Unit-1 Introduction

Turbo Machines :-

Devices in which energy is transferred either to, or from a continuously flowing fluid by the dynamic action of moving blades on the runner.

The word 'turbo' or 'turbinis' is of latin origin and implies that spinning or whirling.

Classification of Turbo Machines :-

a) Open M/c & Enclosed M/c

Open M/c:
- Indefinite quantity of fluid

Eg:- Propellers, windmills & unshrouded fans.

- Comes under the category of aerodynamics.

Enclosed M/c:
- Finite quantity of fluid passes through a casing in unit time.

b) Absorption & Production of Power :-

- Those which absorbs power to inc. the fluid pressure or head.

Eg:- Pump, ducted fans and compressors.

- Those which produces power by expanding fluid to a lower pressure or head.

Eg:- Hydraulic, steam and gas turbines.

c) Type of fluid handled :-

→ Those which handle water.

Eg:- Pumps, and hydraulic turbines.

→ Those which handle steam.

Eg:- Steam turbines.

→ Those which handle air or gas.

Eg:- Ducted fans, compressors and gas turbines.

Hydraulic Machines:-

All Devices / machines handling liquids, It consists of:-

a) Turbomachines:-

Eg:- Pumps & hydraulic turbines generally known as rotodynamic machines.

b) Reciprocating machines:-

Eg:- Reciprocating pumps.

These are known as positive displacement pumps.

c) Various water lifting devices:-

Eg:- Jet pump, air-lift pump, pulsometer pump and hydraulic ram.

d) Pumps transmitting oil:-

Eg:- Gear pumps, constant delivery and variable delivery pumps.

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Principles of Hydraulic Machinery:-

Dynamic Force and Power:-

A stream of fluid entering in a machine such as a hydraulic or steam turbine, a pump or fan, has more or less a defined direction. A force is always required to act upon the fluid to change its velocity either in direction or in magnitude. But according to Newton's third law of motion, an equal and opposite force is exerted by the fluid upon the body that causes the change. This force exerted by virtue of fluid motion is called a Dynamic force and must be distinguished from hydrostatic pressure. Whereas hydraulic pressure implies no motion, dynamic force always involve a change in velocity and thus a change in momentum.

→ A turbine produces power while a pump, compressor or fan consumes power in order to run. The pow is determined from the dynamic force or forces whic are being exerted by the flowing fluid on the boundaries of flow passage and which are due t change of momentum. These are determined by applying Newton's 2nd law of motion.

$$F \propto \frac{d(mv)}{dt} \longrightarrow \text{momentum}$$

Dynamic force

$$\text{Power} = \omega T = \underline{\omega} \times (F \times \underline{r}) = v \times F$$

→ Momentum may be linear or angular. Angular momentum is moment of linear momentum.

$$T \propto \frac{d(I\omega)}{dt} \longrightarrow \text{Angular momentum}$$

Torque

→ If a fluid particle moves in specified direction and strikes a boundary, change of linear momentum will be involved giving rise to force. This force will be responsible for the motion of a turbine runner. The force multiplied by the distance moved by the runner per unit time will give the power developed by the machine. This is the case of tangential flow machine, known as Pelton turbine.

In case of curved ~~[illegible]~~ path of fluid particles, the torque multiplied by angular velocity of the runner will give the power of the machine. The torque may be +ve or −ve depending upon whether it is exerted on the fluid by the body, which is being revolved by some external energy or it is exerted on the body by the fluid to revolve it.

The positive torque multiplied by angular velocity results in power consumed by a machine such as pump, compressor, blower or fan.

The negative torque multiplied by angular velocity will give the power developed by the machine such as turbine, ship and aeroplane propeller including helicopter, windmill and fluid coupling.

→ $$F \propto \frac{d}{dt}(mv)$$

In x-direction,

$$F_x \propto \frac{d}{dt}(mv_x)$$

$$\underbrace{F_x\, dt}_{\text{Impulse}} = \underbrace{m v_x}_{\text{Momentum}} \quad \{\text{proportionality constant} = 1\}$$

∴ Above eqn is called Impulse – momentum eqn.

→ A system refers to a definite mass of material and all other matter around it is known as its surrounding. The boundaries of the system will form a closed surface and this surface may change with time so that it contains the same mass during which the change takes place. The system may contain an infinitesmal mass or a large finite mass of fluid.

<u>Control Volume</u>:- It is a specific region in space, its size and shape being entirely arbitrary, however these are made to coincide with solid boundaries.
The boundary of a control volume is its <u>control surfa</u>
Thus control surface specifies the volume of fluid under consideration.

For a control volume,

$$\Sigma F_x = \frac{m}{t}(V_{x_2} - V_{x_1})$$

$$\Sigma F_x = \dot{m}(V_{x_2} - V_{x_1})$$

$$\Sigma F_x = \rho Q(V_{x_2} - V_{x_1})$$

L.H.S of the eqn will give the total force or resultant force.
So, External forces F_x may be of three kinds:- pressure forces, inertia forces (body forces) and drag forces.

<u>Applications of Linear Momentum Equation</u>:-
Two kinds of application of linear-momentum equ are:-

a) To determine the forces exerted by the flowing fluid on the boundaries of flow passage due to change of momentum.

b) To determine the flow characteristics when there is some loss of unknown quantity of energy in the flow system such as sudden enlargement of a pipe cross-section and hydraulic jump in an open channe flow.

Applications under (a) above are as follows :-

1) Forces caused by a jet of fluid striking a surface.
2) Jet propulsion and Rocket Mechanics.
3) Propeller of Marine and Airships including helicopter
4) Force caused by flow round a pipe-bend.

Basic Equation of Energy Transfer in Rotodynamic Machines :- (or Euler's Energy Equation)

The basic equation of fluid dynamics relating to energy transfer is same for all rotodynamic machin and is a simple form of "Newton's Laws of Motion" applied to a fluid element traversing a rotor.

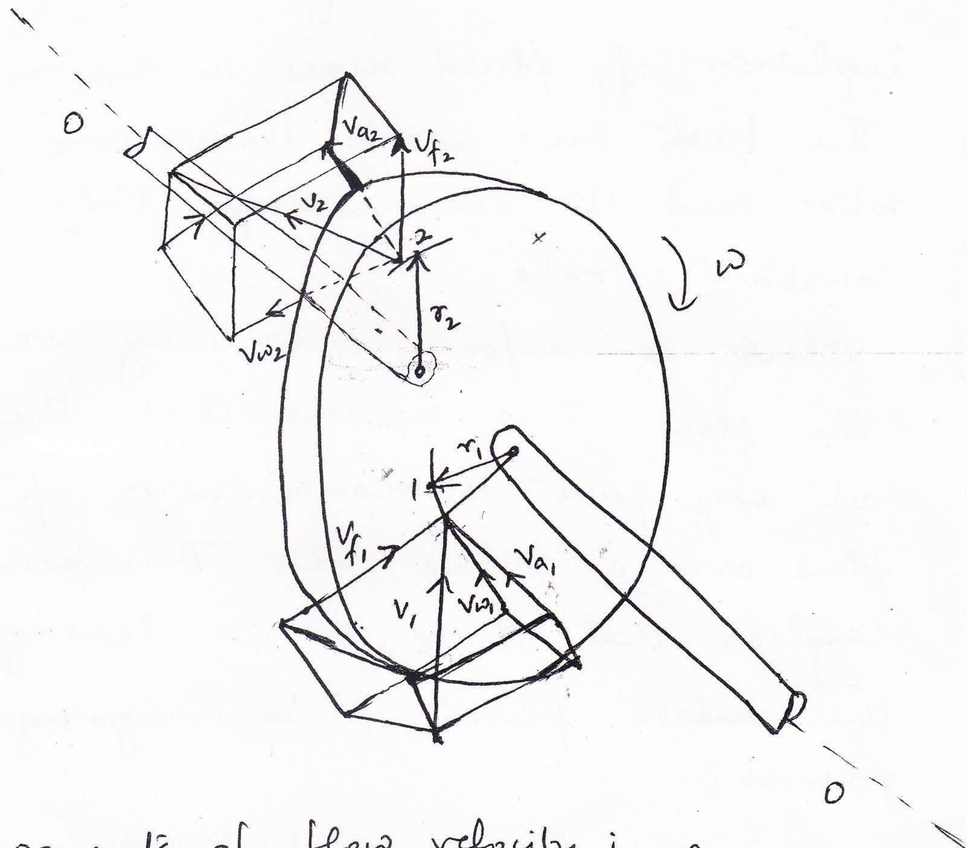


Fig:- Components of flow velocity in a generalised fluid machine

Above fig. represents diagrammatically a rotor of a generalised fluid machine, with O-O the axis of rotation and ω the angular velocity.

Fluid enters the rotor at point 1, passes through the rotor by any path and is discharged at point 2. The points 1 and 2 are at radii r_1 and r_2 from the centre of the rotor, and the directions of fluid velocities at 1 and 2 may be at any arbitrary angles.

For the analysis of energy transfer due to fluid flow in this situation, we assume the following:-

a) The flow is steady, i.e; the mass flow rate is constant across any section (no storage or depletion of fluid mass in the rotor).

b) The heat and work interactions between the rotor and its surroundings take place at a constant rate.

c) velocity is uniform over any area normal to the flow. This means that the velocity vecto at any point is representative of the total flow over a finite area. This condition also implies that there is no leakage loss, and the entire fluid is undergoing the same process.

The velocity at any point (V_1 & V_2) may be resolved into three mutually perpendicular components. The axial component of velocity (V_a) is directed parall to the axis of rotation, the radial component (V_f) is directed radially through the axis of rotation, while the tangential component (V_w) is directed at right angles to the radial direction and along the tangent to the rotor at that part

The change in magnitude of the axial velocity components through the rotor causes a change in the axial momentum. This change gives rise to an axial force, which must be taken by a thrust bearing to the stationary rotor casing.

The change in magnitude of radial velocity causes a change in momentum in radial direction and is carried in a similar manner as a journal load.

The tangential component (V_w) only has an effect on the angular motion of the rotor. The change in magnitude of the tangential velocity and the radius of it's application causes a change in the angular momentum or moment of the momentum which, according to the principle of conservation of angular momentu equals to the torque (T) acting on or exerted by the rotor.

If the inlet and outlet velocities of the fluid at all entry and exit point's are defined as V_1 an V_2 respectively, then, under a steady flow condition with a mass flow rate ($\dot{m}$), we can write

$$T = \frac{d}{dt}(I\omega)$$

for a circular rotor, $I = mr^2$ and $\omega = V/r$

$$\therefore \; T = \frac{d}{dt}(mVr)$$

$\Rightarrow \quad T = \dot{m}\, d(V r)$

$$\boxed{T = \dot{m}\,(V_{w_2} r_2 - V_{w_1} r_1)} \quad - (1)$$

Now, the rate of energy transfer $(\dot{E})$ or power between the fluid and the rotor can be written as:-

$$\dot{E} = \omega T = \dot{m}\,(V_{w_2} r_2 \omega - V_{w_1} r_1 \omega)$$

$P = \frac{E}{t}$

$$\boxed{\dot{E} = \dot{m}\,(V_{w_2} U_2 - V_{w_1} U_1)} \quad - (2)$$

where, ω is the angular velocity of the rotor and $U = r\omega$ represents the linear velocity of the rotor.

Equ^n (2) can also be expressed in the following form:-

$$\frac{\dot{E}}{\dot{m}} = V_{w_2} U_2 - V_{w_1} U_1$$

or $$\frac{E}{m} = V_{w_2} U_2 - V_{w_1} U_1$$

or $$\boxed{\frac{E}{mg} = \frac{V_{w_2} U_2 - V_{w_1} U_1}{g}} \quad - (3)$$

or H

The L.H.S of equ^n (3) is the energy per unit weight of the fluid and thus represents the head gained by the fluid (as in pumps, fans or compressors) or transferred by the fluid depending upon the cases (as in turbines).

Equ^{n}s 1, 2 & 3 are the different forms of the basic equ^n of energy transfer in a fluid machine. All these equ^{n}s are known as Euler's equ^n in relation to

Impact of Jets

The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressu
If some plate (fixed or moving), is placed in the path of the jet, a force is exerted by the jet on the plate. This force is obtained from Newton second law of motion or from impulse-momentum equation.

Thus, impact of jet means the force exerted by the jet on a plate which may be stationary or moving.

