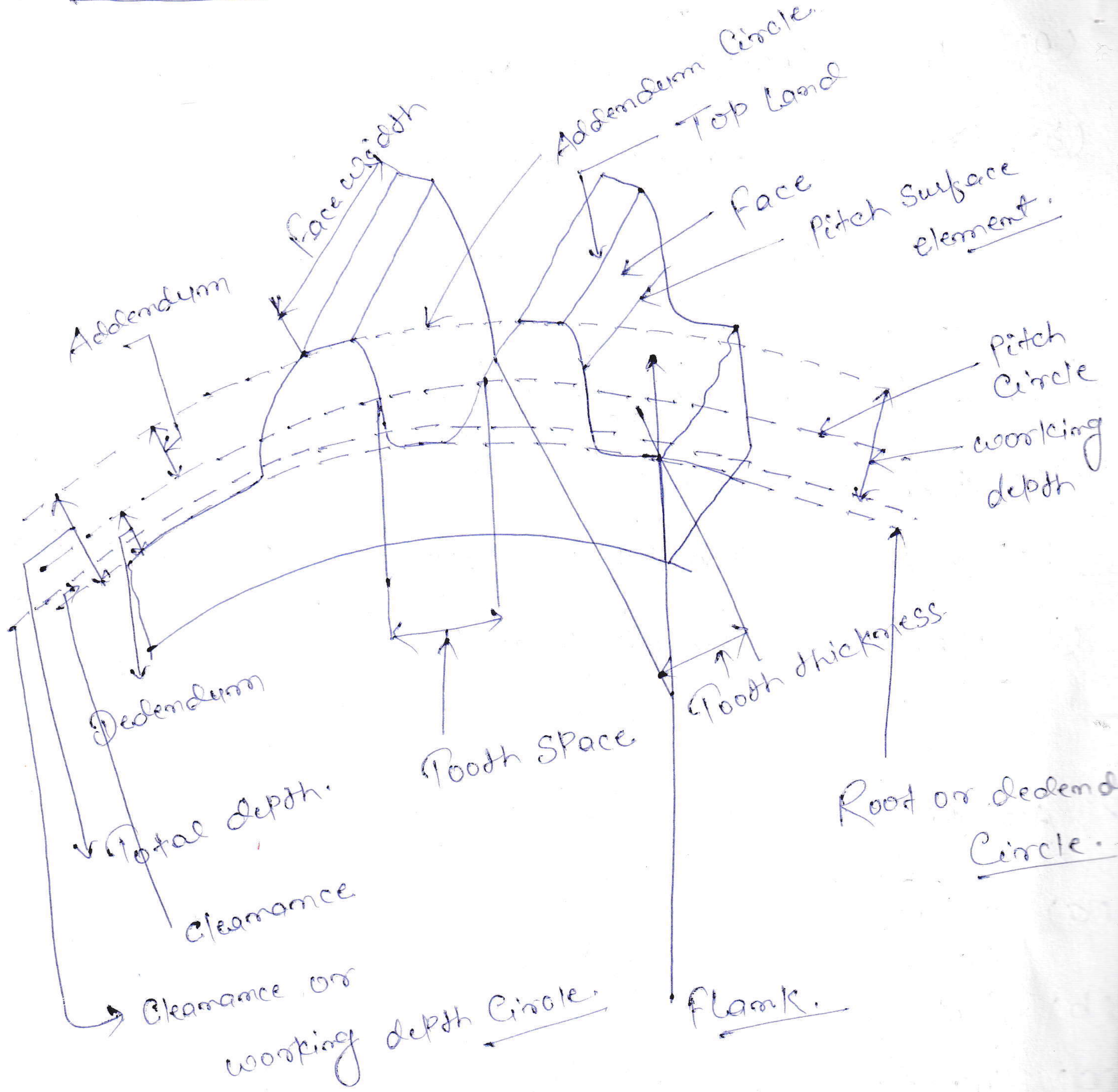
⇒ According to position of teeth on the gear surface:-

(a) Straight.

(b) Inclined

(c) Curved.

⇒ Terms used in Gears:-



!- Terms used in Gear

→ Pitch Circle :- It is an imaginary Circle which by pure rolling action.

→ Pitch Circle diameter :- It is the diameter of the Pitch Circle. The Size of the gear is usually specified by the pitch circle diameter. It is also known as pitch diameter.

NOTE:- Root circle diameter = Pitch circle diameter × $\cos\phi$.

where ϕ = Pressure angle.

→ Circular Pitch :- It is the distance measured on the circumference of pitch circle from a point of one tooth to the corresponding point on the next tooth.

Circular pitch $\boxed{P_c = \frac{\pi D}{T}}$

where, D = Diameter of the pitch circle and

T = Number of teeth on the wheel.

NOTE :- If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively, then for them to mesh correctly.

$$P_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2}$$

$$\Rightarrow \boxed{\frac{D_1}{D_2} = \frac{T_1}{T_2}}$$

→ **Diametral Pitch** :- It is the ratio of the Number of teeth to the Pitch Circle diameter in mm.

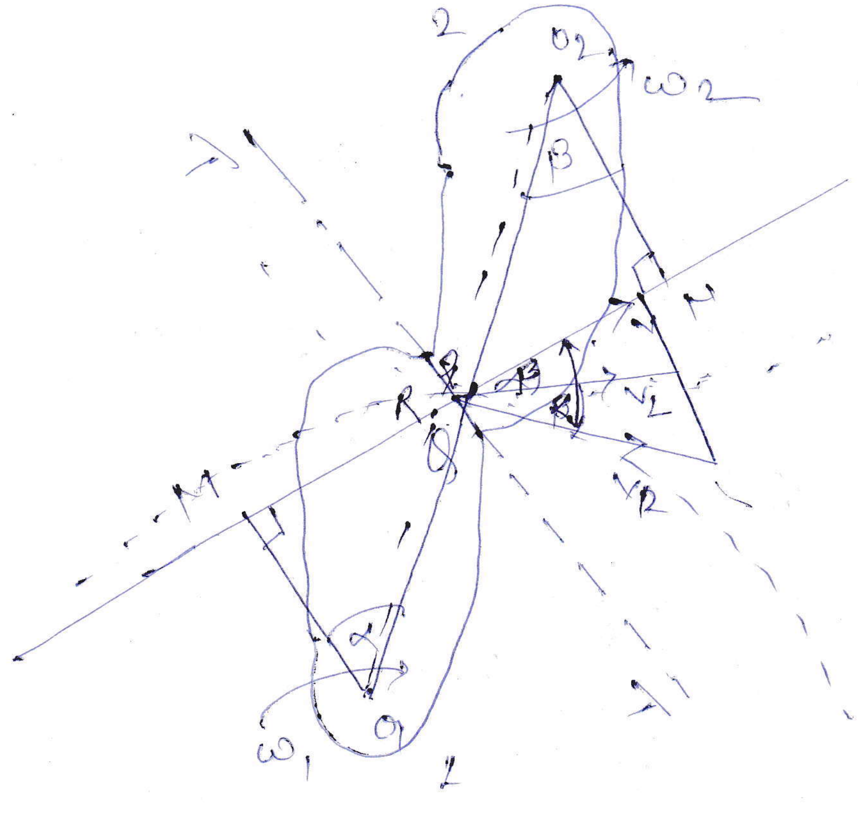
Now,

$$\boxed{P_d = \frac{T}{D} = \frac{\pi}{P_c}}$$

Module :- It is the ratio of Pitch Circle diameter to the number of teeth

$$\boxed{m = \frac{D}{T}}$$

⇒ **Law of Gearing - Condition for Constant velocity ratio of Toothed wheels** :-



Now from given diagram.

$$v_1 \cos\alpha = v_2 \cos\beta$$

$$\Rightarrow (\omega_1 \times O_1Q) \cos\alpha = (\omega_2 \times O_2Q) \cos\beta$$

$$\Rightarrow (\omega_1 \times O_1Q) \frac{O_1M}{O_1Q} = (\omega_2 \times O_2P) \times \frac{O_2N}{O_2P}$$

or, $\omega_1 \times O_1M = \omega_2 \times O_2N$

$$\Rightarrow \boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}} \text{ --- (i)}$$

from Similar triangle O_1MQ & O_2NP

$$\boxed{\frac{O_2N}{O_1M} = \frac{O_2P}{O_1Q}} \text{ --- (ii)}$$

from eqn (i) & (ii), we get

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1Q}}$$

$\Rightarrow$ Velocity of sliding:— Component of v_1 & v_2 along T- $v_1 \sin\alpha$ & $v_2 \sin\beta$ respectively.

Velocity of sliding $= v_1 \sin\alpha - v_2 \sin\beta$

$$= \omega_1 O_1Q \frac{QM}{O_1Q} - \omega_2 . O_2P \frac{PN}{O_2P}$$

$= \omega_1 QM - \omega_2 PN$

$= \omega_1 (MR + RQ) - \omega_2 (RN - RP)$

$= \omega_1 MR + \omega_1 RQ - \omega_2 RN + \omega_2 RP$

Here P & Q points are same.

$= (\omega_1 + \omega_2) RQ + \omega_1 MR - \omega_2 RN$

$= (\omega_1 + \omega_2) RQ \qquad [\omega_2 RN = \omega_1 RM]$.

= Sum of Angular velocities × distance betn the Pitch point and the point of Contact.

Also ΔO_1MR & ΔO_2RN are similar.

So, $\dfrac{O_1R}{O_2R} = \dfrac{O_1M}{O_2N} = \dfrac{RN}{RM}$

or $\boxed{\dfrac{O_2R}{O_1R} = \dfrac{O_2N}{O_1M}} = \dfrac{RN}{RM}$

& $\boxed{\dfrac{O_2N}{O_1M} = \dfrac{\omega_1}{\omega_2}} = \dfrac{RN}{RM}$ Proved

Q.No-1. The number of teeth of a Spur gear is 30 and it rotates at 200 rpm. What will be its circ Pitch and the Pitch line velocity if it has a module of 2 mm?

Soln Given data

$T = 30$

$N = 200$ rpm

$m = 2$ mm

$P = \pi m = \pi \times 2 = 6.28$ mm

$V_p = \omega r = 2\pi N \times \frac{d}{2} = \cancel{2}\pi N \times \frac{mT}{\cancel{2}}$

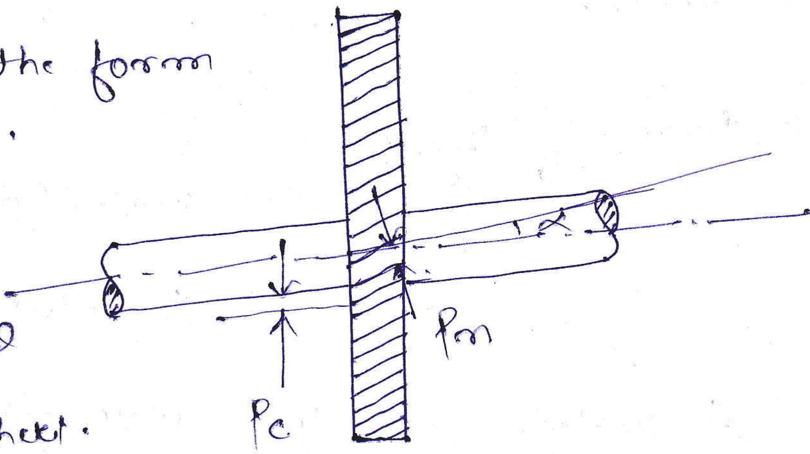
$= \pi \times 200 \times 2 \times 30$

$= 37699$ mm/min

$= 628.3$ mm/s Ans.

⇒ Helical Gear :-

Helical gear has teeth in the form of helix around the gear.



The helix may be right handed on one wheel and left handed on another wheel.

→ Single helical gear
→ Double helical gear.

In case of single helical gears, there are some axial thrust betⁿ teeth, which is a disadvantages. But eliminate this axial thrust, double helical gears are used.

one Double helical gear is equivalent to two helical gear in which equal and opposite thrusts are produced on each gear and the resulting axial thrust is zero.

(i) Normal Pitch (P_N) :→ It is the distance betⁿ similar faces of adjacent teeth, along a helix on the pitch cylinder normal to the teeth.

(ii) Axial Pitch :→ (P_c) The distance measured parallel to the axis, betⁿ similar faces of adjacent teeth.

This is same as circular pitch. If the α is helix angle then circular pitch

$$\boxed{P_c = \frac{P_N}{\cos\alpha}}$$

⇒ Helix angle is also known as Spiral angle of the teeth.

⇒ Form of Teeth:—

(i) cycloidal teeth.

(ii) Involute teeth.

A cycloid is the curve traced by a point on the circumference of a circle which rolls without slippi[ng] on a fixed St. line.

when a circle rolls without slipping on the out side of a fixed circle, the curve traced by a point on the circumference of a circle is known as epi-cycloid.

when a circle rolls without slipping on the in-side of a fixed circle, then the curve traced by [a] point on the circumference of a circle is calle[d] Hypo-cycloid.

Epi-cycloid — Part of the teeth is known as face.

Hypocycloid — Part of the teeth is known a[s] flank.

