Kinematics & Machines (KOM)

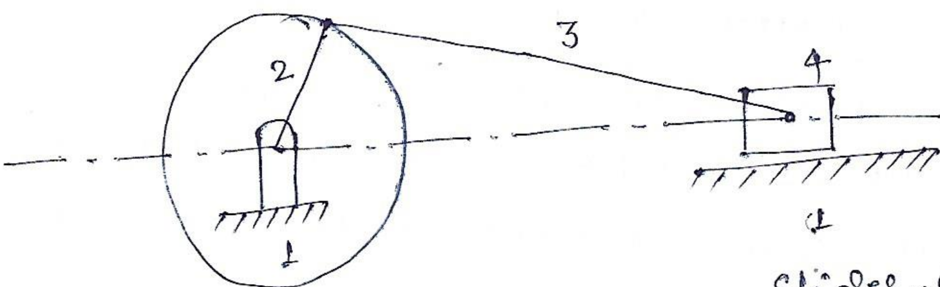
UNIT – I.

Kinematics :— It deals with the relative motions of different parts of a mechanism without taking into consideration of the forces producing the motions. Thus it is the study, from a geometric point of view, to know the displacement, velocity and acceleration of a part of a mechanism.

Dynamics :— It involves the calculations of forces impressed upon different parts of a mechanism. The force can be either static or dynamic.

Mechanism :— If a number of bodies (usually rigid) are assembled in such a way that the motion of one causes constrained and predictable motions to the others, it is known as a mechanism. Thus, mechanism transmits and modifies a motion.

Machine :— A machine is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmit and modifies the available mechanical energy into some kind of desired work.
It is neither a source of energy nor a producer of work but helps in proper utilisation of the same. The motive power has to be derived from external



Slider-Crank mechanism.

A slider-crank mechanism converts the reciprocating motion of a slider into a rotary motion of the crank or vice-versa.
However, when it is used as an automobile engine by adding valve mechanism etc. it becomes a m/c which converts the available energy (Force on the piston) into the desired energy (Torque of the crank-shaft). The torque is used to move a vehicle. Reciprocating compressors, and steam engines are other examples of m/c derived from the slider-crank mechanism. Some other examples are typewriters, clocks, watches, spring toys etc.

Rigid bodies:— A body is said to be rigid if under the action of forces, it does not suffer any distortion or the distances between any two points on it remains constant.

Resistant bodies:— Resistant bodies are those which are rigid for the purposes they have to serve. Apart from rigid body, there are some semi-rigid bodies which are normally flexible, but un- … conditions acts as rigid bod

A belt is rigid when subjected to tensile forces. Therefore, the belt in a belt drive acts a resistant body. Similarly fluid can also act as resistant bodies when compressed as in case of a hydraulic press. For some purpose, springs are also resistant bodies.

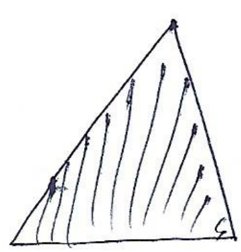
These days resistant bodies are usually referred to as rigid bodies.

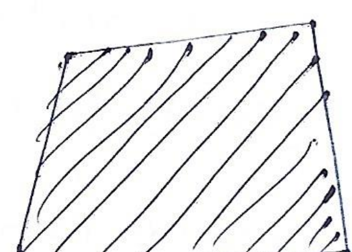
<u>Link</u> :— A mechanism is made of a number of resistant bodies out of which some may have motions relative to the others. A resistant body or a group of resistant bodies with rigid connections preventing their relative movement is known as a link.

A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them. Thus a link may consist of one or more resistant bodies.

A link is also known as kinematic link or Element.

Binary link





The link shown in above fig. are rigid links and there is no relative motion between the joints within the link.

Kinematic Pair :— A kinematic Pair or Simply a Pair is a joint of two links having relative motion between them.

⇒ Types of kinematic Pairs :—

→ Kinematic Pairs according to nature of Contact.

(a) Lower Pair :— A Pair of links having surface or area Contact between the members is known as a Lower Pair. The Contact surface of the two links are similar.

Nut turning on a screw, Shaft rotating in a bearing. All Parts of a slider- Crank mechanism, universal joints etc.

(b) Higher Pair :— When a Pair has a Point or line Contact between the links, it is known as a higher Pair. The Contact surfaces of the two links are dissimilar.

Wheel rolling on a surface, Cam and Follower Pair, Tooth gears, ball and roller bearing etc.

→ Kinematic Pairs according to Nature of Mechanical Constraint.

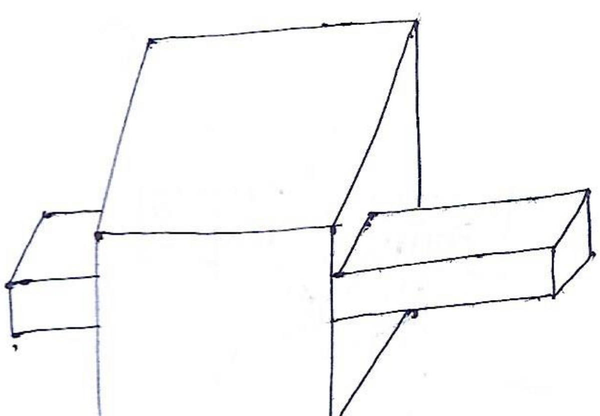
(a) Closed Pair :— When the elements of a Pair are held together mechanically, it is known as a closed Pair.

The lower pairs are self closed pair.

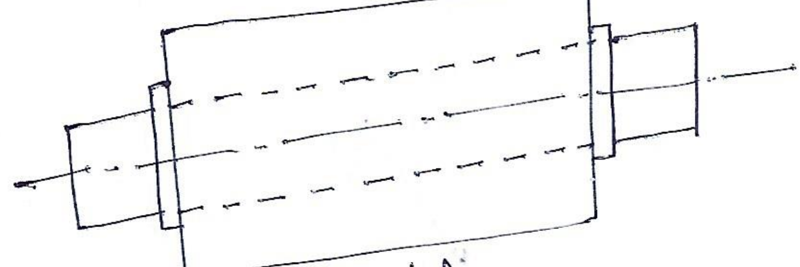
(b) Force-closed pair :- When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a forced-closed pair. The cam and follower is an example of forced-closed pair, as it is kept in contact by the forces exerted by spring and gravity.

→ According to the types of relative motion between the element.

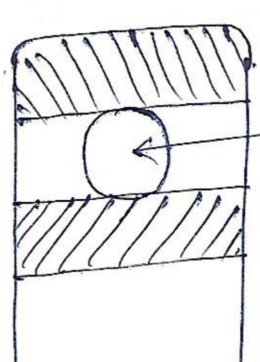
(a) Sliding pair :- If two links have a sliding motion relative to each other, they form a sliding pair. A rectangular rod in a rectangular hole in a prism is a sliding pair.



(a)
Sliding pair



(b)
Turning Pair.

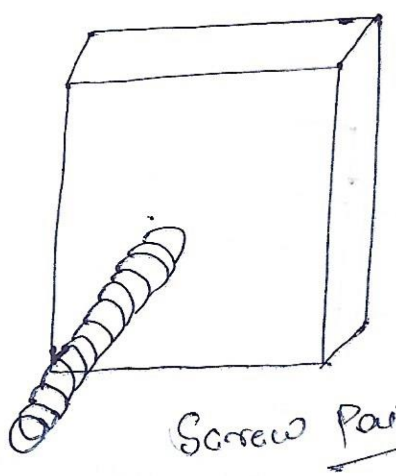


Rolling Pair.

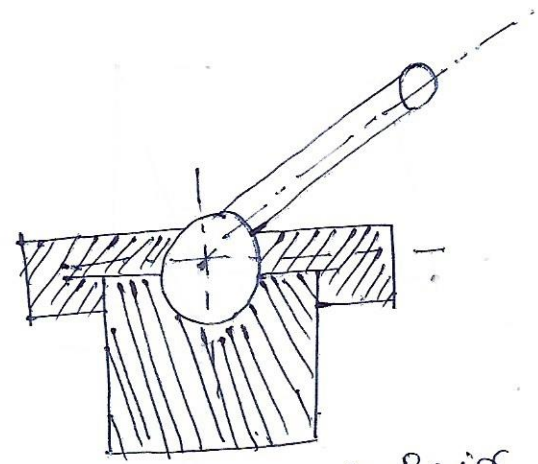
(b) Turning Pair :— When one link has a turning or revolving motion relative to the other, they constitute a turning or revolving pair.

(c) Rolling Pair :— When the links of a pair have a rolling motion relative to each other, they form a rolling pair. e.g a rolling wheel on a flat surface, Ball and roller bearing. In a ball bearing the ball and the shaft constitute one rolling pair, whereas the ball and the bearing is the second rolling pair.

(d) Screw Pair (Helical pair) :— If two mating links have a turning as well as sliding motion between them, they form a screw pair. This is achieved by cutting matching threads on the two links.
The lead screw and the nut of a lathe is a screw pair.



Screw Pair.



Spherical Pair

(e) Spherical Pair :— When one link in the form a sphere turns inside a fixed link, it is a spherical pair.
The ball and socket joint is a spherical pair.

Degree of freedom:— An unconstrained rigid body moving in space can described the following independent motions.

(i) Translational motions along any three mutually perpendicular axes x, y and z.

(ii) Rotational motions about these axes.

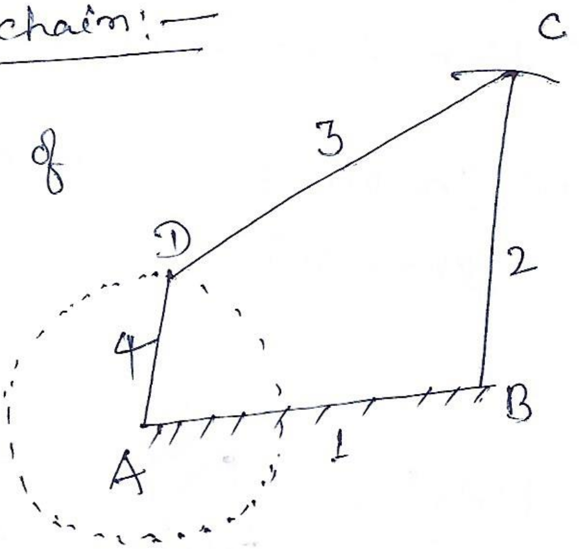
Thus a rigid body possesses six degree of freedom.

The number of restraints can never be zero (joint is disconnected) or six (joint becomes solid).

Degree of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have

$$\boxed{\text{Degree of freedom} = 6 - \text{Number of restraints}}$$

⇒ **Four bar chain or Quadric cycle chain:—**



Four bar chain (kinematic) consists of four links, each of them form a turning pair at A, B, C and D.

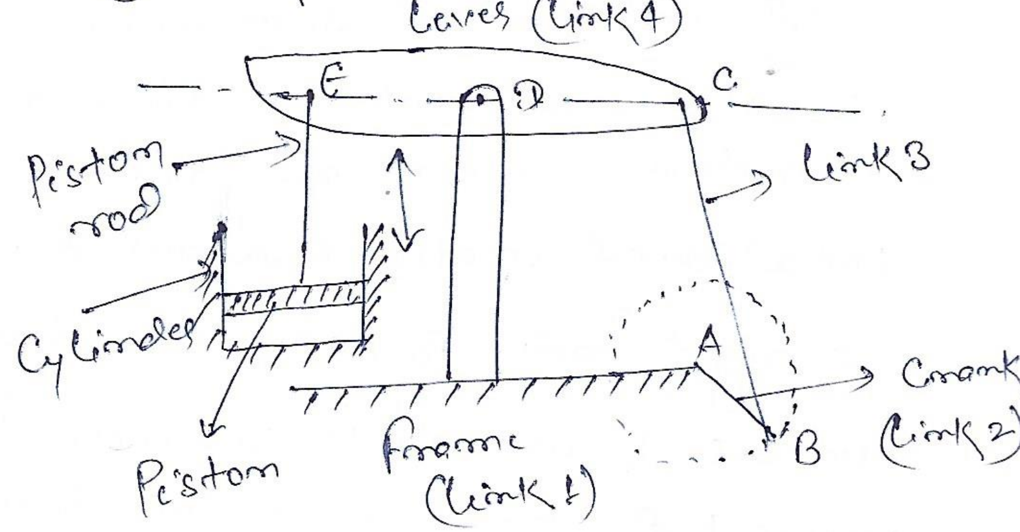
The four links may be of different ~~links~~ lengths.

According to Grashof's law for a four bar mechanism, the sum of the shortest and longest link length should not be greater than the sum of the remaining two link lengths if there is to be continuous

relative motion between two links.

⇒ Inversion of Four bar chain mechanism :—

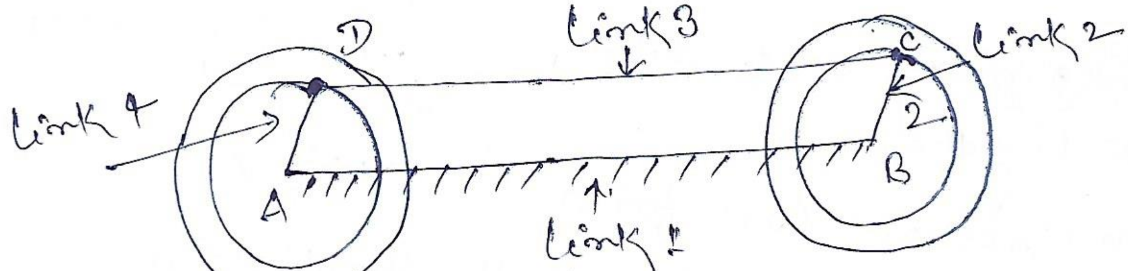
(i) Beam Engine (Crank and Lever mechanism) :—



In this mechanism, when the Beam Engine, Crank rotates about the fixed centre A, the lever oscillate about a fixed (Point) Centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the Crank.

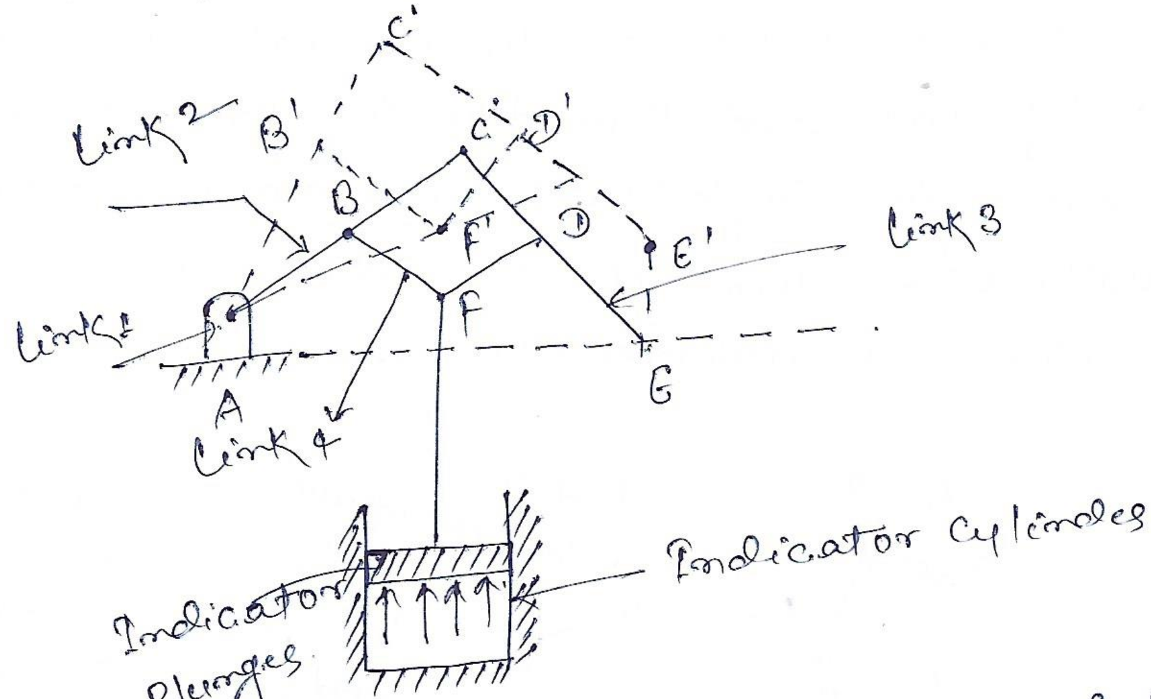
The Purpose of this mechanism is to convert rotary motion into reciprocating motion.

(ii) Coupling rod of a locomotive (Double Crank mechanism).



In this ~~link~~ mechanism link 4 and link 2 acts as a Crank of equal length. The link CD act as a Coupling rod and the link AB is fixed in order to maintain from one wheel to the other wheel.

(iii) Watt's indicator mechanism (Double-Lever mechanism)



Four links, Fixed link at A, link AC, link CE and link BFD. BF and FD form a one link because these two parts have no relative motion between them. The link CE and BFD act as a lever.

