LAMINAR FLOW

in gate sure every year

Reynold's number → It is the ratio of inertia force to viscous force

$$\boxed{R_e = \frac{F_i}{F_v} = \frac{\rho V L}{\mu}}$$

where L is characteristic length. In case of pipe flow problems L is equal to diameter of the pipe. Therefore

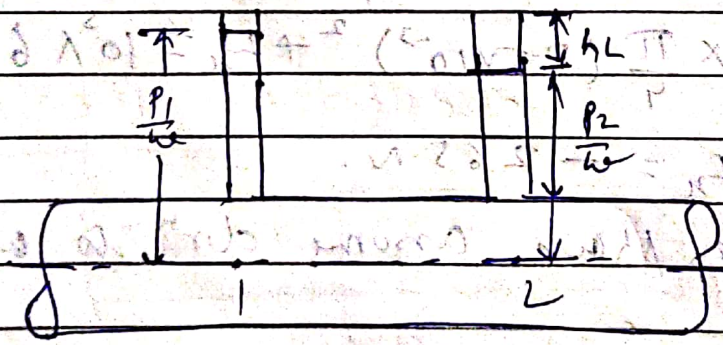
$$\boxed{R_e = \frac{\rho V D}{\mu}}$$

for pipe flow:-

$R_e < 2000 \rightarrow$ Laminar

$R_e > 4000 \rightarrow$ Turbulent

$2000 < R_e < 4000 \rightarrow$ Transition flow



$\cancel{A_1} v_1 = \cancel{A_2} v_2$

$v_1 = v_2$

$$\frac{P_1}{w} + \cancel{\frac{v_1^2}{2g}} + \cancel{z} = \frac{P_2}{w} + \cancel{\frac{v_2^2}{2g}} + h_L$$

$$\boxed{\frac{P_1}{w} = \frac{P_2}{w} + h_L}$$

→ In the direction of flow pressure decreases because the fluid has to overcome frictional losses

$$\boxed{\frac{P_1 - P_2}{w} = h_L} \quad \text{(I)}$$

h_L = head loss due to friction

~~Darcy-Weisbach :-~~

Darcy-Weisbach :→

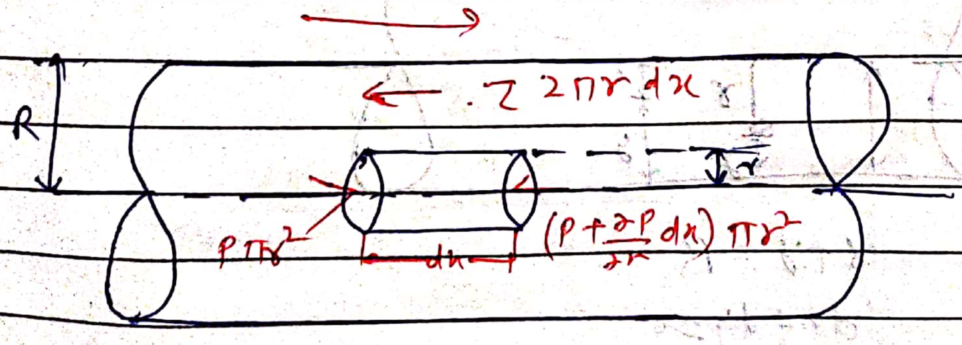
this eqⁿ is use for finding out head loss due to friction i.e

$$h_L = \frac{4 f' L V^2}{2 g D}$$

f' = friction coefficient

$$h_L = \frac{f L V^2}{2 g D} \; ; \; f = \text{friction factor}$$

* Laminar flow through circular pipes :→



$$p \cdot \pi r^2 - \left(p + \frac{\partial p}{\partial x} dx\right)\pi r^2 - \tau\, 2\pi r\, dx = 0$$