Involute Teeth :-

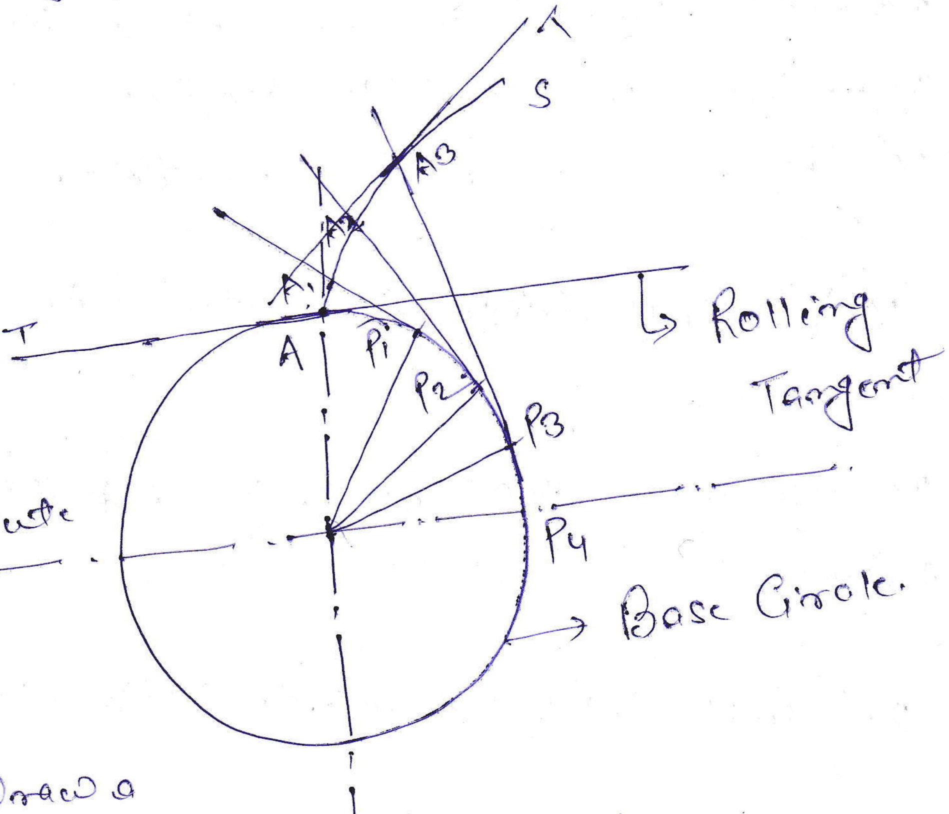
An involute of a Circle is a plane Curve generated by a Point on a tangent, which rolls on a Circle without Slipping or by a Point on a taut String which is unwrapped from a reel. Right

$AP_1 = P_1A_1$

$AP_2 = P_2A_2$

$AP_3 = P_3A_3$.

As is the involute Curve



at Point A3 To draw a tangent A3T, which is Perpendicular to P3A3 and P3A3 is the normal to the involute.

Now, we Can Say that,

Normal at any Point of an involute is a tangent to the Circle.

⇒ System of Gear Teeth :-

(i) 14½° Composite System :-

It is used for general purpose gears. It is stronger but has no interchangeability.
Cycloidal curve at the top & bottom but involute curve at the middle portion. The teeths are produced by milling cutters or hobs.

(ii) 14½° full depth involute System :-

It was developed for use with gear hobs, for ~~spu~~ spur and helical gears.

(iii) 20° full depth involute System :-

It may be cut by hobs. The increase of the pressure angle from 14½° to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base.

(iv) 20° ~~full~~ Stub involute System :-

It has a strong tooth to take heavy loads.

The pitch surfaces of the spiral gears are cylindrical and the teeth have point contact. These gears are only suitable for transmitting small power. Spiral gears may be same hand or opposite hand.

⇒ Minm. No. of Teeth on the Pinion in order to avoid interference.

The addendum Circles for the two mating gears must Cut the Common tangent to the base Circle betn the Point of Tangency.

The limiting Condition reaches, when addendum Circles of Pinion and wheel Passes through Point N and M.

Let t = No. of teeth on the Pinion

T = No. of teeth on the wheel

m = module of the teeth

r = Pitch Circle radius of the Pinion = $mt/2$

G = Gear Ratio = $\left(\frac{T}{t}\right) = \left(\frac{R}{r}\right)$

ϕ = Pressure angle or Angle of obliquity.

from $\Delta\, O_1NP$

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2\sin^2\phi - 2r \cdot R\sin\phi \cdot \cos(90^\circ + \phi)$$

$$[PN = O_2P \sin\phi = R\sin\phi]$$

$$= r^2 + R^2\sin^2\phi + 2r \cdot R\sin^2\phi$$

$$= r^2\left[1 + \frac{R^2\sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right]$$

$$= r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

∴ Limiting radius of the Pinion addendum Circle,

$$O_1N = r\sqrt{1+\frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi}$$

$$= \frac{mt}{2}\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}$$

Let $A_p.m$ = Addendum of the Pinion, where A_P is a fraction by which the standard addendum of one module for the Pinion should be multiplied in order to avoid interference.

Addendum of the Pinion

$$= O_1N - O_1P$$

$$\therefore A_p.m = \frac{mt}{2}\sqrt{1+\frac{T}{t}\left(T/t+2\right)\sin^2\phi} - mt/2$$

$$\Rightarrow A_p.m = mt/2\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}-1\right]$$

$$\therefore A_p = t/2\left[\text{———}\right]$$

$$\therefore t = \frac{2AP}{\sqrt{1+T/t\left(T/t+2\right)\sin^2\phi}-1}$$

$$\boxed{t = \frac{2AP}{\left(1+G(G+2)\sin^2\phi\right)^{1/2}-1}}$$

⇒ Minm number of teeth on the wheel in order to avoid interference.

$$A_W = \frac{T}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{2}+2\right)\sin^2\phi} - 1\right]$$

$$\therefore T = \frac{2A_W}{\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1}$$

$$\text{or,} \quad \frac{2A_W}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}.$$

from this eqn we obtain minm no. of teeth (t).
both side multiplied by t/T, we get.

$$\Rightarrow T\times t/T = \frac{2A_W \times t/T}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

$$\therefore t = \frac{2A_W}{G\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right]}$$

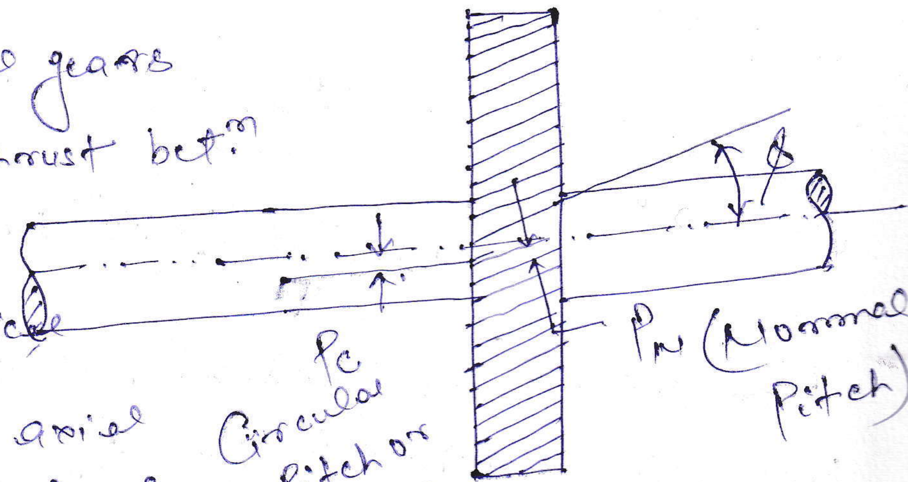
If wheel & pinion have equal teeth $G = 1$ Then

$$\boxed{T = \frac{2A_W}{\sqrt{1+3\sin^2\phi} - 1}}$$

⇒ Minm Number of teeth on a Pinion for involute Rack in order to avoid interference.

$$\boxed{t = \frac{2AR}{\sin^2 \phi}}$$

⇒ Helical gears:-



In Case of Single helical gears there is some axial thrust betn teeth. which is a disadvantage of helical gears. To eliminate axial thrust, double helical gears are used. It is equivalent to two single helical gears in which equal and opposite thrust are produced on each gear and the resulting axial thrust is zero.

→ A distance along the axis of the screw or helix betn one thread or helix to the corresponding point on the next thread or helix. → The axial pitch can also be defined as the circular pitch in the plane of rotation. it is denoted by P_c.

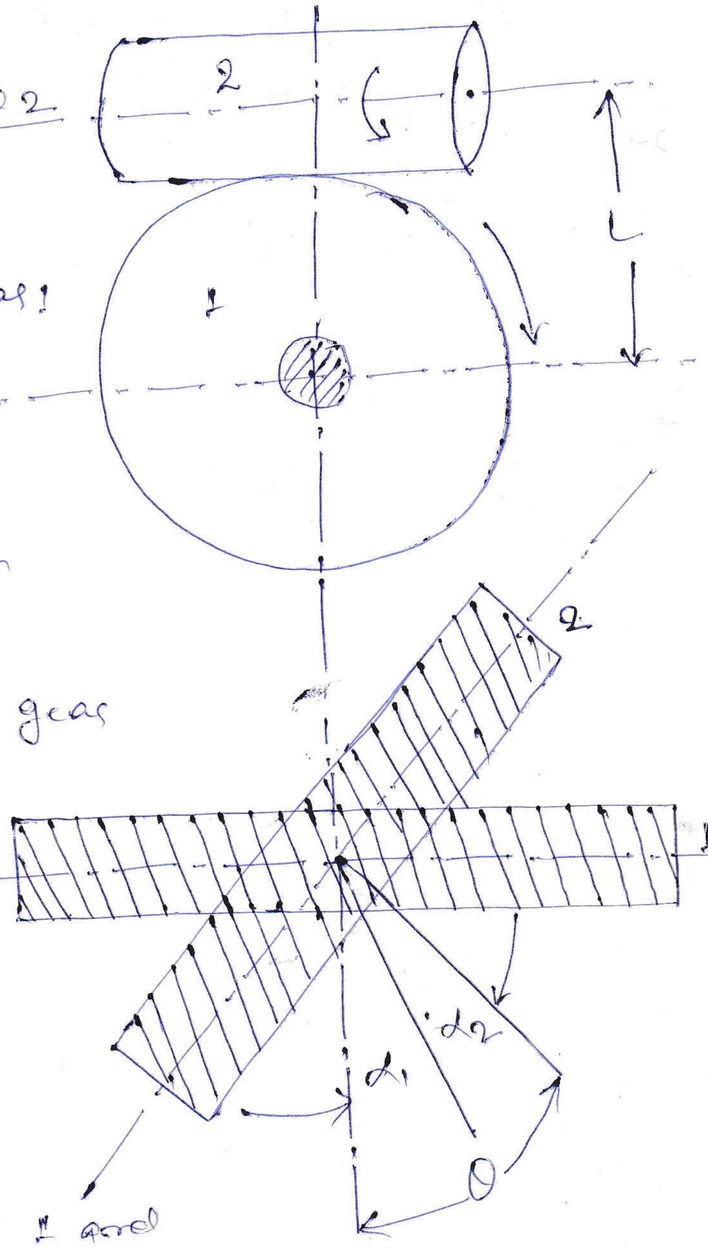
→ Normal Pitch :→ The distance betn point of intersection of the line of action of gear teeth with the working faces of two adjacent teeth. or. It is the distance betn similar faces of adjacent teeth, along a helix on the pitch cylinder normal to the teeth. It is denoted

⇒ Centre distance for a pair of Spiral Gears:-

UNIT - 03

Spiral gears are used to connect and transmit motion betn two non-parallel & non-intersecting shaft.

Let α_1 and α_2 = Spiral angle of gear teeth for 1 and 2 respectively.



P_{c_1} = Circular Pitch of gear 1

P_{c_2} = Circular Pitch of gear 2.

T_1 = Number of teeth on gear 1

T_2 = Number of teeth on gear 2.

d_1 = Pitch Circle diameter of gear 1

d_2 = Pitch Circle diameter of gear 2.

N_1 & N_2 = Speed of gear 1 and 2.

G = Gear Ratio $= \dfrac{T_2}{T_1} = \dfrac{N_1}{N_2}$

P_N = Normal Pitch

L = least Centre distance between the axes of both Shaft of the gear.

※ Here The Normal Pitch is Same for both the Spiral Gears,

$$P_{c_1} = \frac{P_N}{\cos\alpha_1} \quad \text{and} \quad P_{c_2} = \frac{P_N}{\cos\alpha_2}$$

we already know that

$$P_{c_1} = \frac{\pi d_1}{T_1} \quad \text{or,} \quad d_1 = \left(\frac{P_{c_1} \times T_1}{\pi}\right)$$

$$P_{c_2} = \frac{\pi d_2}{T_2} \quad \text{or,} \quad d_2 = \left(\frac{P_{c_2} \times T_2}{\pi}\right)$$

$$\therefore \quad l = \frac{d_1 + d_2}{2} = \frac{\frac{P_{c_1} \times T_1}{\pi} + \frac{P_{c_2} \times T_2}{\pi}}{2}$$

$$= \frac{1}{2}\left[\frac{P_{c_1} \times T_1}{\pi} + \frac{P_{c_2} \times T_2}{\pi}\right]$$

$$= \frac{T_1}{2\pi}\left[P_{c_1} + P_{c_2} \times \frac{T_2}{T_1}\right]$$

$$= \frac{T_1}{2\pi}\left[P_{c_1} + P_{c_2} \cdot G\right]$$

$$= \frac{P_N \times T_1}{2\pi}\left[\frac{1}{\cos\alpha_1} + \frac{G}{\cos\alpha_2}\right]$$

$$\therefore \quad \boxed{l = \frac{P_N \times T_1}{2\pi}\left[\frac{1}{\cos\alpha_1} + \frac{G}{\cos\alpha_2}\right]}$$