The following cases will be considered:-

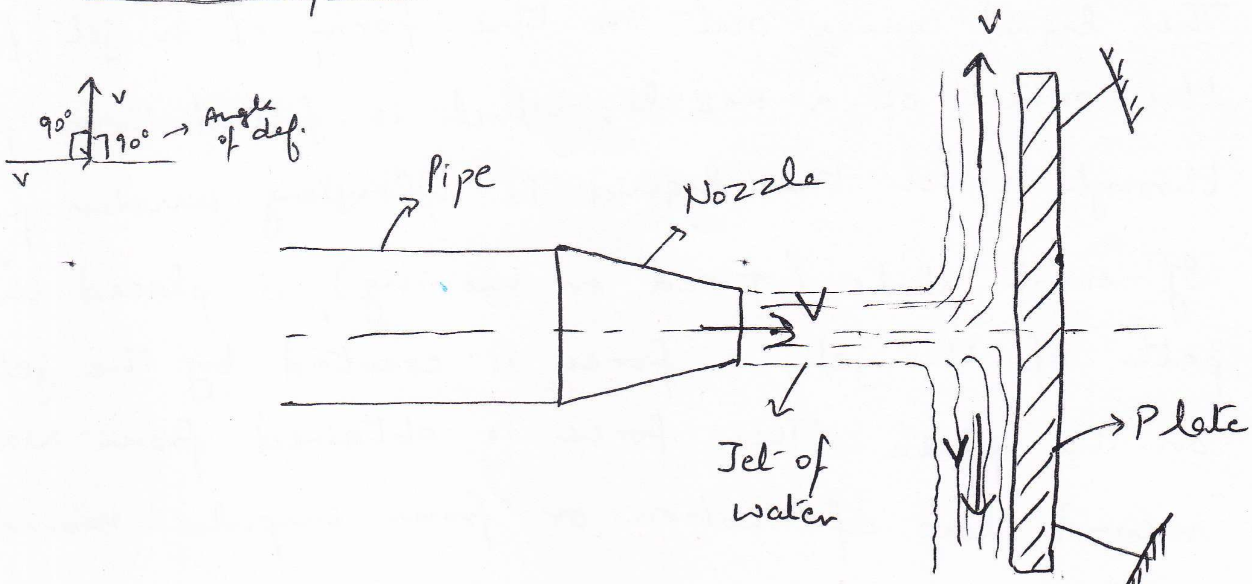
a) Force exerted by the jet on a stationary plat when

1) Plate is vertical to the jet
2) Plate is inclined to the jet
3) Plate is curved.

b) Force exerted by the jet on a moving plate, when

1) Plate is vertical to the jet
2) Plate is inclined to the jet
3) Plate is curved.

Force exerted by the jet on a stationary Vertical plate:—



Consider a jet of water coming out from the nozzle, strikes a flat vertical plate.

Let, V = velocity of the jet

d = diameter of nozzle ejecting the jet

a = area of cross-section of nozzle ejecting jet $= \pi/4 \, d^2$.

The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence, the jet after striking will get deflected through 90°. Hence, the component of the velocity of jet, in the direction of jet, after striking will be zero.

The force exerted by the jet on the plate in the direction of jet,

$$F_x = \text{Rate of change of momentum in the direction of force}$$

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

{ ∵ It is the force by the fluid on the body & its −ve }

$$\therefore \quad F_x = \frac{\text{Mass} \times \text{Initial vel.} - \text{Mass} \times \text{final vel.}}{\text{Time}}$$

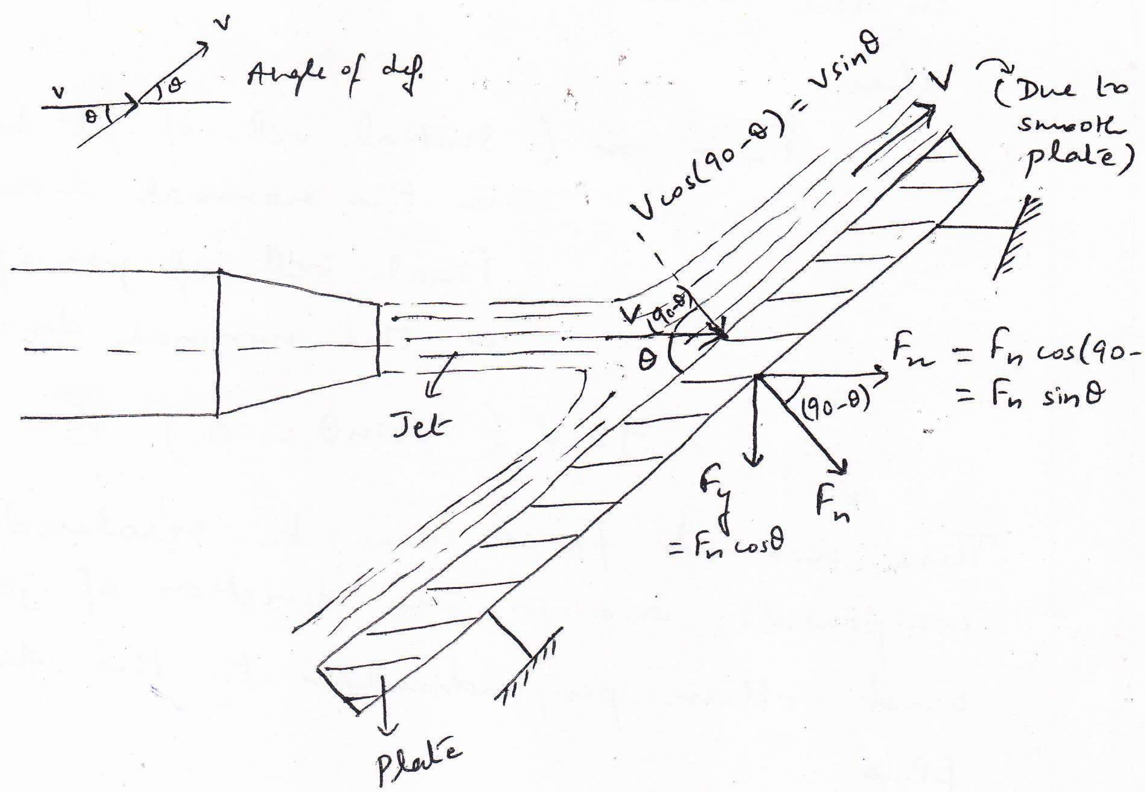
$$= \frac{\text{Mass}}{\text{Time}} \left[\text{Initial vel.} - \text{Final vel.} \right]$$

$$= \rho a v \left[v - 0 \right]$$

$$\boxed{F_x = \rho a v^2} \qquad \boxed{F_y = 0}$$

<u>Note</u>:- If the force by the body on the fluid is to be calculated then it is +ve and we take (final vel. − initial vel.).

<u>Force exerted by a Jet on Stationary Inclined flat plate</u>:-



Let a jet of water coming out from the nozzle, strikes an inclined flat plate.

Let; V = Velocity of jet in the direction of x

θ = Angle b/w jet and plate

a = Area of cross-section of jet

Then, mass of water striking the plate per sec.

$= \rho a V$

Now, if the plate is smooth and if it is assumed that there is no loss of energy (Kinetic) due to impact of the jet, than jet will move over the plate after striking with a velocity equal to initial velocity i.e; V.

Let the force exerted by the jet on the plate in the direction normal to the plate be F_n;

then

$F_n = m$ (Initial vel. of jet before striking in the normal direction − Final vel. of jet after striking in the normal direction)

$$= \rho a V (V \sin\theta - 0) = \rho a V^2 \sin\theta$$

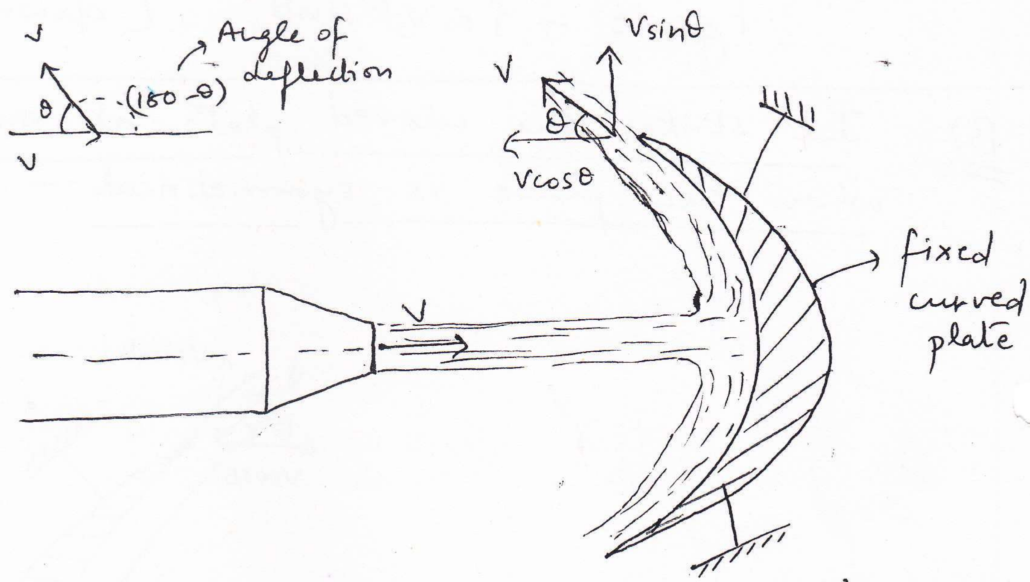
This normal force can be resolved into two components, one in the direction of jet or flow and other perpendicular to the direction of flow.

$$\therefore F_x = F_n \sin\theta = \rho a V^2 \sin^2\theta$$

$$\text{and } F_y = F_n \cos\theta = \rho a V^2 \sin\theta \cos\theta = \left(\frac{\rho a V^2}{2}\right) \sin 2\theta$$

Force exerted by a Jet on Stationary Curved plate :-

A) Jet strikes the curved plate at the centre



Let a jet of water strikes a fixed curved plate at the centre. The jet after striking the plate, come out with the same velocity if the plate is smooth and there is no loss of energy due to impact of jet in the tangential direction of the curved plate.

The velocity at outlet of plate can be resolved into two components, one in the direction of jet and other ⊥'r to the direction of jet.

∴ Component of velocity in the direction of je

$= -V\cos\theta$

∴ Force exerted by the jet in the direction of jet (F_x) = m (Initial vel − Final vel.

$= \rho a V[V - (-V\cos\theta)]$

$= \rho a V[V + V\cos\theta]$

$$\therefore \quad F_x = \rho a V^2 (1 + \cos\theta)$$

and $$F_y = \dot{m} (V_{1y} - V_{2y})$$

$$= \rho a V (0 - V\sin\theta)$$

$$F_y = -\rho a V^2 \sin\theta \quad \text{(downward dir.)}$$

B) Jet strikes the curved plate at one end tangentially when the plate is symmetrical :-

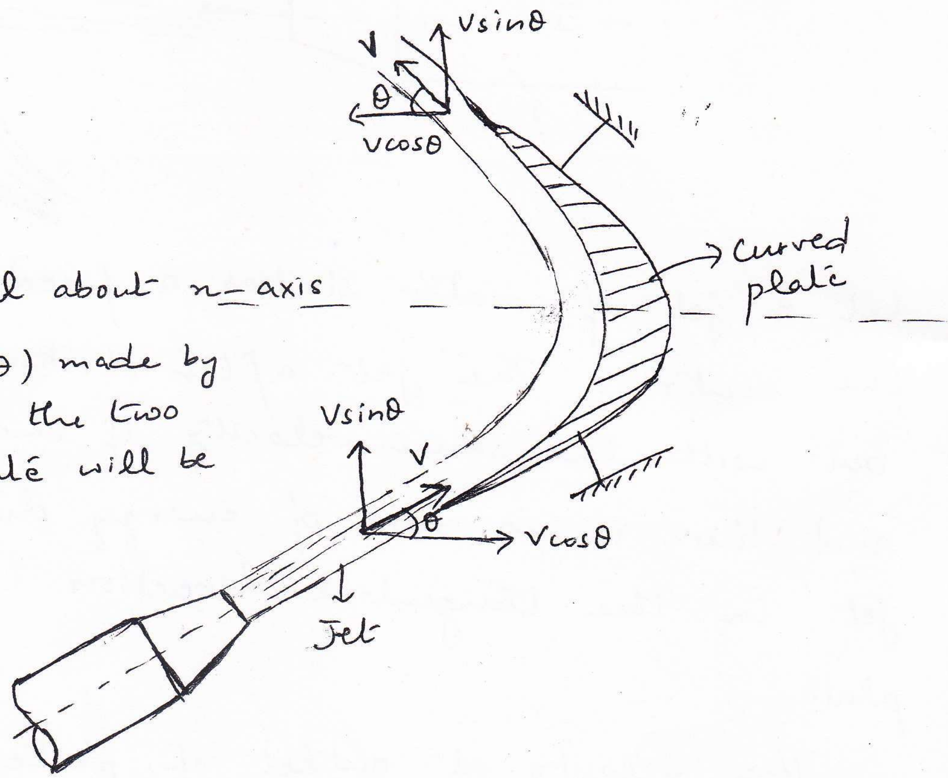


Plate is symmetrical about x-axis

Then the angle (θ) made by the tangents at the two ends of the plate will be same.

