KOM. UNIT – ~~3rd~~ III (FRICTION) A.K.

When one block moves or tends to move tangentially with respect to the suface, on which it rests, the inter locking property of the projecting particles oppose the motion. This opposing force, which acts in the opposite direction of the movement of the upper block is called the force of friction or simply friction.

→ Static friction :– When friction experienced by body, when at rest.

→ Dynamic friction :– When the friction experienced by a body when in motion. It is also called kinetic friction. Dynamic friction is less than static friction.

Dynamic friction.

↓

Sliding friction :– When the friction experienced by a body, when it slides over another body.

Rolling friction. The friction experienced betn the sufaces which has ball's or rollers interposed betn them.

↓

Pivot friction :– The friction experienced by a body due to the motion of rotation as in case of foot step bearing.

Dry or solid friction :– The friction experienced betn two dry or unlubricated surfaces in contact is known as dry or solid friction.

friction betn lubricated Surfaces:-

(i) Boundary friction or greasy friction or Non-viscous friction:-

The friction experienced betn the rubbing surfaces, when the surfaces have a very thin layer of lubricant. The thin layer of lubricant forms a bond betn the two rubbing surfaces. The boundary friction follows the laws of solid friction.

(ii) Fluid friction (film friction or viscous friction) when the surfaces have a thick layer of the lubricant betn rubbing surfaces. The actual surface do not come in contact with each other. It is mainly due to the viscosity and oiliness of the lubricant.

· The lubricant which gives lower force of friction is said to have greater oiliness.

→ When a body just begins to slide over the surface of the other body is known as limiting force of friction or limiting friction.

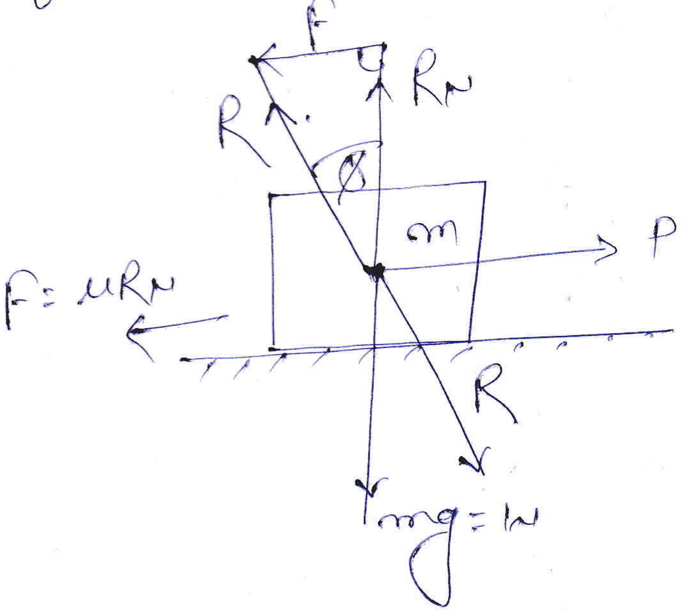
→ When the applied force is less than limiting friction, the body remains at rest. and that friction is called static friction.

→ The force of friction is different for different materials.

⇒ Coefficient of friction :- The ratio of the limiting friction (F) to the normal reaction (R_N) betn the two bodies. it is denoted by μ.

$$\mu = \frac{F}{R_N} \quad \therefore \boxed{F = \mu R_N}$$

⇒ Limiting angle of friction :- (ϕ)



From given diagram

$$\tan\phi = \frac{F}{R_N}$$

$$= \frac{\mu \cancel{R_N}}{\cancel{R_N}} = \mu$$

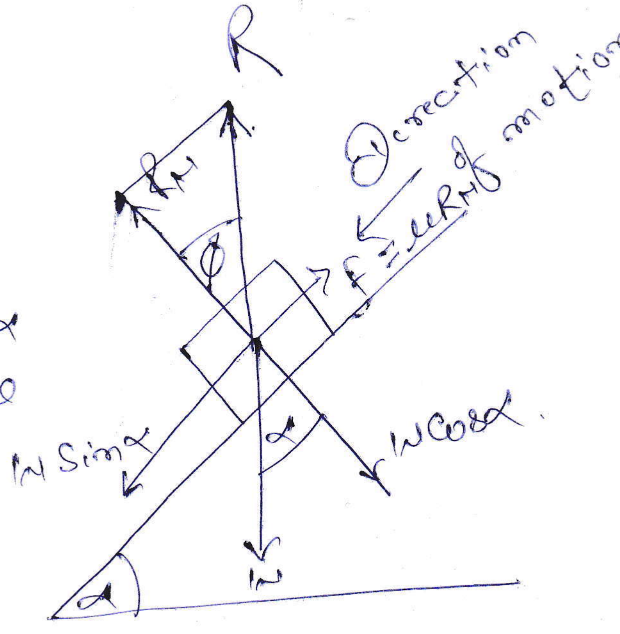
$$\therefore \boxed{\tan\phi = \mu}$$

OR

$$\boxed{\phi = \tan^{-1}(\mu)}$$

⇒ Angle of Repose :-

If the angle of inclination α of the plane to the horizontal is such that the body begins to move down the plane, Then the angle α is known as Angle of Repose.



Body only move on inclined plane due $W\sin\alpha$ Component.

$\Rightarrow W\sin\alpha = F$

$\Rightarrow \quad " \quad = \mu R_N$

$\Rightarrow \cancel{W}\sin\alpha = \mu \cancel{W}\cos\alpha.$

$\Rightarrow \tan\alpha = \mu$

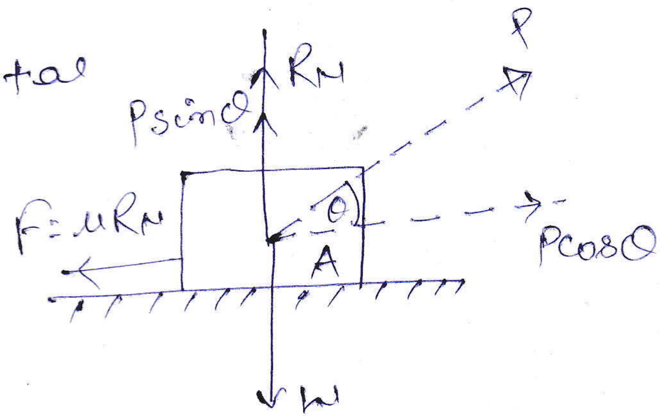
$= \tan\phi$

$\therefore \boxed{\alpha = \phi}$ Angle of Repose = limiting friction ang

For body move in downward direction on the inclined plane.

⇒ Minimum force required to slide a body on a rough Horizontal plane.

When an effort P is applied at an angle θ from horizontal surface such that block A just move.



Now for eqn for the body A.

$$R_N + P\sin\theta = W$$

$$R_N = W - P\sin\theta \quad \text{—— (i)}$$

$$P\cos\theta = F = \mu R_N \quad \text{—— (ii)}$$

from eqn (i) & (ii), we get

$$P\cos\theta = \mu R_N$$

$$= \mu(W - P\sin\theta)$$

$$= \tan\phi\,(W - P\sin\theta)$$

$$\therefore \boxed{P = \frac{W\sin\phi}{\cos(\theta-\phi)}}$$

For P be the minm, $\cos(\theta-\phi)$ will be maxm

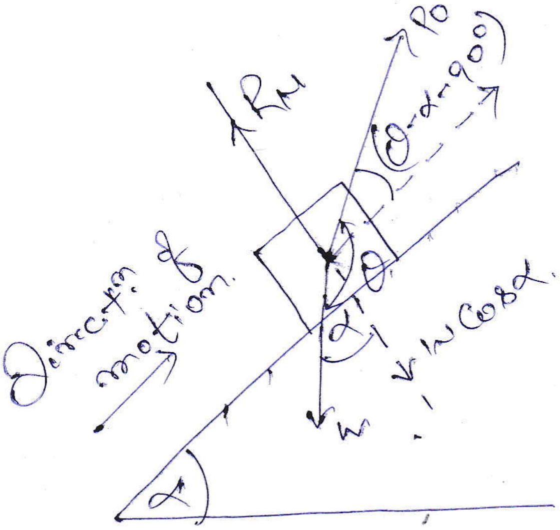
$$\cos(\theta-\phi) = 1$$

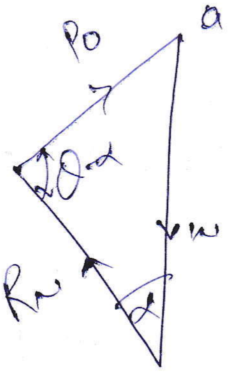
$$\Rightarrow \theta - \phi = 0$$

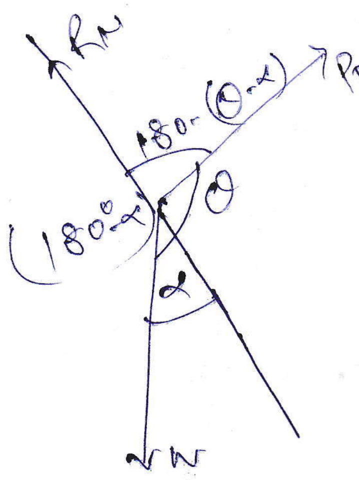
$$\therefore \theta = \phi$$

$$\therefore \boxed{P_{minm} = W\sin\theta}$$

(i) When the friction is neglected. (For motion up the Plane)





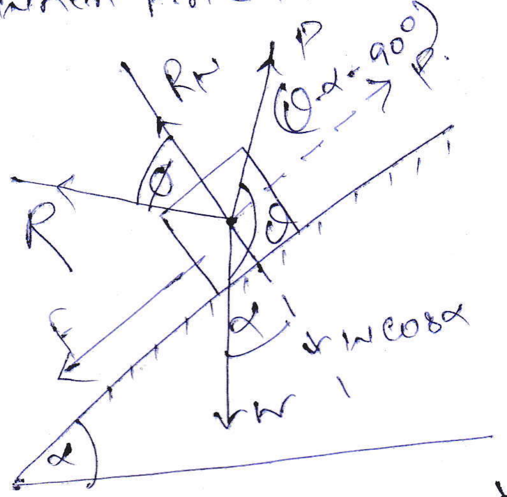


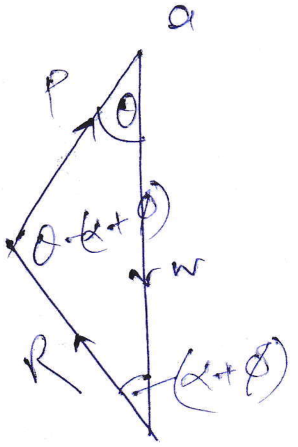
Applying Sine Rule for these three Concurrent forces.