NOTE:- 1. If the Pair of Spiral Gears have teeth of the Same hand, then

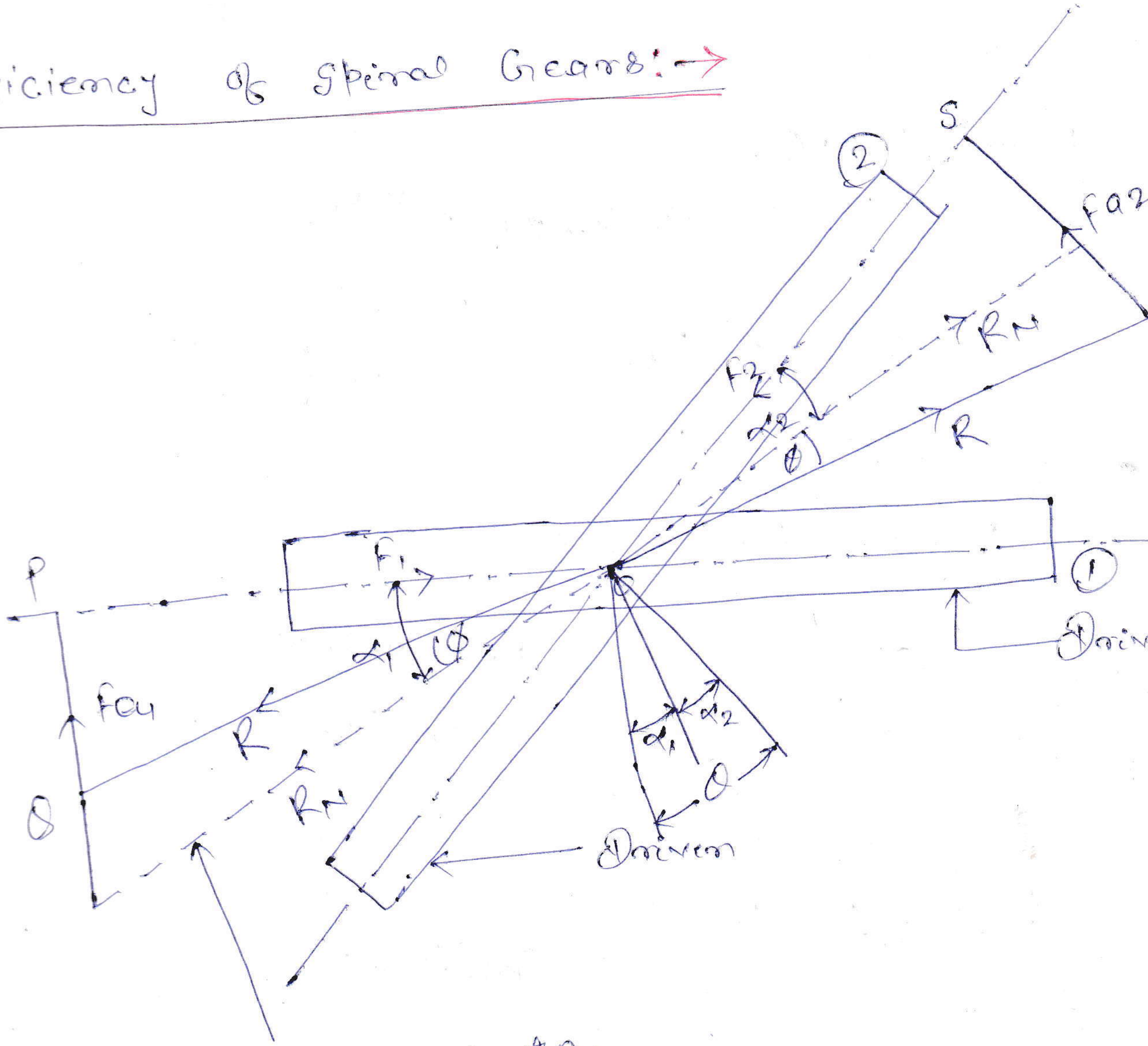
$$\boxed{\theta = \alpha_1 + \alpha_2}$$

2. If the teeth are opposite hand, then

$$\boxed{\theta = \alpha_1 - \alpha_2}$$

3. when $\theta = 90^\circ$, then both the spiral gears must have teeth of the same hand.

⇒ Efficiency of Spiral Gears:→



Common Normal at O.

Let, F_1 = force applied tangentially on the driver

F_2 = Resisting force acting tangentially on the driven,

F_{a1} = Axial or End thrust on the driver.

F_{a2} = Axial or End thrust on the driven

R_N = Normal reaction at the Point of Contact

ϕ = Angle of friction.

R = Resultant reaction at the Point of Contact

and θ = Shaft angle $=(\alpha_1+\alpha_2)$. —— (Both gears are on the same hand)

from Δ OPQ.

$$\cos(\alpha_1-\phi) = \frac{F_1}{R}$$

$$\therefore \boxed{F_1 = R\cos(\alpha_1-\phi)}$$

$\therefore$ work input to the drives

$= F_1 \times \pi d_1 N_1$

$= R\cos(\alpha_1-\phi)\,\pi d_1 N_1$

from Δ OST,

$$\cos(\alpha_2+\phi) = \frac{F_2}{R}$$

$$\therefore F_2 = R\cos(\alpha_2+\phi)$$

$\therefore$ Work output of the driven

$= F_2 \times \pi d_2 N_2$

$= R\cos(\alpha_2+\phi)\,\pi d_2 N_2$.

∴ Efficiency of spiral Gears (η) :-

$$\eta = \frac{\text{Work output}}{\text{Work input}} = \frac{R \cos(\alpha_2+\phi)\, \pi d_2 \cdot N_2}{R \cos(\alpha_1-\phi)\, \pi d_1 \cdot N_1}$$

$$= \frac{\cos(\alpha_2+\phi)\, d_2 N_2}{\cos(\alpha_1-\phi)\, d_1 N_1}$$

$$\therefore \boxed{\eta = \frac{\cos(\alpha_2+\phi)\, d_2 N_2}{\cos(\alpha_1-\phi)\, d_1 N_1}} \quad \text{——— (i)}$$

We already know that

$$d_1 = \frac{P_{c_1} \times T_1}{\pi} = \frac{P_N}{\cos\alpha_1} \times \frac{T_1}{\pi}$$

$$d_2 = \frac{P_{c_2} \times T_2}{\pi} = \frac{P_N}{\cos\alpha_2} \times \frac{T_2}{\pi}$$

$$\therefore \frac{d_2}{d_1} = \frac{\frac{\cancel{P_N}}{\cos\alpha_2} \times \frac{T_2}{\cancel{\pi}}}{\frac{\cancel{P_N}}{\cos\alpha_1} \times \frac{T_1}{\cancel{\pi}}} = \frac{T_2}{T_1} \times \frac{\cos\alpha_1}{\cos\alpha_2}$$

$$\boxed{\frac{d_2}{d_1} = \frac{T_2 \cos\alpha_1}{T_1 \cos\alpha_2}}$$

And. $\boxed{\frac{N_2}{N_1} = \frac{T_1}{T_2}}$

From eqn (i), we get,

$$\eta = \frac{\cos(\alpha_2+\phi)\, d_2 N_2}{\cos(\alpha_1-\phi)\, d_1 N_1}$$

$$= \frac{\cos(\alpha_2+\phi)}{\cos(\alpha_1-\phi)} \times \frac{\cancel{T_2}}{\cancel{T_1}}\frac{\cos\alpha_1}{\cos\alpha_2} \times \frac{\cancel{T_1}}{\cancel{T_2}}$$

$$= \frac{\cos\alpha_1 \cos(\alpha_2+\phi)}{\cos\alpha_2 \cos(\alpha_1-\phi)}$$

$$= \frac{\cos(\alpha_1+\phi+\alpha_2) + \cos(\alpha_1-\alpha_2-\phi)}{\cos(\alpha_2+\alpha_1-\phi) + \cos(\alpha_2-\alpha_1+\phi)}$$

$$\therefore \boxed{\eta = \frac{\cos(\theta+\phi) + \cos(\alpha_1-\alpha_2-\phi)}{\cos(\theta-\phi) + \cos(\alpha_2-\alpha_1+\phi)}}$$

Since, Efficiency will be maxm. when $\cos(\alpha_1-\alpha_2-\phi)$ is maxm because the angle θ and ϕ are Constant.

$$\therefore \cos(\alpha_1-\alpha_2-\phi) = 1$$

$$\text{or, } \cos(\alpha_1-\alpha_2-\phi) = \cos 0$$

$$\Rightarrow \alpha_1-\alpha_2-\phi = 0$$

$$\boxed{\alpha_1 = \alpha_2+\phi} \quad \& \quad \boxed{\alpha_2 = \alpha_1-\phi}$$

Since, $\alpha_1+\alpha_2 = \theta$

$$\therefore \alpha_1 = \theta-\alpha_2 = \theta-\alpha_1+\phi$$

or, $$\boxed{\alpha_1 = \frac{\theta+\phi}{2}}$$

Similarly $$\boxed{\alpha_2 = \frac{\theta-\phi}{2}}$$

Putting the value of $(\alpha_1 = \alpha_2+\phi)$ & $(\alpha_2 = \alpha_1-\phi)$ in the above equation, we get,

$$\eta_{max} = \frac{\cos(\theta+\phi) + \cos(\alpha_2+\phi-\alpha_2-\phi)}{\cos(\theta-\phi) + \cos(\alpha_1-\phi-\alpha_1+\phi)}$$

$$\boxed{\eta_{max} = \frac{\cos(\theta+\phi)+1}{\cos(\theta-\phi)+1}}$$

we already know that

$$R_N = \frac{F_1}{\cos\alpha_1} = \frac{F_2}{\cos\alpha_2}$$

$\therefore$ Axial thrust on the drives, $F_{a_1} = R_N \sin\alpha_1$

$$\boxed{\sin\alpha = \frac{F_{a_1}}{R_N}}$$

$$= \frac{F_1}{\cos\alpha_1} \times \sin\alpha_1$$

$$\therefore \boxed{F_{a_1} = F_1 \tan\alpha_1}$$

$\therefore$ Axial thrust on the driven,

$$F_{a2} = R_N \sin\alpha_2$$

$$\boxed{\sin\alpha_2 = \frac{F_{a2}}{R_N}}$$

$$= \frac{F_2}{\cos\alpha_2} \times \sin\alpha_2$$

$$\therefore \boxed{F_{a2} = F_2 \tan\alpha_2}$$

WORM GEARS

⇒ Worm gear drives are used to transmit power between two non-intersecting shafts, which is in General at right angle to each other.

→ The worm is a threaded Screw, while the worm wheel a toothed gear.

→ The teeth on the worm wheel envelope the threads on the worm and give line Contact between mating parts.

⇒ Advantages of worm Gear is given below.

→ Higher speed reduction. A speed reduction as high as 100:1 Can be obtained with a single pair of worm Gear.

→ The operation is Smooth and Silent.

→ Provision Can be made for self locking operation, where the motion is transmitted only from the worm to worm wheel.

→ This is advantageous in applications like Cranes and lifting devices.

⇒ The drawbacks of worm Gears are given below.

(i) The efficiency is Low as Compared to other types of Gears.

(ii) The worm wheel, in General is made of phosphor bronze, which increases the Cost.

(iii) Considerable amount of heat is generated in worm Gear drives, which is required to be dissipated by a lubricating oil to the housing walls and finally to the Surroundings.

(iv) The Power transmitting capacity is low. worm gear drives are used for upto 100 kW of power transmission.

⇒ Single-threaded worm gives large speed reduction however, the efficiency is low. The large velocity ratio is obtained at the cost of efficiency.

⇒ Multi-threaded worm gives high efficiency, however the speed reduction is low. The high efficiency obtained at the cost of speed reduction.

⇒ <u>Manually operated Intermittent Mechanisms:-</u> In these applications, large mechanical advantage is required and efficiency is of minor importance. The examples of these mechanisms include steering mechanism and opening and closing of gate valves by means of hand wheels.

⇒ <u>Motorized operated Intermittent Mechanisms:-</u> In these applications a small capacity low-cost motor drives the mechanisms and the efficiency is of minor importance. The examples of these mechanisms include drive for small hoists and opening and closing of large gate valves by means of electric-motor.

⇒ <u>Motorized Continuous operations:-</u> In these applications, worm Gear drives are used in place of the other Gear drives due to space limitations and silent operati

The efficiency is more important in these applications. Multi-threaded worms are used in these applications to obtain higher efficiency.
The examples of this type include drives for machine tools and elevators.

⇒ Motorized speed increasing Applications:-

In these applications, worm gear drives are preferred due to high velocity ratio and silent operation.
The efficiency is more important in these applications.
Speed increasing applications include drives for automotive superchargers and centrifugal cream chargers.
In order to increase efficiency, the automotive worm supercharger is provided with six threaded worm having lead angle of about 45°.

⇒ Terminology of worm Gears:-

The meaning of given form is given below.

$$z_1 / z_2 / q / m$$

where, z_1 = Number of starts on the worm.

z_2 = Number of teeth on the worm wheel

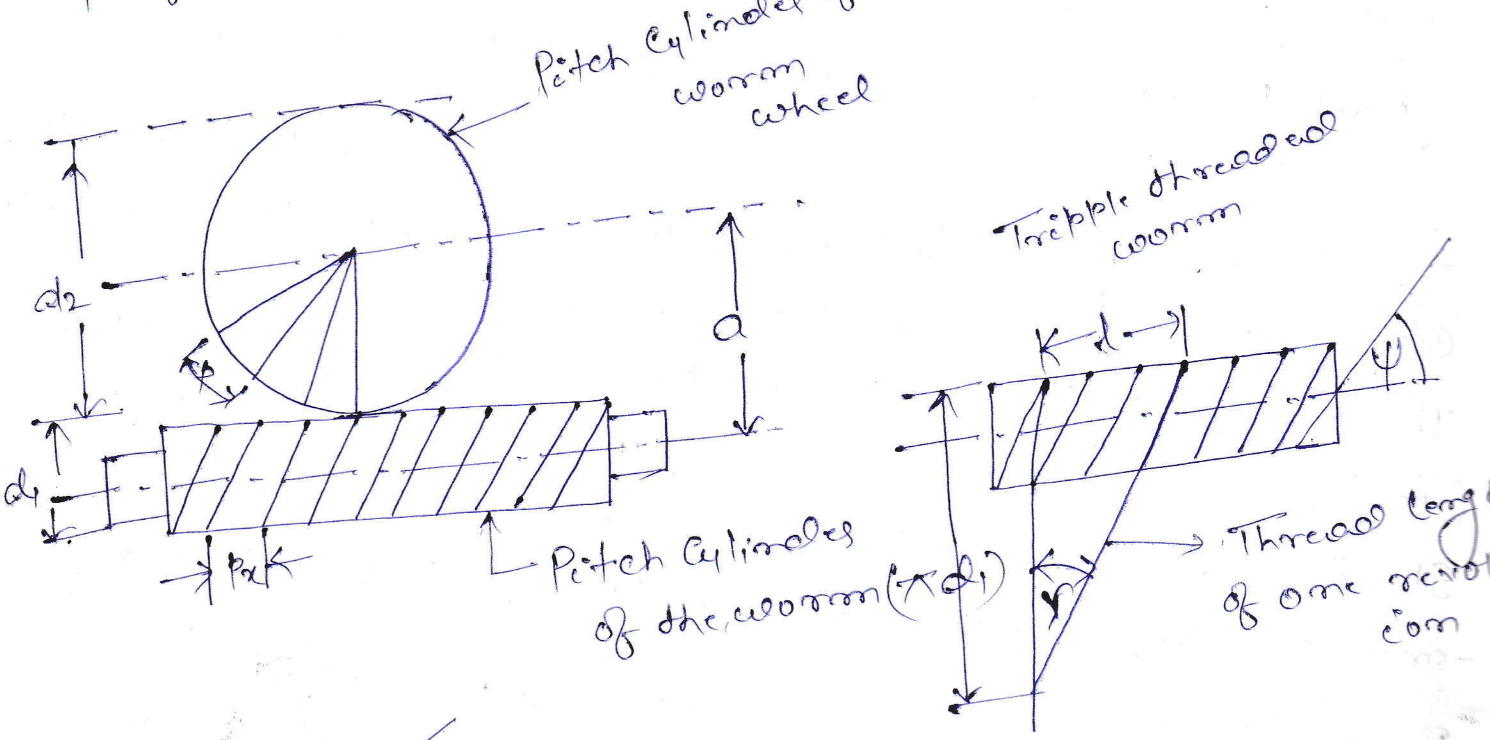
q = diametral quotient

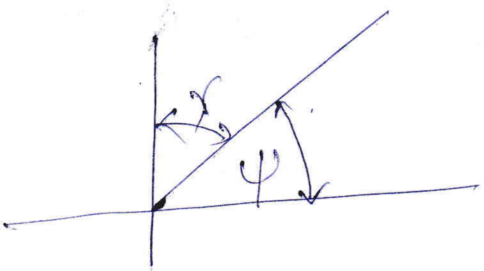
$= \frac{d_1}{m} = \frac{\text{Pitch Circle dia. of the worm}}{\text{module (mm)}}$