Now, $$F_x = \dot{m} (V_{1x} - V_{2x})$$

$$= \rho a V (V\cos\theta - (-V\cos\theta))$$

$$= \rho a V (2V\cos\theta) = 2\rho a V^2 \cos\theta$$

and $$F_y = \dot{m} (V_{1y} - V_{2y})$$

$$= \rho a V (V\sin\theta - V\sin\theta) = 0$$

C Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical :-

When the curved plate is unsymmetrical about x-axis, then angle made by the tangents drawn at the inlet and outlet tips of the plate with x-axis will be different.

Let, θ and ϕ → Angle made by tangent at inlet and outlet tips with x-axis.

Now,
$$F_x = \rho a V [V_{1x} - V_{2x}]$$
$$= \rho a V [V\cos\theta + V\cos\phi]$$
$$= \rho a V^2 (\cos\theta + \cos\phi)$$

and
$$F_y = \rho a V [V_{1y} - V_{2y}]$$
$$= \rho a V [V\sin\theta - V\sin\phi]$$
$$= \rho a V^2 (\sin\theta - \sin\phi)$$

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3 ⇒ Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and the head of water at the centre nozzle is 100 m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.9.

Soln

$d = 100\ mm = 0.1\ m$, $a = \pi/4\, d^2 = 0.007854\ m^2$

$H = 100\ m$

$C_v = 0.95$

$V_{th} = \sqrt{2gH} = 44.294\ m/s$

$V_{act.} = C_v \cdot V_{th} = 42.08\ m/s$

$\therefore F = \rho a V^2 = 10^3 \times 0.007854 \times (42.08)^2 = 13.9\ KN$

Q ⇒ A jet of water of diameter 75 mm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by the jet on the plate

i) in the direction normal to the plate and

ii) in the direction of the jet.

Sol^n

$d = 75\ mm = 0.075\ m$

$a = \pi/_4\, d^2 = 0.004417\ m^2$

$V = 25\ m/s$

$\theta = 60°$

i) $F_n = \rho a V^2 \sin\theta = 10^3 \times 0.004417 \times (25)^2 \times \sin 6$

$= 2390.7\ N = 2.39\ KN$

ii) $F_n = \rho a V^2 \sin^2\theta = 2.07\ KN$

Q ⇒ A jet of water of diameter 50 mm moving with a velocity of 40 m/s, strikes a curved fixed symmetric plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outle of the curved plate.

Sol^n:-

$d = 50\ mm$

$a = \pi/_4\, d^2 = 0.001963\ m^2$

$V = 40\ m/s$

Angle of deflection $= 120°$

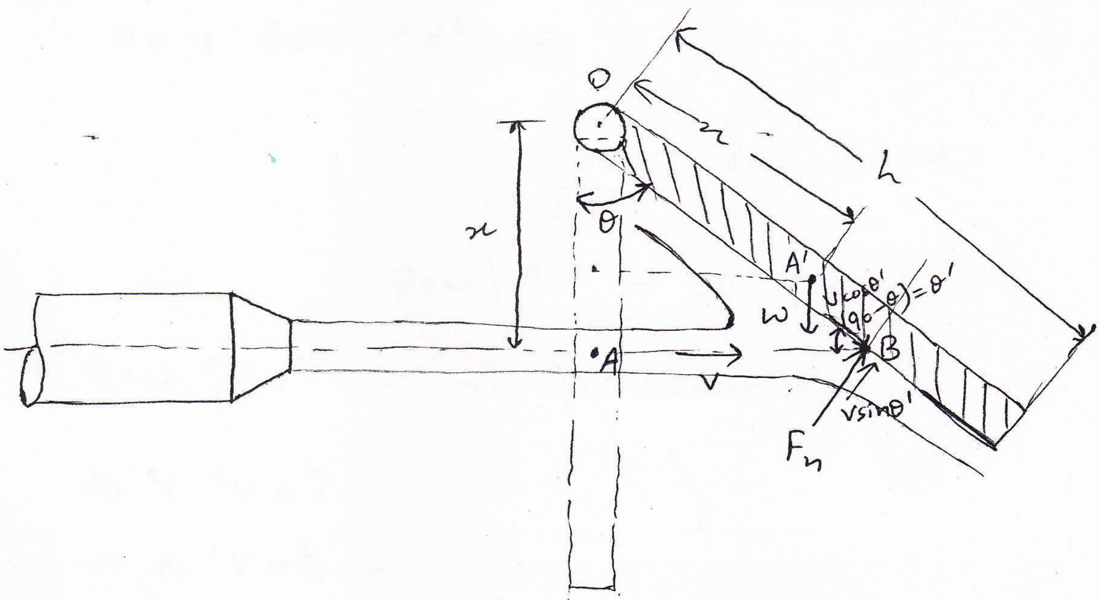
$\therefore \theta = 60°$

$F_n = \rho a V^2 (1 + \cos\theta)$

$= 1000 \times 0.001963 \times (40)^2 \times (1 + \cos 60°)$

$= 4.711\ KN$

Force exerted by a Jet on a Hinged plate :-



Consider a jet of water striking a vertical plate at the centre which is hinged at O. Due to the force exerted by the jet on the plate, the plate will swing through some angle about the hinge.

Let, x = dist^n of the centre of jet from O.

θ = angle of swing about O.

W = weight of plate acting at C.G. of the plate

When the plate is in equilibrium after the jet strike the plate, the moment of all the forces about the hinge must be zero.

Two forces are acting on the plate. They are :-

1) Force due to jet of water normal to the plate

$$F_n = \rho a V^2 \sin\theta'$$

where θ' = Angle b/w jet and plate = $(90° - \theta)$

2) Weight of the plate (W)

Now, moment of force F_n about hinge

$$= F_n \times OB = \rho a v^2 \sin(90-\theta) \times OB$$

$$= \rho a v^2 \cos\theta \times OB$$

Now, $\cos\theta = \frac{OA}{OB}$

$$\Rightarrow OB = OA / \cos\theta$$

$\therefore$ Moment of $F_n = \rho a v^2 \cos\theta \times \frac{OA}{\cos\theta}$

$$= \rho a v^2 \times OA$$

$$= \rho a v^2 \times x \qquad -(1)$$

and Moment of weight W about hinge

$$= W \times OA' \sin\theta$$

$$= W \times x \sin\theta \qquad -(2)$$

for eqn, (1) = (2)

$$\rho a v^2 x = W x \sin\theta$$

$$\Rightarrow \boxed{\sin\theta = \frac{\rho a v^2}{W}}$$

From above eqn, the angle of swing of the plate about hinge can be calculated.

Then, $F_x = F_n \cos\theta = \rho a v^2 \cos^2\theta$

and $F_y = F_n \sin\theta = \rho a v^2 \sin\theta \cos\theta$

Q ⇒ A jet of water of 2.5 cm diameter, moving with a velocity of 10 m/s, strikes a hinged square plate of weight 98.1 N at the centre of the plate. The plate is o/r uniform thickness. Find the angle through which the plate will swing

Solⁿ :-

$d = 2.5 \text{ cm} = 0.025 \text{ m}$

$v = 10 \text{ m/s}$

$W = 98.1 \text{ N}$

$a = \frac{\pi}{4} d^2 = 0.00049 \text{ m}^2$

$\therefore \sin\theta = \frac{\rho a v^2}{W} = \frac{10^3 \times 0.00049 \times (10)^2}{98.1}$

$\sin\theta = 0.499$

$\Rightarrow \theta = 29.96^\circ$

Q ⇒ A square plate of uniform thickness and length of side 300 mm hangs vertically from hinge at its top edge. When a horizontal water jet strikes the plate at its centre, the plate is deflected and comes to rest at angle of 30° to the vertical. The jet is 25 mm in diameter and has a velocity of 6 m/s. Determine the weight of the plate.

Solⁿ :-

$L = 300 \text{ mm} = 0.3 \text{ m}$

$\theta = 30^\circ$

$d = 25 \text{ mm} = 0.025 \text{ m}$

$a = \pi/4 \, d^2 = \pi/4 (0.025)^2 \text{ m}^2$

$v = 6 \text{ m/s}$

$W = ?$

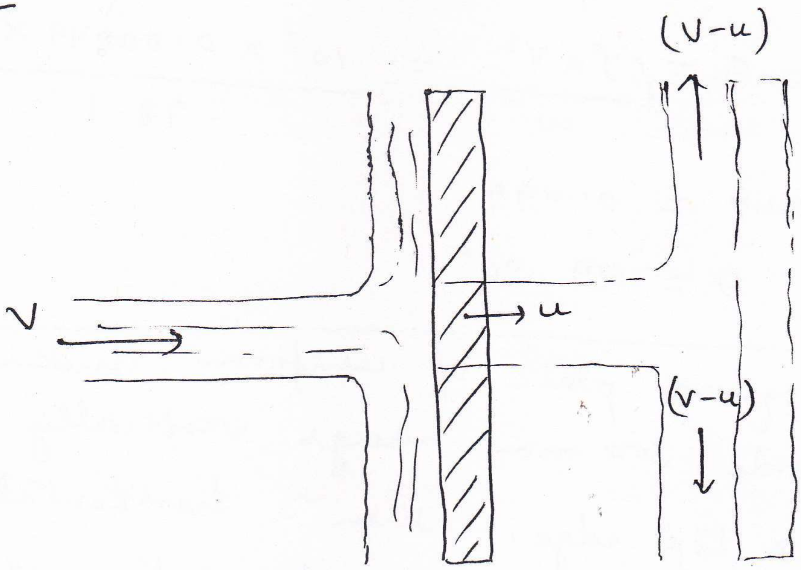
$\Rightarrow \sin\theta = \frac{\rho a v^2}{W} \Rightarrow W = \frac{\rho a v^2}{\sin\theta} = 35.33 \text{ N}$

Force exerted by a Jet on Moving plates :-

The following cases of moving plates will be considered :-

1) Flat vertical plate moving in the direction of jet and away from the jet.
2) Inclined plate moving in the direction of jet
3) Curved plate moving in the direction of jet

Force on flat vertical plate moving in the direction of jet :-



V → vel of jet (absolute)

u → vel. of flat pla[te]

In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity (V−u).

Now mass of water striking the plate per sec.

$$= \rho a (V-u)$$

and $F_x = \rho a (V-u) [(V-u) - 0]$

$$= \rho a (V-u)^2$$

In this case, the work will be done by the jet on th[e] plate as plate is moving. For stationary plates, the work done is zero.

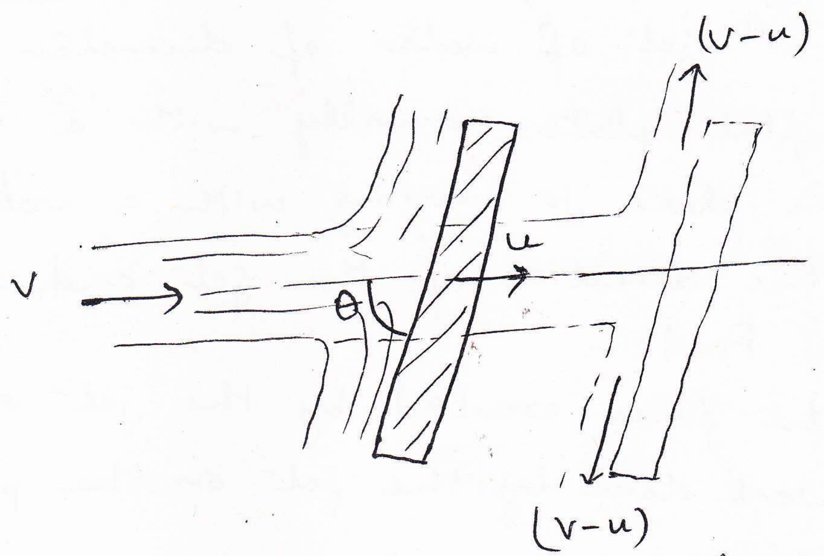
$\therefore$ Work done per sec. by the jet on the plate

$$= \frac{\text{Force} \times \text{Dist}^n \text{ in the direction of force travelled by plate}}{\text{Time}}$$

$= F_n \times u$

$= \rho a (V-u)^2 \times u$ N-m/s or J/s or Watt.