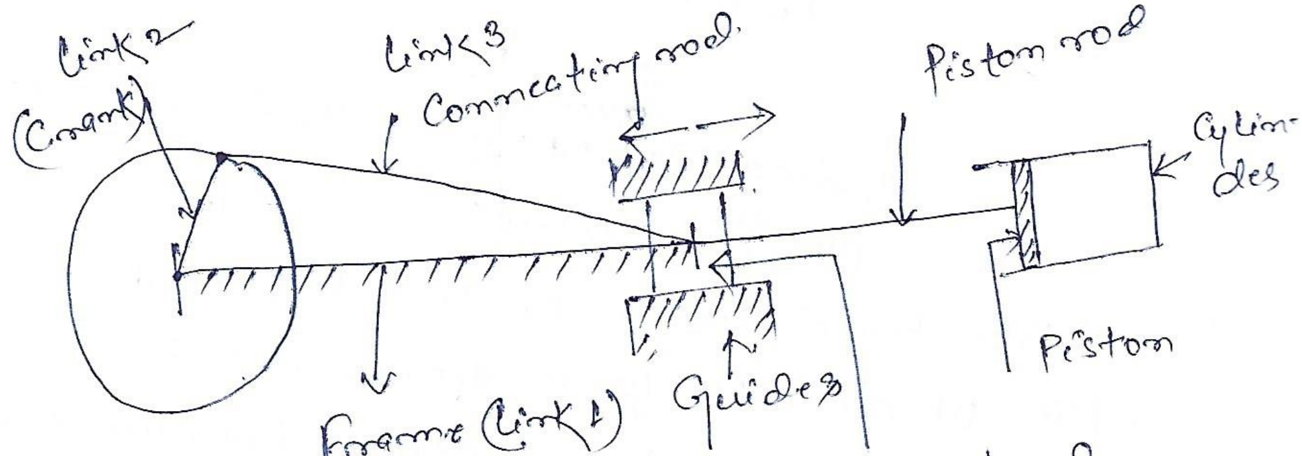
The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts as on the indicator plunger.

On any small displacement of the mechanism, the tracing point E at the end of the link CE ~~act as~~ trace out approximately a straight line.

The position of the whole mechanism is changed when the gas or steam pressure acts on the indicator plunger.

⇒ Single Slider Crank chain: —

A Single Slider Crank chain is a modification of the basic four bar chain. It consist of one sliding pair and three turning pair. In Reciprocating Steam engine Single Slider Crank chain mechanism is usually used. This type of mechanism Converts rotary motion into reciprocating motion and vice versa.



Link 1 Corresponds to the frame of the engine, which is fixed.

Link 2 → Crank

Link - 3 → Connecting rod

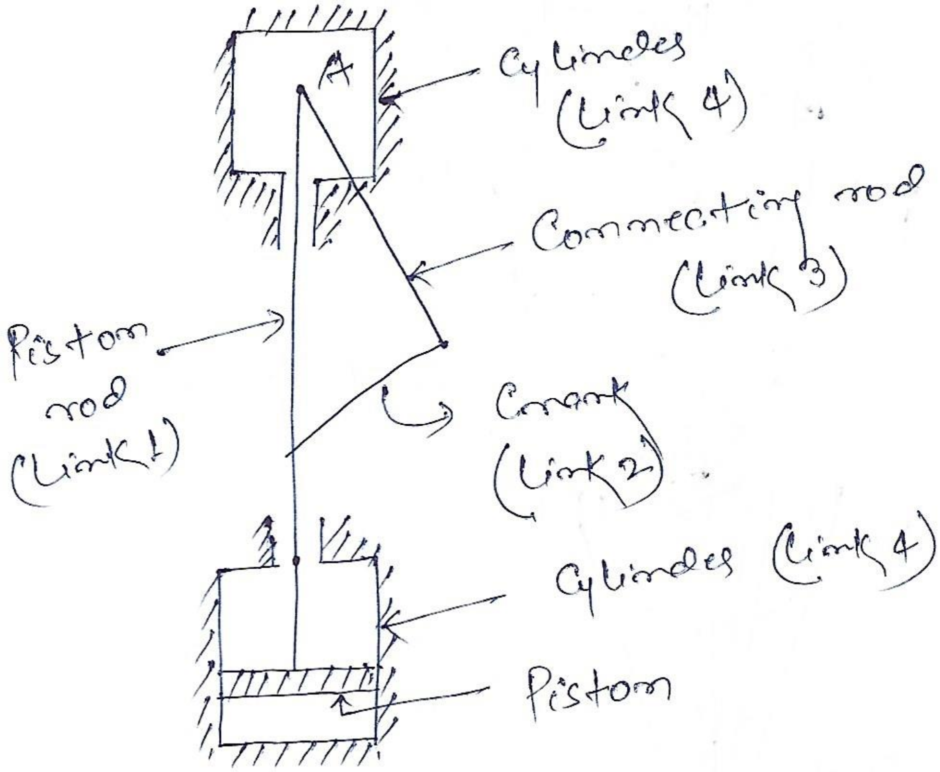
Link 4 → Cross-head.

As the Crank rotates, the Cross-head reciprocates in the guides and thus piston reciprocates in the cylinder.

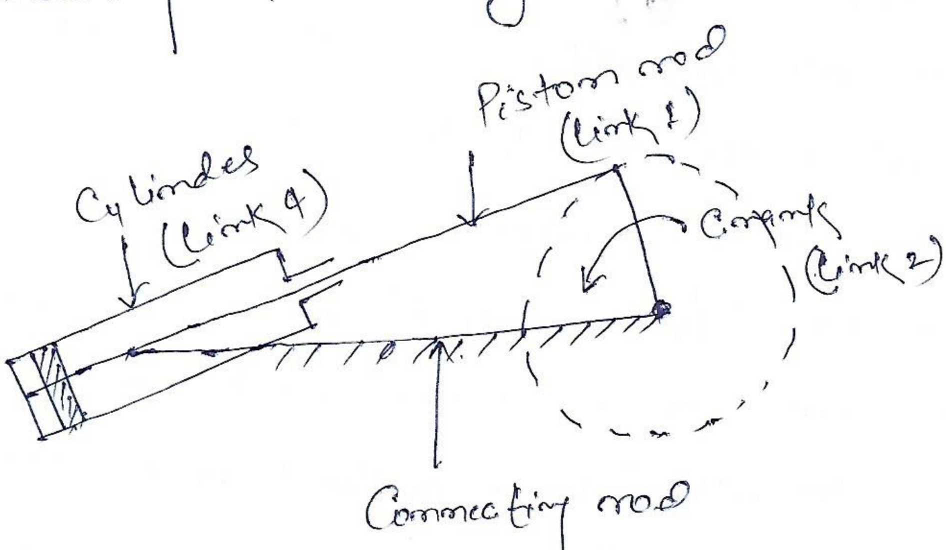
⇒ Inversion of Single Slider Crank chain.

(i) Pendulum Pump or Bull Engine: — In this mechanism the inversion is obtained by fixing the Cylinder or Link 4 (i.e. sliding pair). When the Crank (Link 2) rotates the Connecting rod (Link 3) oscillates about a pin

attached to the Piston rod (Link 1) reciprocates.
The duplex Pump which is used to supply feed water to boiler have two Piston attached to link 1.

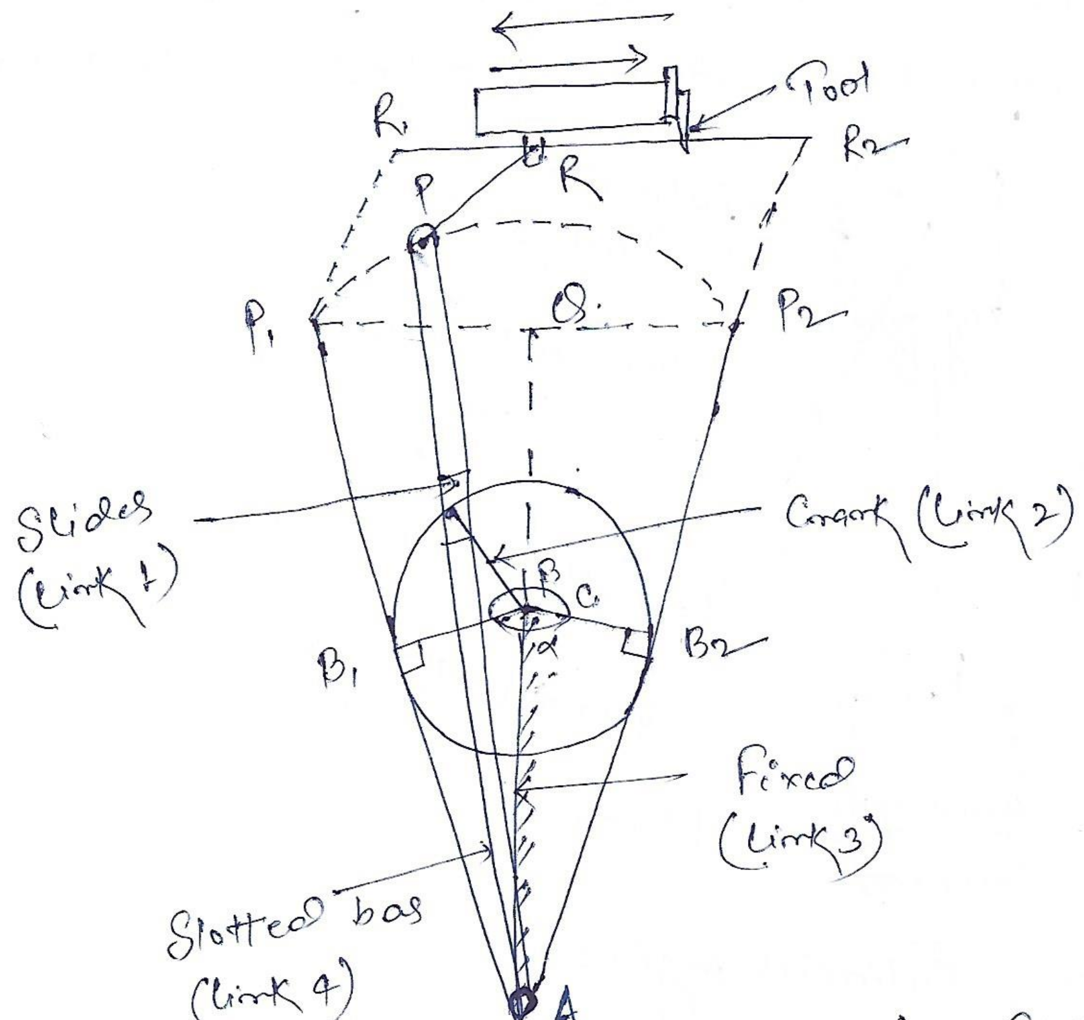


(ii) oscillating Cylinder engine:-



This mechanism is used to Convert reciprocating motion into rotary motion. in this mechanism link 3 is fixed. which is formed Turning Pair. when the crank rotates the Piston attached to Piston rod reciprocates and the Cylinder oscillates about

(iii) Crank and Slotted Lever quick return motion mechanism



AP_1 and AP_2 are Tangential to the given Circle and the Cutting tool is at the end of the stroke.

The forward or Cutting strokes occurs when the crank rotates from CB_1 to CB_2 (form angle β) in the clock wise direction.

The return stroke occurs when crank rotates from the position CB_2 to CB_1 (form angle α) in the clockwise dirn.

$$\frac{\text{Time of Cutting Stroke}}{\text{Time of Return Stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360-\alpha} \text{ or } \left(\frac{360-\alpha}{\alpha}\right)$$

Tool travels a distance of R_1R_2 during cutting and return stroke.

∴ Travel of the Tool or length of stroke $= R_1R_2$

$= P_1P_2$

$= 2P_1Q$

$= 2\,AP_1 \sin \angle P_1AQ$

$= 2AP_1 \sin(90^\circ - \alpha/2) = 2AP\cos(\alpha/2) \quad (AP_1 = AP)$

$= 2AP \times \frac{CB_1}{AC}$

$= 2AP \times \frac{CB}{AC} \quad (CB_1 = CB)$

∴ Travel of the Tool or length of the stroke $= 2AP \times \frac{CB}{AC}$

⇒ The degree of freedom of a locked chain is zero.

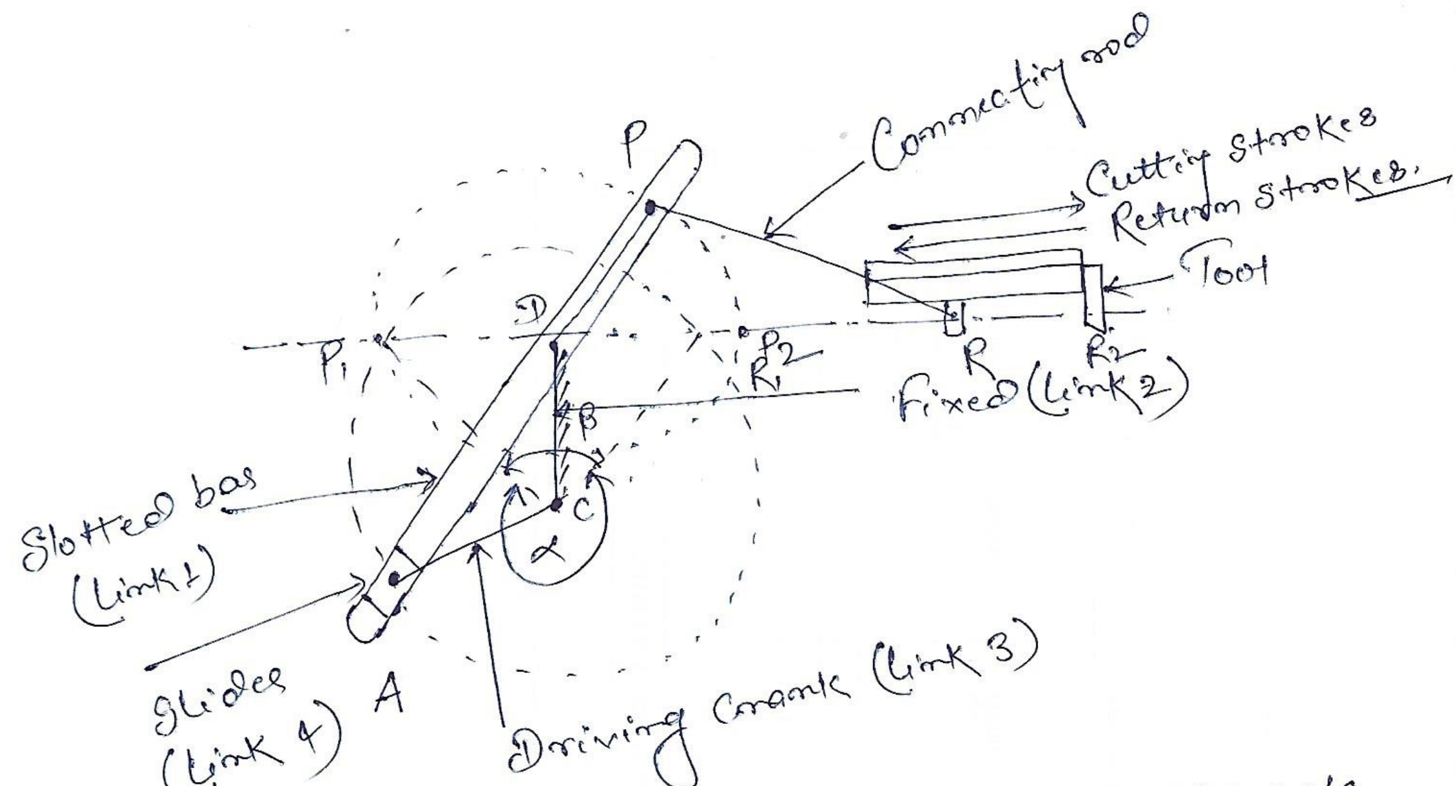
- A structure with -ve degree of freedom is known as a superstructure.

- A linkage is obtained if one of the links of a K.C is fixed to the ground. and motion of any of the movable links results in definite motions of the others. then linkage is known as a mechanism.

→ Redundant chain is also known as kinematic chain.

→ Constrained motion is also known as Definite

(iv) Whitworth quick return motion mechanism :→



This mechanism is mostly used in shaping and slotting m/c. In this mechanism link 2 forming the turning pair is fixed. The driving crank CA (link3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A ~~Connecting~~ slides along the slotted bar PA (link 1), which oscillates at a pivoted point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed.

→ Since, the crank link CA rotates at uniform angular velocity therefore time taken during cutting stroke (forward stroke) is more than time taken during the return stroke. In other words the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke.