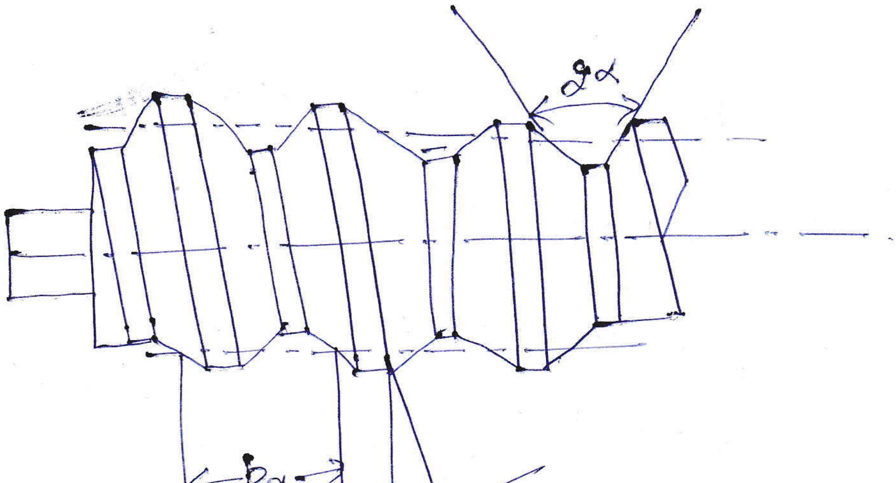
d_2 : Pitch Circle diameter of the worm wheel

→ The worm is similar to a screw with single-start multi-start threads.

→ The threads of the worm have an involute helical profile.







⇒ Single enveloping

(i) Axial pitch :→

The distance measured from a point o[n] one thread to the corresponding point on the adjacen[t] thread, measured along the axis of the worm. It is denoted by (p_x).

(ii) Lead :- The distance that a point on the helica[l] profile will move when the worm is rotated throu[gh] one revolution.

It is the thread advance in one turn. for single-start threads, the lead is equal to the axial pitch for double-start threads, the lead is twice the axi[al] pitch,

$$\boxed{l = p_x \times z_1}$$

Number of starts →	Velocity Ratio
Single-start	20 and above
Double start	12 - 36
Trible - start	8 - 12
Quadruble-start	6 - 12
Sextuble - start	4 - 10

⇒ Module for the worm wheel

$$m = \left(\frac{d_2}{z_2}\right) : \frac{\text{Pitch circle diamet[er] of w.w}}{\text{Number of teeth o[n] the worm wheel}}$$

Now, Axial pitch of the worm is should be equal to the circular pitch of the worm wheel.

$$p_x = \frac{\pi d_2}{z_2} = \frac{\pi (m z_2)}{z_2}$$

$$\boxed{p_x = \pi m}$$

Now from the value of lead is given below.

$$\boxed{l = p_x z_1}$$

$$\Rightarrow l = \pi m \times z_1$$

$$\boxed{l = \pi m z_1}$$

When one thread of the worm is developed, it becomes the hypotenuse of a triangle which is shown in given fig.

The base of the triangle is equal to the lead of the worm.

The latitude is equal to the circumference of the worm. Then the two angles are formed lead angle and Helix angle

(iii) Lead angle :- The lead angle (γ) is defined as the angle between a tangent to the thread at the pitch diameter and a plane normal to the worm axis.

$$\tan\gamma = \frac{l}{\pi d_1} = \frac{\pi m z_1}{\pi d_1} = \frac{m}{\left(\frac{d_1}{z_1}\right)}$$

$$= \frac{m}{q}$$

$$\boxed{\tan\gamma = \frac{m}{q}} \quad \text{or,} \quad \boxed{\gamma = \tan^{-1}\left(\frac{m}{q}\right)}$$

$$\tan \gamma = \frac{\cancel{\pi} \cancel{m} z_1}{\cancel{\pi}(\cancel{m} q)} = \frac{z_1}{q}$$

$$\therefore \boxed{\tan \gamma = \frac{z_1}{q}} \quad \text{or,} \quad \boxed{\gamma = \tan^{-1}\left(\frac{z_1}{q}\right)}$$

(iv) Helix angle:- The helix angle (ψ) is defined as the angle between a tangent to the the thread at the pitch diameter and the axis of the worm.

The worm helix angle is the Complement of the worm lead angle.

$$\boxed{\psi + \gamma = \pi/2}$$

The helix angle should be limited to 60° per thread. For example: if $\psi = 30°$, then the worm should have at least five threads.

(v) Pressure angle:- (α). The tooth Pressure angle (α) is measured in a plane containing the axis of the worm and it is equal to one-half of the thread angle. It is

The pressure angle should not be less than 20° for single and double start worms and 25° for triple and multi-starts worms.

The centre distance is given below

$$a = \frac{1}{2}(d_1 + d_2)$$

$$= \frac{1}{2}(mq + mz_2)$$

$$\boxed{a = \frac{1}{2} m (q + z_2)}$$

when the worm wheel is rotated through one revol-ion, the worm will complete z_2 revolution for sin-start threads.

For double-start threads, The number of revolution the worm will be $\left(\frac{z_2}{2}\right)$.

The speed ratio $(i) = \left(\frac{z_2}{z_1}\right)$

⇒ Single-enveloping worm Gear Drive :- A single-enveloping worm Gear set is one in which the Gear is wrapping around or partially enclose the worm.

→ This result ~~lies~~ in line contact between the threads of the worm and the teeth of the worm wheel.

In this case, the worm is also called "Cylindrical or straight cylindrical worm.

The single-enveloping worm Gear drive is more widely used.

(ii) Double-enveloping worm Gear Drives :- → A double-enveloping gear set is one in which the gear wraps around the worm and the worm also wraps around the gear.

This results in area contact between the threads of the worm and the teeth of the worm wheel. In the case worm is also called 'hourglass' worm. This drive is also called 'cone gearing'.

⇒ The advantages of double-enveloping worm gear drive.

(i) The contact pressure between the threads of the worm and the teeth of the worm wheel is low. This reduces wear.

(ii) The drive occupies less space for a given capacity. Double-enveloping worm gear drives need only about one-third two-third of the space and has about one-third the weight compared with a single-enveloping worm gear drives.

⇒ The main drawback of double-enveloping worm gear drive is the requirement of precise alignment. It is more critical than in case of single-enveloping worm gear drive.

A small deviation from the ~~contact~~ correct centre distance results in the loss of theoretical area of contact.

⇒ Proportions of worm gears:-

For an involute helicoidal tooth form

$$h_a = m$$

$$h_{f1} = (2.2(\cos\gamma - 1))m$$

$$c = 0.2m\cos\gamma$$

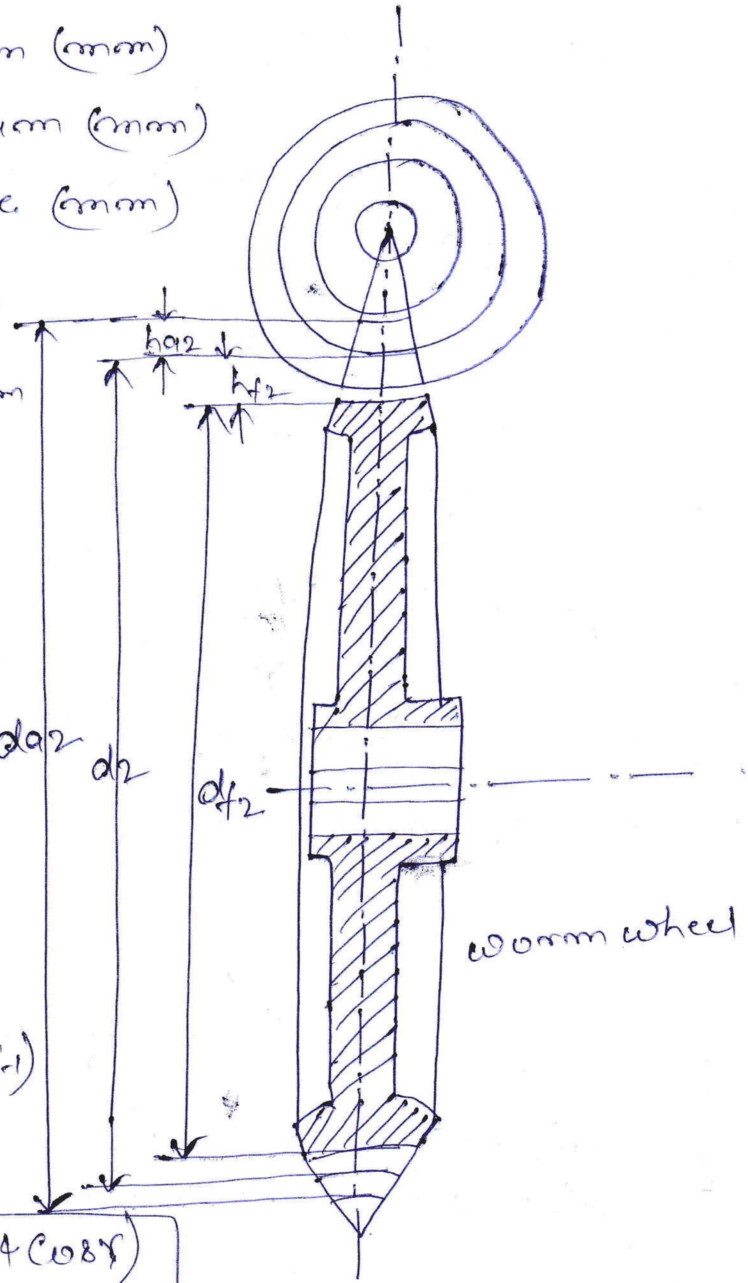
where,

h_{a1} = Addendum (mm)

h_{f1} = Dedendum (mm)

c = Clearance (mm)

The outside and root diameters of the worm are given below



$d_{a1} = d_1 + 2h_{a1}$

$= qm + 2m$

$$\therefore \boxed{d_{a1} = m(q+2)}$$

$d_{f1} = d_1 - 2h_{f1}$

$= qm - 2m(2.2\cos\gamma - 1)$

$$\therefore \boxed{d_{f1} = m(q + 2 - 4.4\cos\gamma)}$$

where d_{a1} = outside diameter of the worm (mm)

d_{f1} = root diameter of the worm (mm)

Similarly, The addendum and dedendum of the worm wheel is given below.

$h_{a2} = m(2\cos\gamma - 1)$ = Addendum at the throat (mm)

$h_{f2} = m(1 + 0.2\cos\gamma)$ = Dedendum in the median plane (mm).

The diamensions of the worm wheel are given below.

$$d_{a2} = d_2 + 2h_{a2}$$

$$= mz_2 + 2m(2\cos\gamma - 1)$$

$$\therefore \boxed{d_{a2} = m(z_2 + 4\cos\gamma - 2)}$$

$$d_{f2} = d_2 + 2h_{f2}$$

$$= mz_2 - 2m(1 + 0.2\cos\gamma)$$

$$\therefore \boxed{d_{f2} = m(z_2 - 2 - 0.4\cos\gamma)}$$

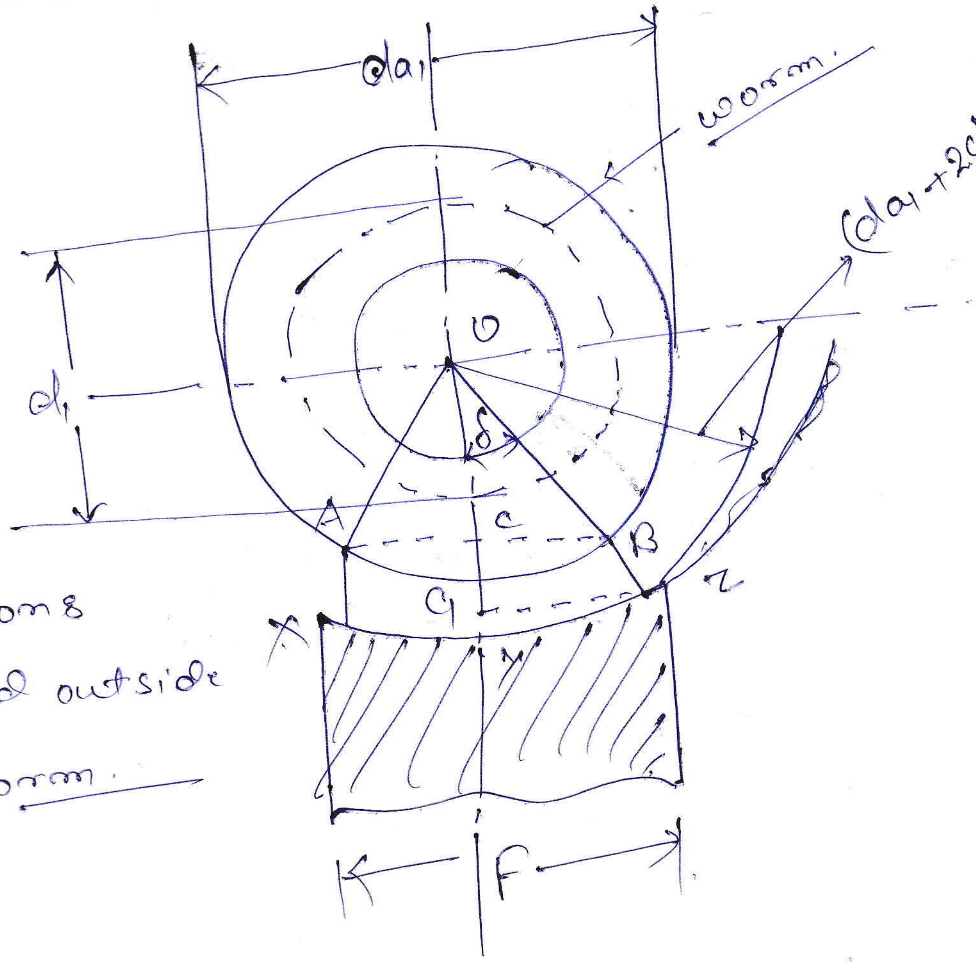
where, d_{a2} = Throat diameter of the worm wheel (mm)

d_{f2} = Root diameter of the worm wheel (mm)

⇒ The effective face width f of the worm wheel are given below.