Force on the inclined plate moving in the direction of the Jet :-



Mass of water striking per second $= \rho a (V-u)$

If the plate is smooth and loss of energy due to impact of the jet is assumed zero, the jet of water will leave the inclined plate with a velocity of (V.

Now, F_n (force exerted by the jet of water on the plate in the normal directi to the plate)

$= \rho a (V-u) \left[(V-u) \sin\theta - 0 \right]$

$= \rho a (V-u)^2 \sin\theta$

Now, this force is resolved into two components

$$F_x = F_n \sin\theta = \rho a (V-u)^2 \sin^2\theta$$

and $F_y = F_n \cos\theta = \rho a (V-u)^2 \sin\theta \cdot \cos\theta$

Now the work done per second by the jet on the plate

$= F_x \times$ Dist^n per sec. in the direction of x

$= F_x \times u$

$= \rho a (V-u)^2 \sin^2\theta \cdot u$ (watt)

Q ⇒ A jet of water of diameter 10 cm strikes a flat plate normally with a velocity of 15 m/s. The plate is moving with a velocity of 6 m/s in the direction of the jet and away from the jet. Find :-

1) the force exerted by the jet on the plate
2) Work done by the jet on the plate per sec.
3) Power and efficiency of the jet.

Sol^n

$d = 10$ cm $= 0.1$ m

$a = \pi/4\, d^2 = 0.007854$ m^2

$V = 15$ m/s

$u = 6$ m/s

1) $F_n = \rho a (V-u)^2 = 10^3 \times 0.007854 \times 9^2$
$= 636.17$ N

2) Work done per sec. $= F_n \times u = 3817.02$ Nm/s

3) Power $=$ work done per sec. $= 3.817$ KW

4) Efficiency $= \dfrac{O/P}{I/P} \times 100\,\% = \dfrac{3817.02}{\frac{1}{2} m V^2} = \dfrac{3817.02}{\frac{1}{2}\rho a V^3}$

Q ⇒ A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate:

i) when the plate is stationary

ii) when the plate is moving with a velocity of 15 m/s and away from the jet.

Also determine the power and efficiency of the jet when the plate is moving.

Solⁿ :-

$d = 7.5 \text{ cm} = 0.075 \text{ m}$

$a = \pi/4 \, d^2 = 0.004417 \text{ m}^2$

Angle b/w plate and jet $(\theta) = 90° - 45°$
$= 45°$

$V = 30 \text{ m/s}$

1) when plate is stationary

$F_n = \rho a V^2 \sin\theta = 10^3 \times 0.004417 \times (30)^2 \times \sin 45$
$= 2810.96 \text{ N}$

2) when plate is moving

$u = 15 \text{ m/s}$

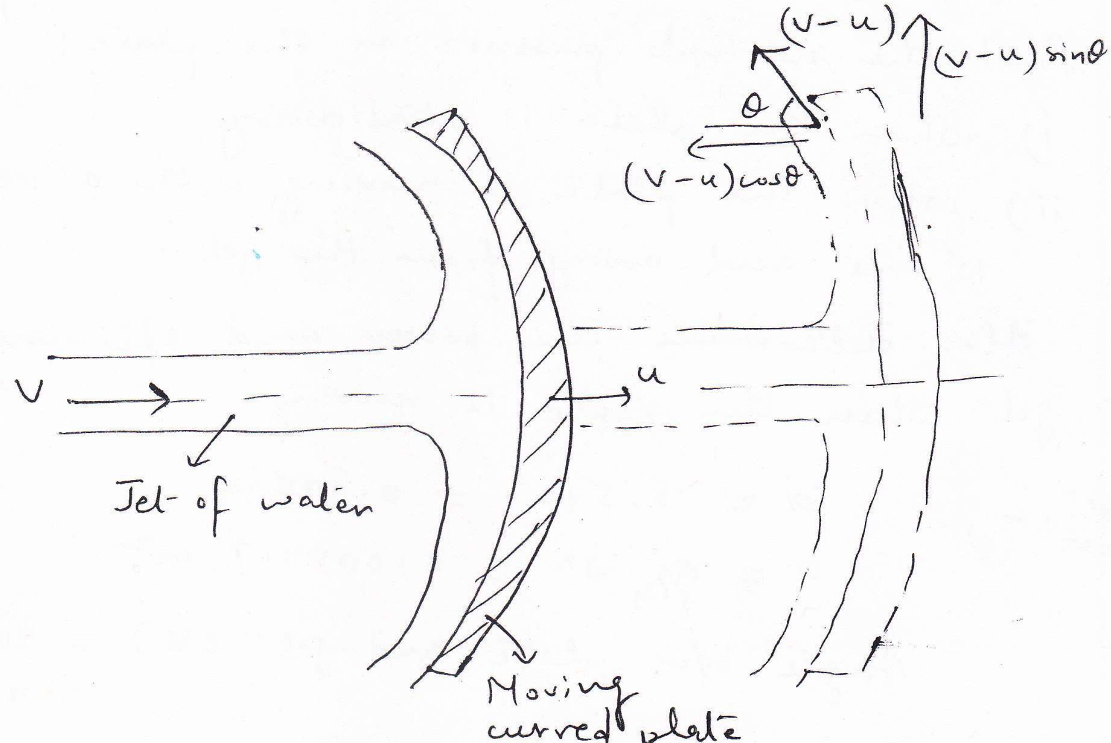
$F_n = \rho a (V-u)^2 \sin\theta = 702.74 \text{ N}$

3) Power = Work done per sec. $= F_x \times u$
$= F_n \sin\theta \times u$
$= 702.74 \times \sin 45° \times 15$
$= 7453.5 \text{ Nm/s}$ or Watt
$= 7.45 \text{ kW}$

Efficiency $= \dfrac{\text{Work done per sec.}}{\text{K.E. of jet per sec.}}$

$= \dfrac{7.45 \times 10^3}{\frac{1}{2} \rho a V^3} = 0.1249 \simeq 12.49\%$

Force on the curved plate when the plate is moving in the direction of Jet :-



Mass of water striking the plate (per sec.) $= \rho a (V-u)$

$$F_x = \dot{m} (V_{1x} - V_{2x})$$

$$= \rho a (V-u) \left[(V-u) - \left[-(V-u)\cos\theta \right] \right]$$

$$= \rho a (V-u) \left[(V-u) + (V-u)\cos\theta \right]$$

$$= \rho a (V-u)^2 \left[1 + \cos\theta \right]$$

and Work done by the jet on the plate per sec.

$$= F_x \times u$$

$$= \rho a (V-u)^2 \times u \left[1 + \cos\theta \right]$$

θ = Angle made by V_{r_1} with direction of motion at inlet also called vane angle at inlet,

V_{w_1} = Component of V_1 in direction of motion of vane, called whirl velocity at inlet.

V_{f_1} = Component of V_1 in direction $\perp$'r to the direction of motion of vane, called velocity of flow at inlet.

V_2, u_2, V_{r_2}, β, ϕ, V_{w_2}, V_{f_2} = corresponding values at outlet.

The Δ's ABD and EGH are called velocity triangles at inlet and outlet.

→ If the vane is smooth, and is having velocity in the direction of motion at inlet & outlet equal then,

$u_1 = u_2 = u$ (assumed)

$V_{r_1} = V_{r_2}$ (due to smooth vane)

Now mass of water striking vane per sec $= \rho a V_{r_1}$

where, a = Area of jet of water

$\therefore$ Force exerted by the jet in the direction of motion

$$F_x = \dot{m}\left[V_{1x} - V_{2x}\right]$$

$$= \rho a V_{r_1}\left[V_{r_1}\cos\theta - (-V_{r_2}\cos\phi)\right]$$

$$= \rho a V_{r_1}\left[(V_{w_1} - \cancel{u_1}) + (\cancel{u_2} + V_{w_2})\right]$$

$$= \rho a V_{r_1}\left[V_{w_1} + V_{w_2}\right] \quad \text{——} \quad (1)$$

Eqn (1) is true only when β is an acute angle

If $\beta = 90°$, then $V_{w_2} = 0$

$$\therefore F_x = \rho a V_{r_1}(V_{w_1})$$

and If β is obtuse angle,

$$F_n = \rho a V_{r_1} [V_{w_1} - V_{w_2}]$$

Thus, in general, $\boxed{F_n = \rho a V_{r_1} [V_{w_1} \pm V_{w_2}]}$

Work done per sec. by the jet on the vane

$$= F_n \times u$$

$$= \rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times u$$

$\therefore$ Work done per second per unit weight of fluid striking per sec.

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times u}{\rho a V_{r_1} \times g} \quad \left(\frac{N\,m/s}{N/s}\right)$$

$$\text{Head} = \frac{(V_{w_1} \pm V_{w_2}) \times u}{g} \quad (m)$$

Work done/sec per unit mass of fluid striking/sec

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times u}{\rho a V_{r_1}} \quad \left(\frac{N\,m/s}{kg/s}\right)$$

$$= (V_{w_1} \pm V_{w_2}) \times u \quad \left(\frac{N-m}{Kg}\right)$$

$$\text{Efficiency of Jet } (\eta) = \frac{O/P}{I/P} = \frac{W.D \text{ per sec. on van}}{K.E. \text{ per sec of je}}$$

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times u}{\frac{1}{2} m V_1^2}$$

$$\eta = \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times u}{\frac{1}{2} (\rho a V_1) \times V_1^2}$$

Q) A jet of water having a velocity of 20 m/s strikes a curved vane which is moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of motion of vane at outlet. Calculate :-

1) Vane angles, so that the water enters and leaves the vane without shock.

2) Work done per sec. per unit weight of water striki[ng] the vane per sec.

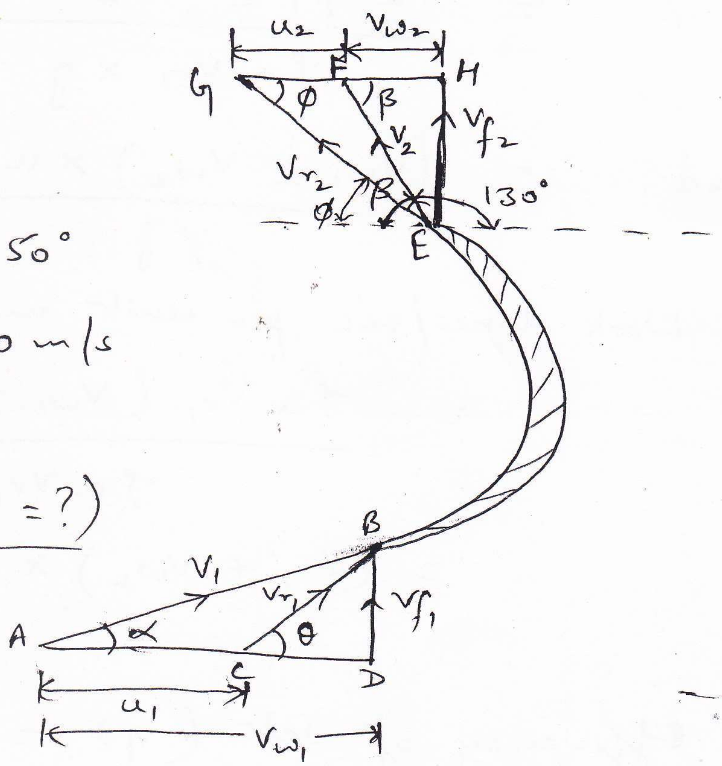
Solⁿ Given:

$V_1 = 20$ m/s

$u_1 = 10$ m/s

$\alpha = 20°$

$\beta = 180° - 130° = 50°$



Here, $u_1 = u_2 = 10$ m/s

$V_{r_1} = V_{r_2}$

1) Vane Angles ($\theta, \phi = ?$)

from $\triangle BCD$,

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}$$

(Acut

Now, $V_{f_1} = V_1 \sin\alpha$

$= 20 \times \sin 20° = 6.84$ m/s

$V_{w_1} = V_1 \cos\alpha = 20 \cos 20° = 18.794$ m/s

$u_1 = 10$ m/s (given)

$$\therefore \tan\theta = \frac{6.84}{18.794 - 10} = 0.7778$$

or $\boxed{\theta = 37.875°}$

Also, $\frac{V_{f1}}{V_{r1}} = \sin\theta$

$$V_{r1} = V_{f1}/\sin\theta = 20/\sin 37.875^\circ = 11.14 \text{ m/s}$$

From ΔEFG, applying sine rule,

$$\frac{V_{r2}}{\sin(180^\circ - \beta)} = \frac{u_2}{\sin(\beta - \phi)}$$

$$\frac{11.14}{\sin 130^\circ} = \frac{10}{\sin(50^\circ - \phi)}$$

$$\sin(50^\circ - \phi) = \frac{10 \sin 130^\circ}{11.14} = 0.6876 = \sin 43.44$$

$$50^\circ - \phi = 43.44^\circ$$

$$\Rightarrow \boxed{\phi = 6.56^\circ}$$

2) Work done/sec. per unit wt./sec.

$$= \frac{1}{g}(V_{w1} + V_{w2}) \times u \quad \left(\frac{Nm}{N}\right)$$

$$V_{w2} = GH - GF$$
$$= V_{r2}\cos\phi - u_2$$
$$= 11.14 \cos 6.56^\circ - 10 = 1.067 \text{ m/s}$$

$\therefore$ W.D per unit wt. $= \frac{1}{9.81} \times (18.794 + 1.067) \times 10$

$$\boxed{\text{"} \quad = 20.24 \ (Nm/N)}$$

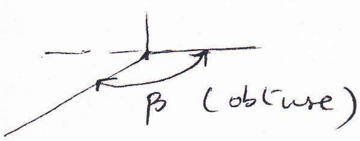
Q→ A jet of water of diameter 50 mm, having a velocity of 20 m/s strikes a curved vane which is moving with a velocity of 10 m/s in the direction of the jet. The jet leaves the vane at an angle of 60° to the direction of motion of vane at outlet.