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \left(\frac{\alpha}{360^\circ-\alpha}\right) \text{ or } \frac{(360^\circ-\beta)}{\beta}$$

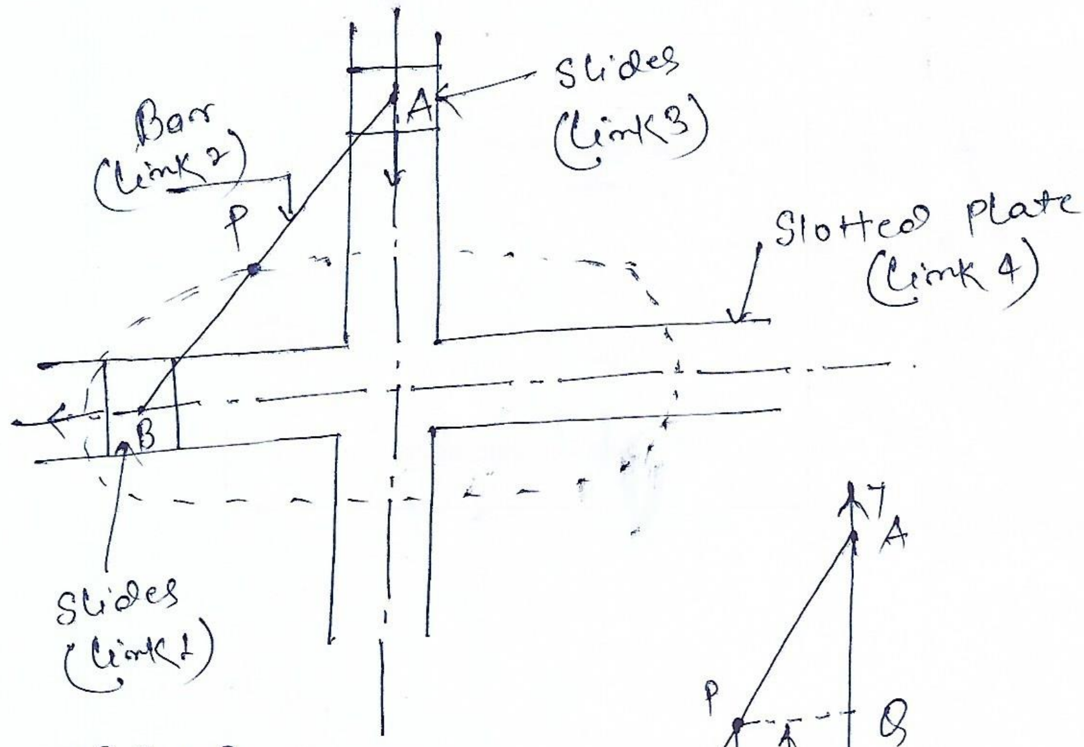
length of effective stroke R_1R_2,

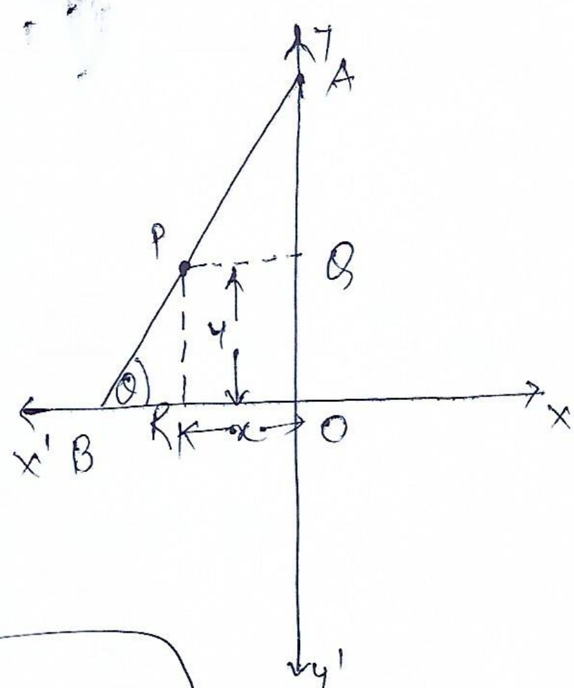
= length of effective

⇒ **Double Slider Crank chain!** → A kinematic chain which Consist of two turning pair and two sliding pair is known as double slider crank chain.

⇒ Inversion of Double Slider Crank chain:—

(i) **Elliptical Trammels:**— It is an instrument used for drawing ellipse. This inversion is obtained by fixing the slotted plate (link 4). The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. The link 1 and link 3 are known as sliders and form sliding pair with link 4. The link 2 (AB) is a bar which form turning pair with link 1 & 3.





$x = PQ = AP\cos\theta$

$y = PR = BP\sin\theta$

$\frac{x}{AP} = \cos\theta$, and $\frac{y}{BP} = \sin\theta$

Squaring and adding.

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1$$

∴ This is the equation of an ellipse. Hence the path traced by point P is an ellipse whose semi major axis is AP and semi-minor axis is BP.

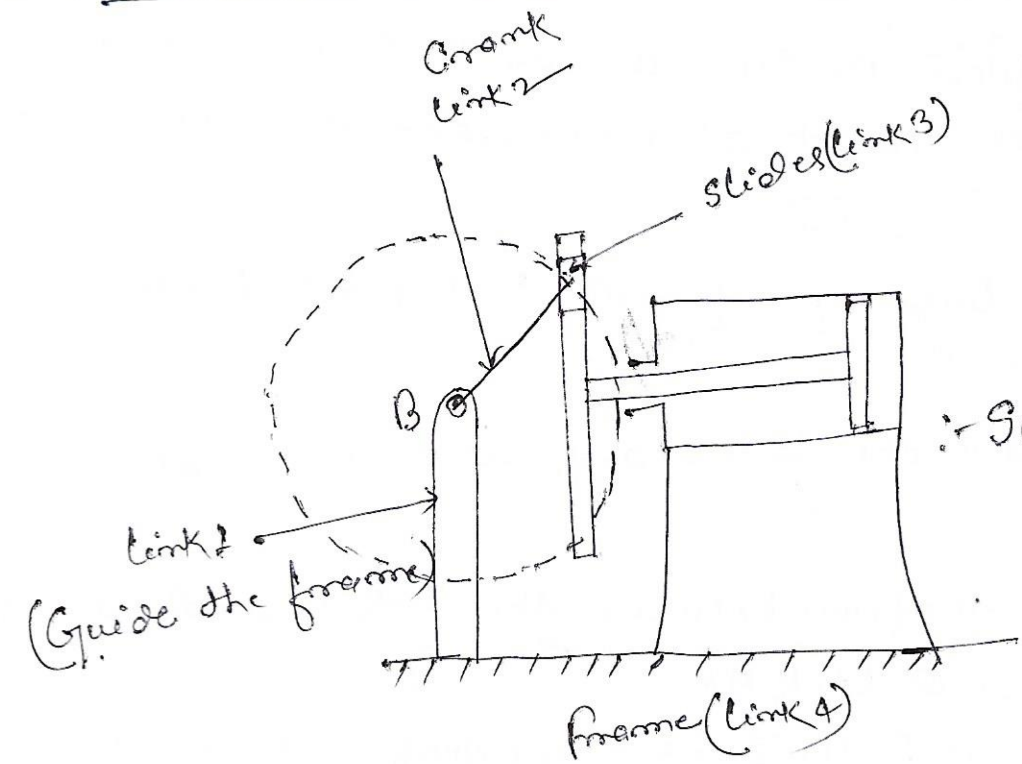
when AP = BP, Then

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(AP)^2} = 1$$

$$\therefore \boxed{x^2 + y^2 = (AP)^2}$$

This is the equation of a circle whose radius is AP. Hence if P is the mid-point of link BA, it will trace a circle.

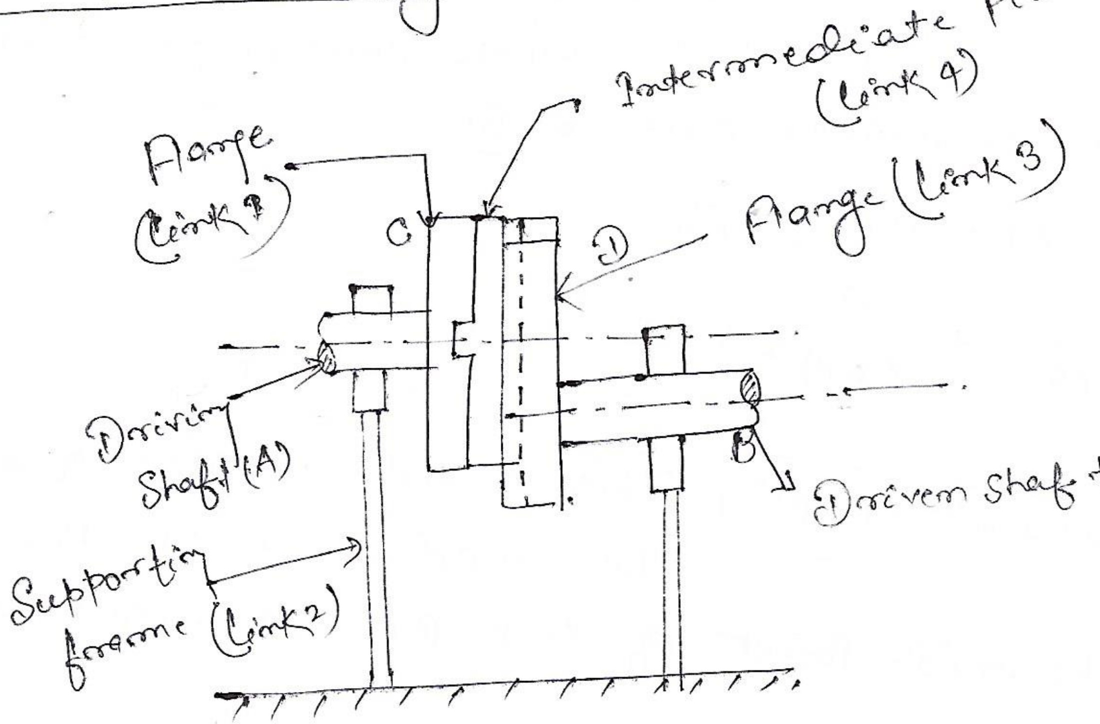
(ii) Scotch yoke mechanism :-



∴ Scoth yoke Mechanism

This mechanism is used for converting rotary motion into a reciprocating motion. This inversion is found by fixing either link 1 or link 3. but in the above diagram link 1 is fixed. when the link 2 rotates about a point B, the link 4 reciprocates.

(iii) Oldham's Coupling:-



An Oldham's Coupling is used for Connecting two parallel Shaft whose axes are at a Small distance apart. The Shafts are Coupled in Such a way that if one shaft rotates, the other shaft also rotates at the Same Speed.

In this inversion link 2 is fixed. link 1 & 3 form Turning Pair with link 2.

link 1, 3 and 4 have the Same angular velocity at every instant.

There is a Sliding motion between the link 4 and Each of the other link 1 and link 3.

If the distance betn the axes of the Shaft is Constant. the Centre of intermediate Piece will describe a Circle of diameter equal to the distance betn the axes of the two shaft. Therefore maxm Sliding speed of each tongue along its slot is equal to the Peripheral velocity of

Grashof's Law :- According to Grashof's law, for the continuous relative motion betn the links of the mechanisms, the sum of length of shortests & longest link should not be greater than the sum of lengths of other two link.

$$\boxed{s + l \leq p + q}$$

Case-I. If law is satisfied $(s + l < p + q)$

If shortest link is fixed $\rightarrow$ Double Crank mechanism

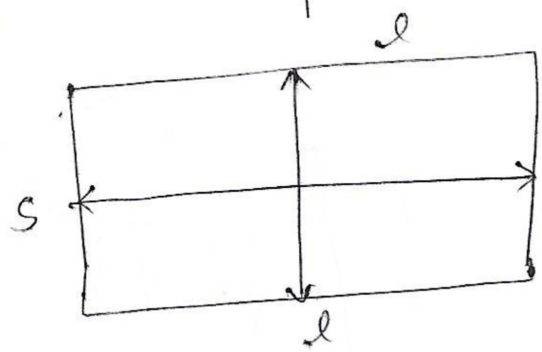
If link adjacent to shortest is fixed $\rightarrow$ Crank & Rocker mechanisms.

Case-II. if $(l + s > p + q)$ $\rightarrow$ laws unsatisfied

(Always Double Rocker Mechanisms)

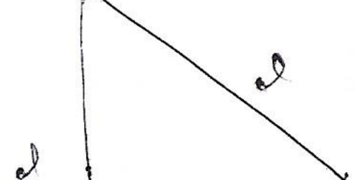
Case-III. $(s + l = p + q)$ $\rightarrow$ law is satisfied having 2,3, 2,3 points of equal length.

(a) Parallelogram linkage.



Always Double Crank Mechanisms.

(b) Deltoid linkage :-



$s \rightarrow$ Fixed : (Double Crank mechanism)

$l \rightarrow$ Fixed (Crank & Rocker

Grabler Criteria :-

we will see only those mechanisms which are having Degree of Freedom (D.O.F) = 1 and They do not have any higher pair.

$n = 1, \quad h = 0$

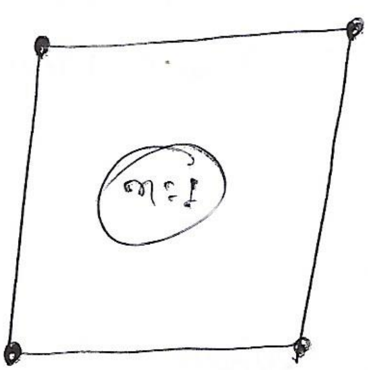
$\Rightarrow \quad n = 3(l-1) - 2J - h$

$\Rightarrow \quad 1 = 3(l-1) - 2J$

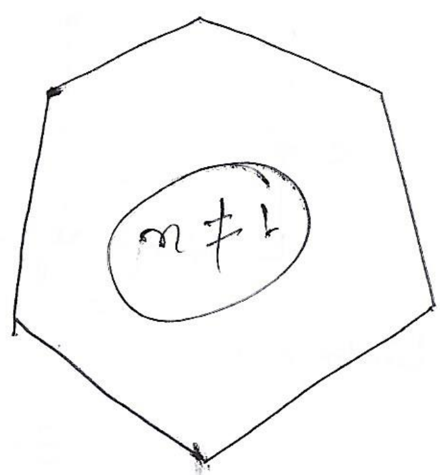
$\Rightarrow \quad \boxed{2J = 3l - 4}$

Since, J & l are whole numbers, So, above Conditions will be Satisfied when l is an even no.

Links	Joints
4	4
6	7
8	10

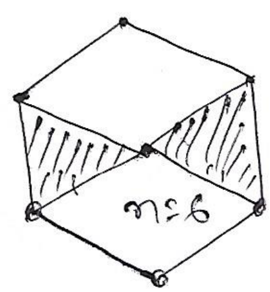


$l = 4$
$J = 4$



$l = 6$
$J = 6$

$\Rightarrow$

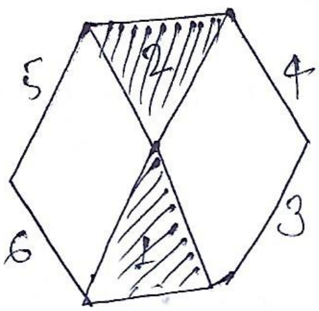


$l = 6$
$J = 7.$

Case:- IV, $\boxed{s+l = p+q}$ law is satisfied having pair not of equal lengths.

1, 5, 2, 4.

Watt's six bar chain

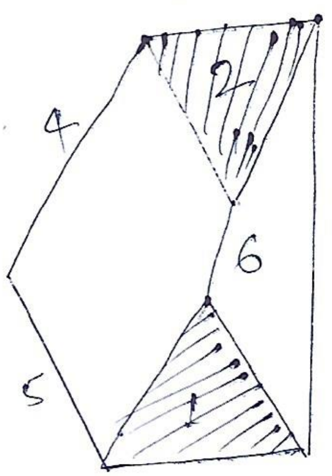


Link 1 & 2 = Ternary links.

Link 4, 3, 5 & 6 = Binary links.

In this dia. both Ternary links are directly attached.

Stephenson's six bar chain.



Link 1 & 2 = Ternary links

Link 3, 4, 5 & 6 = Binary links.

In this dia. both Ternary links is not connected directly.

For linkage having pairs with a single degree of freedom only, $P_2 = 0$.

$$F = 3(N-1) - 2P_1$$

when $F = 1$ = one degree of freedom with one input to any of the links.

$$1 = 3(N-1) - 2P_1$$

$$\boxed{2P_1 = 3N - 4}$$

P_1 & N are to be whole numbers, The relation can be satisfied only if N is even.

(Links) $N = 4$	(Joints) $P_1 = 4$	No excess turning Pair
$N = 6$	$P_1 = 7$	one excess turning Pair
$N = 8$	$P_1 = 10$	Two excess turning Pairs.

L	F (Degree of freedom)	P_1 → No. of joints
1	$N-3$	N
2	$N-5$	$N+1$
3	$N-7$	$N+2$
4	$N-9$	$N+3$
5	$N-11$	$N+4$

where L = Number of loops in a linkage.

Let l = Total no. of links in a mechanism

n = Degree of freedom

P_1 = Numbers of Pairs having one degree of freedom.

P_2 = " " " " Two " " "

No. of movable links $(l-1)$

No. of degree of freedom of $(l-1)$ links is $= 6(l-1)$.

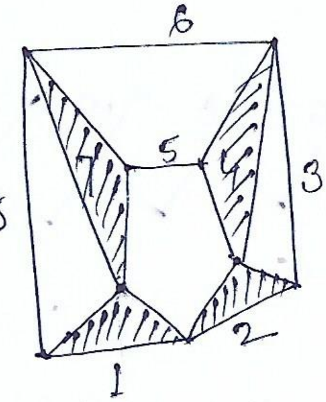
Q. No - 1.

Total no. of link = 8

Total no of Binary joint = 7B + 2T

= 7B + 2×2 B

= 11 Binary joint.



So. $n = 3(l-1) - 2j - h$

Here, $h = 0$.

Then $n = 3(l-1) - 2j = 3(8-1) - 2 \times 11$

$= 21 - 22 = -1$

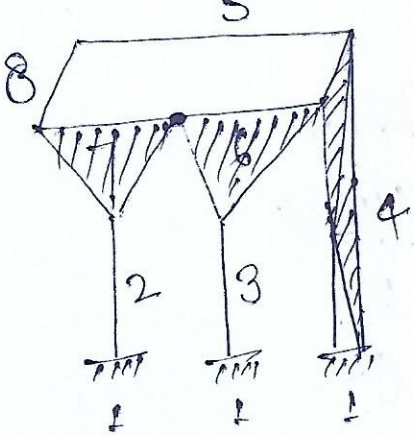
Number of degree of freedom = -1 (Superstructure)

or,

$F = N - (2L+1)$

$= 8 - (2 \times 4 + 1) = 8 - 9 = -1.$

(2)



Binary joint = 10

So, $n = 3(l-1) - 2j$

$= 3(8-1) - 2 \times 10 = 21 - 20$

$= 1.$

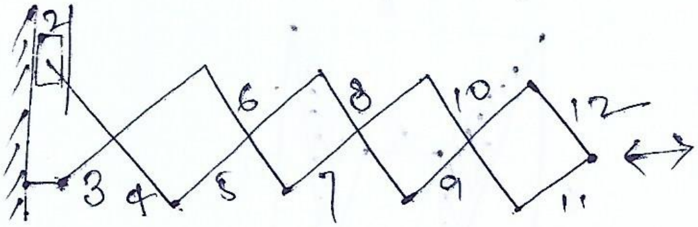
$F = N - (2L+1)$

$= 8 - (2 \times 3 + 1)$

OR, $F = 3(N-1) - 2P_1$

$= 3(8-1) - 2 \times 10$

(3)



Total no. of link = 12

Total no. of Binary joints = 16

Link 1 & 2 makes a sliding pair and one sliding pair is equivalent to one lower pair

we already know that.

lower pair ~~and~~ equivalent to 1 Binary joints.