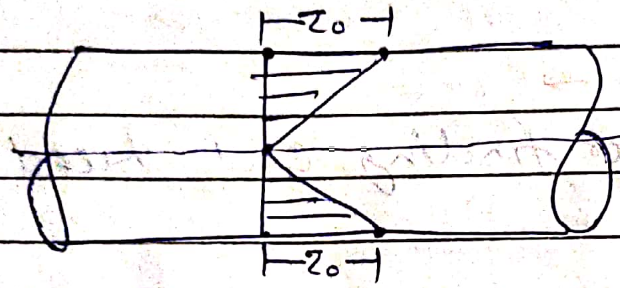
$$pr - \left(p + \frac{\partial p}{\partial x} dx\right) r = 2\tau\, dx$$

$$pr - pr - \frac{\partial p}{\partial x} dx \cdot r = 2\tau dx$$

$$-\frac{\partial p}{\partial x} dx \cdot r = 2\tau dx$$

$$-\frac{\partial p}{\partial x} r = 2\tau$$

$$\boxed{\tau = \left(-\frac{\partial p}{\partial x}\right)\frac{r}{2}} \quad \text{--- (1)}$$



$\tau \propto r$

The shear stress increases linearly from zero at the centre line of the pipe to the maximum at the pipe surface (τ_0).

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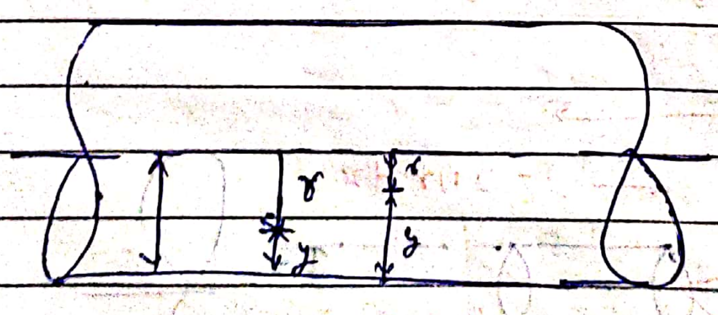
velocity distribution



$$r + y = R$$

$$dr + dy = 0$$

$$dy = -dr$$

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{du}{-dr} \Rightarrow \tau = -\mu \frac{du}{dr}$$

$$\tau = \frac{-\partial p}{\partial x} \cdot \frac{r}{2}$$

$$\tau = -\mu \frac{du}{dr}$$

$$\frac{-\partial p}{\partial x} \cdot \frac{r}{2} = -\mu \frac{du}{dr}$$

$$\frac{\partial p}{\partial x} \cdot \frac{r}{2} = \mu \frac{du}{dr}$$

$$du = \frac{1}{2\mu} \frac{\partial p}{\partial x} \cdot r\,dr.$$

$$\int du = \int \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) r\,dr$$

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\frac{r^2}{2} + C$$

At $r = R$ (at the pipe wall) ; $u = 0$

$$0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2 + C$$

$$C = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2$$

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\frac{r^2}{2} - \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2$$

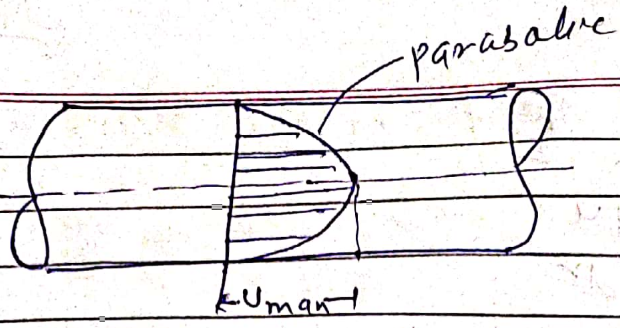
$$u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left[R^2 - r^2\right]$$

$$u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2\left[1 - \frac{r^2}{R^2}\right]$$

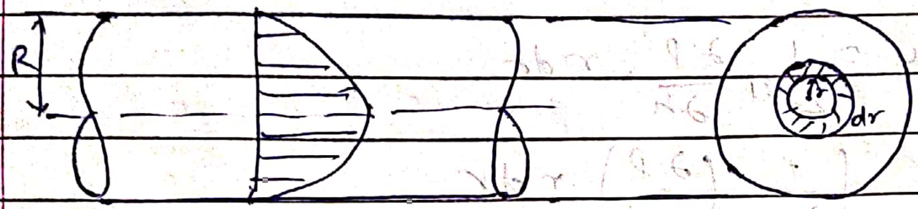
$u = u_{max}$ at the centre, $r = 0$

$$\boxed{u_{max} = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2} \quad (1)$$

$$\boxed{u = U_{max}\left(1 - \frac{r^2}{R^2}\right)} \quad (2)$$



Discharge! → consider flow through an elemental ring of radius r. and thickness dr