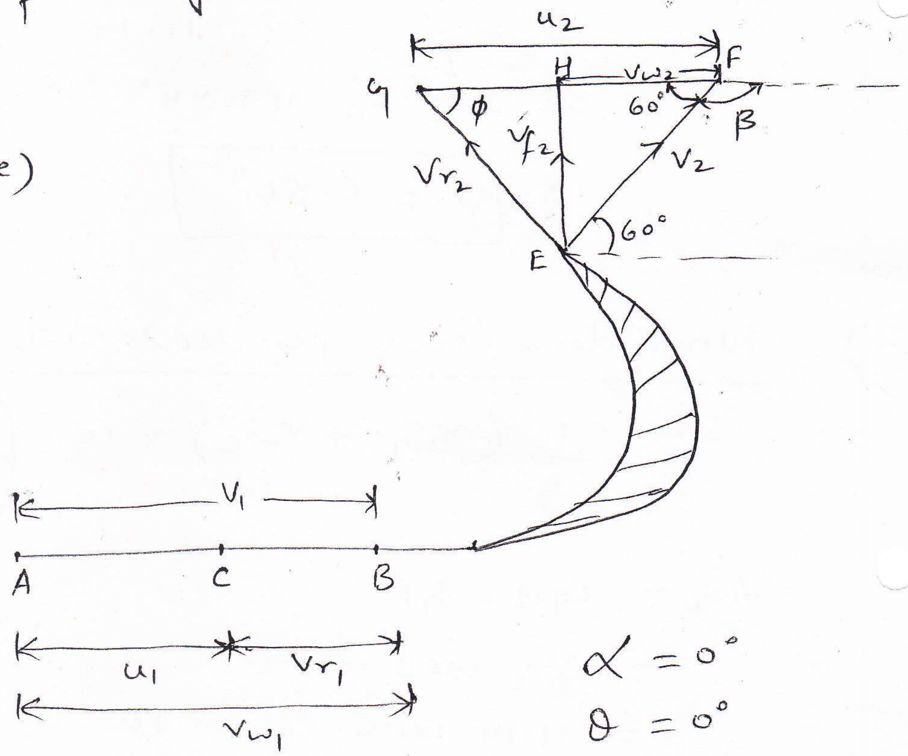
Determine :-

1) The force exerted by the jet on the vane in the direction of motion.

2) Work done per sec. by the jet.

3) Efficiency of the jet





$\alpha = 0°$

$\theta = 0°$

$V_{f_1} = 0$

Solⁿ :- Given

$d = 50 \text{ mm} = 0.05 \text{ m}$

$a = \pi/4 \, d^2 = 0.001963 \text{ m}^2$

$V_1 = 20 \text{ m/s}$, $u_1 = 10 \text{ m/s}$

$\alpha = 0°$, $\beta = 120°$

Here, $u_1 = u_2 = 10 \text{ m/s}$

$V_{r_1} = V_{r_2}$

Now, $V_{r_1} = V_1 - u_1$

$V_{r_1} = 10 \text{ m/s}$

Also, $V_{w_1} = V_1 = 20 \text{ m/s}$

Now from ΔEFG, using sine rule

$$\frac{V_{r_2}}{\sin 60^\circ} = \frac{u_2}{\sin(120^\circ - \phi)}$$

$$\frac{10}{\sin 60^\circ} = \frac{10}{\sin(120^\circ - \phi)}$$

$$\Rightarrow 60^\circ = 120^\circ - \phi$$

$$\Rightarrow \boxed{\phi = 60^\circ}$$

Now, $V_{w_2} = GF - GH = u_2 - V_{r_2} \cos\phi$

$= 10 - 10 \cos 60^\circ$

$= 10 - 5 = 5 \text{ m/s}$

1) Force exerted by the jet (F_x) $= \rho a V_{r_1} (V_{w_1} - V_{w_2})$

$= 10^3 \times 0.001963 \times 10 \times (20-$

$= 294.45 \text{ N}$

2) W.D per sec. by the jet $= F_x \times u$

$= 294.45 \times 10$ Nm/s

$= 2944.5$ W

3) Efficiency $= \dfrac{\text{W.D per sec.}}{\text{K.E. per sec.}} = \dfrac{2944.5}{\text{K.E. per sec.}}$

K.E. per sec. $= \frac{1}{2} (\rho a V_1) V_1^2$

$= \frac{1}{2} \times 10^3 \times 0.001963 \times (20)^3$

$= 7852$

$\therefore \eta = 0.375 = 37.5\ \%$

Q→ A jet of water having a velocity of 15 m/s strikes a curved vane which is moving with a velocity of 5 m/s. The vane is symmetrical and is so shaped that the jet is deflected through 120°. Find the angle of the jet at inlet of the vane so that there is no shock. What is the absolute velocity of the jet at outlet in magnitude and direction and the work done per unit weight of water. Assume the vane to be smooth.

Solⁿ :- Given:

$V_1 = 15$ m/s

$u_1 = 5$ m/s

As vane is symmetrical,

$\theta = \phi$

Angle of def. $= 120°$

Also, $\theta + \phi + 120° = 180°$

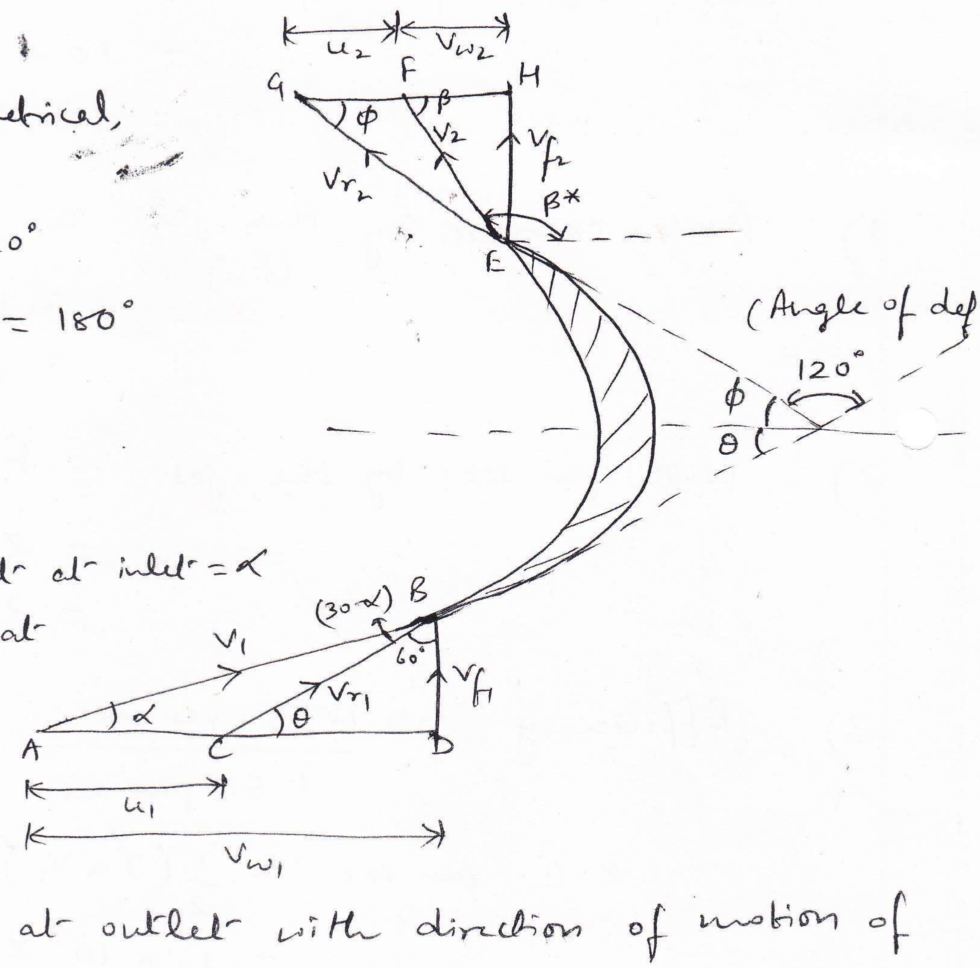
$2\theta = 60°$

$\Rightarrow \theta = 30°$

and $\phi = 30°$

Let, ∠ made by jet at inlet $= \alpha$

Abs. velocity of jet at outlet $= V_2$



Angle made by V_2 at outlet with direction of motion of vane $= \beta^*$

For no shock condition, $u_1 = u_2$

As vane is smooth, $V_{r_1} = V_{r_2}$

Applying sine rule to ΔACB,

$$\frac{AB}{\sin(180^\circ - \theta)} = \frac{AC}{\sin(30^\circ - \alpha)}$$

$$\frac{V_1}{\sin 150^\circ} = \frac{u_1}{\sin(30^\circ - \alpha)}$$

$$\frac{15}{\sin 150^\circ} = \frac{5}{\sin(30^\circ - \alpha)}$$

$$\sin(30^\circ - \alpha) = \frac{5 \sin 150^\circ}{15} = 0.1667 = \sin 9.596^\circ$$

$$30^\circ - \alpha = 9.596^\circ$$

or $\boxed{\alpha = 20.404^\circ}$

Also, $$\frac{V_1}{\sin 150^\circ} = \frac{V_{r_1}}{\sin 20.404^\circ}$$

$$\Rightarrow V_{r_1} = 10.46 \text{ m/s}$$

From velocity ΔHEG at outlet,

$$V_{r_2} \cos\phi = u_2 + V_{w_2}$$

$$10.46 \cos 30^\circ = 5 + V_{w_2}$$

$$\Rightarrow V_{w_2} = 4.06 \text{ m/s}$$

Also, $V_{r_2} \sin\phi = V_{f_2}$

$$\therefore V_{f_2} = 10.46 \sin 30^\circ = 5.23 \text{ m/s}$$

In ΔHFE, $V_2 = \sqrt{V_{f_2}^2 + V_{w_2}^2} = \boxed{6.62 \text{ m/s}}$

$$\tan\beta = \frac{V_{f_2}}{V_{w_2}} = 1.288$$

$$\beta = 52.17^\circ$$

$\therefore$ Angle made by absolute velocity at outlet with direction of motion $(\beta^*) = 180^\circ - \beta = \boxed{127^\circ 49.8'}$