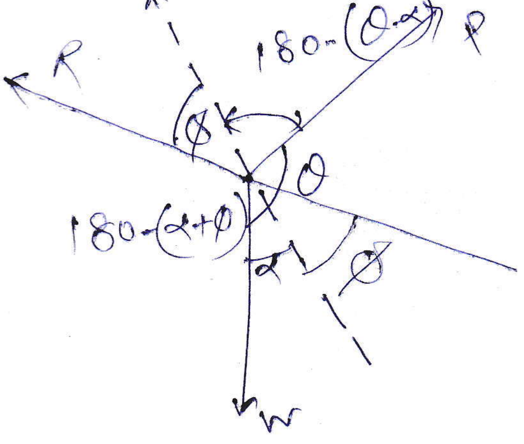
$$\frac{P_0}{\sin(180^\circ - \alpha)} = \frac{W}{\sin[180^\circ - (\theta - \alpha)]}$$

$$\Rightarrow \frac{P_0}{\sin\alpha} = \frac{W}{\sin(\theta - \alpha)} \quad \therefore \boxed{P_0 = \frac{W \sin\alpha}{\sin(\theta - \alpha)}}$$

(ii) When Friction is Considered :-







Apply Sine Rule, we get

$$\Rightarrow \frac{P}{\sin[180^\circ - (\alpha+\phi)]} = \frac{W}{\sin[\theta - (\alpha+\phi)]}$$

$$\Rightarrow \boxed{\frac{P}{\sin(\alpha+\phi)} = \frac{W}{\sin[\theta - (\alpha+\phi)]}}$$

$$\therefore \boxed{P = \frac{W \sin(\alpha+\phi)}{\sin[\theta - (\alpha+\phi)]}}$$

Case - when the effort applied is horizontal $\theta = 90^\circ$.

Then,

$$\boxed{P_0 = W \tan\alpha}$$

$$\& \quad \boxed{P = W \tan(\alpha+\phi)}$$

when the effort applied is along the inclined plan

$$\theta = (90^\circ + \alpha)$$

Then

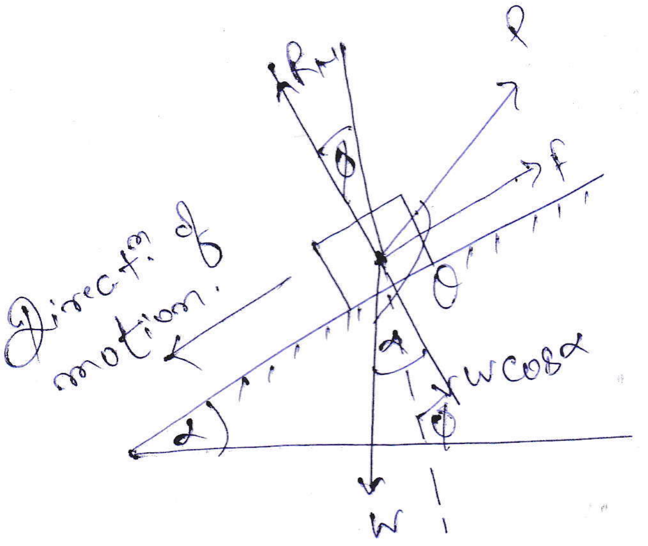
$$\boxed{P_0 = W \sin\alpha}$$

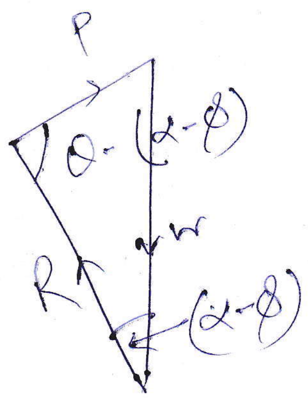
$$\& \quad \boxed{P = W(\sin\alpha + \mu \cos\alpha)}$$

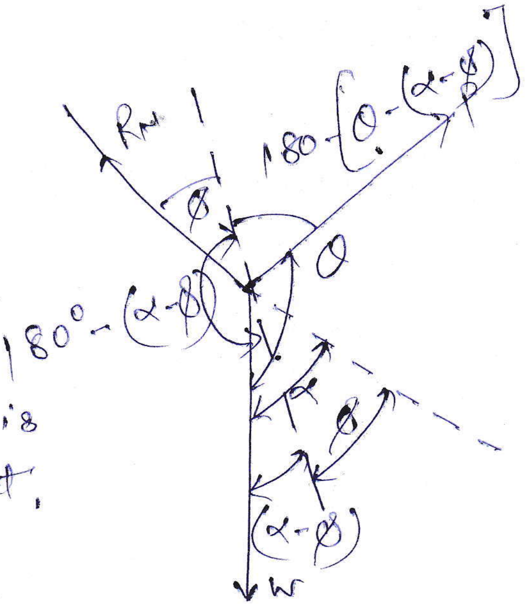
where $\tan\phi = \mu$

⇒ Considering the motion of the body down the plane

$$P_0 = \frac{W \sin\alpha}{\sin(\theta-\alpha)}$$







when the friction is taken into account,

$F = \mu R_N$

Now from Sine rule

$$\frac{P}{\sin[180^\circ-(\alpha-\phi)]} = \frac{W}{\sin[\theta-(\alpha+\phi)]}$$

$$\therefore \boxed{P = \frac{W \sin(\alpha-\phi)}{\sin[\theta-(\alpha-\phi)]}}$$

when the P is applied horizontally $\theta = 90^\circ$.

Then

$$P = \frac{W \sin(\alpha-\phi)}{\sin[90^\circ-(\alpha-\phi)]} = W \tan(\alpha-\phi)$$

When P is applied parallel to inclined plane

then $\theta = (90^\circ + \alpha)$.

Then
$$P = \frac{W \sin(\alpha \pm \phi)}{\sin[(90^\circ + \alpha) - (\alpha \mp \phi)]} = \frac{W \sin(\alpha \pm \phi)}{\cos\phi}$$

$$= W\left(\frac{\sin\alpha \cdot \cos\phi \pm \cos\alpha \cdot \sin\phi}{\cos\phi}\right)$$

$$= W(\sin\alpha \pm \cos\alpha \cdot \tan\phi)$$

$$\boxed{P = W(\sin\alpha - \mu\cos\alpha)}$$

⇒ Efficiency of inclined plane:— The ratio of the effort required neglecting friction (P_0) to the effort required considering friction (P).

$$\eta = \left(\frac{P_0}{P}\right)$$

For the motion of the body up the plane

$$\eta = \frac{P_0}{P} = \frac{W \sin\alpha}{\sin(\theta - \alpha)} \times \frac{\sin[\theta - (\alpha + \phi)]}{W \sin(\alpha + \phi)}$$

$$\boxed{\eta = \frac{\cot(\alpha + \phi) - \cot\theta}{\cot\alpha - \cot\theta}}$$

when the effort applied horizontally $\theta = 90°$

$$\boxed{\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}}$$

when the effort applied parallel to the inclined plane

$$\theta = (90° + \alpha)$$

$$\therefore \boxed{\eta = \frac{\sin\alpha \cdot \cos\phi}{\sin(\alpha+\phi)}}$$

(ii) Motion of the body down the plane.

for this case P will be less than Po.

$$\eta = \left(\frac{P}{P_0}\right)$$

$$= \frac{W \sin(\alpha-\phi)}{\sin[\theta-(\alpha-\phi)]} \times \frac{\sin(\theta-\alpha)}{W \sin\alpha}$$

$$\boxed{\eta = \frac{\cot\alpha - \cot\theta}{\cot(\alpha-\phi) - \cot\theta}}$$

when $\theta = 90°$

$$\therefore \boxed{\eta = \frac{\tan(\alpha-\phi)}{\tan\alpha}}$$

when $\theta = 90° + \alpha$

$$\therefore \boxed{\eta = \frac{\sin(\alpha-\phi)}{\sin\alpha \cdot \cos\phi}}$$

⇒ Screw friction:-

(i) Helix:- The Curve traced by a Particle, while describing a Circular Path at a uniform speed and advancing in the axial direction at a uniform rate.

OR

It is the Curve traced by a Particle, while moving along a Screw thread.

(ii) Pitch:- The distance from a Point of a Screw to Corresponding Point on the next thread. It is measured Parallel to the axis of the Screw.

(iii) Lead:- It is the distance, a Screw thread advances axially in one Turn.

(iv) Depth of thread:- The distance betn Top and bottom surfaces of a thread is known as depth of thread.

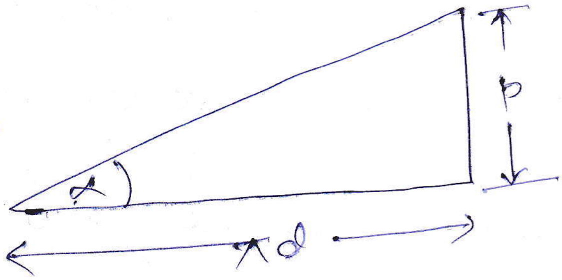
or Distance between Crest and Root of a thread

(v) Multi-threaded Screw:- If more than one thread is Cut in one lead distance of a Screw, it is known as multi-threaded Screw.

$$\boxed{\text{Lead} = \text{Pitch} \times \text{Numbers of threads}}$$

(vi) Single-threaded Screw:- If the lead of a Screw equal to its Pitch, it is known as Single-threaded Screw.

(vii) Helix angle:- Slop or inclination of the thread with the horizontal. is known as helix angle.