$A = \pi r^2$

$$u = u_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$dQ = u \times 2\pi r\, dr$$

$$Q = \int_0^R 2\pi u r\, dr$$

$$Q = 2\pi \int_0^R u r\, dr$$

$$Q = 2\pi \int_0^R u_{max}\left(1 - \frac{r^2}{R^2}\right) r\, dr$$

$$Q = 2\pi u_{max}\left[\int_0^R r\, dr - \frac{r^3}{R^2}\, dr\right]$$

$$Q = 2\pi u_{max}\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

$$Q = 2\pi U_{max}\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]$$

$$Q = \cancel{2}\pi U_{max}\left[\frac{R^2}{\cancel{4}}\right]$$

$$\boxed{Q = \frac{\pi U_{max} R^2}{2}} \quad (4)$$

$$Q = \frac{\pi R^2}{2} \times \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) R^2$$

$$Q = \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4$$

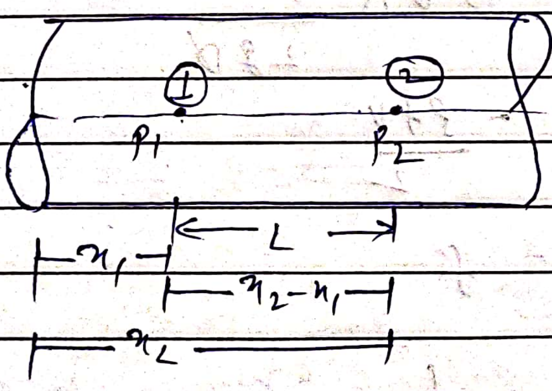
Average velocity (V) → It is the ratio of total discharge by area of flow.

$$Q = AV$$

$$V = \frac{Q}{A} = \frac{\frac{\pi U_{max} R^2}{2}}{\pi R^2} = \frac{U_{max}}{2}$$

$$\boxed{V = \frac{U_{max}}{2}} \quad (5)$$

Pressure drop in a given length →



$$V = \frac{U_{max}}{2}$$

$$V = \frac{1}{2}\left[\frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) R^2\right]$$

$$V = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2$$

$$\frac{8\mu V}{R^2}\,\partial x = -\partial p$$

$$\frac{8\mu V}{R^2}\int_{x_1}^{x_2} \partial x = -\int_{p_1}^{p_2} \partial p$$

$$\frac{8\mu V}{R^2}(x_2 - x_1) = -[P_2 - P_1]$$

$$\frac{8\mu V}{R^2} \times L = P_1 - P_2$$

$$P_1 - P_2 = \frac{8\mu V L}{R^2}$$

$$\boxed{P_1 - P_2 = \frac{32\mu V L}{D^2}} \quad \text{--- (6)}$$

$$P_1 - P_2 = \frac{32\mu V L}{D^2}$$

from eq^n (I) $\quad \frac{P_1 - P_2}{w} = h_L$

$$P_1 - P_2 = w h_L$$

$$P_1 - P_2 = \rho g h_L$$

$$\frac{32\mu V \cancel{L}}{D^2} = \rho \cancel{g} \cancel{L} \cdot \frac{f V^2}{2 \cancel{g} D}$$

$$\frac{32\mu}{D} = \frac{\rho f V}{2}$$

$$\frac{64\mu}{\rho V D} = f$$

$$\frac{64}{\left(\frac{\rho V D}{\mu}\right)} = f$$

$$\boxed{f = \frac{64}{Re}} \quad \text{--- (7)}$$

⇒ → friction factor in laminar flow through circular pipes will depend only on Reynold's number Re.