Now, W.D. per unit weight of water striking

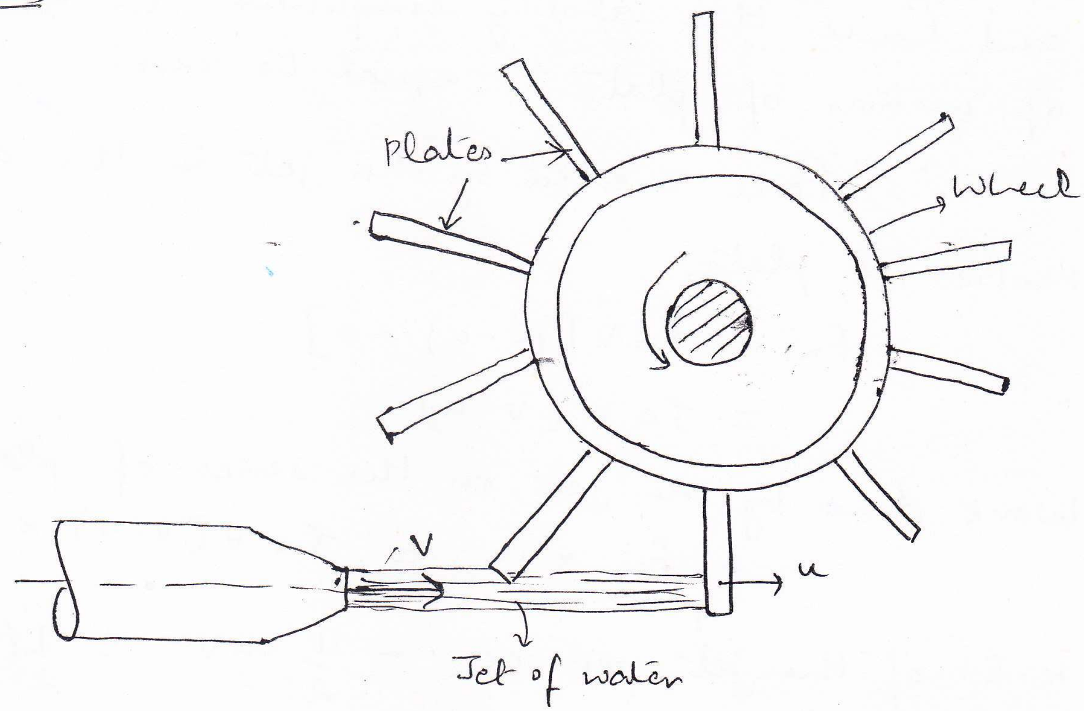
$$= \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u \quad \left(\frac{Nm}{N}\right)$$

$$= \frac{1}{9.81}\left[V_1 \cos\alpha + 4.06\right] \times 5$$

$$= \frac{5}{9.81}\left[15 \cos 20.404^\circ + 4.06\right]$$

$$= 9.225 \left(\frac{Nm}{N}\right)$$

Force exerted by a Jet of water on a Series of vanes:-



The force exerted by a jet of water on a single moving plate (which may be flat or curved) is not practically feasible. This is only a theoretical case. But in actual practice, a large no. of plates are mounted on the circumference of a wheel at a fixed distⁿ apart. So, when jet strikes a plate, it exert some force on the plate and the wheel starts moving and then it continues to strike other consecutive plates.

let; V = velocity of Jet

d = diameter of Jet

a = cross-sectional area of jet $= \frac{\pi}{4} d^2$

u = velocity of vane

In this case, the mass of water coming out from the nozzle per sec is always ~~.....~~ in contact with all the plates ~~.....~~ successively, hence mass of water per sec striking the series of plates $= \rho a V$.

Also, the jet strikes the plate with a velocity $= (V-u)$.
After striking, the jet moves tangential to the plate and hence the velocity component in the direction of motion of plate is equal to zero.

$\therefore$ Force exerted by the jet in the direction of motion of plate,

$$F_x = \rho a V[(V-u) - 0]$$
$$= \rho a V(V-u)$$

Work done by the jet on the series of plates per sec

$$= F_x \times u = \rho a V(V-u) \times u$$

$$K.E. \text{ of the jet per sec.} = \frac{1}{2} \dot{m} V^2 = \frac{1}{2}(\rho a V) V^2$$

$$\therefore \text{Efficiency of jet} = \frac{W.D. \text{ per sec.}}{K.E. \text{ per sec.}} = \frac{\rho a V(V-u) \times u}{\frac{1}{2} \rho a V^3}$$

$$\eta = \frac{2u(V-u)}{V^2}$$

This efficiency can also be called the efficiency of the wheel or series of flat vanes.

Condition for maximum efficiency:-

For a given jet velocity V, the efficiency will be maximum when

$$\frac{d\eta}{du} = 0 \Rightarrow \frac{d}{du}\left[\frac{2u(V-u)}{V^2}\right] = 0 \Rightarrow \frac{d}{du}\left[\frac{2u}{V} - \frac{2u^2}{V^2}\right]$$

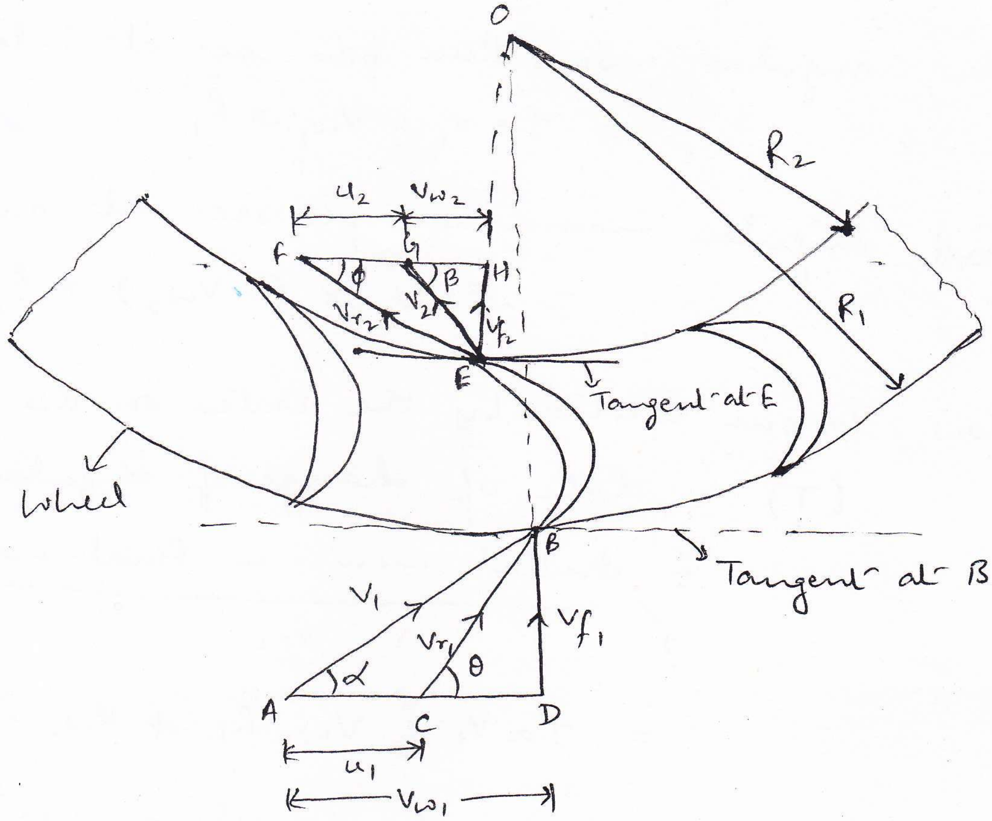
$$\Rightarrow \frac{2}{V} - \frac{2}{V^2} \times 2u = 0$$

$$\Rightarrow 1 - \frac{2u}{V} = 0 \Rightarrow V = 2u \text{ or } \boxed{u = V/2}$$

$$\text{and Max. Efficiency } (\eta_{max.}) = \frac{\cancel{2}\cancel{V} \times \cancel{V}/2}{\cancel{2} \times \cancel{V^2}} = \frac{1}{2} = 50\%$$

(for flat plate serie

Force exerted on a Series of Radial Curved Vanes :-



For a radial curved vane, the radius of the vane at inlet and outlet is different and hence the tangential velocities of the radial vane at inlet and outlet will not be equal.

Let, R_1 = Radius of wheel at inlet of the vane

R_2 = Radius of wheel at outlet of the van

ω = Angular speed of the wheel.

Then, $u_1 = \omega R_1$ and $u_2 = \omega R_2$

The mass of water striking per sec. for a series of vanes = mass of water coming out from nozzle per

$= \rho a V_1$

Momentum of water striking the vanes in tangential direction per sec. at inlet $= \rho a V_1 \times V_{w_1}$

Similarly, momentum of water at outlet per sec.

$$= \rho a V_1 \times (-V_{w_2})$$

Now, angular momentum per sec. at inlet

$$= \rho a V_1 \times V_{w_1} \times R_1$$

and angular momentum per sec. at outlet

$$= \rho a V_1 \times (-V_{w_2}) \times R_2$$

Now, Torque exerted by the water on the wheel

$$(T) = \text{Rate of change of angular momentum}$$

$$= \frac{\text{Initial mom.} - \text{Final mom.}}{\text{sec.}}$$

$$= \rho a V_1 \left[V_{w_1} \cdot R_1 + V_{w_2} \cdot R_2 \right]$$

Work done per sec. on the wheel $= \omega \cdot T$

$$= \rho a V_1 \cdot \omega \left[V_{w_1} R_1 + V_{w_2}$$

$$= \rho a V_1 \left[V_{w_1} \cdot \omega R_1 + V_{w_2} \cdot \omega R_2$$

$$= \rho a V_1 \left[V_{w_1} \cdot u_1 + V_{w_2} \cdot u_2 \right]$$

But if the angle β is an obtuse angle, then

$$\text{W.D/sec.} = \rho a V_1 \left[V_{w_1} \cdot u_1 - V_{w_2} \cdot u_2 \right]$$

$\therefore$ Generally, W.D./sec. on the wheel $= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2$

and if the discharge is radial at outlet, then $\beta = 90$

and $\text{W.D/sec.} = \rho a V_1 (V_{w_1} \cdot u_1)$

Efficiency of the Radial Curved vane :-

W.D. per sec. on the wheel is the o/p of the system whereas the initial kinetic energy per sec. of the jet is the input.

Hence, Efficiency $(\eta) = \dfrac{\text{W.D. per sec.}}{\text{K.E. per sec.}}$

$$= \frac{\rho a V_1 \left[V_{w_1} u_1 \pm V_{w_2} u_2 \right]}{\frac{1}{2} \rho a V_1^3}$$

$$= \frac{2\left[V_{w_1} u_1 \pm V_{w_2} u_2 \right]}{V_1^2}$$

If there is no loss of energy when water is flowing over the vanes, the W.D. on the wheel per sec. is also equal to the change in K.E. of the jet per sec.

W.D per sec. on the wheel = Change in K.E./sec

$$= \frac{1}{2} \dot{m} V_1^2 - \frac{1}{2} \dot{m} V_2^2$$

$$= \frac{1}{2} \dot{m} \left(V_1^2 - V_2^2 \right)$$

$$= \frac{1}{2} \rho a V_1 \left(V_1^2 - V_2^2 \right)$$

Hence, efficiency $(\eta) = \dfrac{\text{W.D per sec.}}{\text{K.E. per sec.}}$

$$= \frac{\frac{1}{2} \rho a V_1 \left(V_1^2 - V_2^2 \right)}{\frac{1}{2} \rho a V_1 \cdot V_1^2}$$