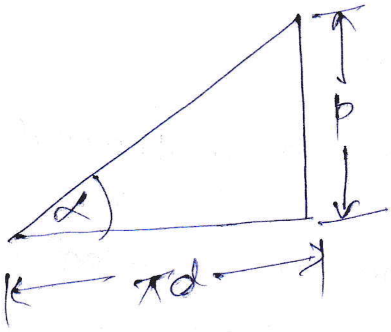


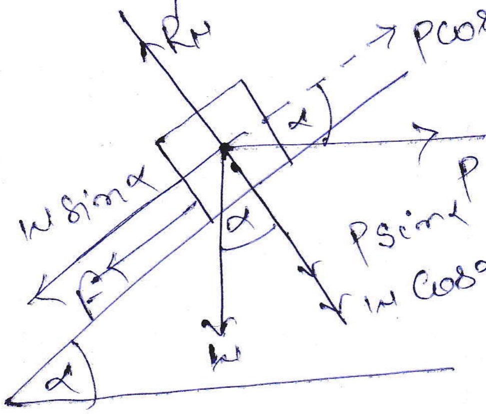
$$\tan\alpha = \frac{\text{Lead of Screw}}{\text{Circumference of Screw}}$$

$$\boxed{\tan\alpha = \frac{p}{\pi d}}$$ (For single threaded screw)

$$\boxed{\tan\alpha = \frac{n.p}{\pi d}}$$ (For multi threaded screw)

→ Torque Required to lift the load by a screw jack.





Force acting on the Screw.

From the given diagram.

$$P\cos\alpha = W\sin\alpha + F$$

$$= W\sin\alpha + \mu R_N$$

$$= W\sin\alpha + \mu \overset{R_N}{\cancel{\cos\alpha}}$$

$$\therefore P\cos\alpha = W\sin\alpha + \mu$$

$$\therefore \boxed{P\cos\alpha = W\sin\alpha + \mu \overset{R_N}{\cancel{\cos\alpha}}} \quad \text{—— (i)}$$

Now,

$$R_N = W\cos\alpha + P\sin\alpha \quad \text{—— (ii)}$$

From eqn (i) & (ii) we get $P = W \times \frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha}$

$= W \times \frac{\sin\alpha + \tan\phi.\cos\alpha}{\cos\alpha - \tan\phi\sin\alpha}$

$$\boxed{P = W\tan(\alpha+\phi)}$$

∴ Torque required to overcome friction betn the screw and Nut.

$$\boxed{T_1 = P\times\frac{d}{2} = W\tan(\alpha+\phi)\cdot\frac{d}{2}}$$

when the axial load is taken up by a thrust collar on a flat surface, so the load can not rotate with the screw.

$$T_2 = \mu_1 W\left(\frac{R_1+R_2}{2}\right)$$

$$\boxed{T_2 = \mu_1 W R}$$

R_1 & R_2 = outside and inside radii of the collar

R = Mean radius of the collar

μ_1 = Coefficient of friction for the collar.

Total Torque $T = T_1 + T_2$

$$\boxed{T = P\times\frac{d}{2} + \mu_1 W R}$$

$$\Rightarrow \boxed{T = P\times\frac{d}{2} = P_1\times l}$$

l = length of the arm or lever.

P_1 = Effort applied at the end of the lever.

Mean diameter of the Screw

$$d_{(mean)} = \frac{d_o + d_c}{2} =$$

$$d_{(mean)} = d_c + \frac{p}{2}$$

$$d_{(mean)} = d_o - \frac{p}{2}$$

Mechanical advantages (M.A) : $\frac{\text{Load lifted}}{\text{Effort applied}}$

$$= \frac{W}{P_1} = \frac{W}{\frac{Pd}{2l}}$$

$$= \frac{2Wl}{Pd} = \frac{\cancel{W} \times 2l}{\cancel{W} \tan(\alpha+\phi) \cdot d}$$

$$\Rightarrow \boxed{M.A = \frac{2l}{\tan(\alpha+\phi) \cdot d}}$$

⇒ Torque required to lower the load by a Screw Jack :→

$$T = P \times \frac{d}{2}$$

$$\boxed{T = W \tan(\phi - \alpha) \times \frac{d}{2}}$$

when $\alpha > \phi$.

Then $\boxed{T = W \tan(\alpha - \phi) \times \frac{d}{2}}$

⇒ Efficiency of a Screw Jack = It is the ratio bet Ideal effort and Actual effort.

$$\eta = \frac{\text{Ideal Effort}}{\text{Actual Effort}}$$

$$\therefore \boxed{\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}}$$

for maxm efficiency of a Screw jack is given below.

$$\boxed{\eta_{max} = \left(\frac{1-\sin\phi}{1+\sin\phi}\right)}$$

⇒ over Hauling and Self locking screws:—

we already know that

$P = W\tan(\phi-\alpha)$. for lowering case (or lower the load.

$$\text{Torque}(T) = P \times \frac{d}{2}$$

$$= W\tan(\phi-\alpha) \times \frac{d}{2}$$

when $\phi < \alpha$ in the above eqn Then the Torque required to lower the load is −ve.

Now, The load will start moving downward without the application of any Torque. Then that condition is known as overhauling of Screw:

If $\phi > \alpha$, Then torque required to lower the load is +ve. and some torque is applied to lower the load. Such condition is known as Self locking of screw.

In case of self locking friction angle > Helix angle

(μ) $\tan\phi$ Coefficient of friction > ($\tan\alpha$)(tangent of helix angle)

Ideal effort :- The effort required to move the load in case of neglecting friction.

Actual effort :- The effort required to move the load in presence of friction.

$$\eta = \frac{P_0}{P} = \frac{\cancel{W} \tan\alpha}{\cancel{W} \tan(\alpha+\phi)}$$

$$\boxed{\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}}$$

Similarly, in Torque form,

$$\eta = \frac{T_0}{T} = \frac{P_0 \times \frac{d}{2}}{P \times \frac{d}{2} + \mu_1 W R}$$

and again efficiency is defined in the form of mechanical advantage & velocity Ratio.

$$M \cdot A = \frac{W}{P_1} = \frac{W \times 2l}{P \times d} = \frac{\cancel{W} \times 2l}{\cancel{W} \tan(\alpha+\phi) \times d}$$

$$\boxed{M \cdot A = \frac{2l}{\tan(\alpha+\phi) \cdot d}}$$

$$\text{Velocity Ratio} = \frac{\text{Distance move by the effort } P_1 \text{, in one revolution}}{\text{Distance moved by the load } W \text{, in one revolution.}}$$

$$= \frac{2\pi l}{p} = \frac{2\cancel{\pi} l}{\cancel{\pi} d \times \tan\alpha} = \frac{2l}{\tan\alpha \cdot d}$$

$$\therefore \eta = \frac{M \cdot A}{V \cdot R} = \frac{\cancel{2l}}{\tan(\alpha+\phi) \cdot \cancel{d}} \times \frac{\tan\alpha \cdot \cancel{d}}{\cancel{2l}}$$

⇒ Efficiency of a self locking screw :-

we already know that

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

for self locking screw

$$\phi \geq \alpha \quad , \quad \text{or} \quad \alpha \leq \phi$$

$$\eta \leq \frac{\tan\alpha}{\tan(\alpha+\phi)} \leq \frac{\tan\phi}{\tan(\phi+\phi)} \leq \frac{\tan\phi}{\tan(2\phi)}$$

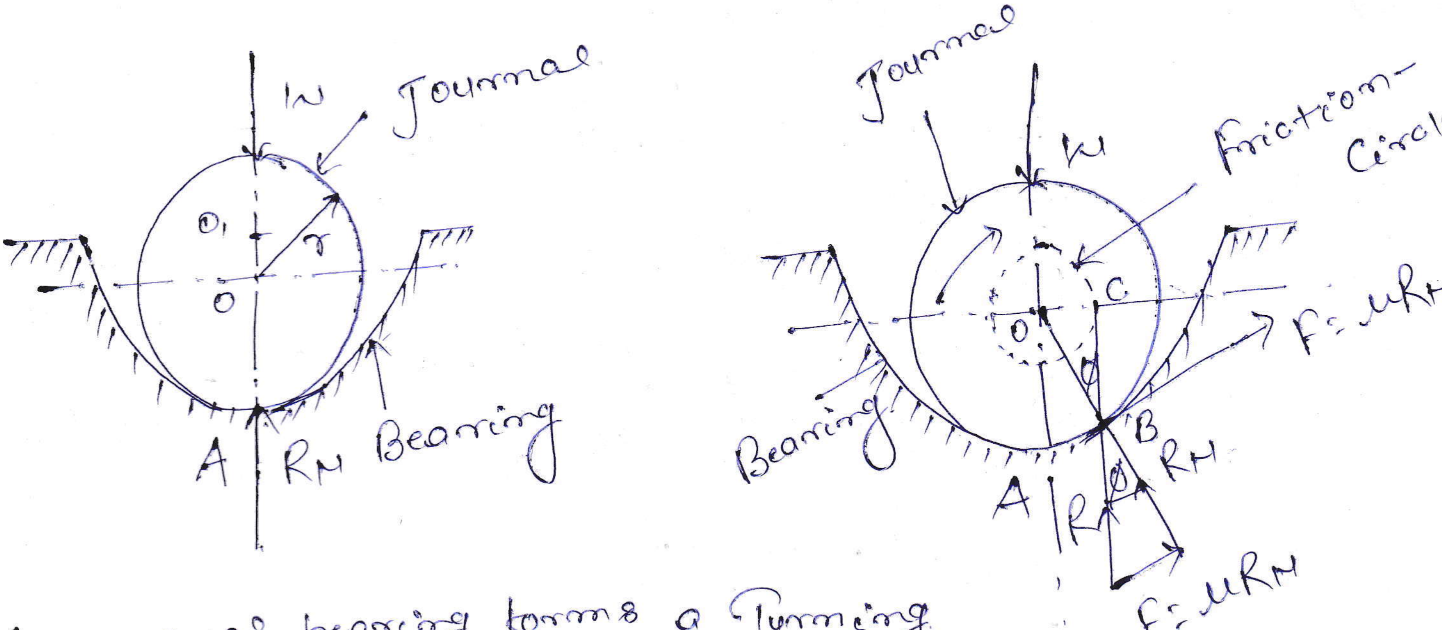
$$\leq \frac{\tan\phi\,(1-\tan^2\phi)}{2\tan\phi}$$

$$\leq \cancel{\frac{\tan\phi}{2\tan\phi}} - \frac{\cancel{\tan^3\phi}\ \tan^2\phi}{2\,\cancel{\tan\phi}}$$

$$\boxed{\eta \leq \frac{1}{2} - \frac{\tan^2\phi}{2}}$$

From the above expression efficiency of self locking screw is less than ½ or 50%. If the η is more than 50%, then screw is known as overhauling screw.