Tangent AB to the pitch circle diameter of the worm.

A and B are the points of intersections of this tangent and outside diameter of the worm.



From ΔOAC,

$$(AC)^2 = (AO)^2 - (OC)^2$$

$$\Rightarrow \left(\frac{f}{2}\right)^2 = \left(\frac{da_1}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2$$

$$= \left[\frac{m(q+2)}{2}\right]^2 - \left[\frac{qm}{2}\right]^2$$

$$\therefore \boxed{f = 2m + \sqrt{(q+1)}}$$

From Δ OZG

$$\sin\delta = \frac{GZ}{OZ} = \frac{F/2}{(da_1+2c)/2}$$

$$\Rightarrow \sin\delta = \frac{f}{(da_1+2c)}$$

$$\therefore \boxed{\delta = \sin^{-1}\left(\frac{f}{da_1+2c}\right)}$$

The length of the root of the worm wheel teeth is the arc XYZ, It is denoted by l_r.

$$l_r = \text{arc } XYZ$$

$$= \left(\frac{2\delta}{2\pi}\right)\left[\pi(da_1+2c)\right]$$

$$= (da_1+2c)\cdot\delta$$

$$\therefore \boxed{l_r = (da_1+2c)\sin^{-1}\left[\frac{f}{(da_1+2c)}\right]}$$

⇒ Efficiency of worm Gearing:-

It is the ratio of work done by the worm gear to the work done by the worm.

$$\therefore \boxed{\eta = \frac{\tan\lambda(\cos\phi - \mu\tan\lambda)}{(\cos\phi \cdot \tan\lambda + \mu)}}$$

where, ϕ = Normal pressure angle
μ = Coefficient of friction.
λ = lead angle.

The efficiency is maximum, when

$$\tan\lambda = \sqrt{1+\mu^2} - \mu.$$

when, we assume square threads, and in which

$$\phi = 0,$$

$$\therefore \cos\phi = \cos 0° = 1$$

Now, the above eqn becomes,

$$\eta = \frac{\tan\lambda(1 - \mu\tan\lambda)}{(\tan\lambda + \mu)}$$

$$= \frac{(1 - \mu\tan\lambda)}{1 + \mu/\tan\lambda}$$

$$= \frac{\tan\lambda}{\tan(\lambda + \phi_1)}$$

where, ϕ_1 = Angle of friction,

then $\tan\phi_1 = \mu$

The value of $\mu = 0.015$ at a rubbing speed $V_r = $

$$= \frac{\pi D_{w} \cdot N_{w}}{\cos \lambda}$$, between 100 and 165 m/min.

for a speed below 10 m/min. take $\mu = 0.015$.

$$\mu = \frac{0.275}{(V_r)^{0.25}}$$, for rubbing speed between 12 and 180 m/min.

$$= \left(0.025 + \frac{V_r}{18000}\right)$$ for rubbing speed more than 180 m/min.

⇒ If the efficiency of worm Gearing is less than 50%. the the worm gearing is said to be Self locking. Then it can not be driven by applying a torque to the wheel.

This property of Self locking is desirable in some applications such as hoisting machinery.

⇒ <u>Strength of worm Gear Tooth:-</u>

It is Safe to assume that the teeth of worm gear are always weaker than the threads of the worm.

In worm gearing two or more teeth are usually in Contact, but due to uncertainty of load distribution among themselves it is assumed that a load is transmitted by one tooth only.

we know that according to lewis equation

$$\boxed{W_T = (\sigma_0 \cdot C_v)\, b \cdot \pi \cdot m \cdot y}$$

where, W_T = Permissible tangential tooth load or bea[m] strength of gear tooth.

σ_0 = Allowable static stress.

C_v = velocity factor.

b = face width.

m = module.

y = Tooth form factor or lewis factor.

⇒ $C_v = \frac{6}{6+v}$, where v is the peripheral velocity of the worm gear in m/s.

$y = 0.124 - \frac{0.912}{T_G}$, for $14\frac{1}{2}^\circ$ involute teet[h]

$= 0.154 - \frac{0.912}{T_G}$, for 20° involute teeth

⇒ The dynamic tooth load on the worm gear is given by

$$W_D = \frac{W_T}{C_v} = W_T\left(\frac{6+v}{6}\right)$$

W_T = Actual tangential load on the tooth

→ The dynamic load need not to be calculated beca[use] it is not so serve due to sliding action between the worm and worm gear.

⇒ The static tooth load or endurance strength of the tooth (W_S) may be also be obtained by the gi relations. which is given below

$$\boxed{W_S = \sigma_e \, b \, \pi . m . y}$$

where, σ_e = flexural endurance limit.

= 84 MPa for Cast iron.

= 168 MPa for phosphor bronze.

⇒ Wear tooth load for Worm Gear:-

The limiting or maximum load for wear i given below.

$$\boxed{W_W = D_G . b . K}$$

where, D_G = Pitch Circle diameter of the wor Gear.

b = face width of the worm Gear.

K = load stress factor or Material Combination factor.

The value of K.

Worm	Worm Gear	K
1. Steel (B.H.N. 250)	Phosphor Bronze	0.415
2. H.S	Cast iron	0.345
3. H.S	Phosphor bronze	0.550

3.	H.S	- Chilled Phosphor bronze	-	0.830
4.	H.S	- Antimony bronze	-	0.830
5.	Cast iron	- Phosphor bronze	-	1.035

This value is only for when lead angle upto when lead angle between 10° and 25° then the value of K is increased by 25 percent and when lead angle is ~~bet~~ greater than 25° the the value of K is increased by 50 percent.

⇒ Thermal rating of worm Gearing:-

The quantity of heat generated (Q_g) is given by

Q_g = power lost in friction in watt

$$\boxed{Q_g = P(1-\eta)}$$

where, P = power transmitted in watt.

η = Efficiency of the worm gearing.

The heat generated must be dissipated through the lubricating oil to the gear box housing and then to atmosphere.

⇒ The heat dissipated capacity depends upon the following factors.

(i) Area of the housing (A)

(ii) Temp. difference betn the housing surface and surrounding air $(t_2 - t_1)$ and.

(iii) Conductivity of the material (K).

The heat dissipating capacity,

$$\boxed{Q_d = A\,(t_2 - t_1)\cdot K}$$

The average value of K may be taken as 378 W/m²/°C

<u>NOTE:-</u> Maximum temp. $(t_2 - t_1)$ should not exceed <u>27°C to 38°C.</u>

The Max^{m} temp. of the lubricant should not exceed 60°C.

According to AGMA recommendations, the limiting input power of a plain worm gear unit from the stand point of heat dissipation, for worm gear speed upto 2000 r.p.m. may be cheked from the following relation.

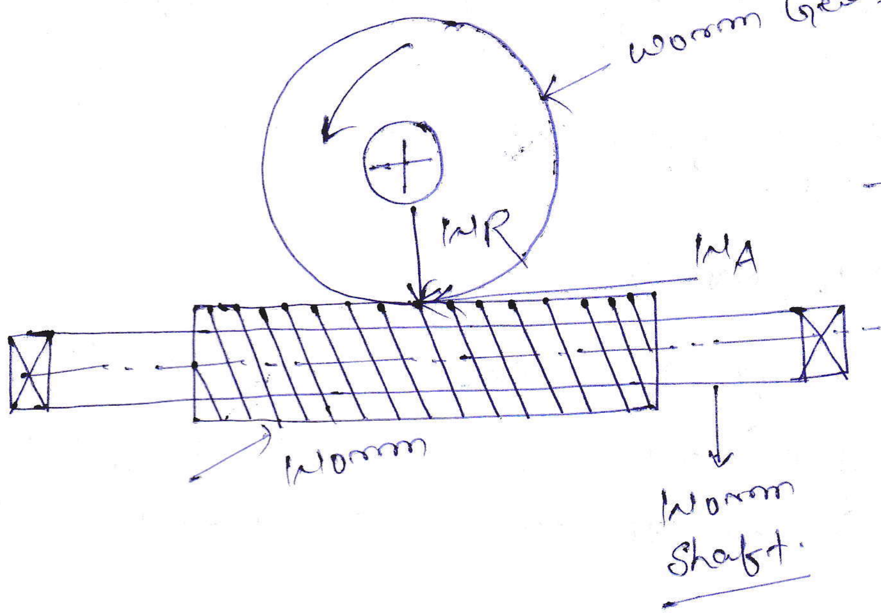
$$\boxed{P = \left(\frac{3650\,x^{1.7}}{V.R + 5}\right)}$$

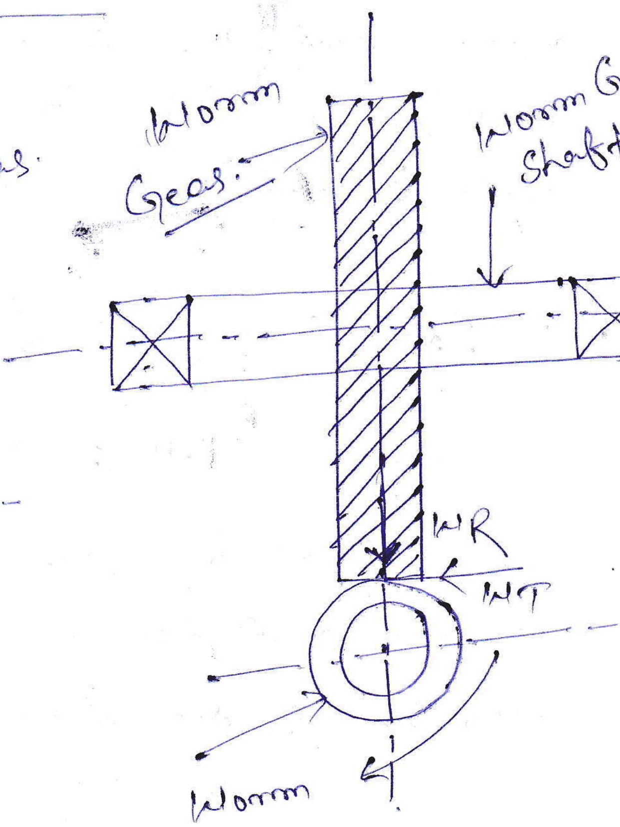
where, P = Permissible input power in KW.

x = Centre distance in m.

V.R = Velocity ratio or Transmission ratio.

⇒ Forces Acting on Worm Gears:-





(i) Tangential force on the worm,

$$W_T = \frac{2 \times \text{Torque on worm}}{\text{Pitch circle diameter of worm } (D_W)}$$

= Axial force or thrust on the worm gea

Tangential force (W_T) on the worm produces a twisti moment of magnitude $\left(W_T \times \frac{D_W}{2}\right)$ and bends th worm in the horizontal plane.

(ii) Axial force or thrust on the worm,

$W_A = W_T / \tan\lambda$ = Tangential force on the worm gear.

$$= \frac{2 \times \text{Torque on Worm gear}}{\text{Pitch circle diameter of worm gear } (D_G)}$$