$$= \frac{V_1^2 - V_2^2}{V_1^2} = 1 - \left(\frac{V_2}{V_1} \right)^2$$

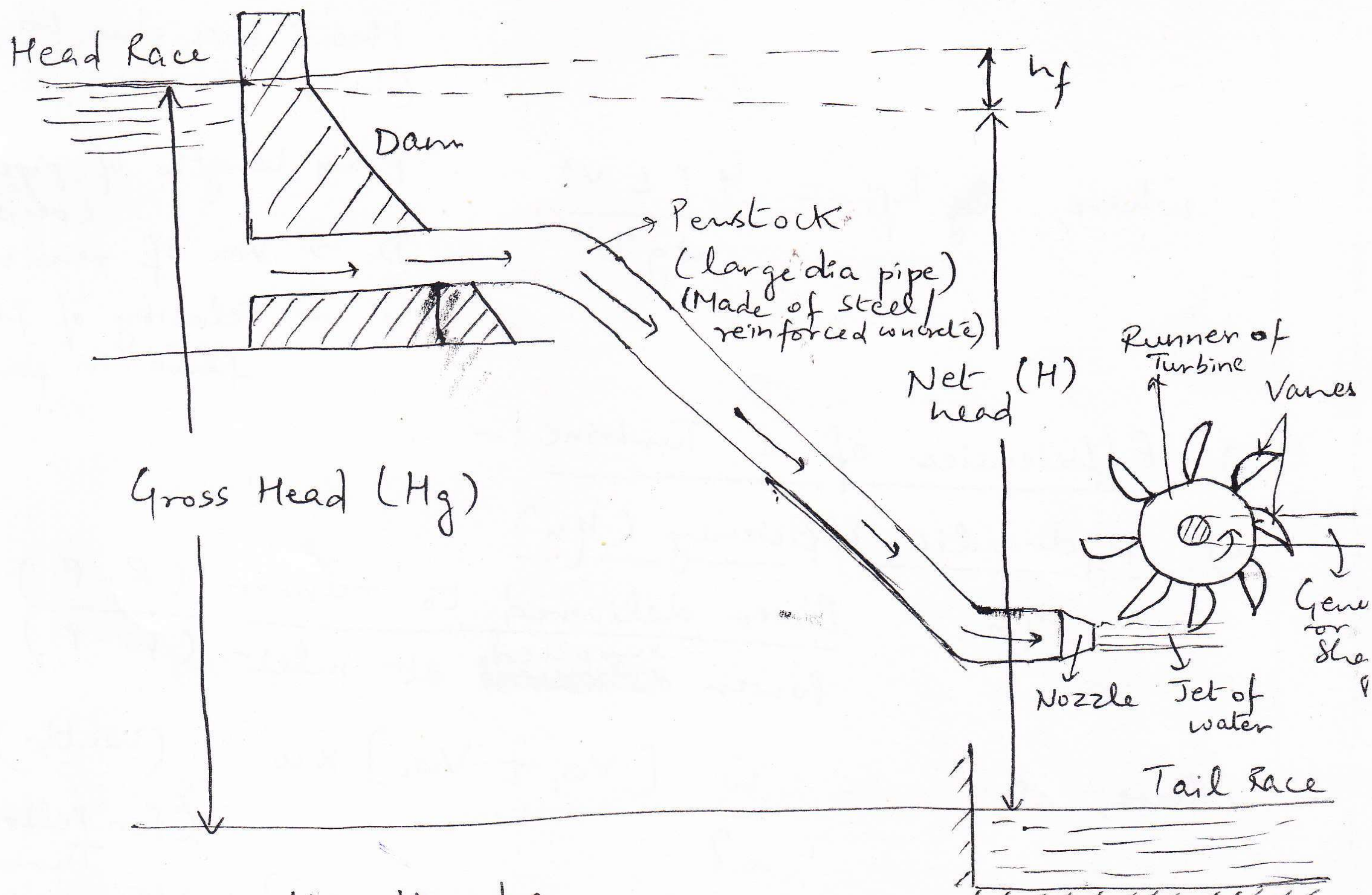
For max. efficiency, $V_2 < V_1$ but $V_2 \neq 0$ as the jet which is coming in has to move out of the vane.

Hydraulic M/c — Turbines

Turbines :-

Those m/c which convert hydraulic energy into mechanical energy. This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine. Thus, the mechanical energy is converted into electrical energy. The electric power generated here is known as Hydro-electric power.

General Layout of a Hydroelectric power plant :-



$$H = H_g - h_f$$

Definitions of Heads and Efficiencies of a Turbine :-

1) Gross Head:- The diffⁿ b/w head race level and tail race level when no water is flowing (i.e; no losses are there) is known as Gross Head. It is denoted by H_g.

2) Net head :- It is also called effective head and is defined as the head available at the inlet of the turbine. When water is flowing from head race to the turbine, a loss of head due to friction b/w the water and penstock occurs.

$$\therefore \text{ Net head } (H) = \text{Gross Head } (H_g) - \text{Head loss due to friction } (h_f)$$

where, $h_f = \dfrac{4fLV^2}{2gD}$

$L \rightarrow$ length of pipe (penstock)
$D \rightarrow$ Dia of penstock
$V \rightarrow$ Velocity of fluid flow in penstock

3) Efficiencies of a Turbine :-

a) Hydraulic Efficiency (η_h)

$$\eta_h = \frac{\text{Power delivered to runner (R.P.)}}{\text{Power } \cancel{\text{observed}} \text{ supplied at inlet (W.P.)}}$$

where, $R.P. = \dfrac{\dot{w}}{g}[V_{w_1} \pm V_{w_2}] \times u$ (Watt.) (for Pelton Turbine)

$= \dfrac{\dot{w}}{g}[V_{w_1}u_1 \pm V_{w_2}u_2]$ (W) (for radial flow turbi.

$\dot{w}$ = wt. of water striking the vanes of turbine per s
$= \dot{m}g = \rho AVg = \rho g Q$

and $W.P. = \dot{w} \times H$ (Watt.)

b) Mechanical Efficiency (η_m)

$$\eta_m = \frac{\text{Power at the shaft of the Turbine}}{\text{Power delivered by water to the runner}}$$

$$= \frac{\text{Shaft Power (S.P.) or Brake Power (B.P.)}}{\text{Runner Power (R.P.)}}$$

$\boxed{S.P. = \omega T}$

c) Volumetric Efficiency (η_v)

$$\eta_v = \frac{\text{Vol}^m \text{ of water actually striking the runner}}{\text{vol}^m \text{ of water supplied to the turbine}}$$

Some of the vol^m of water is discharged to the tail race without striking the runner of turbine.

d) Overall Efficiency (η_o)

$$\eta_o = \frac{\text{Power available at the shaft of Turbine}}{\text{Power supplied at the inlet of Turbine}}$$

$$= \frac{S.P}{W.P} = \eta_h \times \eta_m$$

Classification of Hydraulic Turbines :-

1) According to the type of energy available at inlet :-

a) Impulse Turbine (only K.E. is available)
b) Reaction Turbine (K.E. + Pr. E.)

2) According to the direction of flow through runner :-

a) Tangential flow turbine
b) Radial flow turbine
c) Axial " "
d) Mixed " " (Radial at inlet + Axial at outlet

3) <u>According to the head at the inlet of turbine</u>

a) High head turbine
b) Medium head turbine
c) low head turbine

4) <u>According to the specific speed of the turbine</u>

a) low sp. speed turbine
b) Medium sp. speed turbine
c) High sp. speed turbine

Specific speed $(N_s) = \dfrac{N\sqrt{P}}{H^{5/4}}$ (for turbines)

<u>Pelton Wheel (or Turbine):-</u>

It is a tangential flow impulse turbine. The water strikes the bucket (or runner blade) along the tangent of the runner. The energy available at the inlet of the turbine is only K.E. The pressure at the inlet and outlet of the turbine is atmospheric. This turbine is used for high heads and is named after L.A. Pelton, an American Engineer.

The main components of Pelton turbine are:-

1) Nozzle and flow regulating arrangement (spea
2) Runner and buckets
3) Casing
4) Breaking Jet

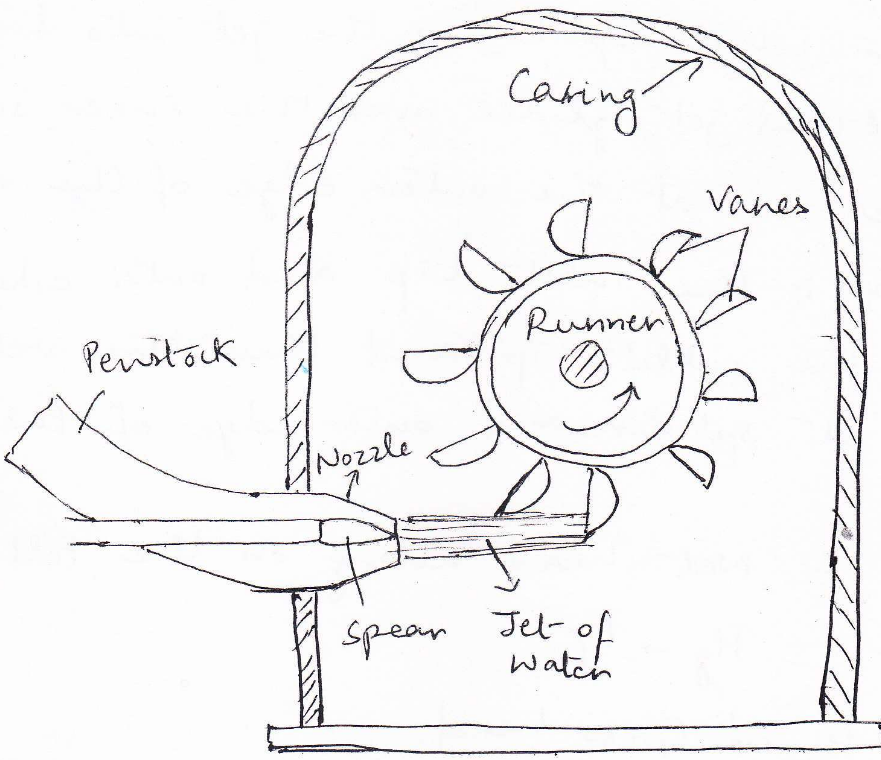
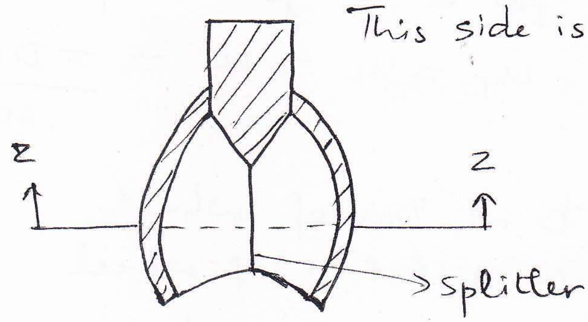
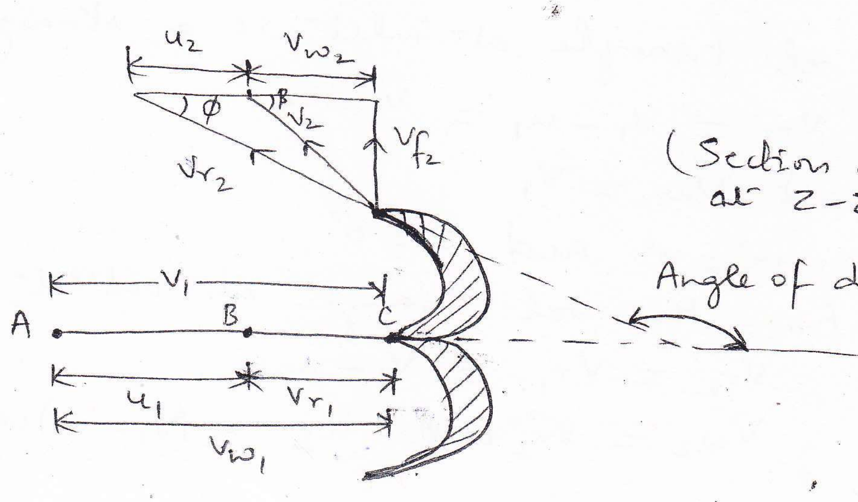


Fig :- Pelton Turbine

Velocity Triangles and Work done for Pelton wheel :-





(Section of Bucket at Z–Z)

The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts. So the splitted jet glides over the inner surfaces and comes out at the outer edge of the vane.

The splitter is the inlet tip and outer edge of the bucket is the outlet tip and thus the velocity Δ's are drawn at splitter and outer edge of the bucket.

Let, H = Net head acting on the Pelton wheel

$= H_g - h_f$

where, $H_g \rightarrow$ Gross head

$h_f = \dfrac{4fLV^2}{2gD^*}$ where $D^* \rightarrow$ Dia of penstock

Then, V_1 (velocity of jet at inlet) $= \sqrt{2gH}$

Also, for a tangential flow turbine,

$$u_1 = u_2 = u = \frac{\pi D N}{60}$$

where, $D \rightarrow$ Dia of wheel

$N \rightarrow$ R.P.M of wheel

and $d \rightarrow$ Dia of jet

The vel. triangle at inlet is a straight line, whe

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

and $V_{w_1} = V_1$

$\alpha = 0°$ and $\theta = 0°$

and from the vel. triangle at outlet,

$$V_{r_2} = V_{r_1} = V_1 - u$$

$$V_{w_2} = V_{r_2}\cos\phi - u_2 = (V_1 - u)\cos\phi - u$$

The force exerted by the jet of water in the direction of motion is given by,

$$F_x = \rho a V_1 [V_{w_1} + V_{w_2}]$$

(for β is acute)

here, $a = \pi/4 \, d^2$
(Area of Jet)

Now, W.D./sec by the jet on the runner

$$= F_x \times u = \rho a V_1 (V_{w_1} + V_{w_2}) \times u \left(\frac{Nm}{s}\right)$$

Power given to the runner by the jet

$$= \rho a V_1 (V_{w_1} + V_{w_2}) \times u \; (W)$$

W.D/sec per unit weight of water striking/sec.

$$= \frac{\rho a V_1 (V_{w_1} + V_{w_2}) \times u}{\rho a V_1 \cdot g}$$

$$= \frac{1}{g} \times u \times (V_{w_1} + V_{w_2}) \; (m)$$

The energy supplied to the jet at inlet is in the form of K.E.