So, $n = 3(l-1) - 2j$

$= 3(12-1) - 2\times 16 = 33 - 32 = 1$ Ans.

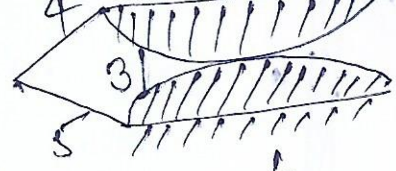
OR

L = no. of loops = 5

$F = N - (2L+1)$

$= 12 - (2\times 5+1) = 12 - 11 = 1$ Ans.

(4) $l = 5$



Total no. of higher pair = 1

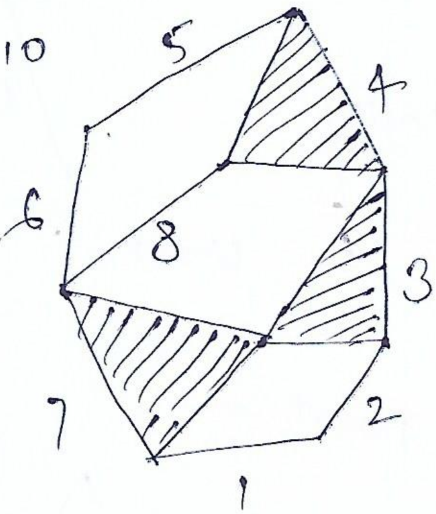
Total no. of Binary joint = 5

$\therefore n = 3(l-1) - 2j - h$

$= 12 - 10 - 1 = 1$ Ans.

$\Rightarrow$ Total No. of Link = 8

Total No. of Binary point = 10



$\therefore n = 3(l-1) - 2J$

$= 3(8-1) - 2 \times 10$

$= 21 - 20 = \underline{\underline{1.}}$

OR $F = N - (2L+1)$

$= 8 - (2 \times 3 + 1)$

$= 8 - (7) = 1$ Ans.

$F = N - (2L+1)$

$P_1 = N + (L-1)$

or

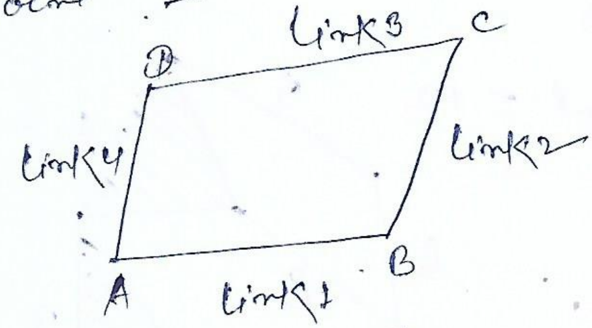
$(N + L - 1)$

where, N – Total No. of Link.

L – No. of Loop.

P_1 – No. of Binary point.

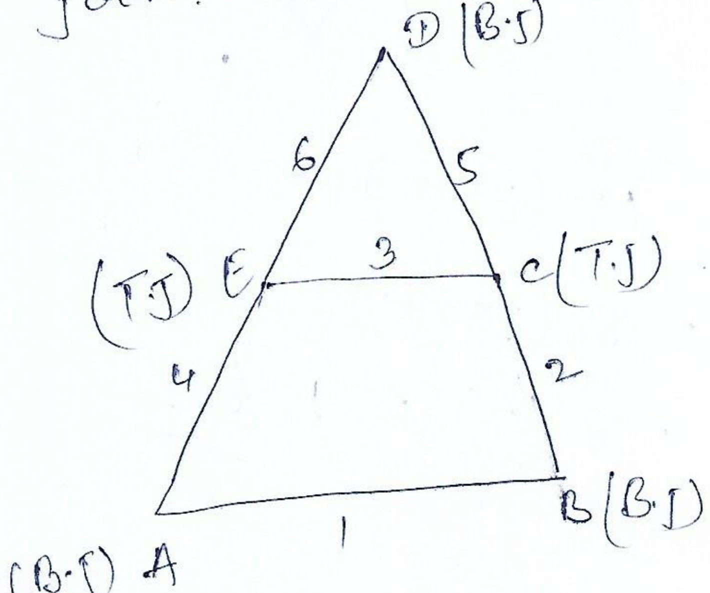
(i) Binary joint :-



J = 4

$\boxed{J = \frac{3}{2}l - 2}$

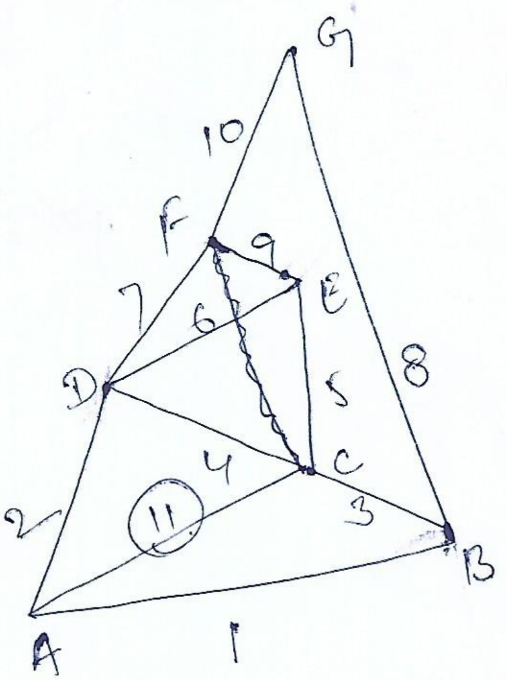
(ii) Ternary joint :-



3 = Binary joint
2 = Ternary joint
= 2×2
= 4 Binary joint.

Total J = 3+2×2
= 7.

(iii) Quaternary joint :-



Total J = 1+4×2+2×3
= 1+8+6 = 15

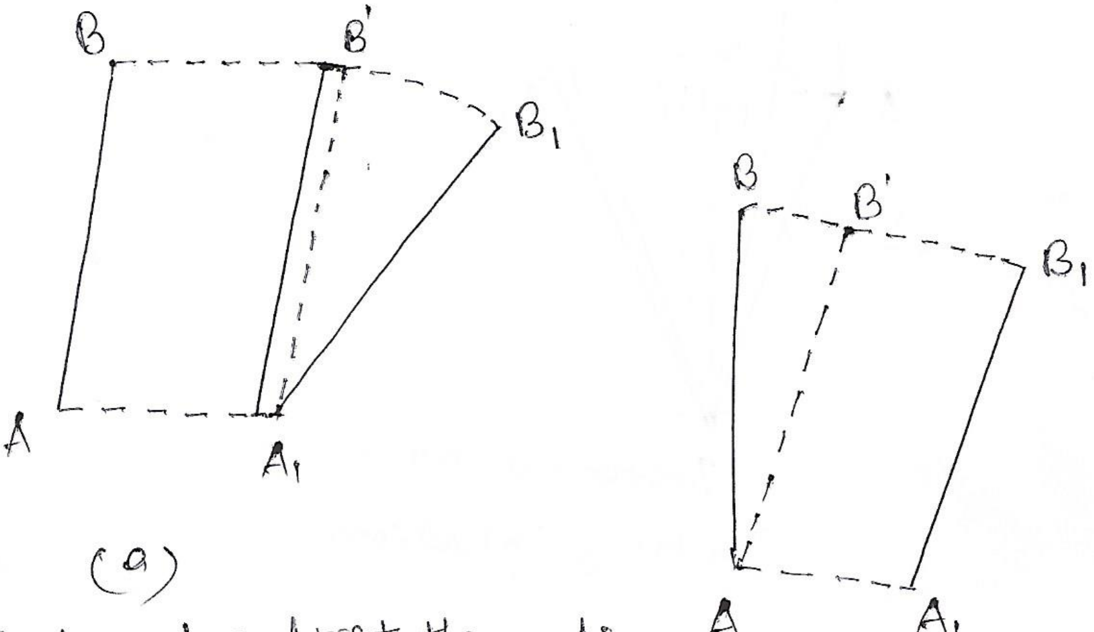
Let ω = Angular velocity of Each shaft in (rad/s)

d = Distance betn the axes of the shafts in (m)

∴ Maximum sliding speed of Each Torque in (m/s) is

$$\boxed{v = \omega \cdot d}$$

⇒ Velocity in Mechanisms (Instantaneous Centre Method) :-



(a)

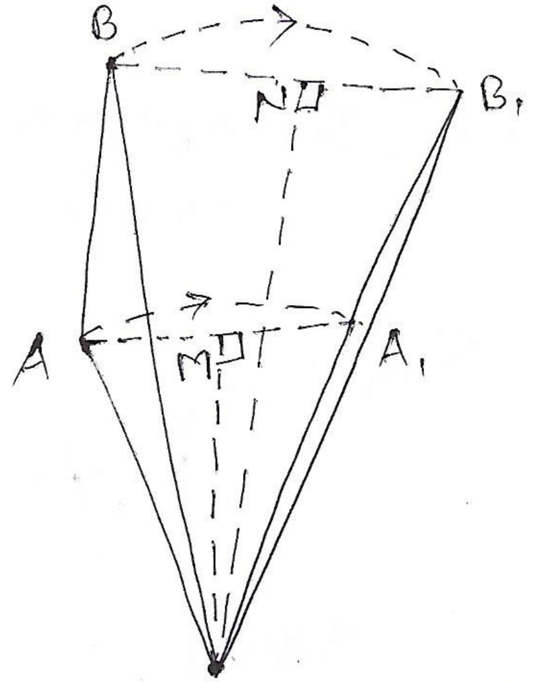
(b)

In fig (a) The link has first the motion of Translation from AB to A_1B' and then the motion of rotation about A_1, till it occupies the final Positions A_1B_1.

In fig. (b). The link AB has first the motion of rotation from AB to AB' about A and then the motion of translation from AB' to A_1B_1.

Such a motion of link AB to A_1B_1 is an example of Comor the motion of Translation.

→ This Combined motion of Rotation and translation of the link AB may be assumed to be a motion of Pure rotation about Some Centre I, known as Instantaneous Centre of Rotation (also Called Centro or virtual Centre).



Instantaneous
Centre of Rotation.

→ The Locus of all Such instantaneous Centre is known as Centrode.

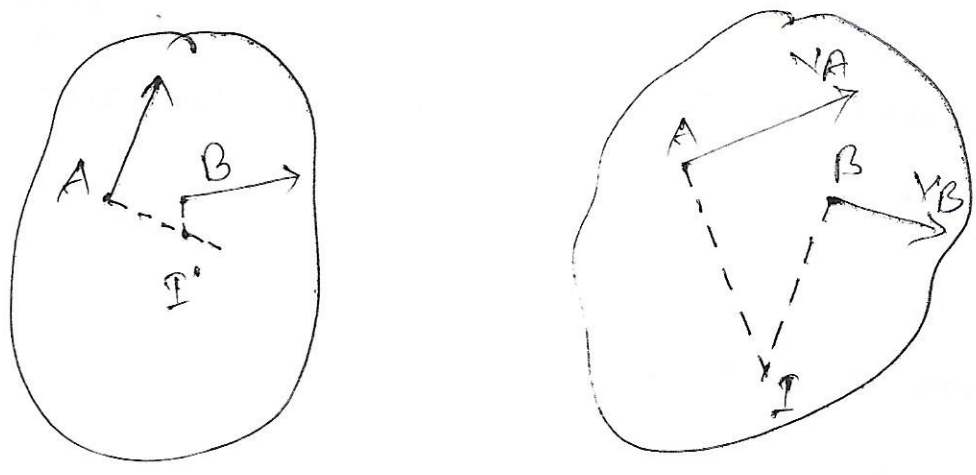
→ A line drawn through an instantaneous Centre & Perpendicular to the plane of motion is called instantaneous axis.

→ The locus of this axis is known as axode.

→ The locus of the instantaneous centre in space during a definite motion of the body is called the space relative to the body itself is called the body Centrode.

⇒ Instantaneous Centre Method :-

Instantaneous Centre of Rotation



In general, motion of any link in mechanism is neither Pure rotation nor Pure translation. It is the Combination of translation and Rotation which is said to be general motion.

But any link at any moment can be assumed to be in Perfect rotation with respect to a Point in space known as instantaneous Centre or rotation. This Centre is also known as virtual Centre.

As the link moves in general motion it's I.C keeps on changing, the locus of I.C of rotation for a link during it's whole motion is known as Centrode.

The line Passing through I.C of the rotation and ⊥r to the Plane of mechanism is known instantaneous axis of rotation. The locus of instantaneous axis

Motion	Centrodes	Axode
General	Curved line	Curved Surface
Pure translation	Line	Plane Surface
Pure Rotation	Point	Line.

No. of instantaneous Centre in a mechanism will be

$$= {}^{n}C_2 = \frac{n(n-1)}{2}$$

where n = No. of link in a mechanism.

Case-I, when $n = 4$

$$\therefore I.C = \frac{n(n-1)}{2} = \frac{4(3)}{2} = 6$$

I_{12} I_{13} I_{14}

I_{23} I_{24}

I_{34}

Case-II, when $n = 6$,

$$I.C = \frac{6(5)}{2} = 15$$

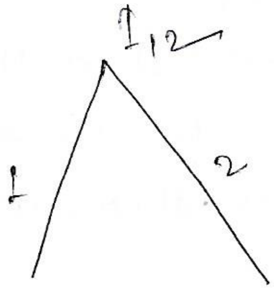
I_{12} I_{13} I_{14} I_{15} I_{16}

I_{23} I_{24} I_{25} I_{26}

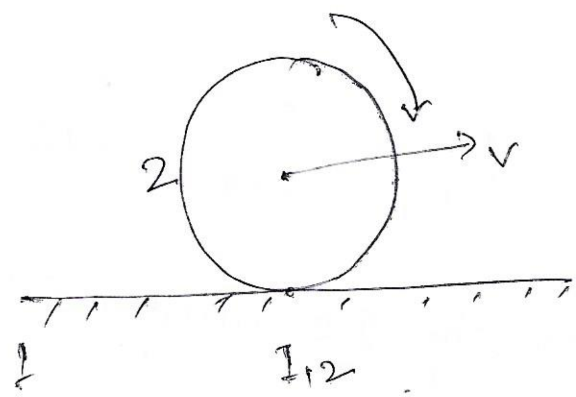
I_{34} I_{35} I_{36}

I_{45} I_{46}

(i) If the two links have turning pair then I.C goes on the pin joint.

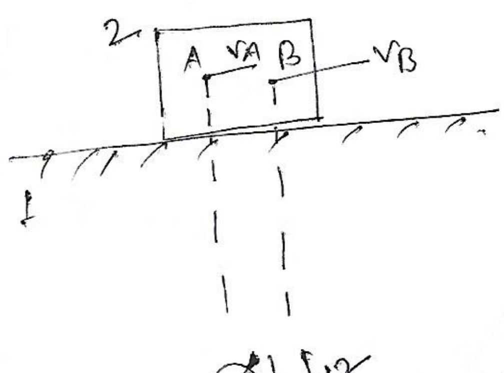


(ii) Rolling Pair:-



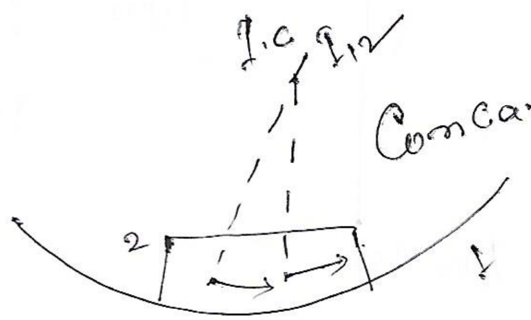
I.C lies on the contact point.

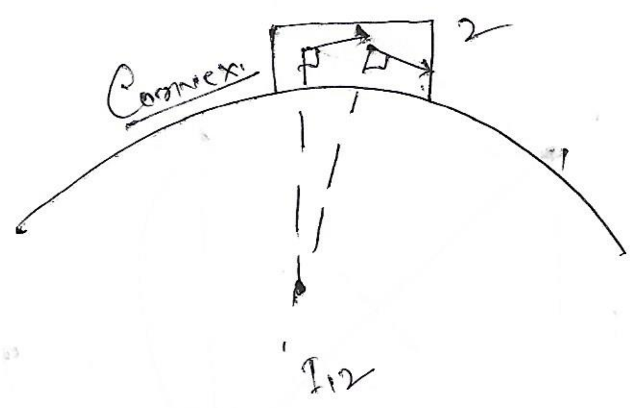
(iii) Sliding pair on plane surface



I.C lies on the infinity.

(iv) Sliding on Curved Surface.