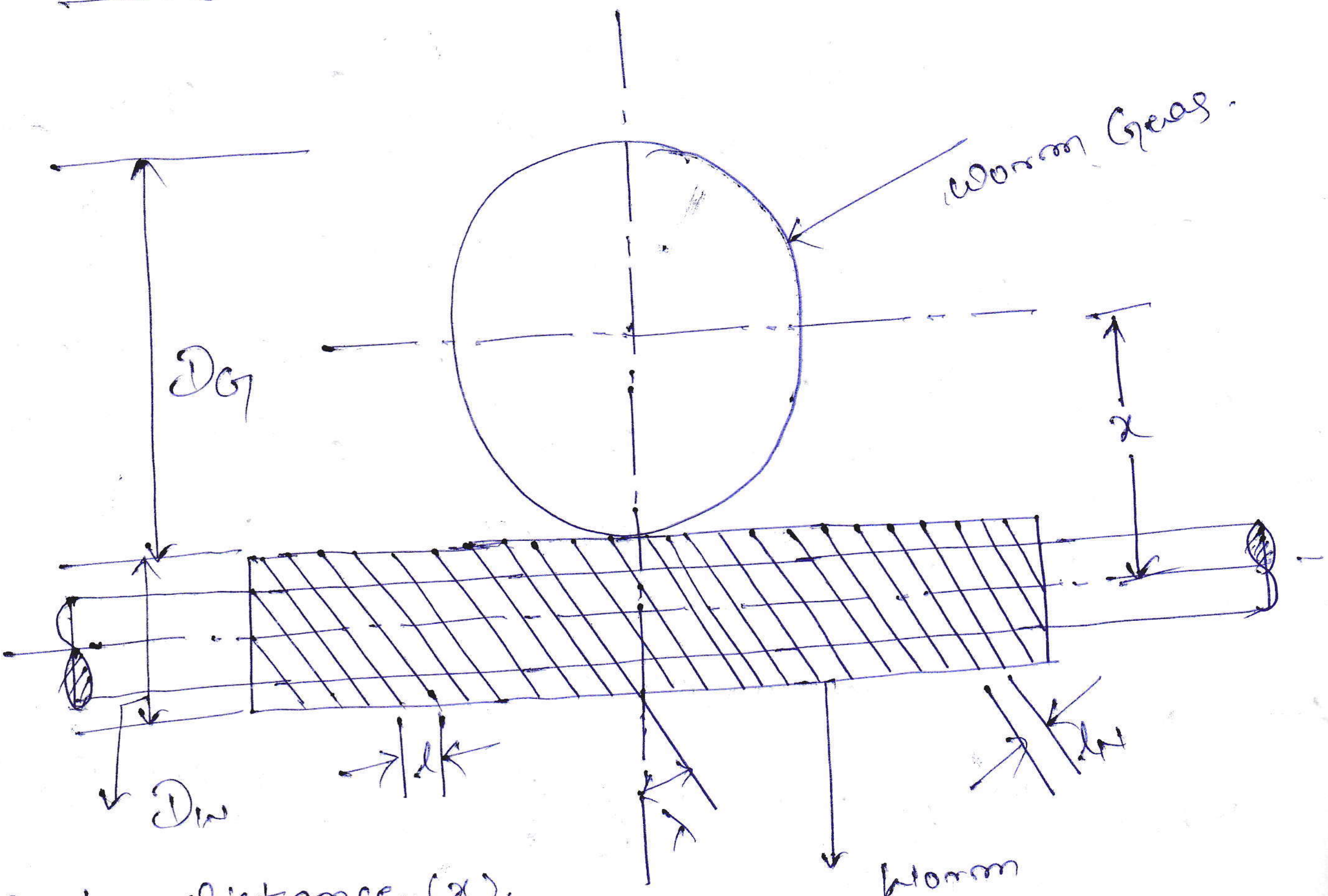
The axial force on the worm tends to move the worm axially, induces an axial load on the bearing and bends the worm in a vertical plane with a bending moment of magnitude $\left(W_A \times \frac{D_W}{2}\right)$

(iii) Radial or seperating force on the worm.

$$W_R = W_A \cdot \tan\phi$$

The radial or seperating force tends to force the worm and worm gear out of mesh. This force also bends the worm in the vertical plane.

⇒ Design of Worm Gearing:-



Centre distance (x).

$$x = \frac{D_W + D_G}{2}$$

The centre distance may be expressed in terms of the axial lead (l), lead angle (λ) and velocity ratio (V.R).

Then

$$\boxed{x = \frac{l}{2\pi}\left(\cos\lambda + V.R\right)}$$

In terms of Normal lead ($l_N = l\cos\lambda$)

$$x = \frac{l_N}{2\pi}\left[\frac{1}{\sin\lambda} + \frac{V.R}{\cos\lambda}\right]$$

$$\Rightarrow \boxed{\frac{x}{l_N} = \frac{1}{2\pi}\left[\frac{1}{\sin\lambda} + \frac{V.R}{\cos\lambda}\right]} \text{ ——— (A)}$$

NOTE:- The lowest point on the curve may be determined by differentiating the above eqn (A) with respect to λ and equating to zero. i.e

$$\Rightarrow \frac{(V.R)\sin^3\lambda - \cos^3\lambda}{\sin^2\lambda \cdot \cos^2\lambda} = 0$$

or, $\boxed{V.R = \cot^3\lambda}$

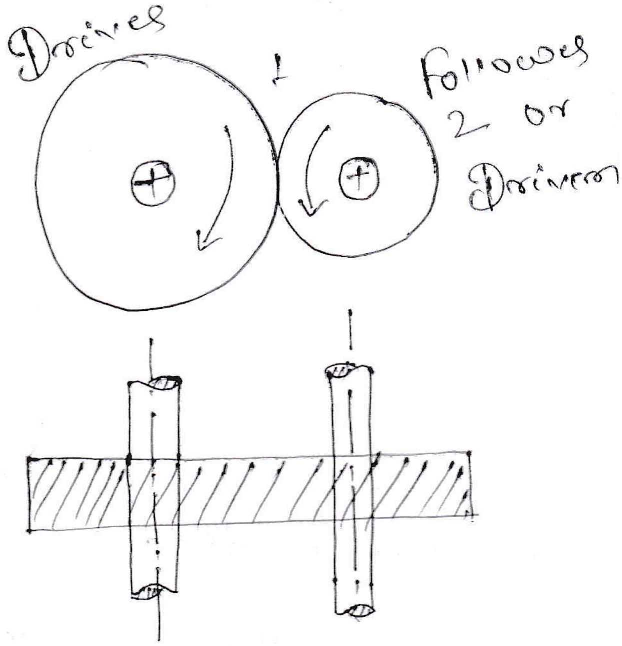
UNIT – IV Gear Train :–

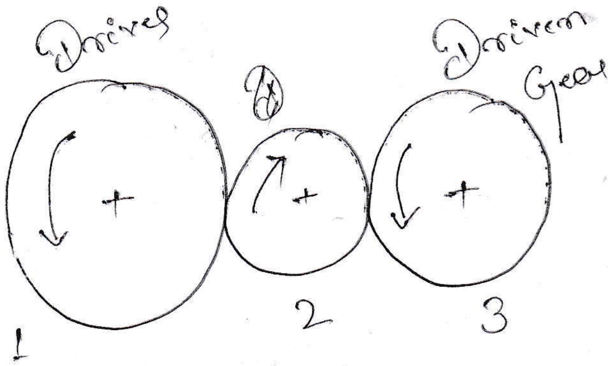
When two or more gears are made to mesh with each other to transmit power from one shaft to another. Then the combination is called Gear Train or Train of toothed wheels.

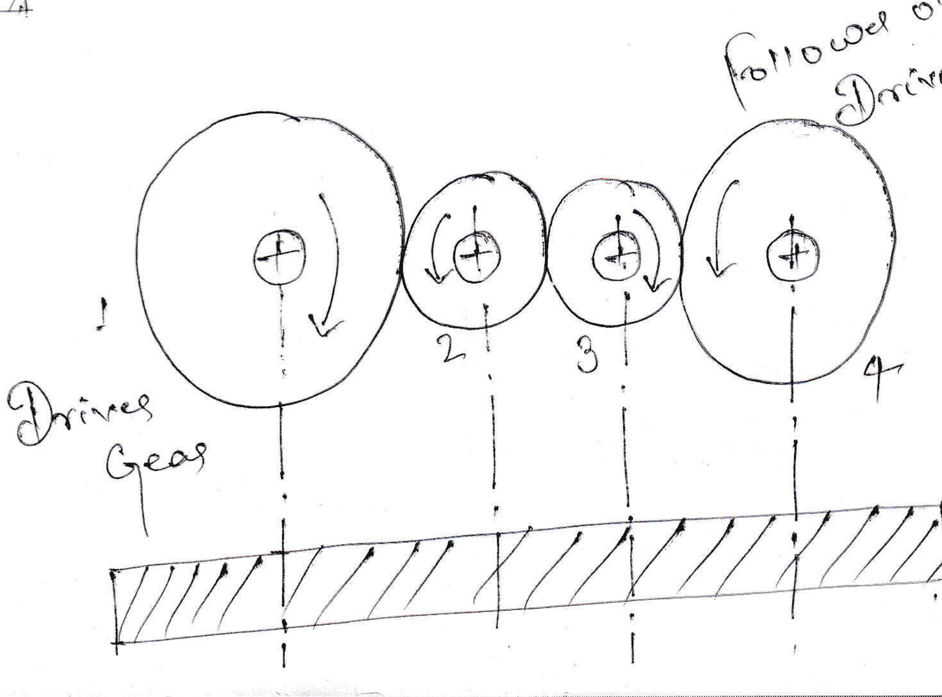
Types of Gear Train :–

→ Simple gear Train :–

When there is only one gear on each shaft. it is known as Simple gear train. The gears are represented by their Pitch Circles.







Let N_1 = Speed of gear 1 or Speed of Driver.
N_2 = " " " 2 " " " follower or Driven.

T_1 = Number of teeth on gear 1 and
T_2 = Number of teeth on gear 2.

Speed Ratio or velocity ratio of gear Train =

$$= \frac{\text{Speed of Driver}}{\text{Speed of Driven}} = \frac{\text{Number of Teeth on the follower}}{\text{Number of teeth on the driver.}}$$

$$\boxed{S.R = \frac{N_1}{N_2} = \frac{T_2}{T_1}}$$

$$\text{Train value} = \frac{\text{Speed of Driven}}{\text{Speed of Driver}} = \frac{N_2}{N_1}.$$

$$= \frac{\text{Number of Teeth on the driver}}{\text{Number of Teeth on the follower}}$$

$$\boxed{T.V = \frac{N_2}{N_1} = \frac{T_1}{T_2}}$$

(Train value)

when the distance betn the two gears is large.

→ By Providing the large size gear – This is very in-Convenient and uneconomical method.

→ By Providing one or more intermediate gears:- It is very Convenient and Economical.

when the Number of intermediate gear is odd, the motion of driver & driven gear is Same or like. But when number of intermediate gear is even, the motion of driver and driven gear is in opposite direction.

⇒ For Gear 1 & 2.

$$\text{Speed ratio (S.R)} = \frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{— (i)}$$

for gear 2 & 3.

$$\text{Speed Ratio (SR)} = \frac{N_2}{N_3} = \frac{T_3}{T_2} \quad \text{— (ii)}$$

from eqn (i) × (ii) we get.

$$\frac{N_1}{\cancel{N_2}} \times \frac{\cancel{N_2}}{N_3} = \frac{\cancel{T_2}}{T_1} \times \frac{T_3}{\cancel{T_2}}$$

$$\Rightarrow \boxed{\frac{N_1}{N_3} = \frac{T_3}{T_1}}$$

ie Speed Ratio $= \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of Teeth on driven}}{\text{No. of Teeth on driver}}$

and Train value $= \frac{\text{Speed of driven}}{\text{Speed of driver}}$

$= \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$

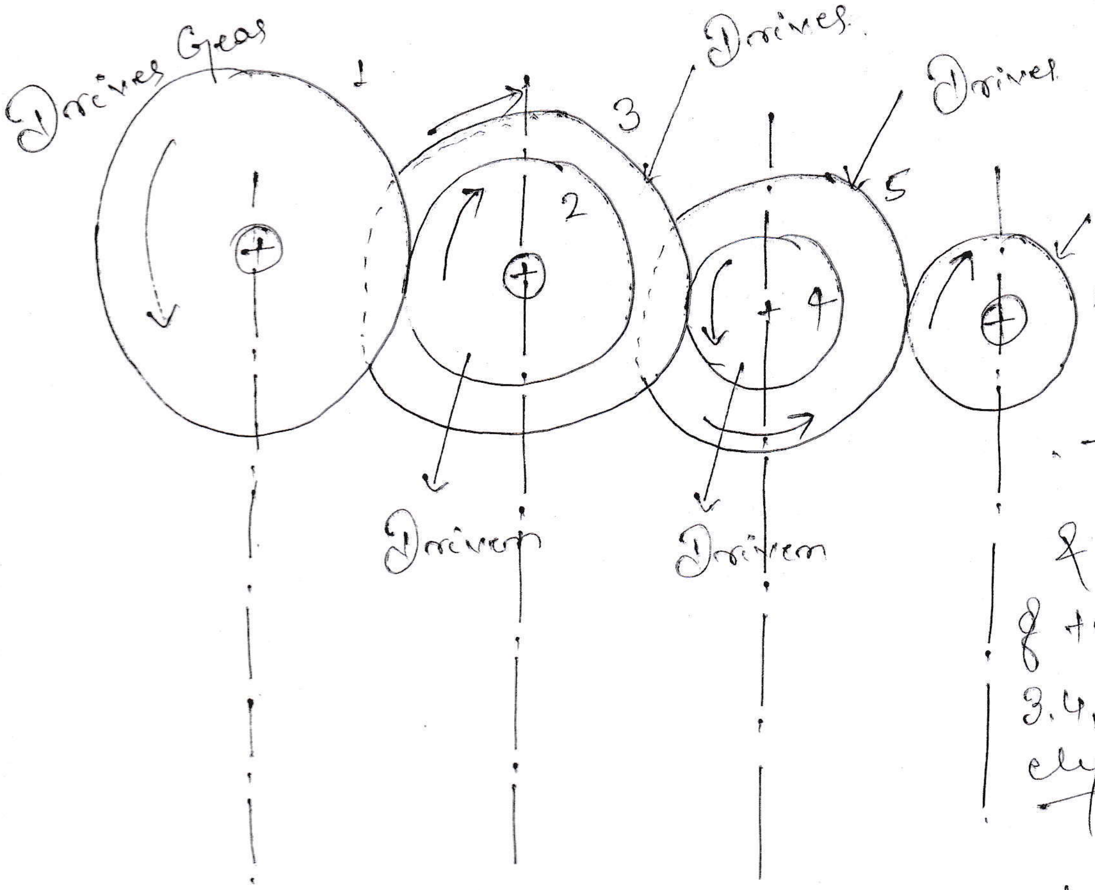
In a Simple gear Train Speed ratio is independent of intermediate number of gear. The intermediate gear is also known as idle gear. They donot effect the Speed ratio or Train value of the System.

* To Connect gears where a large Centre distance is required.

* To obtain the desired direction of motion of the driven gear (Either in Clockwise or in Anti Clockwise or Counter Clockwise direction).

⇒ Compound Gear Train :-

When there are more than one gear are mounted on a shaft is called a Compound train of gears.



T1, T2, T3, T4, T5 & T6 is the No. of teeth on gear 1, 2, 3, 4, 5 & 6 respectively.

Gear 2 & Gear 3 is Compound gear because it is mounted on the Same Shaft.

$$\therefore \boxed{N_2 = N_3} \quad \text{———} \quad (A)$$

Gear 4 & Gear 5 is also a Compound gear because it is also mounted on same another Shaft of Gear 2 & Gear 3.

$$\therefore \boxed{N_4 = N_5} \quad \text{———} \quad (B)$$

Now we obtained a speed ratio with the help of a pair of a gears.