$\therefore$ K.E. of jet /sec. $= \frac{1}{2}(\rho a V_1) \times V_1^2$

$\therefore$ Hydraulic efficiency $(\eta_h) = \frac{R.P.}{W.P.}$

$$= \frac{\rho a V_1 \times u \times (V_{w_1} + V_{w}}{(\rho a V_1 g) \cdot H}$$

$$= \frac{\rho a V_1 \times u \times (V_{w_1} + V_{w}}{\frac{1}{2}(\rho a V_1) \times V_1^2}$$

$$= \frac{2(V_{w_1} + V_{w_2}) \times u}{V_1^2}$$

$$\therefore \eta_h = \frac{2\left[V_1 + (V_1 - u)\cos\phi - u\right] \times u}{V_1^2}$$

$$= \frac{2\left[(V_1 - u) + (V_1 - u)\cos\phi\right] \times u}{V_1^2}$$

$$= \frac{2(V_1 - u) \times u \times \left[1 + \cos\phi\right]}{V_1^2}$$

For max. efficiency, $\frac{d}{du}(\eta_h) = 0$

$$\Rightarrow \frac{(1+\cos\phi)}{V_1^2} \cdot \frac{d}{du}\left[2(V_1 - u).u\right] = 0$$

$$\Rightarrow \frac{(1+\cos\phi)}{V_1^2}\left[2V_1 - 4u\right] = 0$$

$$\Rightarrow 2V_1 = 4u \Rightarrow \boxed{u = V_1/2}$$

and $$\text{Max.}(\eta_h) = \frac{2\left[V_1 - \frac{V_1}{2}\right] \cdot \frac{V_1}{2} \cdot (1+\cos\phi)}{V_1^2}$$

$$= \frac{1+\cos\phi}{2}$$

Points to be Remember (for Pelton wheel)

1) Vel. of jet at inlet (V_1) $= C_v \cdot \sqrt{2gH}$
(Abs. vel. or actual vel.)

$C_v \rightarrow$ Coeff^n of velocity $= 0.98$ or 0.99

2) The vel. of wheel $(u) = \phi \cdot \sqrt{2gH}$
where, $\phi \rightarrow$ Speed ratio $= 0.43$ to 0.48

$$\left\{\because \phi = \frac{u}{\sqrt{2gH}} = \frac{u}{V_1} = \frac{V_1/2}{V_1} \simeq 0.5\right\}$$

3) $\angle$ of deflection of jet through buckets is taken as $165°$ (if no angle of deflection is given).

4) The mean dia or the pitch dia (D) of the pelton wheel is given by,

$$u = \frac{\pi D N}{60} \quad \text{or} \quad D = \frac{60 \cdot u}{\pi N}$$

5) <u>Jet ratio (m)</u>: It is defined as the ratio of pitch dia (D) of Pelton wheel to the dia of jet (d).

$$m = D/d = 12 \text{ (for most cases)}$$

6) No. of buckets on a runner is given by,

$$Z = 15 + \frac{D}{2d} = 15 + 0.5m$$

7) $$\text{No. of Jets} = \frac{\text{Total flow rate through the turbine } (Q)}{\text{flow rate through a single jet } (q)}$$

<u>Design of Pelton Wheel</u>:-

In designing of Pelton wheel, the following data is to be determined:-

1) Dia of jet (d)
2) Dia of wheel (D)
3) Width of buckets = 5 × d
4) Depth of buckets = 1.2 × d

(size of bucket)

5) No. of buckets on the wheel (Z).

Q ⇒ A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 lt/sec. under a head of 30 m. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and hydraulic efficiency of the turbine. Assume, coeffn of velocity as 0.98.

Soln:- Given,

Speed of bucket or vane $(u) = u_1 = u_2 = 10 \text{ m/s}$

$Q = 700 \text{ lt/sec.} = 700 \times 10^{-3} \text{ m}^3/\text{sec.}$

$(= a V_1) \quad = 0.7 \text{ m}^3/\text{sec.}$

$H = 30 \text{ m}$

Angle of def. $= 160°$

$\therefore \phi = 180° - 160° = 20°$

$C_v = 0.98$

Power = ?

Efficiency (η_h) = ?

$V_1 = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 30} = 23.77 \text{ m/s}$

$V_{r_1} = V_1 - u_1 = 13.77 \text{ m/s}$

$V_{w_1} = V_1 = 23.77 \text{ m/s}$

Also, $V_{r_2} = V_{r_1} = 13.77 \text{ m/s}$

$V_{w_2} = V_{r_2} \cos\phi - u_2$

$= 13.77 \cos 20° - 10 = 2.94 \text{ m/s}$

$\therefore$ Power $= \text{W.D/sec.} = \rho a V_1 [V_{w_1} + V_{w_2}] \times u$

$= 10^3 \times 0.7 \times 10 [23.77 + 2.94]$

$= 186.97 \text{ kW}$

and $\eta_h = \dfrac{2[V_{w_1} + V_{w_2}) \times u}{V_1^2} = 94.54\%$

Q→ A Pelton wheel is to be designed for the followi[ng]
specifications :-

Shaft power = 11,772 KW

Head = 380 m

Speed = 750 rpm

Overall efficiency = 86 %

Jet Diameter is not to exceed one-sixth of the wheel diameter. Determine :

1) The wheel dia
2) No. of jets required
3) Dia of the jet

Take $K_{v_1} = 0.985$ and $K_{u_1} = 0.45$.

Solⁿ :- Given :-

$S.P = 11,772$ KW

$H = 380$ m

$N = 750$ rpm

$\eta_o = 0.86$

$d/D = 1/6$

Coeffⁿ of velocity (K_{v_1}) or $(C_v) = 0.985$

Speed ratio (K_{u_1}) or $(\phi) = 0.45$

Now, $V_1 = C_v \cdot \sqrt{2gH} = 85.05$ m/s

$u = u_1 = u_2 = \phi \cdot \sqrt{2gH} = 38.85$ m/s

But, $u = \dfrac{\pi D N}{60}$

$38.85 = \dfrac{\pi D \times 750}{60} \Rightarrow D = 0.989$ m

and $d = 0.165$ m

Discharge of one jet $(q) = \dfrac{\pi}{4} d^2 \times V_1 = 1.818\ m^3/s$

Now, $\eta_o = \dfrac{S.P.}{W.P} = \dfrac{11772}{\dfrac{\rho g Q H}{10^3}}$

$$\Rightarrow \quad 0.86 = \frac{11772 \times 10^3}{10^3 \times 9.81 \times Q \times 380}$$

$\therefore$ Q (total discharge) $= 3.672 \ m^3/s$

$$\therefore \text{No. of Jets} = \frac{\text{Total discharge} \ (Q)}{\text{Discharge of one jet} \ (q)}$$

$$= \frac{3.672}{1.818} = 2 \text{ jets.}$$

Governing of Turbines :-

The governing of a turbine is defined as the operation by which the speed of the turbine is kept constant under all conditions of working. It is done automatically by means of a governor, which regulates the rate of flow through the turbines according to the changing load conditions on the turbine.

→ Governing of a turbine is necessary as a turbine is directly coupled to an electric generator, which is required to run at constant speed under all fluctuating load conditions. So, the speed of generator will be constant, when the speed of the turbine is constant.

(↑) Speed of generator (shaft) (Change due to fluctuating load) (↓) ⟹ Speed of turbine (runner) (↑) (due to change of speed of generator shaft)

⇓

Jet of water impinging on runner blades or flow rate (↓)

⟸

$\boxed{\text{Governer}}$
(to control flow rate)

Governing of Pelton Turbine :-

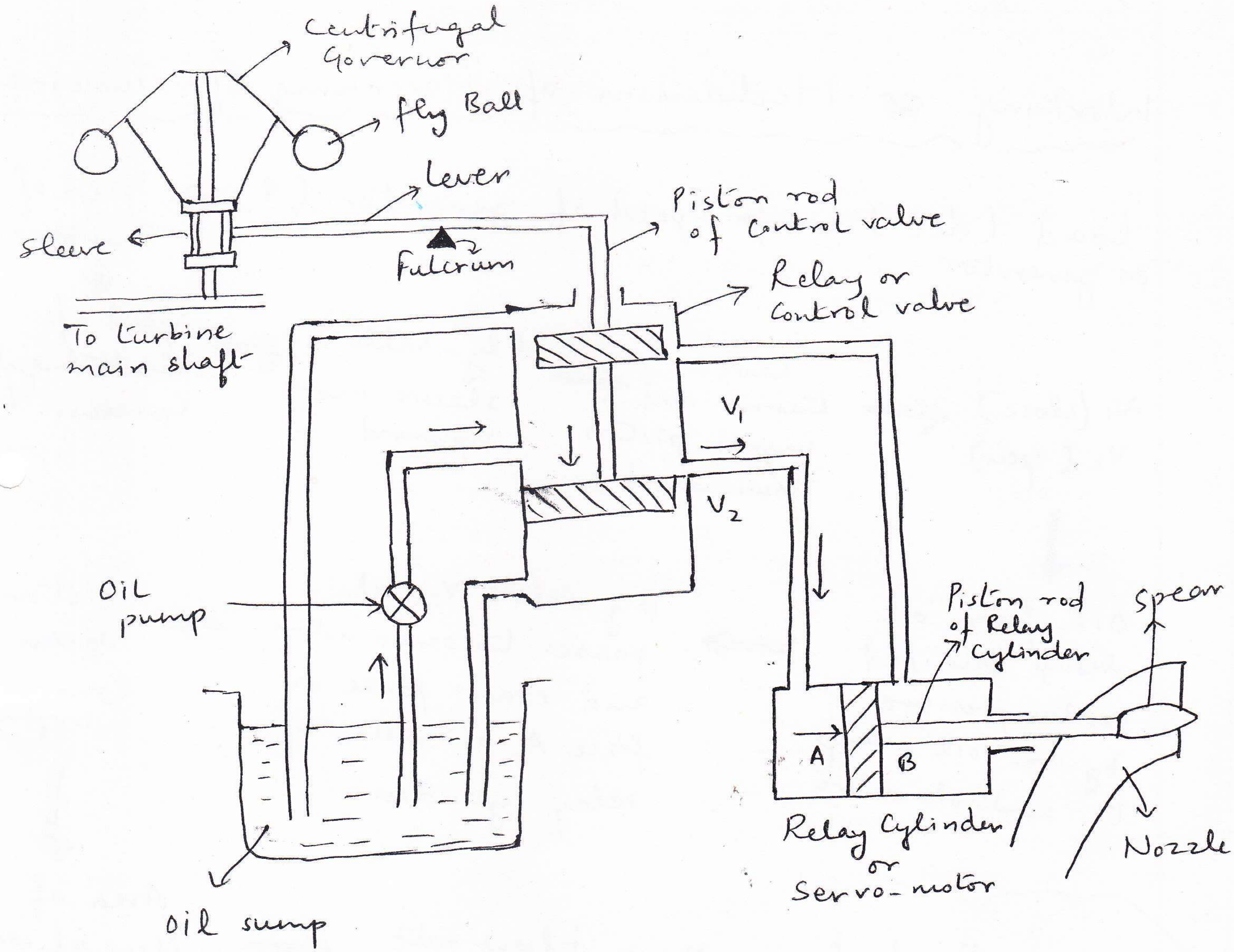


Fig:- Governing of Pelton Turbine

Governing of Pelton turbine is done by means of oil pressure governor, which consists of the following parts:-

1) Oil sump
2) Gear pump / oil pump (driven by the power obtained from turbine shaft).
3) Serromotor also called relay cylinder.
4) Control valve / distribution valve / relay valve
5) Centrifugal governor or pendulum (driven by belt or gear from the turbine shaft)

6) Pipes connecting the oil sump with the control valve and control valve with servomotor

7) Spear rod or needle.

Working Mechanism of Governing of Turbine:

Load (dec.) on generator $\Rightarrow$ Speed of generator (↑) $\Rightarrow$ Speed of Turbine (↑)

$\Downarrow$

Speed of Centrifugal Governor (↑) $\Rightarrow$ Fly-balls & sleeve move upward $\Rightarrow$ Lever turns and moves piston downward $\Rightarrow$ V_1 (close), V_2 (open)

$\Downarrow$

Oil from oil sump pumped under pressure by an oil pump to control valve $\Rightarrow$ By valve V_2, oil passes to servomotor and exert force on face A of piston of relay cylinder $\Rightarrow$ Piston & spear (moves right)

$\Downarrow$

Area of flow of water at outlet of nozzle (↓) $\Rightarrow$ Flow rate (↓) $\Rightarrow$ Speed of Turbine (↓)

$\Downarrow$

When speed of Turbine becomes normal then fly-balls, sleeve, lever and piston-rod of control valve come to its normal position.

→ The same can be explained for the case, when the load on the generator increases.