Shear Velocity (V_*) :→

$$\tau = \left(-\frac{\partial P}{\partial x}\right)\frac{r}{2}$$

$$\tau_0 = \left(-\frac{\partial P}{\partial x}\right)\cdot\frac{R}{2}$$

$$\tau_0 = -\frac{(P_2 - P_1)}{x_2 - x_1}\cdot\frac{D}{4}$$

$$\tau_0 = \frac{P_1 - P_2}{L}\cdot\frac{D}{4}$$

$$\tau_0 = \frac{(P_1 - P_2)D}{4L}$$

$$\tau_0 = \frac{w h_L \cdot D}{4L}$$

$$\tau_0 = \frac{\rho g D}{4L} \times \frac{f L v^2}{2 g D}$$

$$\boxed{\tau_0 = \frac{\rho f v^2}{8}}$$ (8)

$$\frac{\tau_0}{\rho} = \frac{f v^2}{8}$$

$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f}{8}}\cdot v$$

$\sqrt{\frac{\tau_0}{\rho}}$ = shear velocity = V_*

$$\boxed{V_* = \sqrt{\frac{f}{8}}\cdot V}$$

Equations to be remembered for laminar flow in circular pipes

(1) $\tau = \left(\frac{-\partial p}{\partial x}\right) \cdot \frac{r}{2}$

(2) $U_{max} = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) R^2$

(3) $u = u_{max}\left(1 - \frac{r^2}{R^2}\right)$

(4) $Q = \frac{\pi R^2 U_{max}}{2}$

(5) $V = \frac{U_{max}}{2}$

(6) $P_1 - P_2 = \frac{32 \mu V L}{D^2}$

(7) $f = \frac{64}{Re}$

(8) $\tau_0 = \frac{\rho f V^2}{8}$

Q. The velocity profile in a laminar flow through a pipe of diameter D is given by $u = u_0\left(1 - \frac{4r^2}{D^2}\right)$ where r is radial distance from the centre. then the pressure drop in a length L is given by ?

Ans
$$U = U_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$u = u_{max}\left(1 - \frac{4r^2}{D^2}\right)$$

Ans $\frac{16 \mu v_0 L}{D^2}$

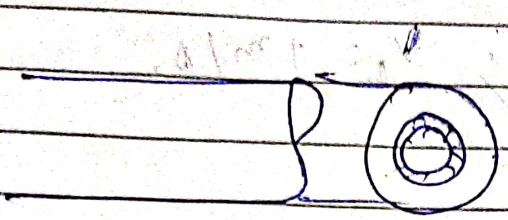
$$u = U_0\left(1 - \frac{4r^2}{D^2}\right)$$

$$u_{max} = u_0$$

$$P_1 - P_2 = \frac{32 \mu V L}{D^2} \Rightarrow \frac{\overset{16}{32} \mu U_0}{\cancel{2}} \frac{L}{D^2}$$

Momentum correction factor :→ (β)

It is the ratio of actual momentum to the average momentum.



Actual momentum	Average momentum
mu	mV
$\rho A u \cdot u$	$\rho AV \cdot V$
$\rho A \cdot u^2$	$\rho A V^2$
$\int \rho u^2 dA$	

$$\beta = \frac{P_{act}}{P_{avg}} = \frac{\cancel{m/s}\, V_{act}}{m/s\, V_{avg}}$$

$$\beta = \frac{\int \cancel{\rho} u^2 dA}{\cancel{\rho} A V^2}$$

$$\boxed{\beta = \frac{1}{AV^2}\int u^2 dA}$$

Kinetic energy correction factor (α) :→ It is the ratio of kinetic energy based on actual velocity to the kinetic energy based on average velocity.

$$\boxed{\alpha = \frac{\int u^3 dA}{AV^3}}$$

Q. find the momentum correction factor for Laminar flow through circular pipes

$$\beta = \frac{\int u^2 dA}{AV^2}$$

$\int u^2 dA$ $\qquad u = u_{max}\left(1-\frac{r^2}{R^2}\right)$

$$\int u^2 dA = \int_0^R \left[u_{max}\left(1-\frac{r^2}{R^2}\right)\right]^2 \cdot 2\pi r\,dr$$

$$\int u^2 dA = 2\pi U_{max}^2 \int_0^R \left(1-\frac{r^2}{R^2}\right)^2 r\,dr$$

$$\int u^2 dA = 2\pi U_{max}^2 \int_0^R \left(1+\frac{r^4}{R^4}-\frac{2r^2}{R^2}\right) r\,dr$$

$$\int u^2 dA = 2\pi U_{max}^2 \left[\int_0^R \left(r+\frac{r^5}{R^4}-\frac{2r^3}{R^2}\right)\right] dr$$