⇒ Friction in Journal Bearing - Friction Circle :-



A Journal bearing forms a Turning pair. The fixed outer element of a turning pair is called a bearing and that portion of the inner element (shaft) which fit in the bearing is called a Journal.
The diameter of Journal is slightly less than the dia of given bearing.

→ when the bearing is not lubricated (or the journal stationary), then their is a line contact between the two elements which is given in 1st diagram.
The load W on the journal and normal reaction R_N of the bearing acts through the centre. The reaction R_N acts vertically upward at point A. This point is known as Seat or Point of Pressure.

For uniform motion, The resultant force acting the shaft must be zero and the resultant turning moment on the shaft must be zero.

$\therefore\ R = W,\ \ \& \ \ T = W \times OC = W \times OB \sin\phi$

$= Wr \sin\phi$

ϕ is very small then $\sin\phi \to \tan\phi$

$= Wr \tan\phi$

$= \mu W r$

$\therefore \boxed{T = \mu W r}$ N-m

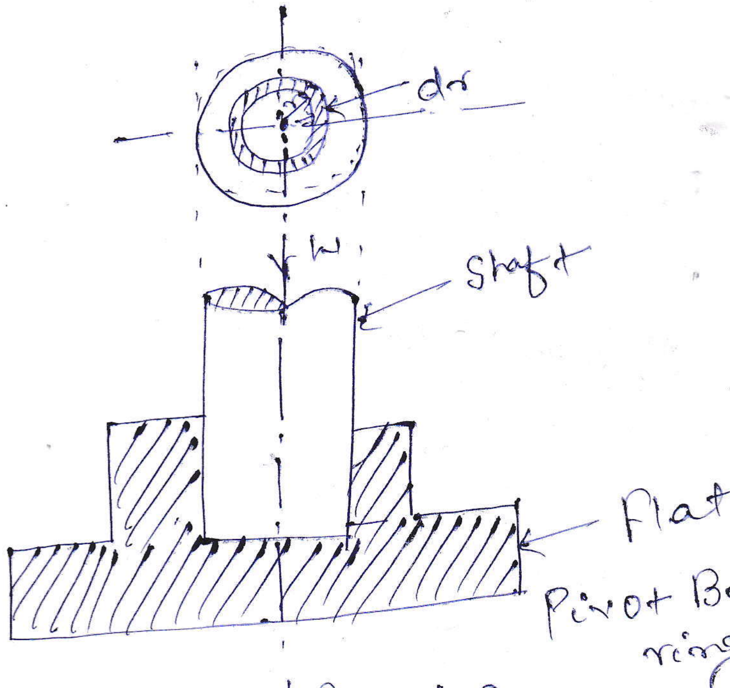
again, Power wastes in friction

$P = T \times \omega$

$= T \times \frac{2\pi N}{60}$ watt

where N = speed of the shaft in r.p.m.

⇒ Flat Pivot Bearing :-



Considering uniform Pressure

Total frictional torque,

$$\boxed{T = \frac{2}{3}\mu W R}$$ N-m

Power lost in friction

$P = T \times \omega = \frac{2\pi N}{60}$ watt. OR footstep Bearing.

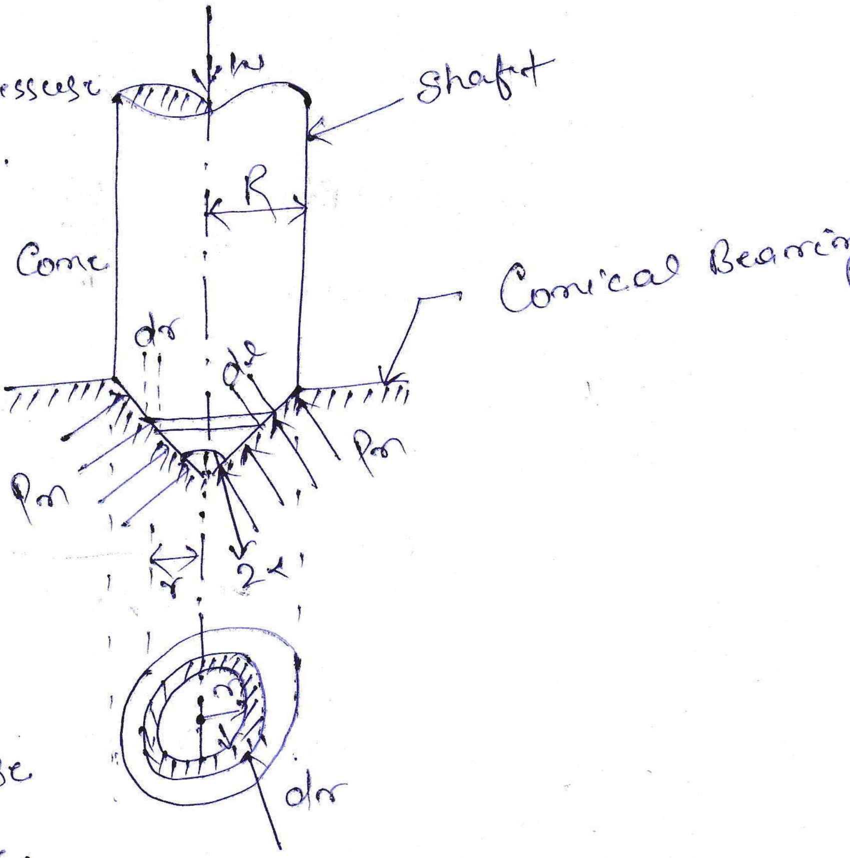
Considering uniform wear :-

Total frictional Torque $\boxed{(T) = \frac{1}{2}\mu W R}$ N-m

& $\boxed{P = T \times \omega}$ watt

$= T \times 2\pi N/60$ watt.

⇒ Conical Pivot Bearing :-



Let p_n = Intensity of Pressure normal to the Cone.

α : Semi angle of the Cone

μ = Coefficient of friction betn shaft & Bearing.

R = Radius of the shaft.

Considering uniform Pressure

Total frictional torque

$$\boxed{T = \frac{2}{3} \mu W R \operatorname{cosec}\alpha}$$

→ If slant length (l) of the cone is known then

$$\boxed{T = \frac{2}{3} \mu W l}$$

Considering uniform wear :-

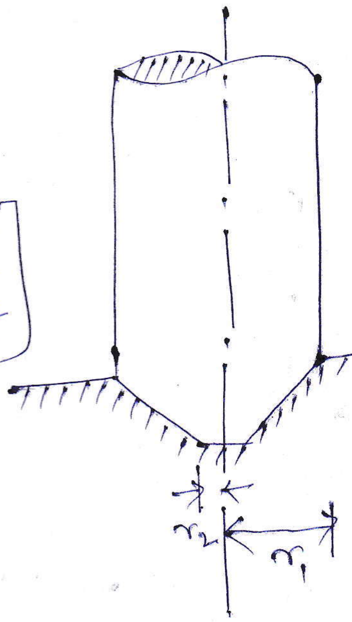
Total frictional torque

$$\boxed{T = \frac{1}{2} \mu W R \operatorname{cosec}\alpha}$$

OR

$$\boxed{T = \frac{1}{2} \mu W l}$$

⇒ Trapezoidal or Truncated Conical Pivot Bearing



Total torque acting on the bearing

Considering uniform pressure

$$T = \frac{2}{3}\,\mu W \cdot \operatorname{cosec}\alpha \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

Consider uniform wear

$$T = \frac{1}{2}\,\mu W \left(\frac{r_1 + r_2}{2}\right) \operatorname{cosec}\alpha$$

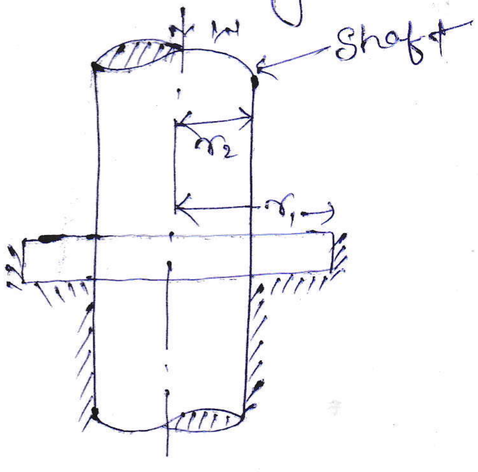
$$\boxed{T = \mu W R \operatorname{cosec}\alpha}$$

where R = Mean radius of the bearing.

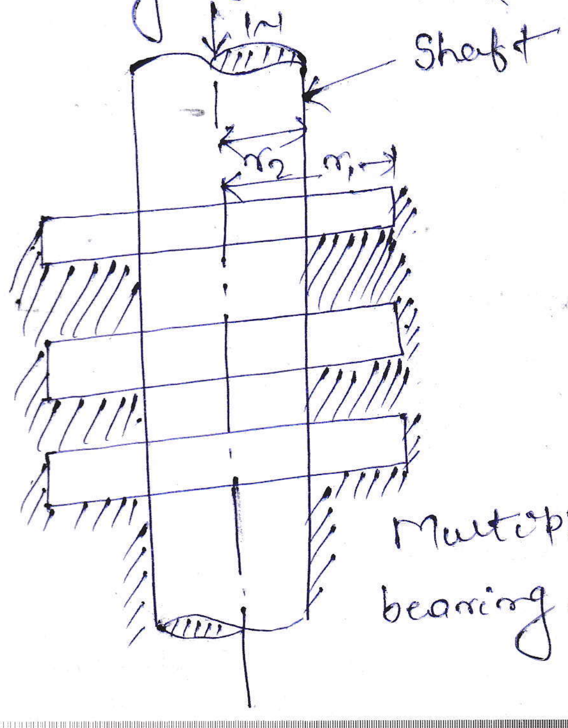
$$= \left(\frac{r_1 + r_2}{2}\right)$$

⇒ Flat Collar bearing :-

The collar bearing are used to take the axial thrust of the rotating shaft. It is mainly single collar and multiple collar bearing. It is also known as thrust bearing.



Single Collar bearing



Multiple Collar bearing.

Let r_1 = External radius of the collar
r_2 = Internal radius of the collar.

Area of the bearing surface

$$\boxed{A = \pi (r_1^2 - r_2^2)}$$

Considering uniform pressure.