For Gear 1 & 2.

$$\text{Speed Ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{--- (i)}$$

For Gear 3 & 4

$$\text{Speed ratio} = \frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \text{--- (ii)}$$

For Gear 5 & 6.

$$\text{Speed ratio} = \frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \text{--- (iii)}$$

From eqn (i) × (ii) × (iii) we get speed ratio of a Compound gear train.

$$\Rightarrow \frac{N_1}{\cancel{N_2}} \times \frac{\cancel{N_3}}{\cancel{N_4}} \times \frac{\cancel{N_5}}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$$[\because N_2 = N_3 \; \& \; N_4 = N_5]$$

$$\therefore \boxed{\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}}$$

$$SR = \frac{\text{Speed of Driver (1st)}}{\text{Speed of Last driven}} = \frac{\text{Product of No. of teeth on Driven Gear}}{\text{Product of No. of teeth on Driver Gear}}$$

$$\text{Train value} = \frac{1}{\text{Speed Ratio}}$$

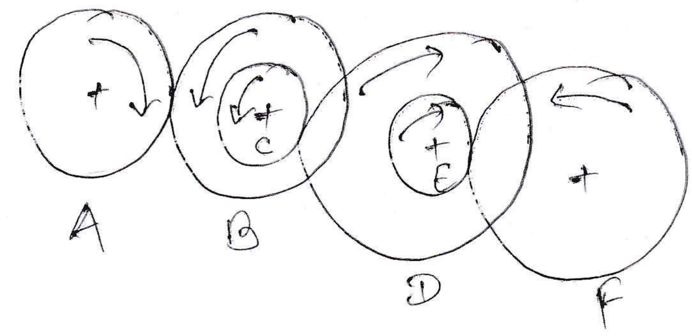
$$= \frac{\text{Speed of last driven}}{\text{Speed of first driver}}$$

$$= \frac{\text{Product of Total No. of Teeth on driver gear}}{\text{Product of Total No. of teeth on driven gear.}}$$

* The advantage of a Compound train over a Simple gear train is that a much larger Speed reduction from the first to the last shaft Can be obtained with the help of a Small gears.

Q. No-1. The gearing of a m/c tool is shown in given. The motor Shaft is Connected to gear A and rotate at 975 r.p.m. The gear wheels B, C, D and E are fixed to Parallel Shaft rotating together. The final gear is fixed on the out Put Shaft. what is the Speed of gear F? The Number of teeth on Each gear are given below.

Gear :-	A	B	C	D	E	F
No. of Teeth :-	20	50	25	75	26	65.



Soln :- Given $N_A = 975$ rpm.

$T_A = 20$, $T_B = 50$, $T_C = 25$, $T_D = 75$, $T_E = 26$

and $T_F = 65$.

Driver Gear — A, C and E

Driven " — B, D and F

we already know that.

$$\text{Speed Ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven}} = \frac{\text{Product of No. of teeth on drivens}}{\text{Product of No. of teeth on drivers}}$$

$$\Rightarrow \frac{975}{N_F} = \frac{T_B \times T_D \times T_F}{T_A \times T_C \times T_E}$$

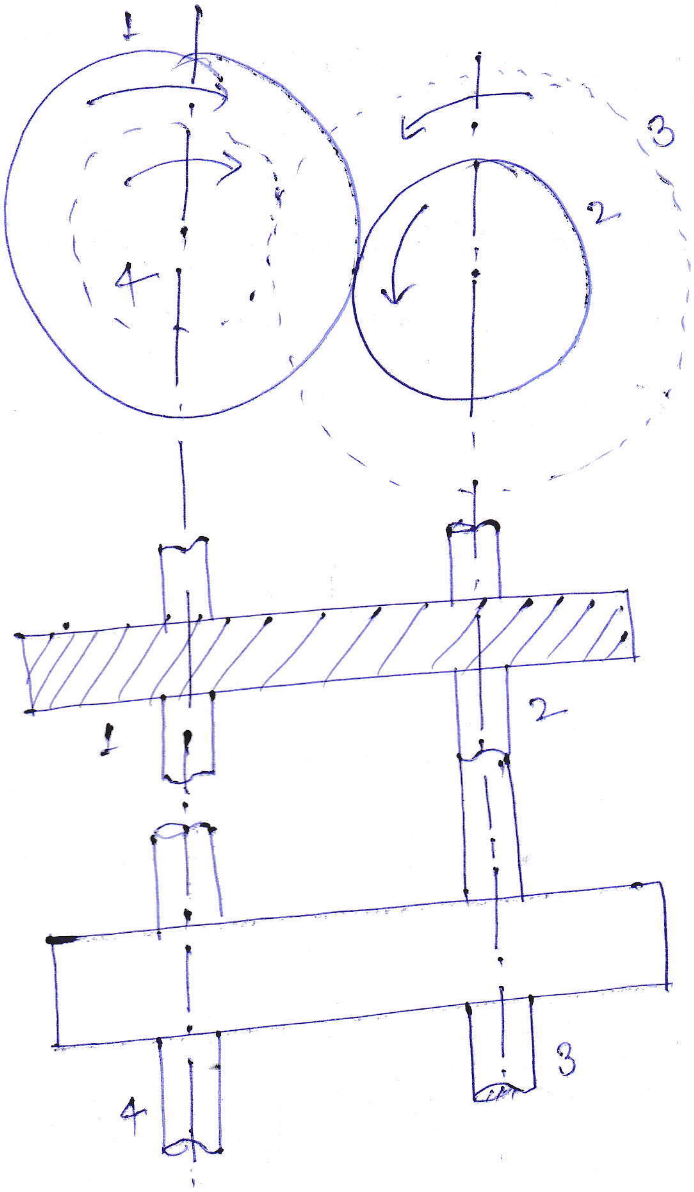
$$\Rightarrow \frac{975}{N_F} = \frac{50 \times 75 \times 65}{20 \times 25 \times 26}$$

$$\therefore N_F = \frac{975 \times \cancel{20}^{4} \times \cancel{25} \times \cancel{26}^{13}}{\cancel{50}_{2} \times 75 \times \cancel{65}_{5}} = 52 \text{ r.p.m}$$

$$\boxed{N_F = 52 \text{ rpm}}$$ Ans.

⇒ Reverted Gear Train:-

When the axes of the first gear (first driver) and the last gear (last driven or follower) are Co-axial then the gear train is known as Reverted gear Train.



Let

T_1 = Total number of teeth on gear 1

r_1 = Pitch Circle radius of gear 1

N_1 = Speed of gear 1.

Similarly T_3, T_4, T_2 = Total Number of teeth on 2, 3, 4 respectively

r_2, r_3 & r_4 = Pitch Circle radius of gear 2, 3 & 4 respectively

N_2, N_3 & N_4 : Speed of gear 2, 3, & 4 respectively

The Centre distance betⁿ Shaft 1 & 2 and Shaft 3 & 4 is Same. which is given below

$$\boxed{r_1 + r_2 = r_3 + r_4} \quad \text{——— (i)}$$

or

$$\boxed{d_1 + d_2 = d_3 + d_4}$$

when we assumed Circular Pitch and module of gear is Same. Then the Number of teeth on each gear is directly Proportional to its Circumference or Radius.

$$\boxed{T_1 + T_2 = T_3 + T_4} \quad \text{——— (ii)}$$

Speed Ratio = $\dfrac{\text{Product of Number of teeth on drivens}}{\text{Product of Number of teeth on drivers}}$

$$\boxed{S.R \quad \frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}} \quad \text{——— (iii)}$$

We already Know that

Circular Pitch $= \dfrac{\pi D}{T}$

$P_c = \dfrac{2\pi r}{T} = \pi m$ or $r = \dfrac{mT}{2}$

Similarly for $r_1 = \dfrac{mT_1}{2}$, $r_2 = \dfrac{mT_2}{2}$

$r_3 = \dfrac{mT_3}{2}$ & $r_4 = \dfrac{mT_4}{2}$

From eqn (i), we get

$$r_1 + r_2 = r_3 + r_4$$

$$\Rightarrow \frac{mT_1}{2} + \frac{mT_2}{2} = \frac{mT_3}{2} + \frac{mT_4}{2}$$

$$\therefore \boxed{T_1 + T_2 = T_3 + T_4}$$

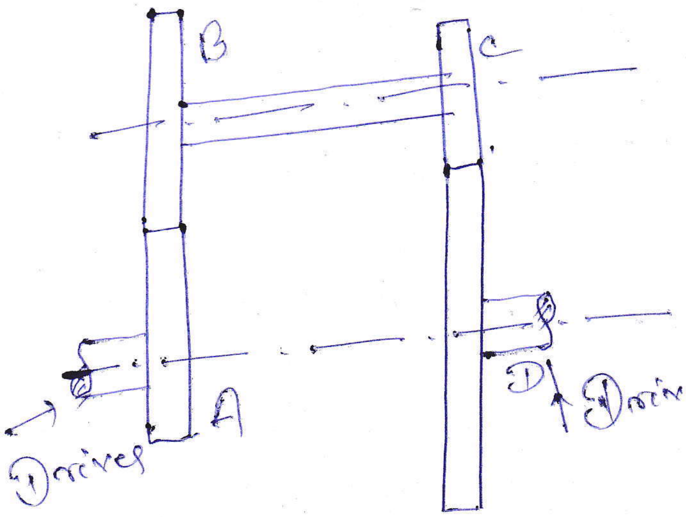
Q. No-1. The speed ratio of the reverted gear train which is shown in fig. is to be 12. The module pitch of gears A and B is 3.125 mm & gears C & D is 2.5 mm. Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth.

Soln: Given data:-

$$\frac{N_A}{N_D} = 12$$

$$m_A = m_B = 3.125 \text{ mm}$$

$$m_C = m_D = 2.5 \text{ mm}$$



Speed ratio for gear A & B is

$$S.R = \frac{N_A}{N_B} = \sqrt{12}$$

and for gear C & D

$$S.R = \frac{N_C}{N_D} = \sqrt{12}$$

'6' i.e $S.R = \frac{N_A}{N_B} = \frac{N_C}{N_D} = \sqrt{12} = 3.464$

and we already know that,

$$S.R = \frac{N_A}{N_B} = \frac{T_B}{T_A}$$

$$\frac{N_C}{N_D} = \frac{T_D}{T_C}$$

So, we get

$$\boxed{\frac{T_B}{T_A} = \frac{T_D}{T_C} = 3.464} \text{ --- (i)}$$

$$r_A + r_B = r_C + r_D = 200$$

$$\Rightarrow \frac{m_A T_A}{2} + \frac{m_B T_B}{2} = \frac{m_C T_C}{2} + \frac{m_D T_D}{2} = 200$$

$$\Rightarrow \frac{m_A T_A}{\cancel{2}} + \frac{m_B T_B}{\cancel{2}} = \frac{m_C T_C}{\cancel{2}} + \frac{m_D T_D}{\cancel{2}} = 400$$

$$\Rightarrow m_A (T_A + T_B) = m_C (T_C + T_D)$$

$$\Rightarrow 3.125 (T_A + T_B) = 2.5 (T_C + T_D) = 400$$

So, $T_A + T_B = \frac{400}{3.125}$ --- (ii)

$T_C + T_D = \frac{400}{2.5}$ --- (iii)

from above, with the help of all equation we obtained

$T_A = 28$, $T_B = 100$, $T_C = 36$ & $T_D = 124$ Ans.

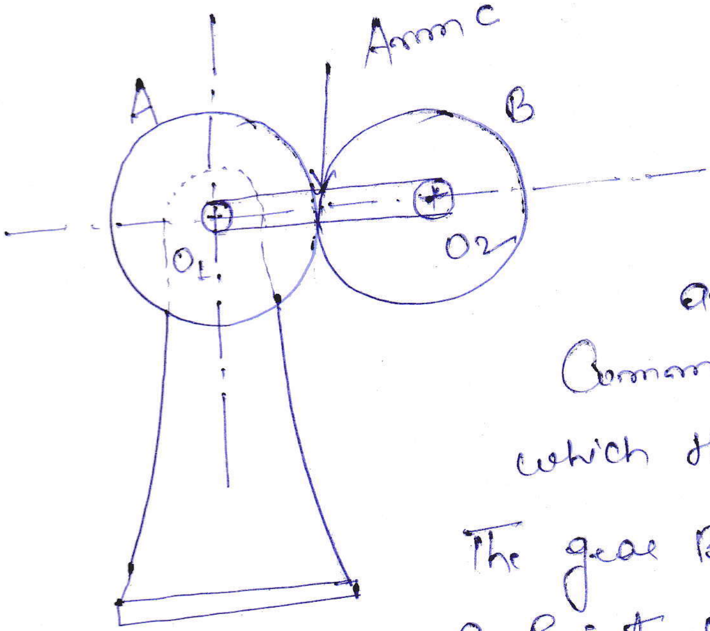
Now,

Speed ratio for reverted gear train is given below.

$$\frac{N_A}{N_D} = \frac{T_B \times T_D}{T_A \times T_C} = \frac{100 \times 124}{28 \times 36} = 12.3$$

⇒ Epicyclic Gear Train:-

The axes of the Shaft, over which the gears are mounted, may move relative to a fixed axis. The simple gear train is given below.



In this epicyclic gear train gear A and the arm C have a common axis at O_1 about which they can rotate. The gear B is also rotate abo[ut] a point O_2 which is situat[ed] on the arm C.

If Arm C is fixed, then the gear train is a Examp[le] of Simple gear train. Gear A drives gear B and Vice-Versa.

When gear A is fixed and arm C is rotated ab[out] the axis of gear A (O_1) then the gear B is forced [to] rotate upon and around gear A. Then this ty[pe] of motion is called Epicyclic.