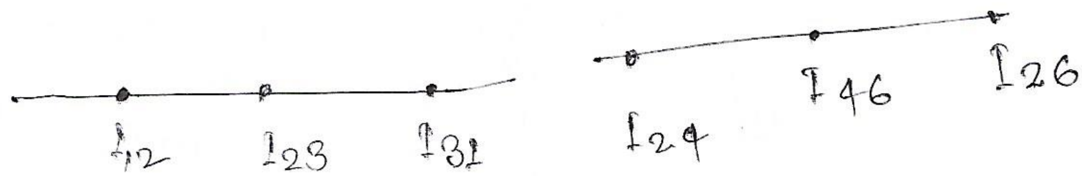




⇒ Kennedy's Theorem:-

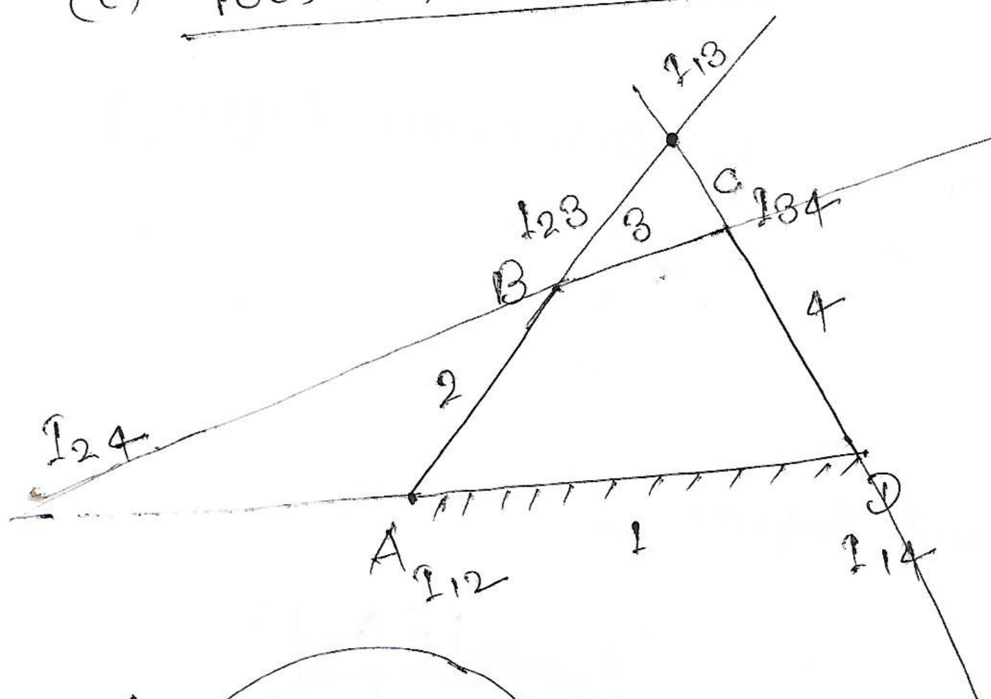
If three links are having relative motion with each other, then their relative Instantaneous Centres must lie on a straight line. This is known as Kennedy's theorem or three Centre inline theorem.

Let link 1, 2, 3   2 4 6



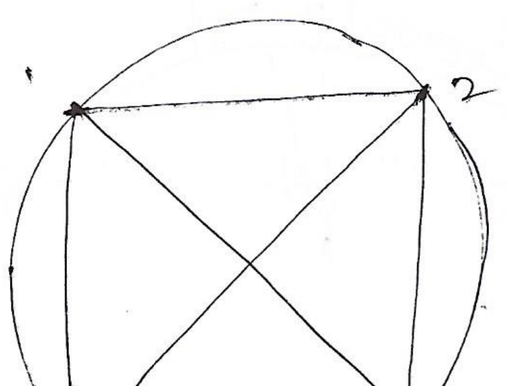
⇒ Velocity calculation by I.C method:-

(i) Four bar chain mechanism.



Total no. of link
$n = 4$
$\therefore N = 6.$

I_{12} I_{13} I_{14}
I_{23} I_{24}
I_{34}



$I_{13} \rightarrow I_{12}\ I_{23}$

$I_{24} \rightarrow I_{23}\, I_{34}$

$I_{24} \rightarrow I_{21}\, I_{14}$

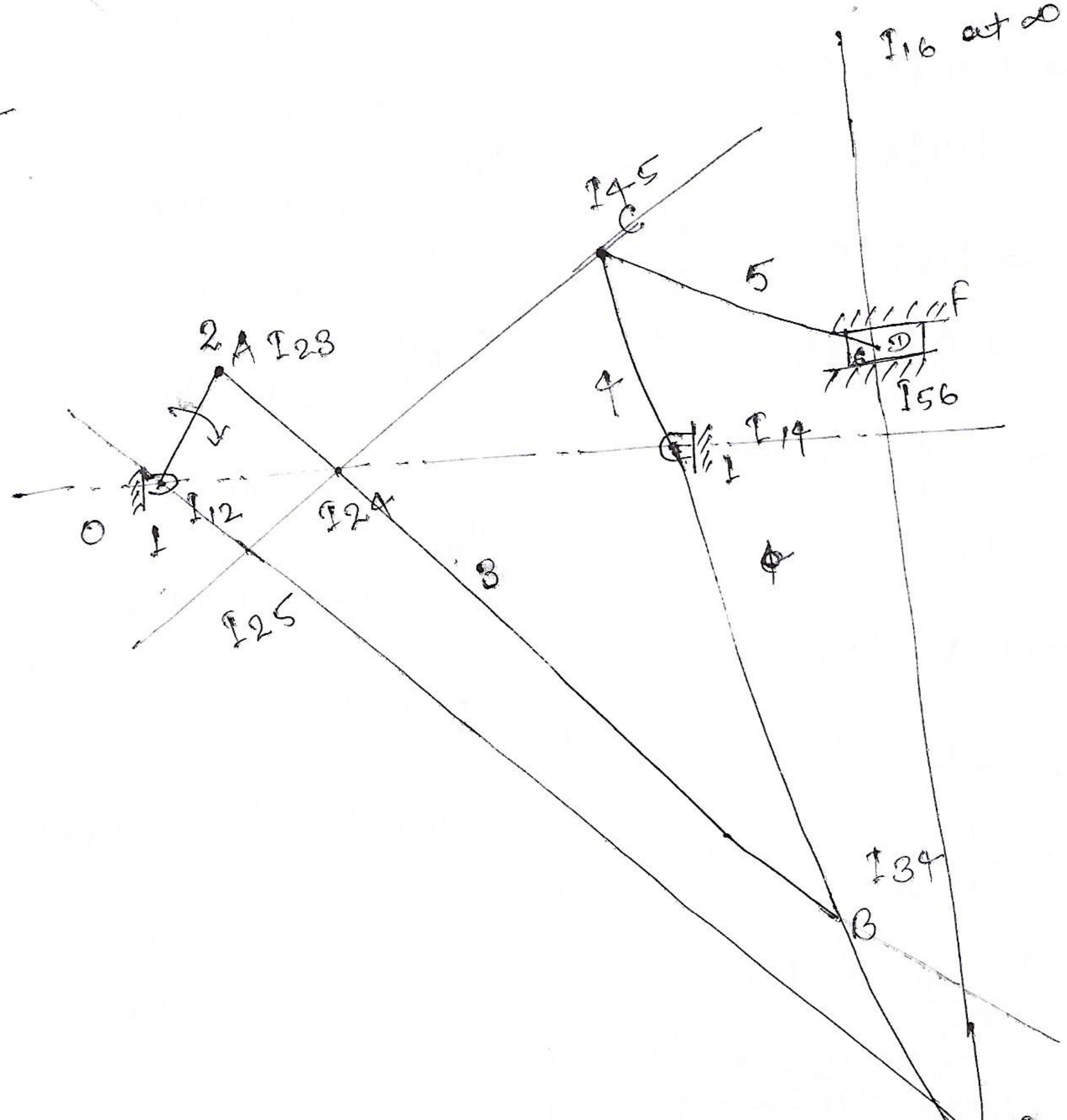
Link 3 (BC) $\rightarrow I_{13}$

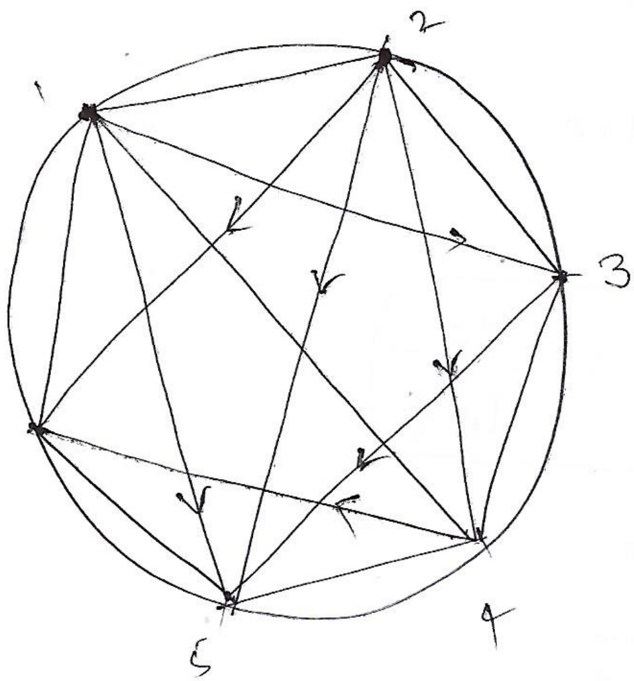
$$\boxed{\omega_{BC} = \frac{V_B}{I_{13}B} = \frac{V_C}{I_{13}C}}$$

Link 4 (CD) $\rightarrow I_{14}$

$$\omega_{CD} = \frac{V_C}{I_{14}C}$$

Q.





$n = 6$

$\therefore N = \frac{6 \times 5}{2}$

$= 15$

$I_{13} \to I_{12}\, I_{23}$, $\; I_{14}\, I_{43}$

$I_{24} \to I_{23}\, I_{34}$, $\; I_{21}\, I_{14}$

$I_{15} \to I_{14}\, I_{45}$, $\; I_{16}\, I_{65}$

$I_{25} \to I_{21}\, I_{15}$, $\; I_{24}\, I_{45}$

$I_{35} \to I_{34}\, I_{45}$, $\; I_{32}\, I_{25}$

$I_{26} \to I_{21}\, I_{16}$, $\; I_{25}\, I_{56}$

$I_{36} \to I_{35}\, I_{56}$, $\; I_{31}\, I_{16}$

$I_{46} \to I_{45}\, I_{56}$, $\; I_{41}\, I_{16}$.

Link 3 $\to AB \to I_{13}$

$\omega_3 = \omega_{AB} = \frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B}$

Link 5 $\to CD \to I_{15}$

$\omega_5 = \omega_{CD} = \frac{V_C}{I_{15}C} = \frac{V_D}{I_{15}D}$

Link 4 $\to BC \to I_{14}$

$\omega_4 = \omega_{BC} = \frac{V_B}{I_{14}B} =$

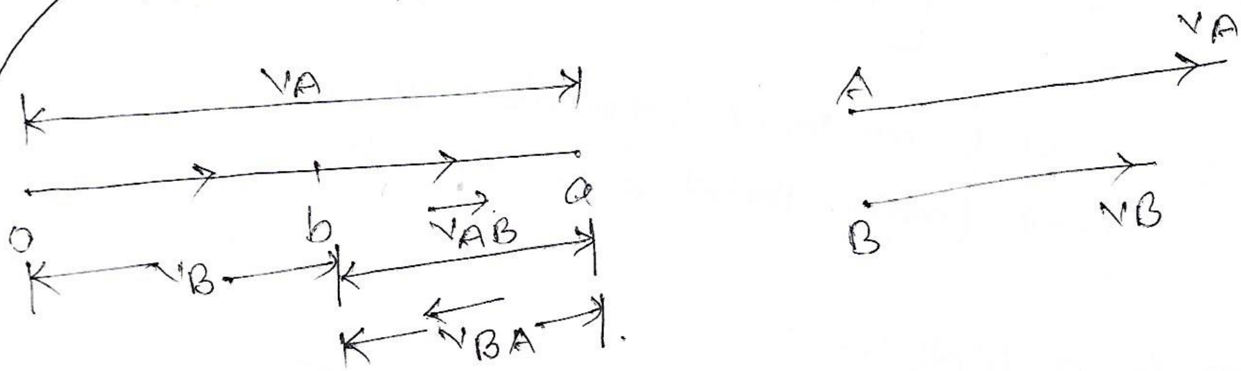
$\frac{V_C}{I_{14}C}$

⇒ Relative velocity of Two bodies moving in Straight Lines

Two bodies A & B moving along parallel line in the same direction with absolute velocities V_A and V_B.
In which ($V_A > V_B$).

The relative velocity of A w.r.t B,

$V_{AB} = \vec{V_A} - \vec{V_B}$



$$\boxed{\vec{ba} = \vec{oa} - \vec{ob}}$$

Similarly, The relative velocity of B w.r.t A is

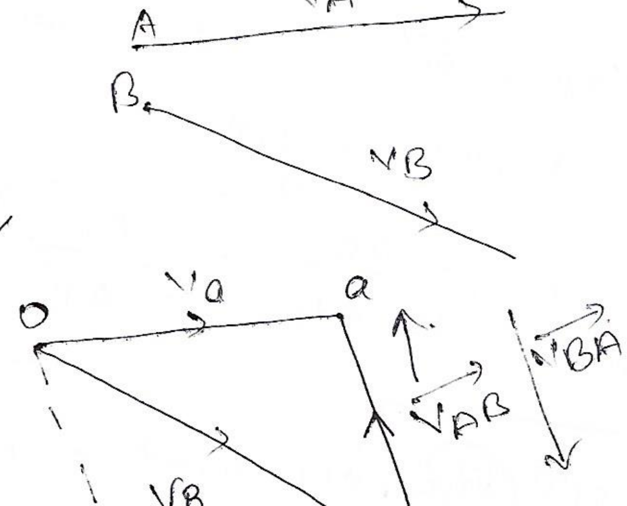
$\vec{V_{BA}} = \vec{V_B} - \vec{V_A}$

$$\boxed{\vec{ab} = \vec{ob} - \vec{oa}}$$

⇒ For Inclined direction.

$\vec{V_{AB}} = \vec{V_A} - \vec{V_B}$

$$\Rightarrow \boxed{\vec{ba} = \vec{oa} - \vec{ob}}$$



Similarly, The relative velocity of B w.r.t A is given below.

$$\vec{V_{BA}} = \vec{V_B} - \vec{V_A}$$

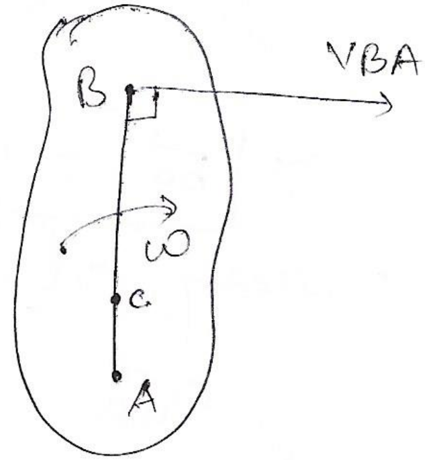
$$\boxed{\vec{ab} = \vec{ob} - \vec{oa}}$$

We find that $\boxed{V_{AB} = -V_{BA}}$ or $\boxed{\vec{ba} = -\vec{ab}}$

V_{AB} - Start from Point b towards a and

V_{BA} - Start from Point a towards b..

⇒ Motion of a link :-



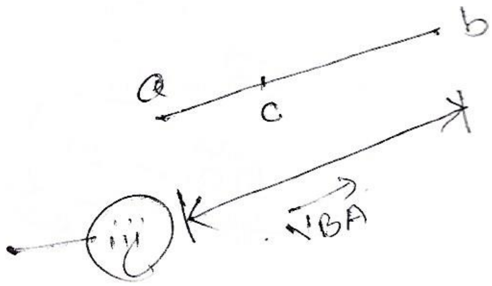
Velocity of any Point on a link with respect to another Point on the same link is always ⊥r to the line joining these Points, on the given Configurations.