Total frictional torque

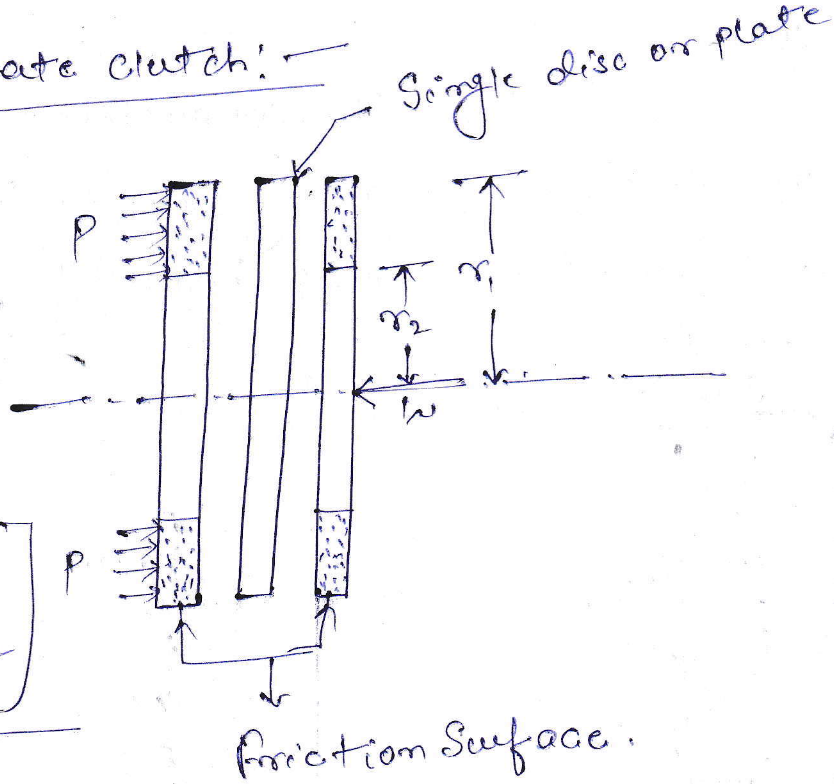
$$T = \frac{2}{3} \times \mu W \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right)$$

Considering uniform wear

$$T = \frac{1}{2} \times \mu W (r_1 + r_2)$$

$$\boxed{T = \mu W R}$$

⇒ Single Disc or Plate Clutch :-



Considering uniform pressure :-

Total frictional Torque

$$T = \frac{2}{3} \times \mu W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

Considering uniform wear.

Total frictional torque $T = \frac{1}{2} \mu W (r_1 + r_2) = \mu W R.$

Total frictional torque acting on the frictional surface (or on the clutch)

$$\boxed{T = n \mu W R}$$

$n \rightarrow$ Number of Pairs of friction or Contact Surfa

$$R = \boxed{\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}} \quad \text{for uniform Pressure}$$

$$R = \frac{r_1 + r_2}{2} \quad \text{for uniform wear.}$$

For Single disc or Plate clutch has two Pairs of Surfa in Contact $n = 2$.

Intensity of Pressure is maxm at the inner radius r_2

$$P_{max} \times r_2 = C$$

$$\therefore \boxed{P_{max} = C/r_2}$$

Intensity of Pressure is minm at outer surface r_1

$$P_{minm} \times r_1 = C$$

$$\therefore \boxed{P_{minm} = \frac{C}{r_1}}$$

Average Pressure $(P_{av.})$ on the friction or Contact Surface

$$P_{av} = \frac{\text{Total force of friction surface}}{\text{Cross-sectional area of frictional surface}}$$

$$P_{av} = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

The uniform pressure theory gives a higher frictional torque than the uniform wear theory.

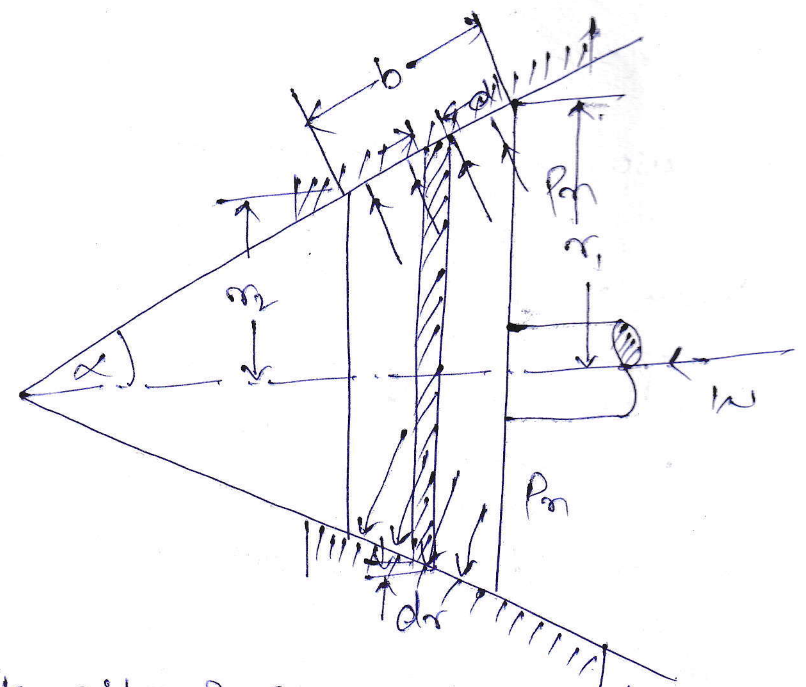
For multi disc clutch

Total frictional torque

$$T = n \cdot \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right] \quad \text{for uniform pressure}$$

$$T = n \cdot \mu W R \quad \text{for uniform wear.}$$

⇒ Cone clutch :-



Let

P_n = Intensity of pressure with which the conical friction surface are held together.

r_1 & r_2 = outer and inner radius of friction surfaces respectively.

R = Mean radius of the friction surfaces.

α = Semi angle of the Cone. (The angle of the frictio Suface with the axis of the clutch.

μ = Coefficient of friction betn Contact Surfaces.

b = width of the Contact surfaces (face width or clutch face).

Considering uniform Pressure :-

Total frictional torque

$$T = \frac{2}{3} \times \mu W \operatorname{Cosec} \alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right].$$

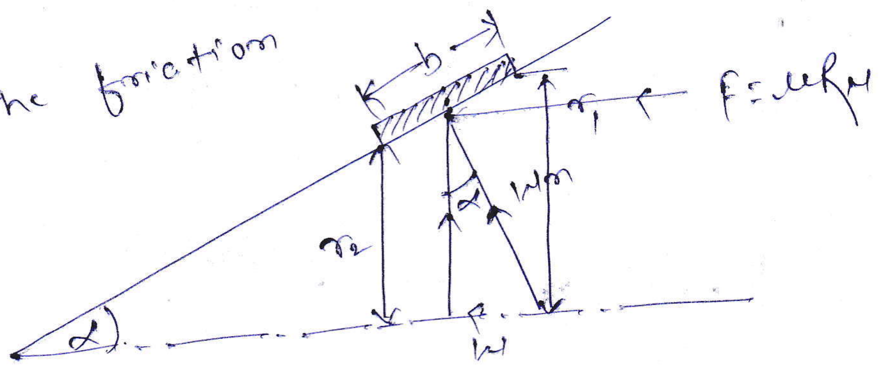
Considering uniform wear :-

Total frictional torque

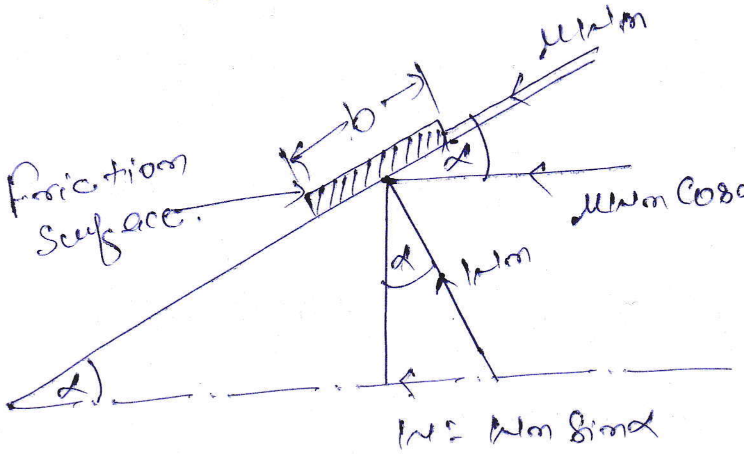
$$\boxed{T = \mu W R \operatorname{Cosec} \alpha}$$

For steady operation of the Clutch :-

The forces on the friction Surfaces.



→ During Engagement of the clutch.



The normal force acting on the friction surface is given below

$$\sin\alpha = \frac{W}{W_n}$$

$$\therefore \boxed{W_n = W/\sin\alpha}$$

∴ Axial force required for engaging the clutch

$$W_e = W + \mu W_n \cos\alpha$$

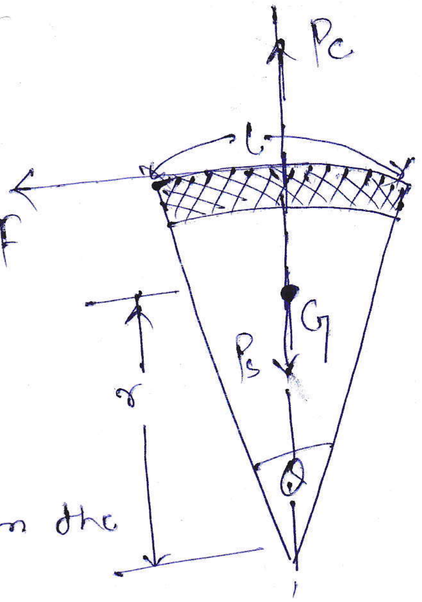
$$= W_n \sin\alpha + \mu W_n \cos\alpha$$

$$\boxed{W_e = W_n(\sin\alpha + \mu\cos\alpha)}$$

The axial for required to disengage the clutch is given by

$$\boxed{W_d = W_n(\mu\cos\alpha - \sin\alpha)}$$

⇒ Centrifugal Clutch :-



1. Mass of the shoes

Let m = Mass of each shoes

n = Number of shoes

r = Distance of centre of gravity of the shoe from the centre of the spider.