Epi - upon → Epicyclic gear train.
cyclic - around

and the gear trains arranged in Such a manner tha[t] one or more of their member move upon and aroun[d] another member are known as Epicyclic Gear Train:-

These are two method to find velocity Ratio for of Epicyclic gear Train:-

(i) Tabular method

(ii) Algebraic method

(i) Tabular method :-

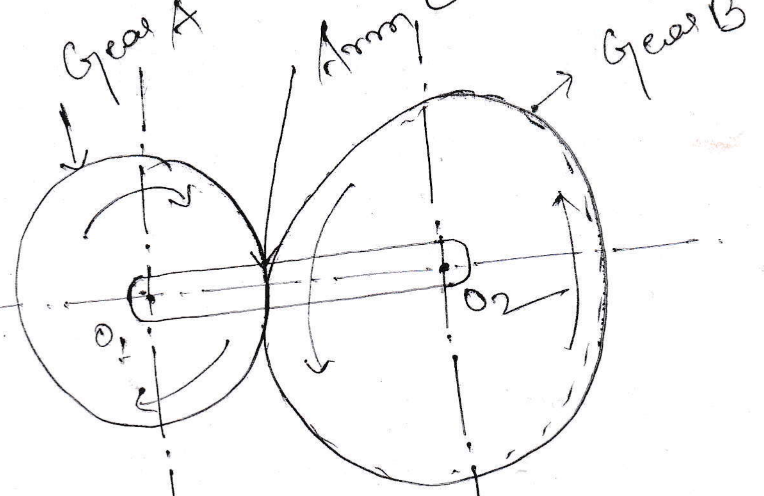
Let

N_A = Speed of Gear A

N_B = Speed of Gear B

and O_1O_2 = Arm C.

when Arm C is fixed Then Gear A and Gear B make a relative motion betn each other in opposite direction.



when Gear A rotate in Clockwise direction then gear B is driven by gear A in Anti-Clockwise direction.

when Gear A is fixed then Arm C is rotate either in Clockwise or in anticlockwise direction about fixed Point or axes of gear A. Then it form a whole mechanism which is known as Epicyclic gear train.

Table Contain Three Row and a Single row of a Total motion.

First Row.

when Arm C is fixed. Gear A make a +1 revolution in Anticlockwise direction then Gear B make $\left(-\frac{TA}{TB}\right)$ revolution in clockwise direction

Sign :-

Anticlockwise direction — +ve

Clockwise direction — -ve.

For Gear A & B.

$$S.R = \frac{NB}{NA} = \frac{TA}{TB}$$

$$\Rightarrow \boxed{NB = \frac{TA}{TB}}$$ But Sign in -ve.

Second Row.

+x revolution make by gear A and

$\left(-x\frac{TA}{TB}\right)$ revolution make by Gear B.

Third Row.

+y revolution added to Gear A then

$(x+y)$

and For Gear B. — $+y - x\left(\frac{TA}{TB}\right)$

Fourth Row :-

Addition of all motion or Total motion.

Table

S.No	Condition of Motion	Revolutions of Elements: Arm C	Gear A	Gear B
1.	Arm fixed - A rotates through +1 revolutions i.e 1 Revolution Acw	0	+1	$-\frac{T_A}{T_B}$
2.	Gear A rotates through +x revolutions	0	+x	$-x\frac{T}{T_B}$
3.	Add +y revolutions to all elements.	+y	x+y	$y-x\left(\frac{T_A}{T_B}\right)$
4.	Total motion	+y	x+y	$y-x\frac{T_A}{T_B}$

Q.NO-2 In a Simple train of wheels the motion is to be transmitted from one shaft to another in the same direction of rotation with a velocity ratio of 12. If the follower has 12 teeth, find the number of teeth on the driver and the minimum number of intermediate wheels required. If the driving shaft rot at 80 rpm, what will be speed of the driven shaft?

Soln Number of teeth on follower = $T_F = 12$

velocity ratio = 12

Speed of driving shaft = 80 r.p.m

$$\Rightarrow \text{velocity ratio} = \frac{T_{driver}}{T_{driven}} = \frac{T_D}{12}$$

$$\Rightarrow 12 = \frac{T_D}{12} \quad \therefore \underline{T_D = 144.}$$

Again

$$\text{Velocity ratio} = \frac{N_{Driven}}{N_{Driver}}$$

$$\Rightarrow 12 = \frac{N_{Driven}}{80}$$

$\therefore$ N_{Driven} (Speed of driven shaft) =

960 r.p.m. Ans.

Q.NO-3 In a Compound train of wheels, the drivers have 30, 60, 90 and 120 teeth and the followers have 12, 40, 50 and 80 teeth. If the driving shaft rotates at 120 r.p.m. find speed of the driven shaft.

Soln

Velocity ratio of Compound gear train –

$$V.R\ (\text{Speed ratio}) = \frac{\text{Product of Numbers of teeth on drivers}}{\text{" " " " driven}}$$

$$\Rightarrow V.R = \frac{30 \times 60 \times 90 \times 120}{12 \times 40 \times 50 \times 80}$$

$$= 10.125$$

Again,

$$V.R = \frac{\text{Speed of driven shaft}}{\text{Speed of driver shaft}}$$

$$\Rightarrow 10.125 = \frac{N_{Driven}}{120}$$

$$\therefore N_{Driven} = 120 \times 10.125$$

$$= 1215 \text{ r.p.m Ans.}$$

Q.NO-4 The lead of a lathe has a right handed single start thread of 0.5 cm pitch. The smallest change wheel has 20 teeth, the largest 100 teeth and the numbers of teeth on intermediate size increases in steps of 5 teeth. find a suitable gear train for connecting the spindle and the lead screw if 11 thread per cm has to be cut.

Soln. The required screw is right hand and has 11 ~~teeth~~ thread per cm. it follows the spindle must make 11 revolution while the saddle moves 1 cm towards the head stock while the lead screw moves 2 revolutions since it has a pitch of 0.5 cm.

Hence,

Speed of spindle (N_s) = 11 revolutions

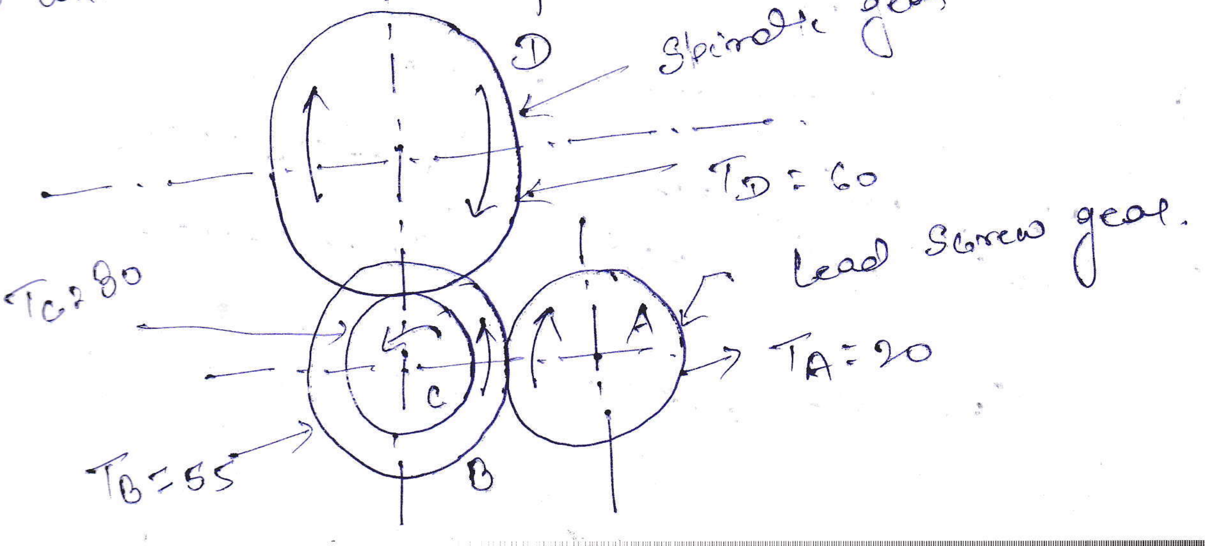
.. lead screw (N_L) = 2 "

$$\therefore \frac{N_L}{N_s} = \frac{2}{11} = \frac{5}{55} \times \frac{2}{1}$$

wheel with 55 teeth has been choosen since this is the only available in given set of wheel whose factor is 11.

$$\therefore \frac{N_L}{N_s} = \frac{20}{55} \times \frac{30}{60}$$

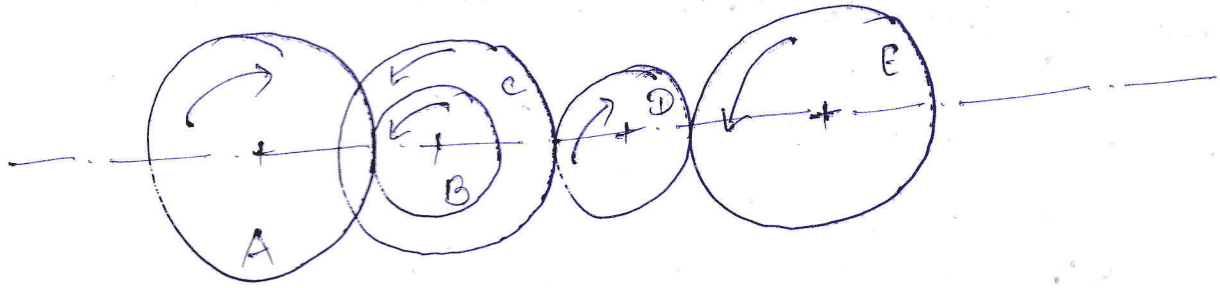
Now wheel would be arranged in given form.



Q.No (4) Fig. shows a compound gear train in which wheel A is fixed to the driving shaft, B-C are compounded, D is idle and E is fixed on the driven shaft. If gear A rotates at 1000 r.p.m. find the speed and direction of E.

(Take $T_A = 20$, $T_B = 15$, $T_C = 30$, $T_D = 15$ and $T_E = 40$.

Soln:-



Speed ratio for gear A & B

$$\frac{N_A}{N_B} = \frac{T_B}{T_A} \quad \text{--- (i)}$$

for gear C & D

$$\frac{N_C}{N_D} = \frac{T_D}{T_C} \quad \text{--- (ii)}$$

and for gear

$$\frac{N_D}{N_E} = \frac{T_E}{T_D} \quad \text{--- (iii)}$$

from eqn (i) x (ii) x (iii) we get,

$$\Rightarrow \frac{N_A}{\cancel{N_B}} \times \frac{\cancel{N_C}}{\cancel{N_D}} \times \frac{\cancel{N_D}}{N_E} = \frac{T_B \times T_D \times T_E}{T_A \times T_C \times T_D}$$

($N_B = N_C$ due to compound gear train)

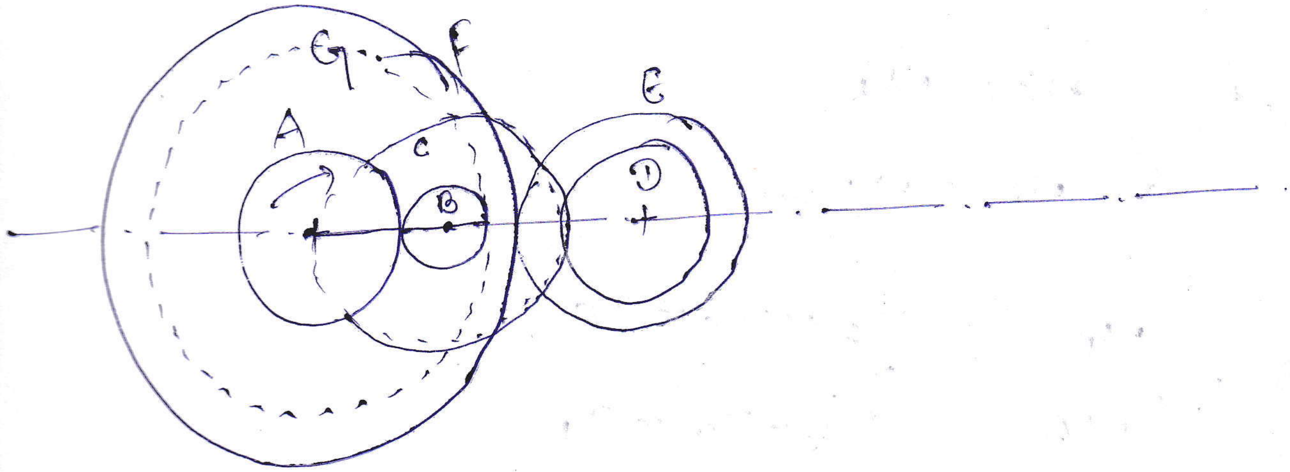
$$\Rightarrow \frac{N_A}{N_E} = \frac{T_B \times T_D \times T_E}{T_A \times T_C \times T_D}$$

$$\Rightarrow \frac{1050}{N_E} = \frac{\cancel{15} \times \cancel{15} \times \cancel{40}\ 20}{\cancel{20} \times \cancel{30} \times \cancel{15}}$$

$$\therefore \boxed{N_E = 1050 \text{ r.p.m}}$$

Q.No.5 Fig. shows a Compound gear train, wheel A drives the wheel B. B and C are Compounded. wheel E & D are Compounded. wheel E drives the wheel F and G is an internal wheel which is driven by wheel B. If A runs at 480 r.p.m. find the speed of gear G & E. Gear G & F have the same axis as A but they are mounted on different shafts.

Take – $T_A = 40$, $T_B = 20$, $T_C = 50$, $T_D = 20$, $T_E = 40$ and $T_F = 90$.



Since T_G is not given.

Now, $R_A + D_B = R_G$

or, $D_A + 2D_B = D_G$

or, $T_A + 2T_B = T_G$ (Since Pitch of mating gear is same)

$\Rightarrow$ $40 + 2\times 20 = T_G$

$\therefore \underline{T_G = 80}$.

For motion of A-B-G

$\Rightarrow$ $\frac{N_G}{N_A} = \frac{T_A}{T_B} \times \frac{T_B}{T_G}$

$\Rightarrow$ $\frac{N_G}{480} = \frac{\cancel{40} \times \cancel{20}}{\cancel{20} \times \cancel{80}_2}$

$\therefore N_G = 240$ r.p.m Ans

Now, Consider the motion of gear (A-B-C-D-E-F).

$$\frac{N_B}{N_A} \times \frac{N_D}{N_C} \times \frac{N_F}{N_E} = \frac{T_A}{T_B} \times \frac{T_C}{T_D} \times \frac{T_E}{T_F}$$

But $N_C = N_B$

& $N_D = N_E$

$\therefore \frac{N_F}{N_A} = \frac{T_A \times T_C \times T_E}{T_B \times T_D \times T_F}$

$\Rightarrow$ $\frac{N_F}{480} = \frac{40 \times 50 \times 40}{20 \times 20 \times 90}$ $\therefore N_F = 1066.7$

$= 1067$ r.p.m