Let ω = Angular velocity of the link AB about A.

$$\vec{V_{BA}} = \vec{ab} = \omega \cdot AB \quad \text{------ (i)}$$

Similarly, $\vec{V_{CA}} = \vec{ac} = \omega \cdot AC \quad \text{------ (ii)}$

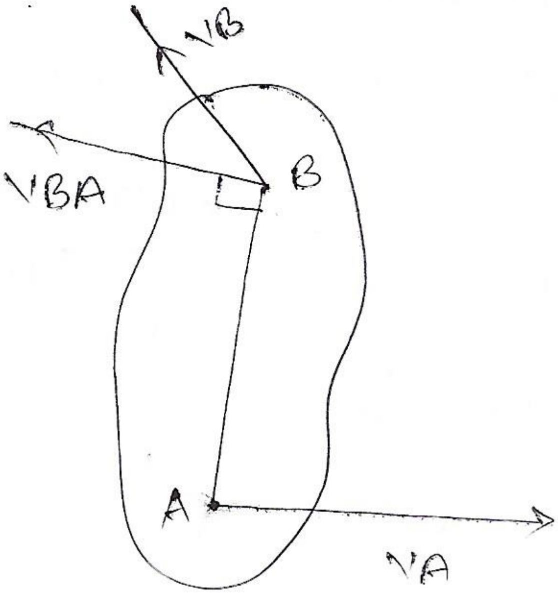
From eqn (i) & (ii) we get,

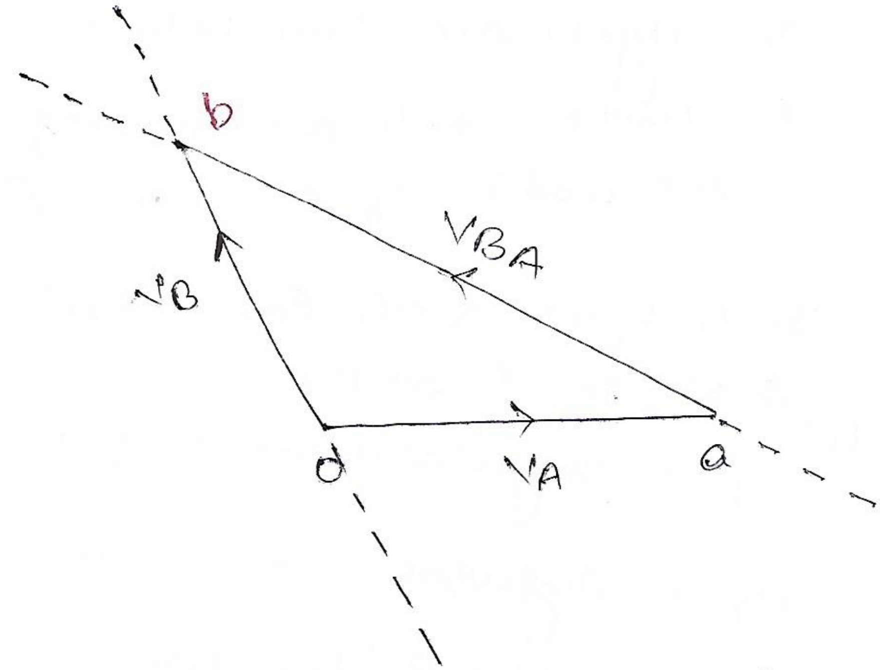


$$\boxed{\frac{\vec{V_{CA}}}{\vec{V_{BA}}} = \frac{\omega \cdot AC}{\omega \cdot AB} = \frac{AC}{AB}} \quad \text{------ (iii)}$$

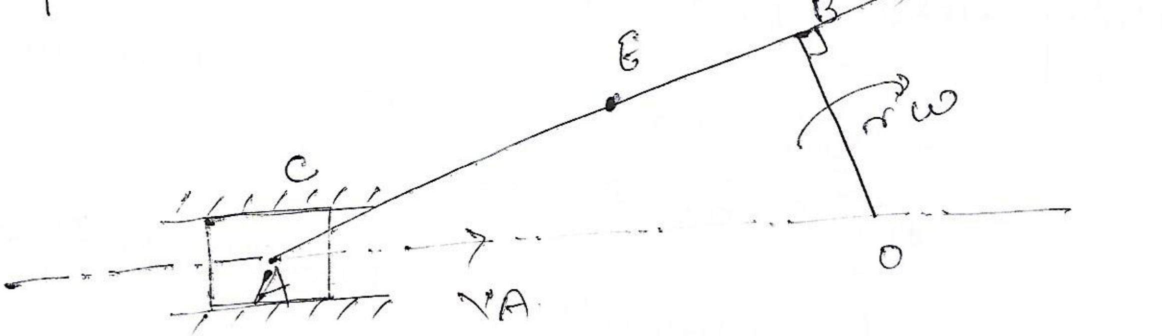
From above eqn (iii), the Point c on the vector ab divides

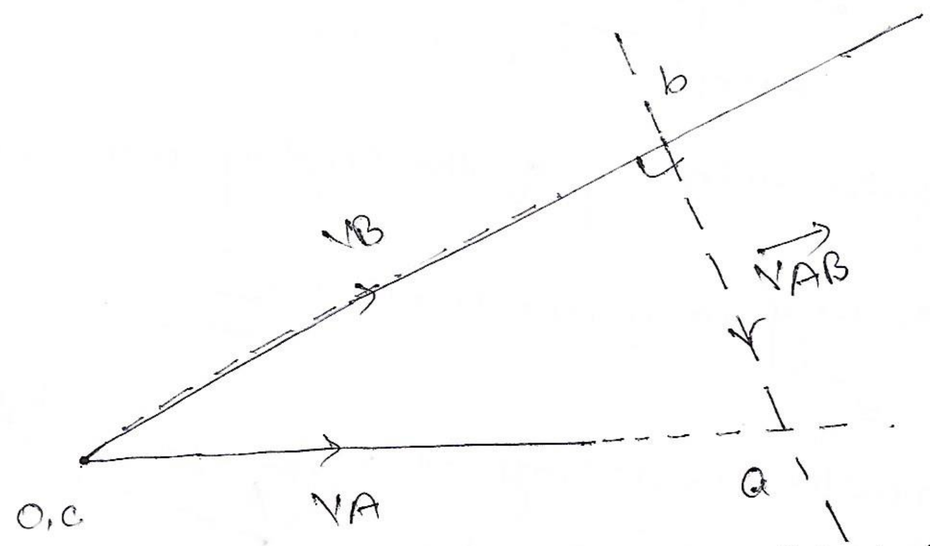
⇒ Velocity of a point on a link by Relative velocity Method.





⇒ Velocity in slider crank mechanism:—



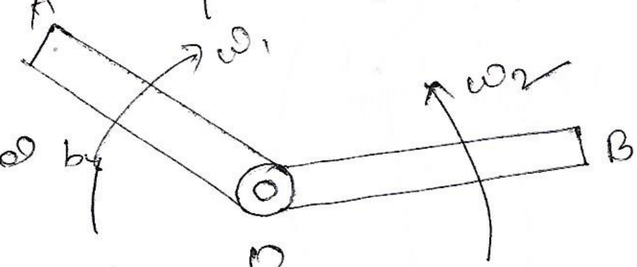


The angular velocity of the connecting rod (AB) (ω_{AB}) is determined by

$$\boxed{\omega_{AB} = \frac{V_{BA}}{AB} = \frac{ab}{AB}}$$

⇒ Rubbing velocity at a Pin joint.

The algebraic sum between the angular velocities of the links which are connected by pin joints, multiplied by the radius of the pin.



Let link OA & OB connected by a pin joint at O.

Let,

ω_1 = Angular velocity of the link OA

ω_2 = Angular " " " OB

r = Radius of the pin.

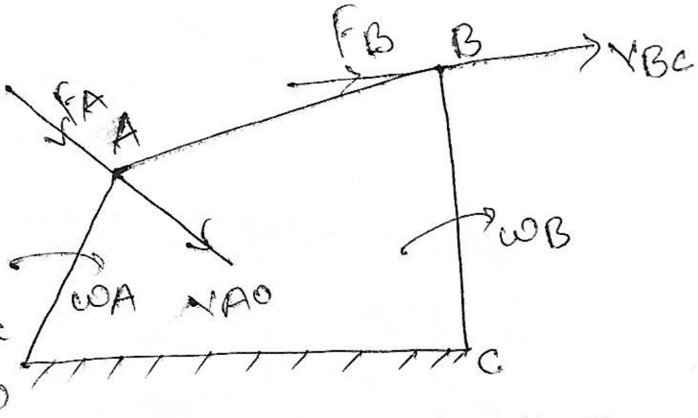
Rubbing velocity at the pin joint O

$= (\omega_1 - \omega_2) \cdot r$, when the link moves in the same direction.

$= (\omega_1 + \omega_2) \cdot r$, when the link move in the opposite direction.

⇒ The angular velocity of the sliding member is zero.

⇒ Forces Acting in a mechanism :-



Let, FA newton is acting at the joint A in the direction of VAO.

FB newton is transmitted to the joint B in the direction of

Mechanism

Neglect the effect of friction and the change of K.E of the link.

A/c to the Principle of Conservation of Energy

Input work per unit time = output work per unit time

∴ work supplied to the joint A = work transmitted by the joint B.

$$\Rightarrow \quad F_A \cdot V_A = F_B \cdot V_B$$

$$\therefore \boxed{F_B = \frac{F_A \cdot V_A}{V_B}} \quad \& \quad \therefore \boxed{F_A = \frac{F_B \cdot V_B}{V_A}}$$

$$\eta = \frac{\text{output}}{\text{Input}} = \frac{F_B \cdot V_B}{F_A \cdot V_A}$$

$$\therefore \boxed{F_B = \frac{\eta \cdot F_A \cdot V_A}{V_B}}$$ when the considers of the effect of friction.

Driving torque (T_A) by FA and the resisting torque T_B by FB.

$$\therefore T_A \cdot \omega_A = T_B \cdot \omega_B$$

$$\& \boxed{\eta = \frac{T_B \cdot \omega_B}{T_A \cdot \omega_A}}$$

ω_A – Angular velo. of the link DA

" " " CB.

⇒ Mechanical Advantages:-

The ratio of the load to the effort is known as mechanical advantages. The link DA is called driving link.
Link CB is driven link.

FA acting at A is the effort
FB acting at B is the load or resistance to over-come.

From the Energy Conservation Equation we know that

$$F_A \cdot V_A = F_B \cdot V_B$$

$$\text{or, } \boxed{\frac{F_B}{F_A} = \frac{V_A}{V_B}}$$

$$\boxed{(\text{Mechanical advantage})_{ideal} = \frac{F_B}{F_A} = \frac{V_A}{V_B}}$$

$$\boxed{(M.A)_{Actual} = \eta \times \frac{F_B}{F_A} = \eta \times \frac{V_A}{V_B}}$$

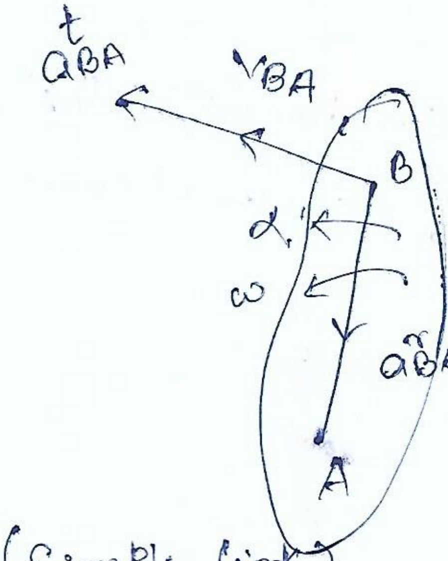
In the form of Torque.

$$\boxed{(M.A)_{ideal} = \frac{T_B}{T_A} = \frac{\omega_A}{\omega_B}}$$ when neglecting the effect of friction

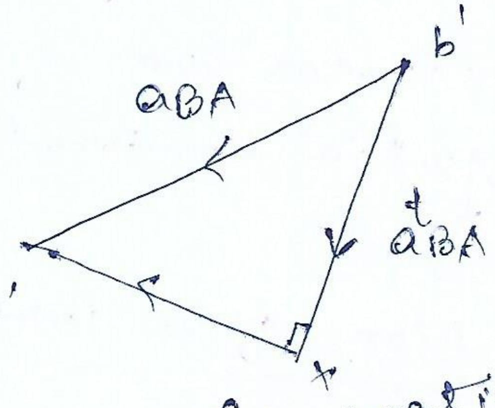
$$\boxed{(M.A)_{Actual} = \eta \cdot \frac{T_B}{T_A} = \eta \times \frac{\omega_A}{\omega_B}}$$ when the Considering the effect of friction.

⇒ Acceleration in mechanism :-

→ Acceleration diagram for a link is given below.



(Simple link)



(Acceleration diagram.)

Acceleration of a particle whose velocity changes both in magnitude & direction at any instant has two component

(i) Centripetal or Radial Component – This component is always perpendicular to the velocity of the particle at any instant, and it is also acts towards given link BA.

$$a^r_{BA} = \omega^2 \times \text{length of link} = \omega^2 \cdot AB$$

$$\boxed{a^r_{BA} = \omega^2 \cdot AB}$$

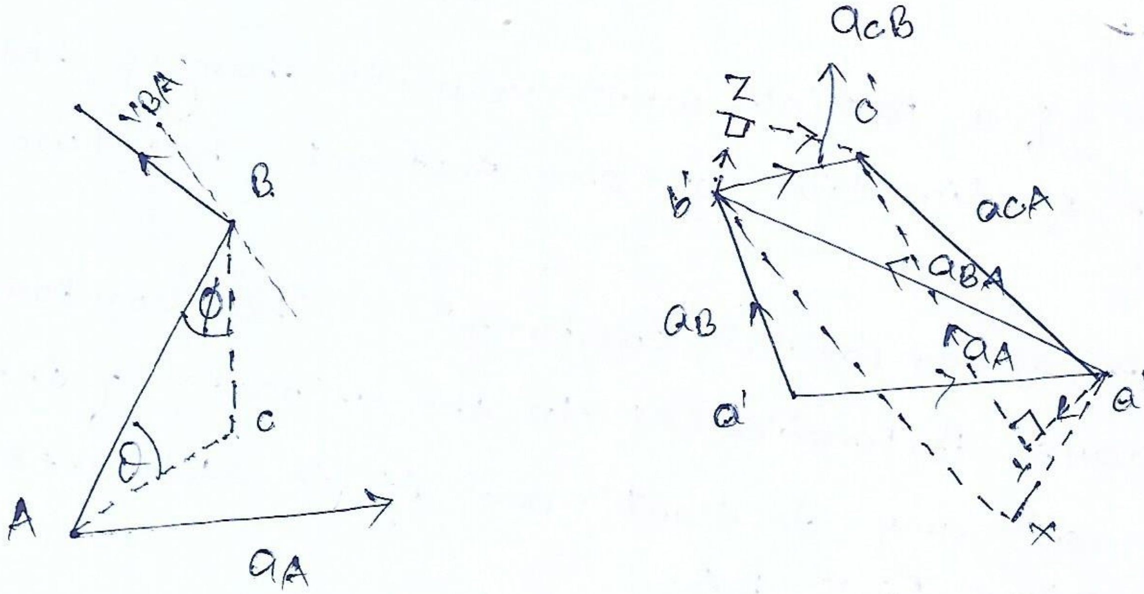
(ii) Tangential Component – It is parallel to the velo. of the particle at given instant and it is also perpendicular to the link AB.

→ Vector b'a' represents the total acceleration of B w.r.t A (a_{BA}).

→ Vector b'a' is also known as acceleration image of the link AB.

→ It is the vector sum of radial component (a^r_{BA}) and Tangential Component (a^t_{BA}) of acceleration.

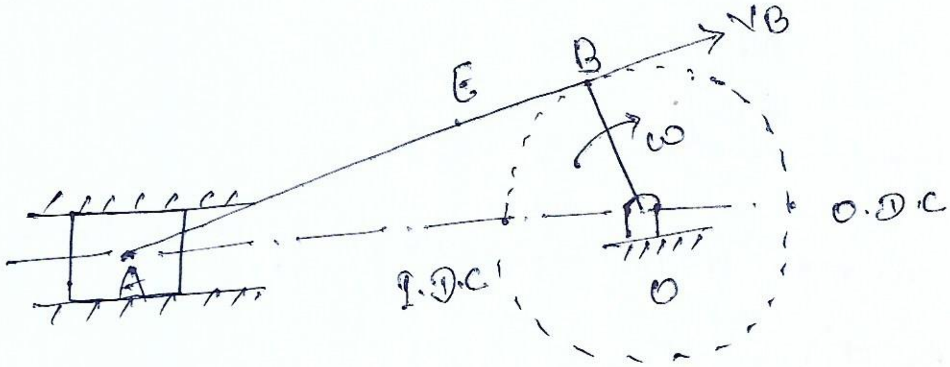
⇒ Acceleration of a Point on a link :→



Two Points A & B on the rigid link is given. Let the acceleration of the Point a_A is known in magnitude & direction & The direction of Path of B is given.

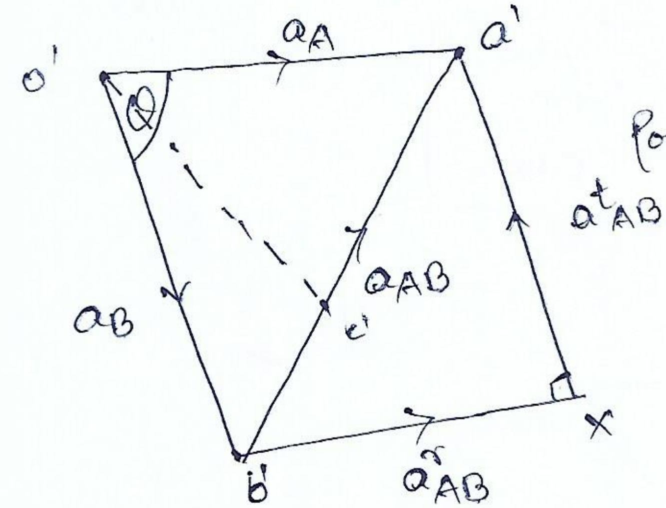
The acceleration of the Point B is determined in magnitude and direction by drawing the accen diagram is given below.

⇒ Acceleration in the slider Crank mechanism :-



Slider Crank mechanism.

Then Acceleration diagram is given below.



$a^r_{BO} = \omega^2 \cdot OB$

=

where ω is given.

Then magnitude of Radial Component of Point B towards O is given or find out by given data & formula.

Due to uniform angular velocity. The Tangential Component of accⁿ is Zero.

The acceleration of any other point on AB such as E may be obtained by dividing the vector b'a' at e' in the same ratio as E divides AB.

$$\boxed{\frac{a'e'}{a'b'} = \frac{AE}{AB}}$$

Velocity of B w.r.t O or velocity of B (because O is a fixed point)

$V_{BO} = V_B = \omega_{OB} \times OB$ (Tangentially at B)

$a^t_{BO} = 0$ (due to constant angular velocity)

$$a^r_{BO} = a^r_B = \omega^2_{BO} \times OB = \frac{\vec{V}^2_{BO}}{(OB)^2} \times OB$$

$$= \left(\frac{\vec{V}^2_{BO}}{OB}\right)$$

$$\boxed{a^r_B = a^r_{BO} = \frac{\vec{V}^2_{BO}}{OB}}$$

⇒ Whitworth Quick Return motion mechanism:-

Time taken during the Cutting stroke is more than the time taken during the return stroke.