$$\int u^2 dA = 2\pi U_{max}^2 \left[\frac{r^2}{2}+\frac{r^6}{6R^4}-\frac{2r^4}{4R^2}\right]_0^R$$

$$\int u^2 dA = 2\pi U_{max}^2 \left[\frac{R^2}{2}+\frac{R^6}{6R^4}-\frac{2R^4}{4R^2}\right]$$

$$\int u^2 dA = 2\pi U_{max}^2 \left[\frac{R^2}{6}\right]$$

$$\int u^2 dA = \frac{\pi}{3} U_{max}^2 R^2$$

$$\beta = \frac{\int u^2 dA}{AV^2} \qquad V = \frac{U_{max}}{2}$$

$$\beta = \frac{\frac{\pi}{3} U_{max}^2 R^2}{\pi R^2 \times \frac{U_{max}^2}{4}}$$

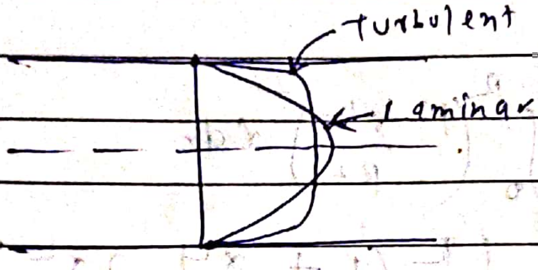
$$\boxed{\beta = 4/3}$$

β → Laminar → $\frac{4}{3} = 1.33$

Turbulent → 1.2

α → Laminar → 2

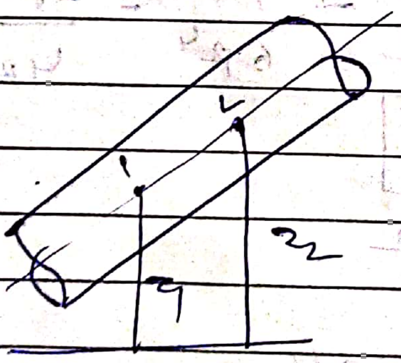
Turbulent → $\frac{4}{3} = 1.33$



→ The lower correction factor for turbulent flow indicate that the velocity distribution is more uniform ~~than in~~ in turbulent than in laminar flow

equations to be remember for inclined pipe

(1)



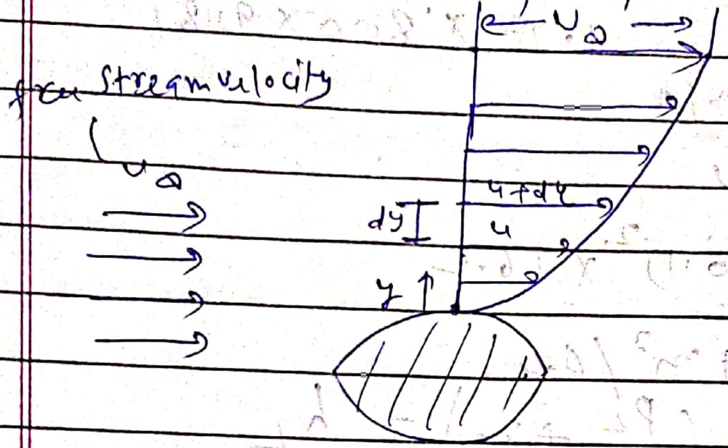
$$\frac{P_1}{w} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{w} + z_2 + \frac{v_2^2}{2g} + h_L$$

$$\boxed{\left(\frac{P_1}{w} + z_1\right) - \left(\frac{P_2}{w} + z_2\right) = h_L}$$