R = Inside radius of the pulley rim

N = Running speed of the pulley in r.p.m

ω = Angular running speed of the pulley in rad/s

$= \frac{2\pi N}{60}$ rad/s.

ω_1 = Angular speed at which the engagement begins to take place,

μ = Coefficient of friction betn the shoe and Rim.

$P_c = m\omega^2 r$

$P_s = m\omega_1^2 r$

Net radial outward force $(F_c) = P_c - P_s$

Frictional force acting tangentially on each shoe

$F = \mu(P_c - P_s)$

Frictional torque

$= F \times R = \mu(P_c - P_s) \times R$

∴ Total frictional torque transmitted

$T = \mu(P_c - P_s) \times R \times n = n.F.R$

2. Size of the shoes:-

let l = Contact length of the shoes

b = width of the shoes

R = Contact radius of the shoes.

θ = Angle subtended by the shoes at the centre of the spides

p = Intensity of pressure exerted on the shoe.

we already know.

$$\theta = l/r \quad \therefore \; l = \theta . r$$

Contact area of the shoe

$$A = l.b$$

force with which the shoe presses against the rim

$$= A \times p = \underline{l.b.p.}$$

The force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$

$$\therefore \boxed{l.b.p = (P_c - P_s)}$$

From the above eqn we find the width of the shoe (b).

⇒ **Velocity Ratio of Belt Drive:–**

Length of the belt passes over the driver pulley in one minute $= \pi d_1 N_1$ ——— (i)

Similarly on follower or driven pulley

$= \pi d_2 N_2$ ——— (ii)

From eqn (i) & (ii), we get,

$$\cancel{\pi} d_1 N_1 = \cancel{\pi} d_2 N_2$$

$$\Rightarrow \boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$

When the thickness of the belt (t) is considered then velocity ratio.

$$\boxed{\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}}$$

⇒ **Slip of Belt:–** Some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called slip of the belt. It is generally expressed as a percentage.

Now, $\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$

OR $\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \left(1 - \frac{S}{100}\right)}$

Where S = Total Percentage of slip.

→ **Creep of Belt:–** When the belt passes from the slack side to tight side, a certain portion of the belt extends and it contracts again when the belt

Passes from tight side to slack side. Due to these change of length, there is a relative motion betn the belt & the pulley surfaces. This relative motion is known as creep.

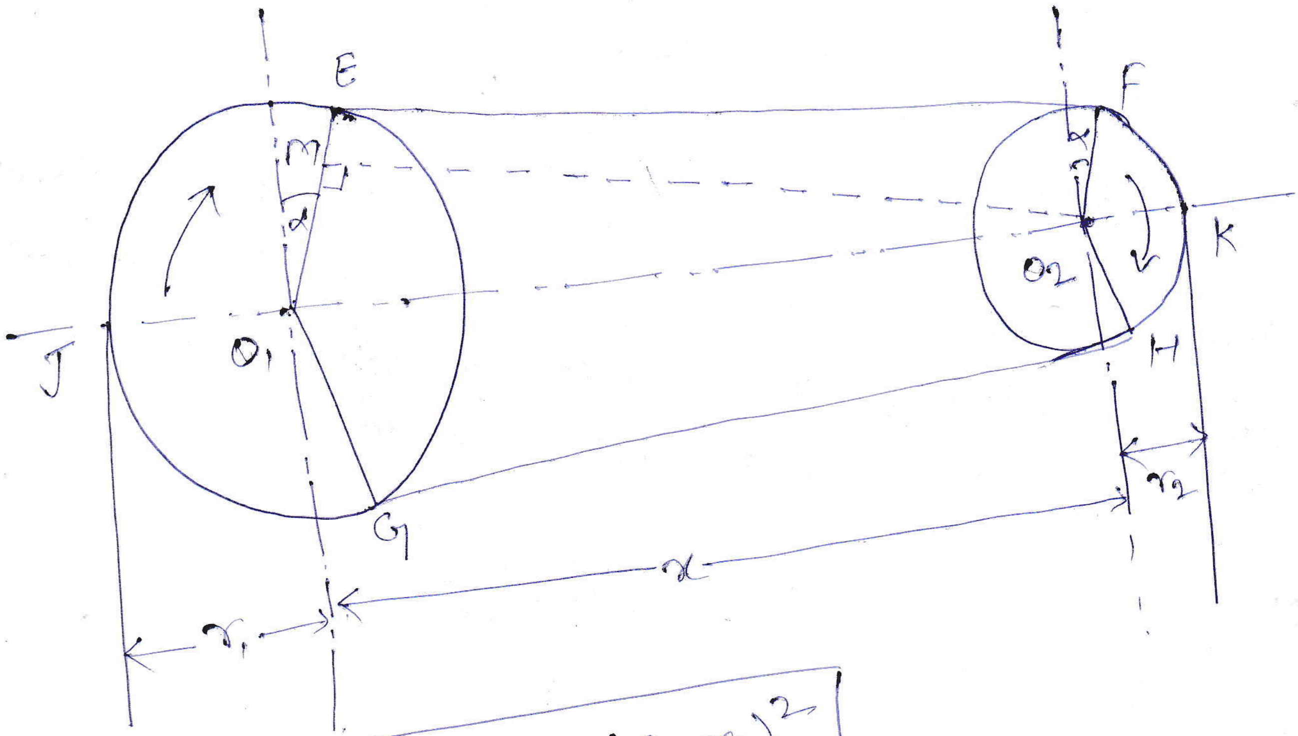
Velocity ratio $= \boxed{\dfrac{N_2}{N_1} = \dfrac{d_1}{d_2} \times \dfrac{E+\sqrt{\sigma_2}}{E+\sqrt{\sigma_1}}}$

σ_1 = Stress in the belt on the ~~slack~~ Tight side.

σ_2 = " " " " " " slack " .

E = young's modulus for the material of the belt

⇒ Length of an open belt drive :-

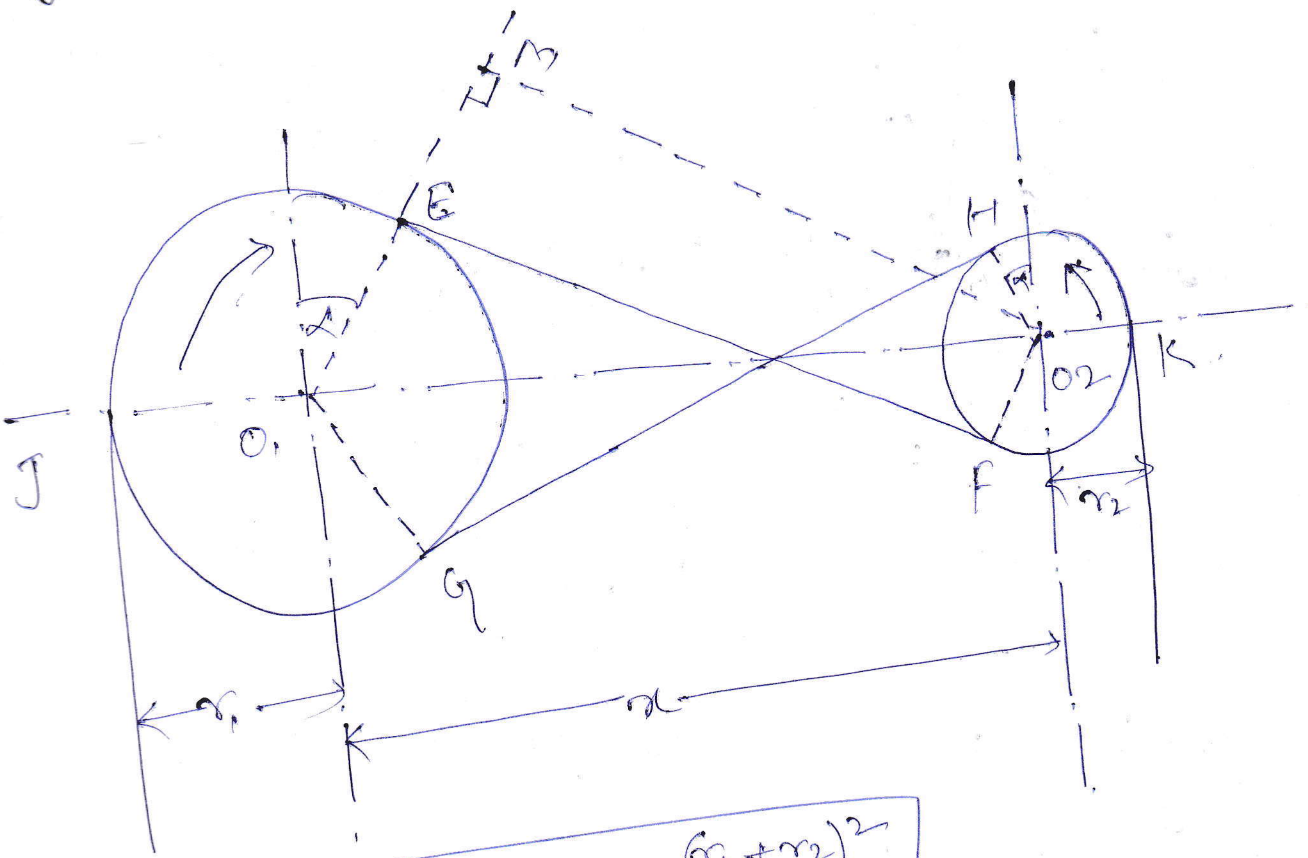


$$\boxed{L = \pi(r_1+r_2) + 2x + \frac{(r_1-r_2)^2}{x}}$$

OR

$$\boxed{L = \frac{\pi}{2}(d_1+d_2) + 2x + \frac{(d_1-d_2)^2}{4x}}$$

⇒ Length of Cross Belt Drive :-



$$\boxed{L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}}$$

OR

$$\boxed{L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}}$$

⇒ Power transmitted by the belt :→

The effective turning (driving) force at the circumference of the follower is $(T_1 - T_2)$.