Mean Speed of Ram during cutting stroke is

$$V_C = \frac{\text{distance travelled}}{\text{time}} = \frac{2AE}{\left(\frac{\alpha}{\omega}\right)} = \frac{2AE\omega}{\alpha}$$

$$\boxed{V_C = \frac{2AE\omega}{\alpha}} \quad \text{---} (i)$$

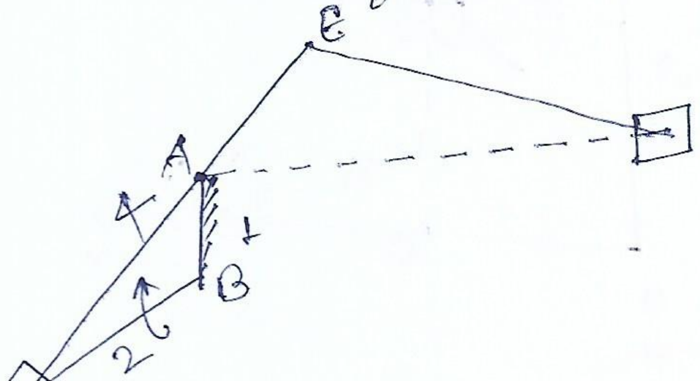
Similarly, Mean Speed of Ram during Return stroke is

$$\boxed{V_R = \frac{2AE\omega}{\beta}} \quad \text{---} (ii)$$

$\boxed{V_C < V_R}$ Because $\alpha > \beta$.

The ratio betn time taken during the cutting & Return stroke is given by

$$\frac{\text{Time of Cutting Stroke}}{\text{Time of Return Stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{(360^\circ - \alpha)} \text{ OR } = \left(\frac{360^\circ - \beta}{\beta}\right)$$

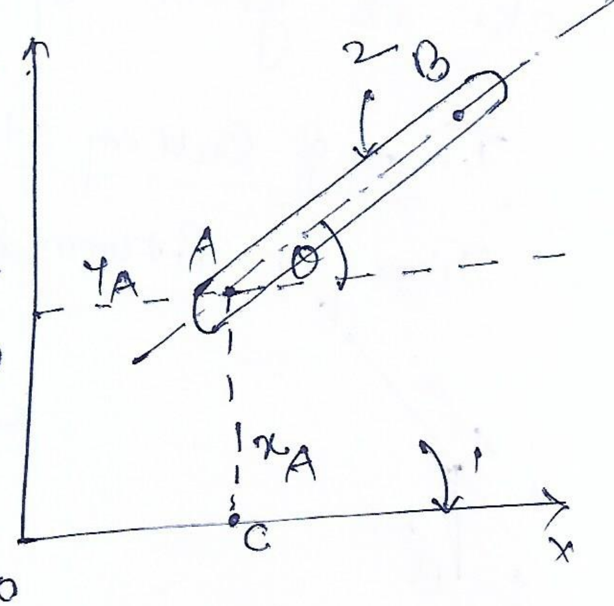


Link 1 = Crank AB (Fixed)
Link 2 = Driving Crank
Link 3 = Slides

⇒ Difference between Machine and Mechanism is given below :-

MACHINE	MECHANISM
(i) It is like a human body used for transmitting energy into useful work.	(i) It is like skeleton and has definite motion betn various links.
(ii) It is the practical development of mechanism.	(ii) It is the model of a machine.
(iii) It is related to Energy.	(iii) It is related to motion only.
(iv) It has many link.	(iv) It also has many link.
Ex: Lathe, Shaper in workshop etc.	Ex. Engine indicator. Typewriter etc.

⇒ Degree of freedom :-



Two link 1 & 2 is given below.

Link 1 is fixed. The link 2 has a point A over it and translated by coordinates x_A & Y_A. It can be written as A (x_A, Y_A).

Suppose there is another point B of link 2. Let us say the line joining points A & B makes angle θ with the fixed link 1 (OX).

Thus link 2 is completely specified by three variables (x_A, y_A, θ).

In other words it can be stated that an unconstrained rigid link in the plane has three degree of freedom. Two of these are because of translatory motions and third one due to angular motion.

⇒ If there is an assembly of n links, they have 3n degree of freedom before joining their ends.

→ If point A on link 2 is hinged with point C on fixed link 1, the two variables x_A & y_A for point A will be fixed. The position of link 2 will now be determined by a single variables θ only. It means that link 2 which was earlies having three degree of freedom, now it will have only one degree of freedom.

It shows that degree of freedom is lost by two for every lower pair in the mechanism.

When a mechanism having n links. Since one of the link is ... fixed. So the number of the total numbes of degree of freedom will be $3(n-1)$ before they are connected to any other link.

Let P be the numbes of simple hings in the mechanism. Simple hings are those links which connect two links.

Now, $\boxed{F = 3(n-1) - 2P}$ → Kutzbach equation.

n = No. of links, F = No of Degree of freedom.

P = No. of lowes Pairs or Simple hings.

when,

F = 1, The mechanism is known as Simple degree of freedom System.

= 2, Two degree of freedom System

= 0, The assembly is known as Structure or. There is no relative motion betn its link.

= -1, or less, Intermediate Structure. Most of the mechanism available are of when, $f = 1$. Then the above equation can be written as

$$1 = 3(n-1) - 2P$$

$$\Rightarrow 3n - 3 - 2P = 1$$

$$\Rightarrow 2P - 3n = -4$$

$$\Rightarrow \boxed{2P - 3n + 4 = 0} \rightarrow \text{Grubler's Equation.}$$

⇒ Effect of odd/Even Numbers of links on degree of freedom.

we already know that

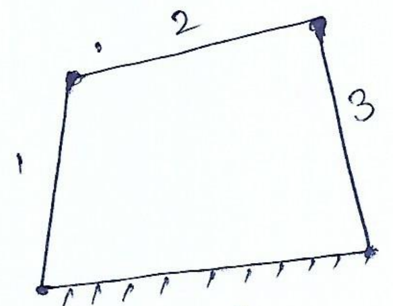
$$\Rightarrow F = 3(n-1) - 2P$$

$$\therefore P = \left[\frac{3(n-1)}{2} - \frac{F}{2}\right]$$

Since, Total no. of turning pairs (P) must be an integer (1, 2, 3, 4, ...) it is possible only when either $(n-1)$ and F should both be odd or even.

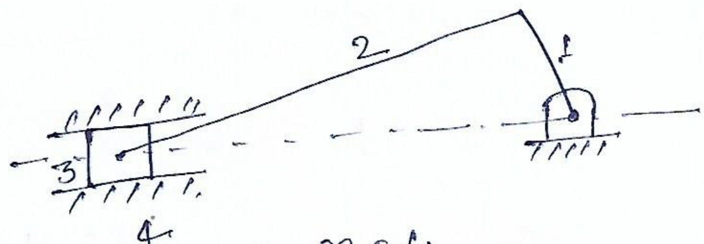
it means that if n is odd, F will be even.

That means degree of freedom (F) of the system will be even numbers (2, 4, 6, 8, ...). If the links (n) are odd in numbers (1, 3, 5, ...) and vice versa is for given drawing,



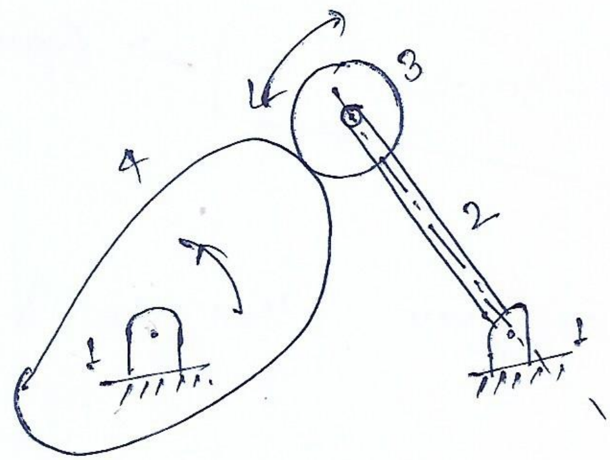
$p = 4$
$n = 4$

(a)



$n = 4$
$P = 4$

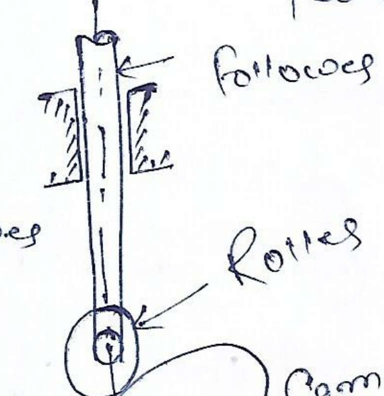
(b)



(c)

from four bar mechanism of fig. (a) & (b)

$n = 4$, No. of lower pair $= 4$, No. of higher Pair (h) $= 0$.



Now, from given fig.

This cam and Roller follower arrangement forms higher Pair. If the follower is not connected to any link, it is having three degree of freedom. It is assumed when it is brought in contact with the cam, it is having only translatory and slipping motion. It mean it is having two degree of freedom after connecting.

→ Thus the degree of freedom is reduced by one in case of higher pair.

If h is the number of higher pairs, since one higher pairs reduces the degree of freedom by one, the degree of freedom of the mechanism can be determined by the given equation.

$$\boxed{f = 3(n-1) - 2P - h}$$

from fig (a) of four bar mechanism is given by

$$f = 3(n-1) - 2P - h$$

$$= 3(4-1) - 2\times 4 - 0 = 9 - 8 = 1.$$

Both the mechanism (a) & (b) have constrained motion. i.e $f = 1$, one degree of freedom.

from fig (c), cam & follower arrangement is given

Three different conditions are possible.

i. $P = 3$, $h = 1$, $n = 4$

(ii) If link 2 and 3 Constitute only one link or 3 is welded to 2 ~~w~~ them.

$$p = 2, \quad h = 1, \quad n = 3$$

$$\therefore F = 3(n-1) - 2p - h$$

$$= 3(3-1) - 2 \times 2 - 1 = 6 - 4 - 1 = 1$$

(iii) The Contact betn Cam and followers is rolling only (no slipping).

$$n = 4, \quad p = 4, \quad h = 0$$

$$\therefore F = 3(n-1) - 2p - h$$

$$= 3(4-1) - 2 \times 4 - 0$$

$$= 9 - 8 = 1$$ (There is a Constrained motion)

⇒ Degree of freedom of a mechanism in Space Can be determined with the help of given relation.

$$\boxed{F = 6(n-1) - 5f_1 - 4f_2 - 3f_3 - 2f_4 - 1f_5}$$

where F = Degree of freedom.

n = Total no. of link.

f_1 = Number of pairs having one degree of freedom.

when it is brought in contact with the cam, it is having only translatory and slipping motion. It mean it is having two degree of freedom after connecting.

→ Thus the degree of freedom is reduced by one in case of higher pair.

If h is the number of higher pairs, since one higher pairs reduces the degree of freedom by one, The degree of freedom of the mechanism can be determined by the given equation.

$$\boxed{F = 3(n-1) - 2P - h}$$

From fig (a) of four bar mechanism is given by

$$F = 3(n-1) - 2P - h$$
$$= 3(4-1) - 2\times 4 - 0 = 9-8 = 1.$$

Both the mechanism (a) & (b) have constrained motion. i.e $F = 1$, one degree of freedom.

From fig (c), cam & follower arrangement is given

Three different conditions are possible.

i) $P = 3$, $h = 1$, $n = 4$

f_3 = " " " " Three degree of freedom.

Similarly f_5 = " " " " Five " " "

Since in a mechanism, one link is fixed. So number of movable links = $(n-1)$.

NOTE:- A link in a space has six motions & one of the link is fixed in the mechanism so degree of freedom will be $6(n-1)$.

⇒ Types of joint:-

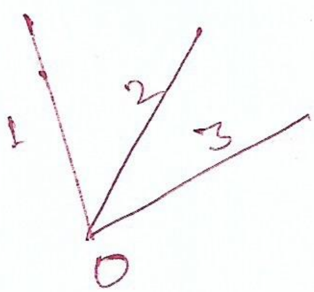
(i) Binary joint :- If two links are joint at the same point, it is called a binary joint. Each link has two end for connections.

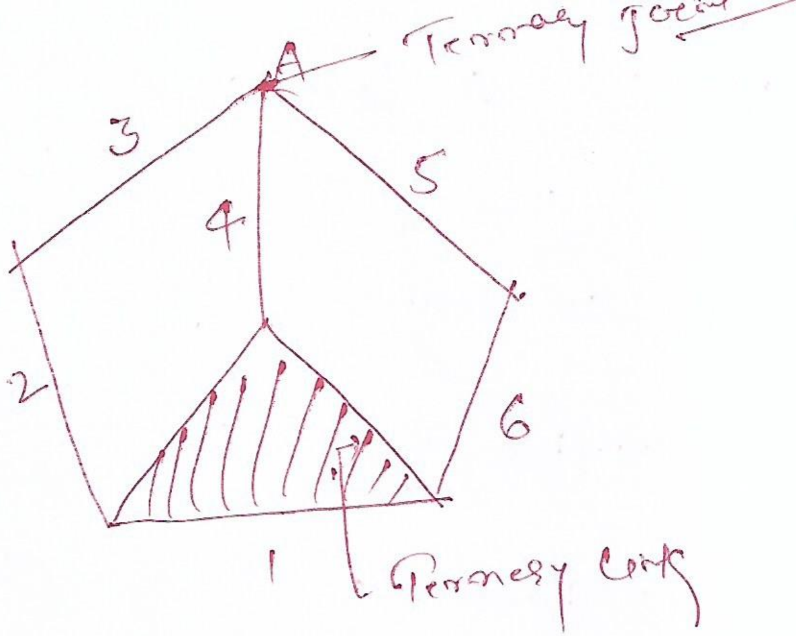
$$\boxed{J + \frac{h}{2} = \left(\frac{3}{2}l - 2\right)}$$

... chain with binary joint.

A link is to which two links are connected is known as binary link. in the given fig all links is a binary link.

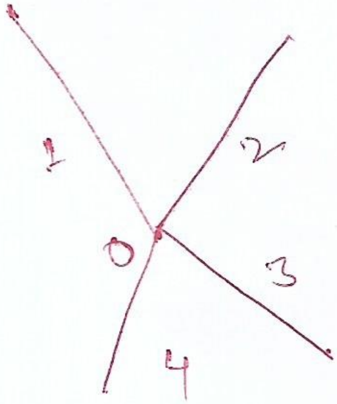
(ii) Ternary joint:- A link to which three links are connected ~~to~~ is known as ternary link.



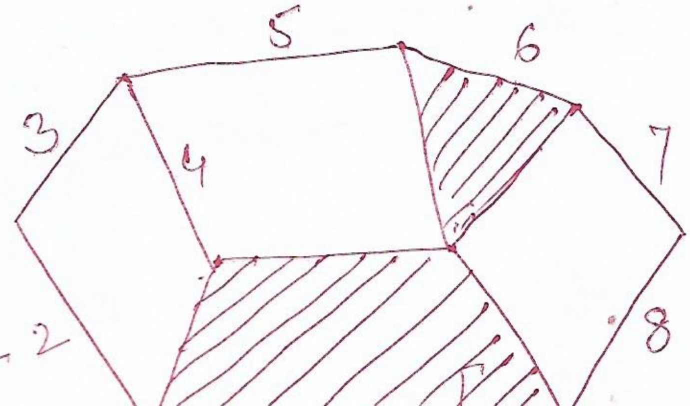


(iii) Quaternary joints:-

A link to which four links are connected is known as an quaternary link.

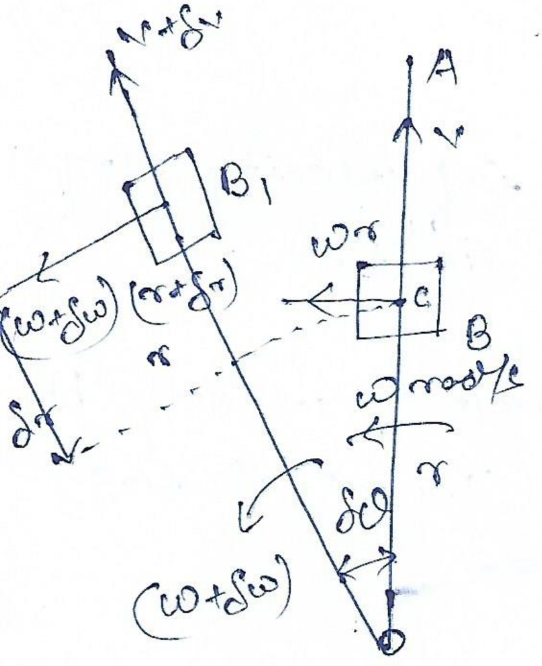


Kinematic chain with Quaternary joint

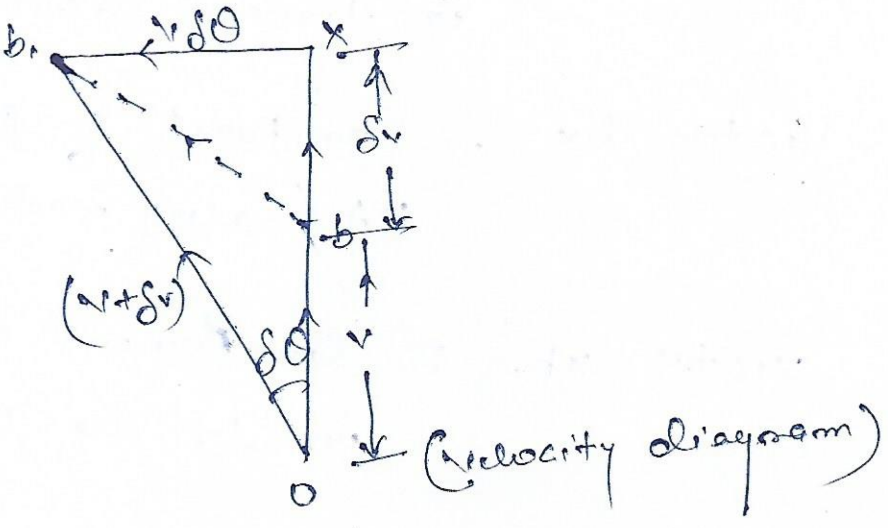


Coriolis Component of Acceleration :—

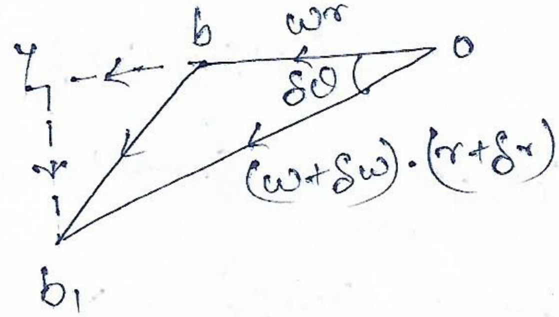
When a Point on one link is sliding along another link, such as in quick return motion mechanism, then the Coriolis Component of acceleration must be Calculated.