where $h_L = \frac{fLv^2}{2gD}$

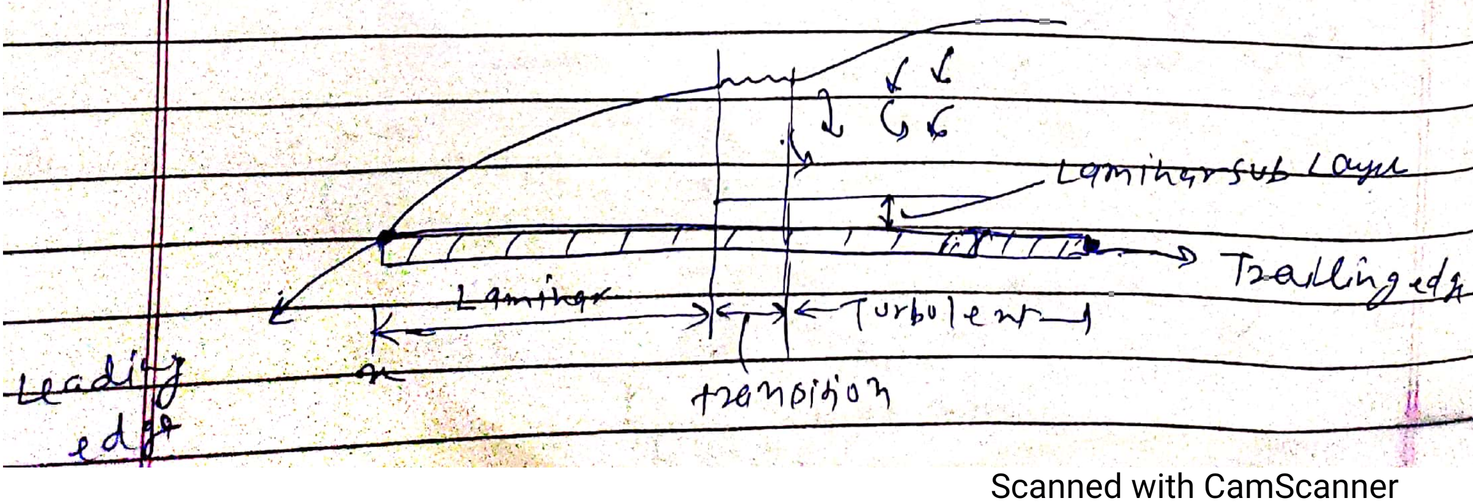
BOUNDRY LAYER THEORY

This theory was proposed by prandtl



When a real fluid flows past a solid boundry a fluid on the surface will have same velocity as that of the surface if the boundry is static the fluid will also have zero velocity. Away from the boundry the fluid velocity gradually increases and velocity gradient $\frac{du}{dy}$ exists in a region closed to the boundry this region is known as Boundry layer region. these velocity gradient will give rise to shear stress and hence the flow is viscous in Boundry layer region.

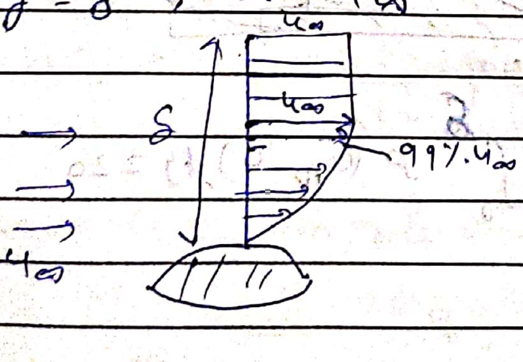
Development of a Boundry Layer along a flat plate



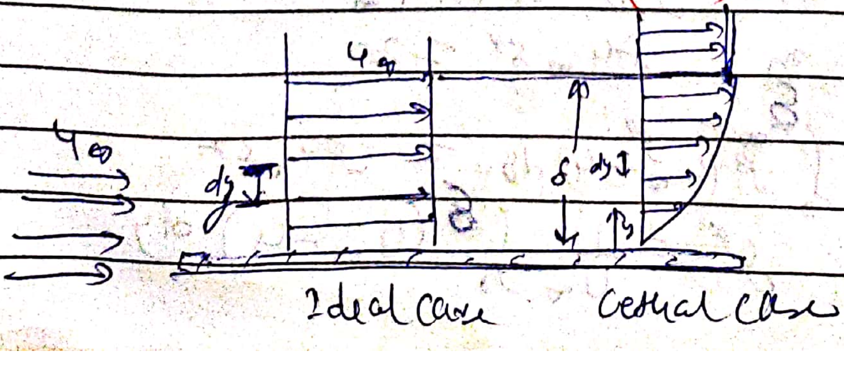
when a fluid flows passed a flat plate the velocity at the leading edge is zero the retardation of fluid particles increase as more and more of the plate is exposed to the flow. hence the boundry layer thickness increases as the distance from the leading edge increases. upto certain distance from the leading edge it is found that the flow in the boundry layer is laminar and as the laminar boundry layer grows instability occurs and the flow changes to laminer to turbulent through transition. It is found that even in turbulent boundry layer closed to the plate the flow is found to be laminar and this region is known as laminar sub-layer region.