∴ Work done per second $= (T_1 - T_2) \cdot v$ N-m/s

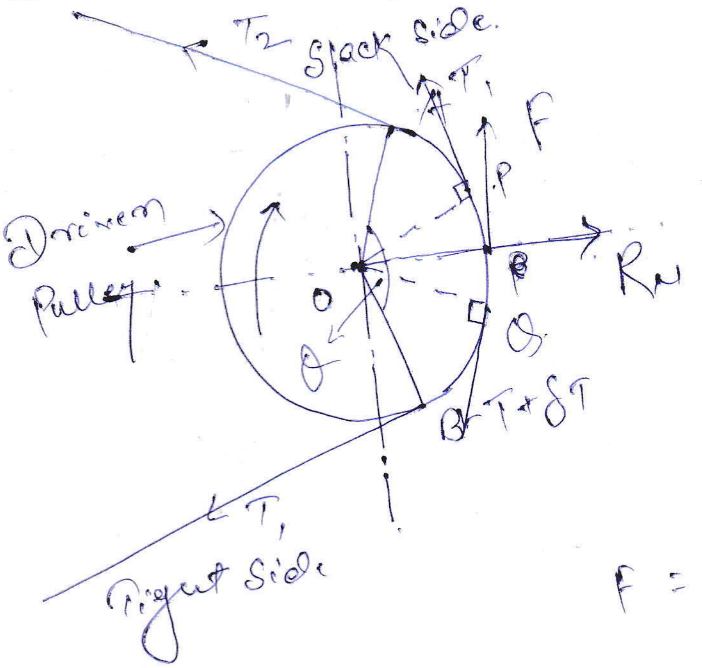
Power transmitted, $\boxed{P = (T_1 - T_2) \cdot v}$ W.

Because $(1 \text{ N-m/s} = 1 \text{ W})$.

Torque exerted on driving pulley $= (T_1 - T_2) \cdot r_1$

" " " Driven (follower) " $= (T_1 - T_2)\, r_2$.

⇒ Ratio of Driving Tension for Flat Belt drive :-



T = Tension in the belt at Point P

$T+\delta T$ = Tension in the belt at Point Q.

R_N = Normal Reaction Force

F = Frictional force.

$$\log_e\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\Rightarrow \boxed{\left(\frac{T_1}{T_2}\right) = e^{\mu.\theta}}$$

OR, $$\boxed{2.3\log_{10}\left(\frac{T_1}{T_2}\right) = \mu\theta}$$

→ Determinat of Angle of Contact. :→

From open belt drive.

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1G - ME}{O_1O_2} = \left(\frac{r_1 - r_2}{x}\right)$$

∴ Angle of Contact or Lap

$$\boxed{\theta = (180 - 2\alpha)\frac{\pi}{180} \text{ rad.}}$$

From Cross-Belt drive.

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1G + ME}{O_1O_2} = \left(\frac{r_1 + r_2}{x}\right)$$

∴ Angle of Contact or Lap $= (180 + 2\alpha)\frac{\pi}{180}$ rad

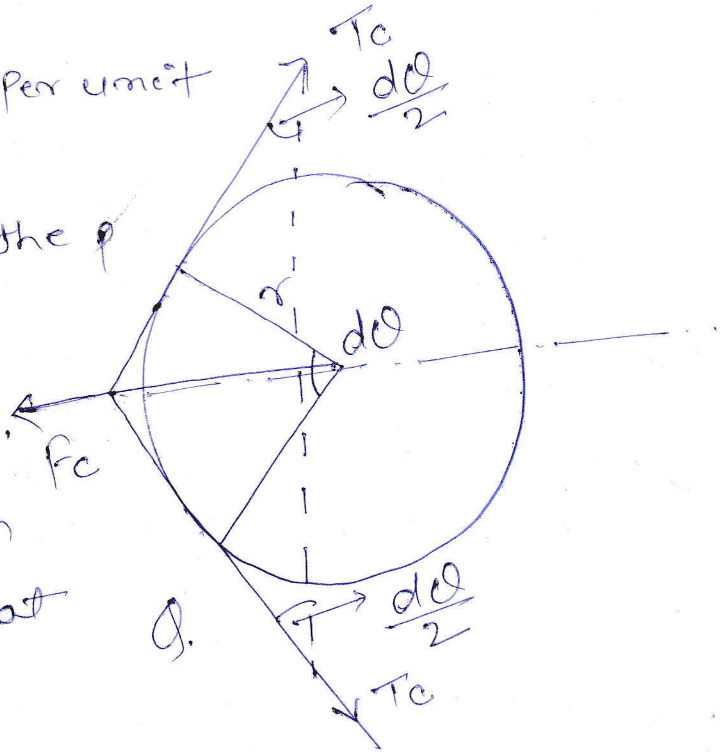
⇒ **Centrifugal Tension :-** The tension caused by centrifugal force is known as centrifugal tension. It is increased the tension in the belt in both side Tight as well as slack side.

Let m = mass of the belt per unit length in kg.

v = linear velocity of the belt in m/s.

r = Radius of the pulley.

T_c = Centrifugal tension acting tangentially at point P & Q.



Length of the belt PQ = r. dθ.

mass of the PQ length belt = m.r. dθ

Centrifugal force acting on belt PQ is.

$$F_c = (m.\cancel{r}.d\theta)\frac{v^2}{\cancel{r}} = m v^2 d\theta$$

Now, $T_c \sin\left(\frac{d\theta}{2}\right) + T_c \sin\left(\frac{d\theta}{2}\right) = F_c = m.d\theta.v^2$

$$\Rightarrow \cancel{2}T_c \frac{\cancel{d\theta}}{\cancel{2}} = m.\cancel{d\theta}.v^2 \quad \left[\begin{array}{l} d\theta \text{ is very small} \\ \therefore \sin\frac{d\theta}{2} \rightarrow \frac{d\theta}{2} \end{array}\right.$$

$$\therefore \boxed{T_c = m v^2}$$

∴ Total tension in tight side belt is

$$\boxed{T_{t1} = T_1 + T_c}$$

Total tension in slack side is.

$$\boxed{T_{t2} = T_2 + T_c}$$

Power transmitted,

$$P = (T_{t1} - T_{t2}) \cdot v$$

$$= \left[(T_1 + \cancel{T_c}) - (T_2 + \cancel{T_c})\right] v$$

$$= (T_1 - T_2) \cdot v \text{ watt.}$$

Ratio of driving tensions may also be written as

$$\boxed{2.3 \log_{10}\left(\frac{T_{t1} - T_c}{T_{t2} - T_c}\right) = \mu\theta}$$

where, T_{t1} = Maxm or Total tension in the belt

⇒ Maxm Tension in the belt :-

Let, σ = Maxm safe stress in N/mm².

b = width of the belt in mm.

t = Thickness of the belt in mm.

Maxm Tension in the belt (T) is equal to the total tension in tight side of the belt (T_{t1}).

$$T = \sigma \times b \times t$$

$$\boxed{T = \sigma b t}$$

When centrifugal tension is neglected

$$\boxed{T \text{ or } (T_{t1}) = T_1}$$

When centrifugal tension is considered

$$\boxed{T \text{ or } (T_{t1}) = T_1 + T_c}$$

⇒ Condition for the Transmission of maxm Power :-

$$\boxed{P = (T_1 - T_2) \cdot v} \text{ watt}$$

$$\boxed{T = 3T_c}$$

when the Power transmitted is maxm, 1/3rd of the maxm tension in absorbed as Centrifugal tens

we already know that.

$$T = T_1 + T_c$$

$$\therefore (T_1 = T - T_c)$$

and for maxm power. $\left(T_c = \frac{T}{3}\right)$

Now,

$$T_1 = T - T_c$$

$$= T - \frac{T}{3}$$

$$= \frac{3T - T}{3} = \frac{2T}{3}$$

$$\boxed{T_1 = \frac{2T}{3}}$$

Again, velocity of the belt for the maxm power.

$$\boxed{v = \sqrt{\frac{T}{3m}}}$$

⇒ Initial Tension in the belt :- When the pulleys are stationary, the belt is subject to some tension, is called initial Tension.

When the drives start rotating, it pulls the belt from one side and deliveres it to the other side. The increased tension in one side of the belt is called tension in tight side.

and the decreased tension in the other side of the b-
is called tension in the slack side.

Let, T_0 = Initial Tension in the belt

T_1 = Tension in the tight side.

T_2 = " " " slack ".

α = Coefficient of increase of the belt length per unit force.

Increase in Tension in Tight side of the belt.

$$= (T_1 - T_0) \quad \text{---} (i)$$

Increase in length of the belt in Tight side.

$$= \alpha (T_1 - T_0) \quad \text{---} (ii)$$

Decrease Tension in slack side of the belt

$$= (T_0 - T_2) \quad \text{---} (iii)$$

Decrease in the length of the belt in slack side

$$= \alpha (T_0 - T_2) \quad \text{---} (iv)$$

Belt material is perfectly elastic so length of the belt remains constant.

From eqn (iii) & (iv), we get,

$$\Rightarrow \cancel{\alpha} (T_1 - T_0) = \cancel{\alpha} (T_0 - T_2)$$

$$\Rightarrow 2T_0 = T_1 + T_2$$

$$\therefore \boxed{T_0 = \frac{T_1 + T_2}{2}}$$

when we consider centrifugal tension.

$$\boxed{T_0 = \frac{T_1 + T_2 + 2T_c}{2}}$$

Q. In a flat belt drive the initial tension in 2000 N. The coefficient of friction betn the belt and the pulley 0.3 and the angle of lap on the smaller pulley is 150 The smaller pulley has a ~~dia~~ radius of 200 mm and rotates at 500 r.p.m. Find ~~and~~ the power in KW transmitted by the belt.

Soln

$T_0 = 2000$ N

$\mu_0 = 0.3$

$\theta = 150^\circ = 150 \times \frac{\pi}{180} = 2.618$ rad.

$r_2 = 200$ mm or $d_2 = 400$ mm

$N_2 = 500$ r.p.m.

Velocity of the belt

$$v = \frac{\pi d_2 N_2}{60} = \frac{\pi \times 0.4 \times 500}{60} = \underline{10.47 \text{ m/s}}$$

T_1 : Tension in tight side.

T_2 = " " slack " .