(a)



(b)



(c)

Let we Consider a link OA and a Slider B. The Slider B moves along the link OA. The Point C is the Coincident Point on the link OA.

Let, ω = Angular velocity of the link OA at time t seconds

v = velocity of the slider along the link OA at time t seconds.

$(\omega + \delta\omega)$, $(v + \delta v)$ and $(\omega + \delta\omega) \cdot (r + \delta r)$: Corresponding values at time $(t + \delta t)$ seconds.

Now, we have to find Accelⁿ of the slider B. w.r.t O and its Coincident Point C.

In velocity diagram (b) - bb_1 represent the change in velocity in time δt seconds.

vector bx = Component of change of velocity along OA (along radial directⁿ)

vector xb_1 = Component of change of velocity bb_1 in ⊥r to OA (Tangential dirⁿ)

bx : $ox - ob$

$= (v + \delta v)\cos\delta\theta - v$ ↑

$\delta\theta$ is very small ∴ $\cos\delta\theta \simeq 1$.

$bx = (v + \delta v) - v = \delta v$

$\boxed{bx = \delta v}$ ↑ Radially outward.

Now,

$xb_1 = (v + \delta v)\sin\delta\theta$

$\delta\theta$ is very small ∴ $\sin\delta\theta \simeq \delta\theta$

∴ $xb_1 = (v + \delta v)\,\delta\theta = v\,\delta\theta + \delta v \cdot \delta\theta$

In the velocity dia. c, when ωr and $(\omega+\delta\omega).(r+\delta r)$ are considered.

Vector bb_1 represent change in velocity.

Vector yb_1 = Component of change of velocity along OA (Along radial dirn)

Vector by = Component of change of velocity ⊥r to OA (Tangential dirn towards left)

$$yb_1 = (\omega+\delta\omega).(r+\delta r) \sin\delta\theta \downarrow$$

$$= (\omega.r + \omega.\delta\omega + \delta\omega.\delta r. + \delta\omega.r) \sin\delta\theta$$

$\delta\theta$ is very small

$$\therefore \sin\delta\theta = \delta\theta.$$

$$yb_1 = (\omega r + \omega.\delta\omega + \delta\omega.\delta r + r.\delta\omega).\delta\theta$$

$$\boxed{yb_1 = \omega r.\delta\theta} \downarrow$$

Remaining parts are neglected.

$$by = oy - ob$$

$$= (\omega+\delta\omega).(r+\delta r) \cos\delta\theta - \omega r$$

$$= (\cancel{\omega r} + \omega\delta r + \delta\omega.r + \delta\omega.\delta r). \quad - \cancel{\omega r}$$

$\leftarrow$

Total Component of change of velocity along Tangential direction

$$= xb_1 + by$$

$$= v.\overleftarrow{\delta\theta} + \overleftarrow{(\omega.\delta r + r.\delta\omega)}$$

$\therefore$ Tangential Component of accn of the slider B w.r.t O. which is Perpendicular to OA.

$$a_{BO}^{t} = Lt \frac{v\delta\theta + (\omega.\delta r + r.\delta\omega)}{\delta t}$$

$$= v.\frac{d\theta}{dt} + \omega.\frac{dr}{dt} + r.\frac{d\omega}{dt}$$

$$= v\omega + \omega v + r.\alpha$$

$$\boxed{a_{BO}^{t} = (2v.\omega + r.\alpha)}$$

$\leftarrow$ Towards Left & $\perp$r to given Link OA.

Now,

Radial Component of accn of the Coincident Point C w.r.t. O acting a direction from C to O.

$$\boxed{a_{CO}^{r} = \omega^2 r \uparrow}$$

Tangential Component

$$\boxed{a_{CO}^{t} = \overleftarrow{\alpha.r}}$$

In the velocity dia. c, when ωr and $(\omega+\delta\omega).(r+\delta r)$ are considered.

Vector bb_1 represent change in velocity.

Vector yb_1 = Component of change of velocity along OA (Along radial dirn)

Vector by = Component of change of velocity $\perp$ to OA (Tangential dirn towards left)

$$yb_1 = (\omega+\delta\omega).(r+\delta r)\sin\delta\theta \downarrow$$

$$= (\omega.r + \omega.\delta\omega + \delta\omega.\delta r + \delta\omega.r)\sin\delta\theta$$

$\delta\theta$ is very small

$$\therefore \sin\delta\theta = \delta\theta.$$

$$yb_1 = (\omega r + \omega.\delta\omega + \delta\omega.\delta r + r.\delta\omega).\delta\theta$$

$$\boxed{yb_1 = \omega r.\delta\theta} \downarrow$$

Remaining parts are neglected.

$$by = oy - ob$$

$$= (\omega+\delta\omega).(r+\delta r)\cos\delta\theta - \omega r$$

$$= (\cancel{\omega r} + \omega\delta r + \delta\omega.r + \delta\omega.\delta r). - \cancel{\omega r}$$

$\leftarrow$

Total Component of change of velocity along radial direction

$$= bx - yb_1$$

$$= (\delta v - \omega r . \delta\theta) \uparrow \text{ Acting radialy outward.}$$

$\therefore$ Radial Component of the accen of the slides B w.r.t O on the link OA.

$$a^r_{BO} = Lt \frac{\delta v - \omega . r . \delta\theta}{\delta t} = \frac{dv}{dt} - \omega . r . \frac{d\theta}{dt}$$

$$\boxed{a^r_{BO} = \frac{dv}{dt} - \omega^2 r}$$

Now,

Radial Component of the slides B w.r.t. Point C on the link OA. Acting Radialy outwards.

$$a^r_{BC} = a^r_{BO} - a^r_{BO}$$

$$= \left(\frac{dv}{dt} - \omega^2 r\right) - \left(-\omega^2 r\right)$$

$$= \frac{dv}{dt} - \cancel{\omega^2 r} + \cancel{\omega^2 r}$$

$$\boxed{a^r_{BC} = \frac{dv}{dt}} \uparrow$$

Tangential Component of the slider B. w.r.t C is given below.

$$a^t_{BC} = a^t_{BO} - a^t_{CO} = (2\omega v + \alpha/r) - \alpha/r$$

$$\boxed{a^t_{BC} = 2\omega . v}$$

This tangential Component of accen of the slider B. w.r.t Coincident Point C on the link OA is Known as Coriolis Component of Acceleration. which is always Perpendicular to the link.

∴ Coriolis Component of the accen of B w.r.to C.

$$\boxed{a^c_{BC} = a^t_{BC} = 2\omega v}$$

Kliem's Construction:-

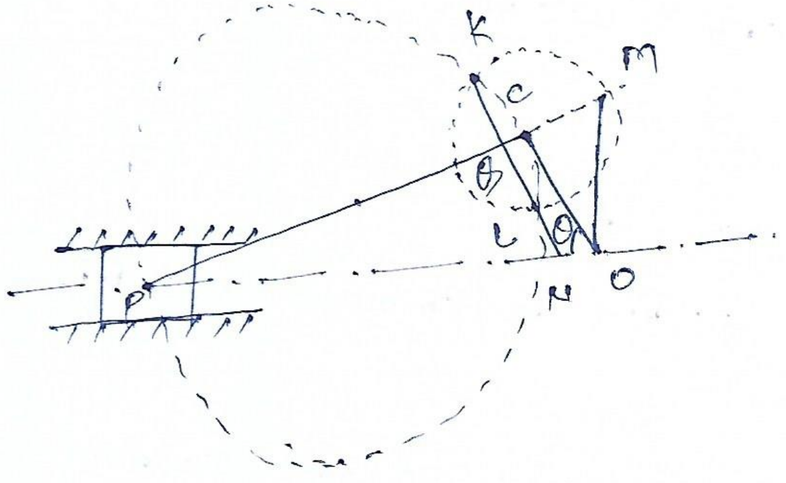
The velocity and acceleration of the reciprocating Part's of the steam engine or internal Combustion engine may be defined by graphical methode or Analytical method. The graphical method may be determined by one of the following Construction

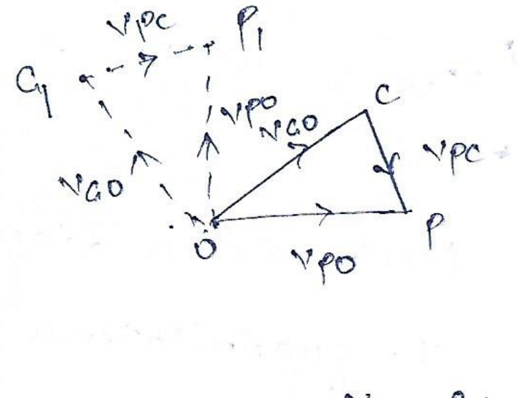
(i) Kliem's Construction.

(ii) Ritteshaus's Construction.

(iii) Bennett's Construction.

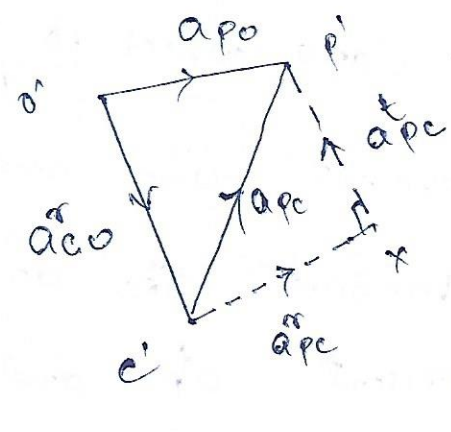
(i) Kliem's Construction:-





velocity diagram:

Kliem's acceleration diagram.



Triangle OCM in fig. (a) is known as Klein's velocity diagram.

From velocity diagram :-

OC_1 represent V_{CO}.

OP_1 represent V_{PO}

C_1P_1 represent V_{PC}.

ΔOC_1P_1 & ΔOCM are similar. Therefore,

$$\frac{OC_1}{OC} = \frac{OP_1}{OM} = \frac{C_1P_1}{CM} = \omega$$

or, $$\frac{V_{CO}}{OC} = \frac{V_{PO}}{OM} = \frac{V_{PC}}{CM} = \omega$$

$$\therefore \boxed{V_{CO} = \omega \times OC, \quad V_{PO} = \omega \times OM \quad \& \quad V_{PC} = \omega \times CM}$$

From Acceleration diagram.

The quadrilateral CQNO is known as Klein's acceleration diagram.

when we observe Klein's acceleration diagram and Acceleration diagram. Then we have to obtained various data which is given below.

$o'c'$ represent a^r_{CO} and is parallel to CO.

$c'x$ represent a^r_{PC} and parallel to CP or CQ

xp' represent a^t_{PC} and parallel to QN (because

O'P' represent a_{PO} and is Parallel to PO and NO.

So, Quadrilateral O'C'xP' is Similar to quadrilateral CQNO.

$$\therefore \frac{O'C'}{OC} = \frac{C'x}{CQ} = \frac{xP'}{QN} = \frac{O'P'}{NO} = \omega^2$$ (which is Constant.)

$$\text{or, } \frac{a^r_{CO}}{OC} = \frac{a^r_{PC}}{CQ} = \frac{a^t_{PC}}{QN} = \frac{a_{PO}}{NO} = \omega^2$$

$$\therefore a^r_{CO} = \omega^2 \times OC$$

$$a^r_{PC} = \omega^2 \times CQ.$$

$$a^t_{PC} = \omega^2 \times QN \text{ and } \underline{a_{PO}} = \underline{\omega^2 \times NO.}$$

Now, Acceleration of Piston w.r.t Crank Pin C (a_{PC}) is obtained by

$$\frac{O'P'}{CN} = \omega^2 \Rightarrow \frac{a_{PC}}{CN} = \omega^2$$

$$\therefore \boxed{a_{PC} = \omega^2 \times CN}$$

→ Find velocity of any Point D which is on the PC Connecting rod. divide CM at D_1 in the Same Ratio as D divides CP ... CD

$\therefore$ velocity of D, $v_D = \omega \times OD_1$

To find accel^n of Point D. For this we draw a Parallel line from D to Po which intersect to CN at D_2.

$\therefore$ Acceleration of of D, $a_D = \omega^2 \times OD_2$

⇒ The accel^n of Piston P is zero (0) and its velocity is max^m, when N coincides with O.

⇒ The accel^n of Piston P is zero when the crank OC and connecting rod PC are at right angle to each other.

Let, α : Angle of inclination of the driving shaft and driven shaft.

ω : Angular velocity of the driving shaft : $\frac{d\theta}{dt}$

ω_1 : Angular velocity of the driven shaft : $\frac{d\phi}{dt}$

$$\therefore \boxed{\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}}$$

or, in the form of R.P.M

$$\boxed{\frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}}$$

→ Max^m and Min^m speeds of driven shaft

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

$$\omega_1 = \frac{\omega\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

ω_1 will be max^m for a given value of α. Then denominators will be ~~min~~ min^m

when $\cos^2\theta = 1$

$\therefore \theta = 0°, 180°, 360°$ etc.

$$(\omega_1)_{max^m} = \frac{\omega\cos\alpha}{1-\sin^2\alpha} = \frac{\omega\cos\alpha}{\cos^2\alpha}$$

$$= \frac{\omega}{}$$

$$\therefore \boxed{(\omega_1)_{maxm} = \frac{\omega}{\cos\alpha}}$$

or,

$$\boxed{(N_1)_{maxm} = \frac{N}{\cos\alpha}}$$

For minm speed, The denominator will be maxm.

when $(\cos^2\theta . \sin^2\alpha)$ is maxm.

$$\cos^2\theta = 0,$$

$$\therefore \theta = 90°, 270° \text{ etc.}$$

$$\therefore \boxed{(\omega_1)_{min} = \omega \cos\alpha}$$

$$\boxed{N_{1(min)} = N \cos\alpha}$$

⇒ Conditions for equal speeds of the driving and driven shaft.

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \cos^2\theta . \sin^2\alpha}$$

For, $\omega = \omega_1$

$$\Rightarrow \cos\alpha = 1 - \cos^2\theta . \sin^2\alpha$$

$$\cos^2\theta = \frac{1-\cos\alpha}{\sin^2\alpha} \quad \text{--- (i)}$$

Now, $\sin^2\theta = 1-\cos^2\theta = 1 - \frac{1-\cos\alpha}{\sin^2\alpha} = 1 - \frac{1-\cos\alpha}{1-\cos^2\alpha}$

$$= 1 - \frac{\cancel{(1-\cos\alpha)}}{(1+\cos\alpha)\cancel{(1-\cos\alpha)}} = \frac{\cancel{1}+\cos\alpha-\cancel{1}}{1+\cos\alpha} = \frac{\cos\alpha}{1+\cos\alpha}$$

$$\therefore \sin^2\theta = \frac{\cos\alpha}{1+\cos\alpha} \quad \text{--- (ii)}$$

from eqn (ii) ÷ (i), we get,

$$\tan^2\theta = \frac{\cos\alpha}{1+\cos\alpha} \times \frac{\sin^2\alpha}{1-\cos\alpha} = \frac{\cos\alpha \times \cancel{\sin^2\alpha}}{\cancel{(1-\cos^2\alpha)}} = \cos\alpha$$

$$\tan^2\theta = \cos\alpha$$

$$\therefore \boxed{\tan\theta = \pm\sqrt{\cos\alpha}}$$

⇒ Maxm fluctuation of speed :-

Maxm speed of driving shaft.

$$\omega_1(max) = \omega/\cos\alpha$$

~~Maxm~~ Minm speed of the driven shaft

$$\omega_1(minm) = \omega\cos\alpha$$

∴ Maxm fluctuation of speed of the driven shaft,

$$= \omega\left(\frac{1}{\cos\alpha} - \cos\alpha\right) = \omega\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right) = \frac{\omega\sin^2\alpha}{\cos\alpha}$$

$$\boxed{q = \omega \tan\alpha \cdot \sin\alpha}$$

Since, α is a small angle, then we put

$\cos\alpha = 1$

$\sin\alpha = \alpha$

$$\therefore \boxed{\text{Max}^m \text{ Fluctuation } (q) = \omega\alpha^2}$$

⇒ The max^m fluctuation of speed of the driven shaft approximately varies as the square of the angle betn the two shaft.

⇒ We already know that the velocity of the driven shaft is not constant, but it varies from max^m to minm values. For a constant velocity ratio of the driving and driven shafts, an intermediate shaft with a Hooke's joint at each end which is give in fig.