Boundry layer thickness or Nominal Boundry layer thickness (δ)! → It is the distance from the Boundry to the point in the y-direction. where the velocity is 99% of free stream velocity for all calculation purpose at

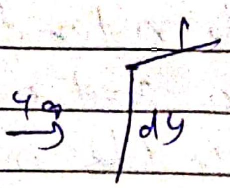
$y = \delta$, $u = u_\infty$

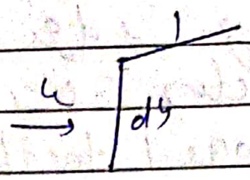


Displacement thickness (δ^*)! —



Ideal case | actual case





$\dot m_{ideal} = \rho\, dy \times 1 \times u_\infty$

$\boxed{\dot m_{ideal} = \rho u_\infty dy}$

$\dot m_{actual} = \rho\, dy \times 1 \times u$

$\dot m_{actual} = \rho\, dy \times 1 \times u$

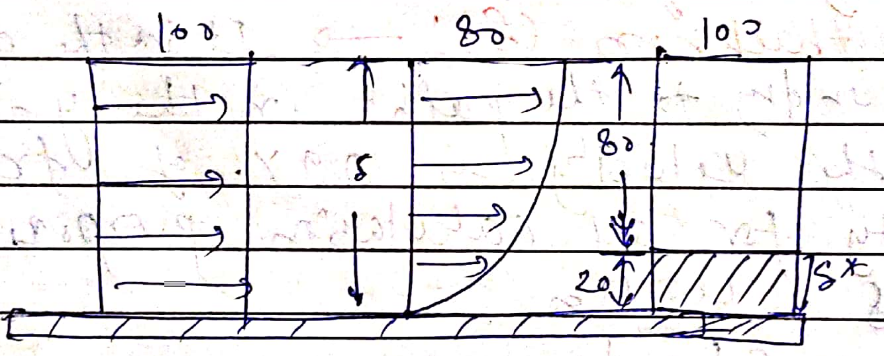
$\dot m_{actual} = \rho u\, dy$

Reduction in mass flowrate $= \dot m_{ideal} - \dot m_{act}$

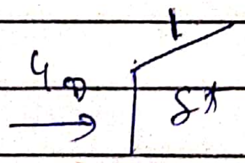
$= \rho U_\infty dy - \rho u\, dy$

$= \rho (u_\infty - u)\, dy$

Total mass reduction $= \int_0^\delta \rho (u_\infty - u)\, dy$



Total mass reduction $= \int_0^\delta \rho (u_\infty - u)\, dy = 20$.



$\dot m = \rho \delta^* \times 1 \times u_\infty$

$\dot m = \rho u_\infty \delta^* = 20$

$\int_0^\delta \rho (u_\infty - u)\, dy = \rho u_\infty \delta^*$

$\delta^* = \frac{1}{U_\infty} \int_0^\delta (U_\infty - u)\, dy$

$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy}$$

Displacement thickness:–

It is the distance by which the boundry should be displaced in order to compensate for the reduction in mass flow rate due to Boundry layer growth.

Momentum thickness (θ):– It is the distance by which the boundry should be displace in order to compensat for the reduction in momentum due to boundry layer growth.

$$\theta = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$$

Energy thickness (δ_e):→ It is the distance by which the boundry should be displace in order to compensat for the reduction in kinetic energy due to boundry layer growth.

$$\delta_E = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u^2}{u_\infty^2}\right) dy$$

Q The velocity distribution in boundry layer is given by $\frac{u}{u_\infty} = \frac{y}{\delta}$ then find the shape factor $\left(\frac{\delta^*}{\theta}\right)$

A
$$\delta^* = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

$$= y\Big|_0^\delta - \frac{y^2}{2\delta}\Big|_0^\delta = \delta/2$$

$$\theta = \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy = \frac{1}{\delta}\left[\left|\frac{y^2}{2}\right|_0^\delta - \left|\frac{y^3}{3\delta}\right|_0^\delta\right]$$