Now, $T_0 = \frac{T_1 + T_2}{2}$

$$\therefore \boxed{T_1 + T_2 = 2T_0 = 4000 \text{ N}} \quad \text{(i)}$$

we already know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu \cdot \theta$$

$$\Rightarrow \quad 2.3 \log\left(\frac{T_1}{T_2}\right) = 0.3 \times 2.618$$

$$\Rightarrow \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.7854}{2.3} = 0.3415$$

$$\therefore \left(\frac{T_1}{T_2}\right) = 2.2 \longleftarrow \text{(ii)}$$ (Taking antilog of 0.3415)

from eqn (i) & (ii)

$T_1 = 2750$ N

$T_2 = 1250$ N

$\therefore$ Power transmitted, $(P) = (T_1 - T_2) \cdot v$

$= (2750 - 1250) \times 10.47$

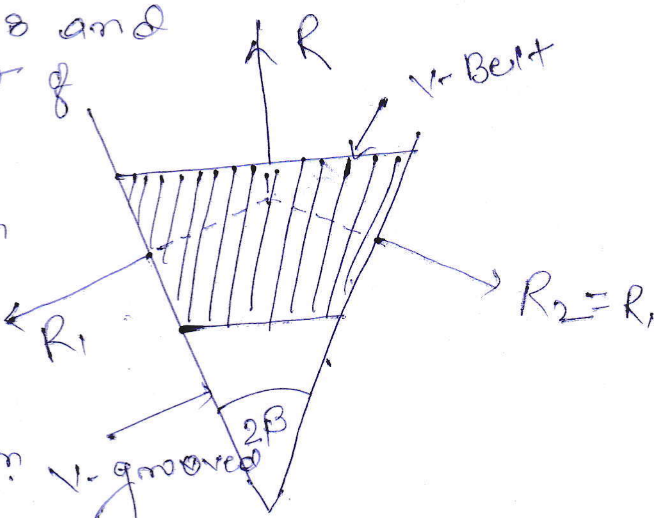
$= 1500 \times 10.47$

$= 15700$ W $= \underline{15.7 \text{ KW}}$

$$\boxed{P = 15.7 \text{ KW}}$$

⇒ Ratio of Driving Tension for v-Belt :-

v-belt is mostly used in factories and workshops where a great amount of Power is to be transmitted from one Pulley to another when the two Pulleys are very near to each other.



Let, R_1 = Normal reaction betn the grooves and belt.

R = Total reaction in the Plane of the groove.

2β = Angle of the groove.

μ = Coefficient of friction betn belt and sides of the groove.

Now,

$$R = R_1 \sin\beta + R_1 \sin\beta$$

$$R = 2R_1 \sin\beta$$

$$\boxed{R_1 = \frac{R}{2\sin\beta}}$$

frictional force $= 2\mu R_1 = 2\mu \times \frac{R}{2\sin\beta} = \frac{\mu R}{\sin\beta}$

$$= \mu R \operatorname{cosec}\beta$$

Relation betn T_1 and T_2 for the v-belt drive will be

$$\boxed{2.3 \log\left(\frac{T_1}{T_2}\right) = \mu\theta \cdot \operatorname{cosec}\beta}$$

Q. Power is transmitted using a v-belt drive. The included angle of v-groove is 30°. The belt is 20 mm deep and maxm width is 20 mm. If the mass of the belt is 0.35 kg per m length and maxm allowable stress is 1.4 MPa. Determine the maxm Power transmit when the angle of lap is 140°. $\mu = 0.15$.

Soln $2\beta = 30^\circ \therefore \beta = 15^\circ$, $t = 20\,mm$, $b = 20\,mm$.

$m = 0.35\,kg/m$, $\sigma = 1.4\,MPa$,

$= 1.4 \times 10^6\,N/m^2$, $\theta = 140^\circ = 140 \times \frac{\pi}{180} = 2.4$ rad

$\mu = 0.15$.

Maxm Tension in the belt

$$T = \sigma b t$$

$$= 1.4 \times 10^6 \times 0.02 \times 0.02$$

$$= 560\,N.$$

Velocity of the belt for maxm Power transmission.

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{560}{3 \times 0.35}} = 23.1\,m/s.$$

Now,

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu\theta \cdot \operatorname{Cosec}\beta$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.416}{2.3}$$

$$\therefore \boxed{\frac{T_1}{T_2} = 4.13} \quad \text{—— (i)}$$

Centrifugal tension $T_c = \frac{T}{3} = \frac{560}{3} = 187\,N$

$$T_1 = T - T_c$$

$$= 560 - 187 = 373\,N.$$

$$T_2 = \frac{T_1}{4.13} = \frac{373}{4.13} = 90.3\,N.$$

$\therefore$ Power transmitted by the belt is

$$P = (T_1 - T_2)v = (373 - 90.3) \times 23.1 = 6530\,watt = 6.53\,K$$

Ans.

⇒ Rope drive:—

(a) fibre ropes → when the pulleys are about 60 m apart.

(b) Wire ropes → when the pulleys are upto 150 m apart.

→ Manila ropes are more durable and stronger than Cotton ropes.

→ The Cotton ropes are Costlier than manila ropes.

→ The diameters of manila and cotton ropes usually ranges from 38 mm to 50 mm.

The size of rope is usually designated by its Circumference or "Girth".

→ The groove angle of the pulley for rope drives is usually 45°.

→ The fibre ropes donot rest at the bottom of the groove.

⇒ Ratio of Driving Tensions for Rope drive:—

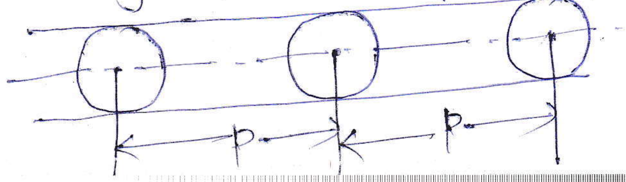
Ratio of driving Tension = $\boxed{2.3 \log\left(\frac{T_1}{T_2}\right) = \mu\theta \cdot \text{Cos}}$

⇒ Chain Drives:—

Teeth are fit into the Corresponding Recesses.

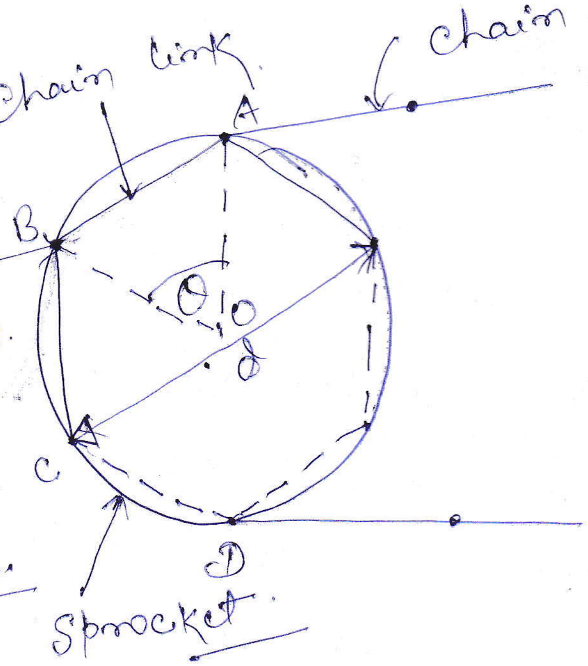
Toothed wheels are known as sprocket wheels or sprockets.

→ Pitch of the chain:— It is the distance betn the hinge Centre of a link and the corresponding hinge Centre of the adjacent link. It is usually denoted by p.



(ii) Pitch Circle diameter of the chain sprocket :-

The Circle drawn through hinge Center is known as pitch Circle ~~diameter~~ and it's diameter is known as Pitch Circle diameter of the given chain sprockets.



Let, d = Diameter of the Pitch Circle

T = Number of teeth on the sprocket.

Now, Pitch of the chain $p = AB = 2AO \sin\frac{\theta}{2}$

$= 2 \times \frac{d}{2} \sin\left(\frac{\theta}{2}\right) = d \sin\left(\frac{\theta}{2}\right)$

We already know that

$$\theta = \frac{360}{T}$$

$$\therefore p = d \cdot \sin\left(\frac{360}{2T}\right)$$

$$\Rightarrow p = d \cdot \sin\left(\frac{180}{T}\right)$$

$$\therefore \boxed{d = p \operatorname{cosec}\left(\frac{180}{T}\right)}$$

⇒ Length of chain :-

For open chain drive System, $\boxed{L = \pi(r_1+r_2) + 2x + \frac{(r_1-r_2)^2}{x}}$ ← (i)

we already know that

Pitch Circle diameter (d) $= p \operatorname{Cosec}\left(\frac{180^\circ}{T}\right)$

$$\Rightarrow \quad 2r = p \operatorname{Cosec}\left(\frac{180^\circ}{T}\right)$$

$$\boxed{r = \frac{p}{2} \operatorname{Cosec}\left(\frac{180^\circ}{T}\right)}$$

For larger sprocket,

$$r_1 = \frac{p}{2} \operatorname{Cosec}\left(\frac{180^\circ}{T_1}\right)$$

For smaller sprocket

$$r_2 = \frac{p}{2} \operatorname{Cosec}\left(\frac{180^\circ}{T_2}\right)$$

Now,

$\pi(r_1+r_2)$ is equal to half the sum of the circumfe-
rences of the pitch circles.

∴ Length of the chain corresponding to

$$\pi(r_1+r_2) = \frac{p}{2}(T_1+T_2)$$

Now from eqn (i)

$$L = \frac{p}{2}(T_1+T_2) + 2x + \frac{\left[\frac{p}{2}\operatorname{Cosec}\left(\frac{180^\circ}{T_1}\right) - \frac{p}{2}\operatorname{Cosec}\left(\frac{180^\circ}{T_2}\right)\right]^2}{x}$$

If $x = m.p.$

Then,

$$L = \frac{p}{2}(T_1+T_2) + 2mp + \frac{\left[\frac{p}{2}\operatorname{Cosec}\left(\frac{180^\circ}{T_1}\right) - \frac{p}{2}\operatorname{Cosec}\left(\frac{180^\circ}{T_2}\right)\right]^2}{m.p}$$

$$L = P\left[\frac{(T_1+T_2)}{2} + 2m + \frac{\left[\operatorname{Cosec}\left(\frac{180^\circ}{T_1}\right) - \operatorname{Cosec}\left(\frac{180^\circ}{T_2}\right)\right]^2}{4m}\right]$$

$$\boxed{L = P \cdot K.}$$

Multiplying factor (K) may not be a complete integer

The length of the chain must be equal to an integer number of times of the pitch of the chain.