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$$= \frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\Big|_0^\delta = \frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2} = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$$

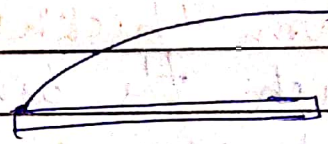
$$\frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = 3$$

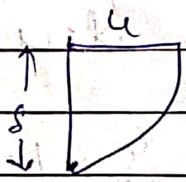
Boundry conditions! →

(1) $x = 0;\ \delta = 0$

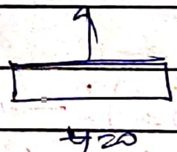
(2) $y = 0;\ u = 0$

(3) $y = \delta;\ u = u_\infty$





$$\tau = \mu \frac{du}{dy}$$



$$\tau_0 = \mu \frac{du}{dy}\Big|_{y=0}$$

Q Assume that the shear stress distribution varies linearly in laminar boundary layer such that $\tau = \tau_0\left(1 - \frac{y}{\delta}\right)$ than find momentum thickness and displacement thickness in terms of δ!

Ans

$$\tau = \mu \frac{du}{dy}.$$

$$\tau = \tau_0\left(1 - \frac{y}{\delta}\right)$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy.$$

$$\tau = \mu\frac{du}{dy} = \tau_0\left(1 - \frac{y}{\delta}\right)$$

$$du = \frac{\tau_0}{\mu}\left(1 - \frac{y}{\delta}\right) dy.$$

$$u = \frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right) + c.$$

at $y = 0$; $u = 0$.

$$\Rightarrow c = 0$$

$$u = \frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right)$$

at $y = \delta$; $u = u_\infty$

$$u_\infty = \frac{\tau_0}{\mu}\left(\delta - \frac{\delta^2}{2\delta}\right)$$

$$u_\infty = \frac{\tau_0}{\mu}\left(\delta - \frac{\delta}{2}\right)$$

$$u_\infty = \frac{\tau_0}{\mu}\cdot\frac{\delta}{2}$$

$$\frac{u}{u_\infty} = \frac{\frac{\cancel{\tau_0}}{\mu}\left(y - \frac{y^2}{2\delta}\right)}{\frac{\cancel{\tau_0}\,\delta}{2\cancel{\mu}}}$$

$$\frac{u}{u_\infty} = \frac{2}{\delta}\left(y - \frac{y^2}{2\delta}\right)$$

$$\frac{u}{u_\infty} = \frac{2y}{\delta} - \frac{\cancel{2}y^2}{\cancel{2}\delta^2}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy$$

$$= y - \frac{\cancel{2}y^2}{\cancel{2}\delta} + \frac{y^3}{3\delta^2}\Big|_0^\delta$$

$$= \delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2} = \frac{\delta}{3}$$

$$\theta = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy.$$

$$\theta = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right] dy.$$

$$\boxed{\theta = \frac{2}{15}\delta}$$

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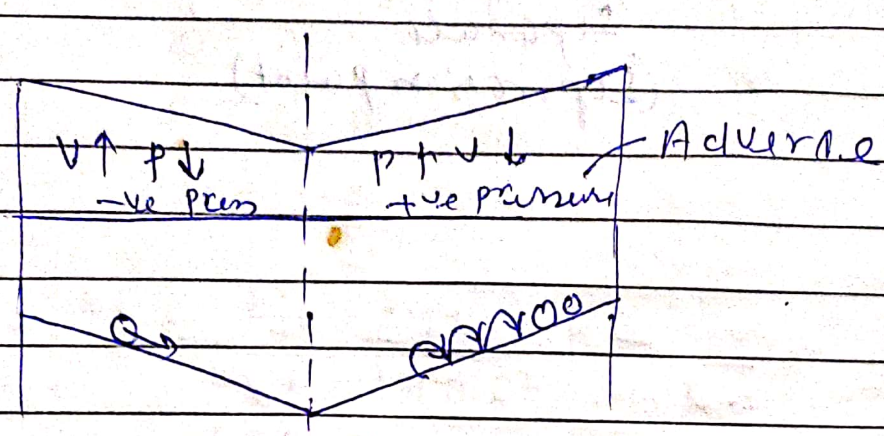
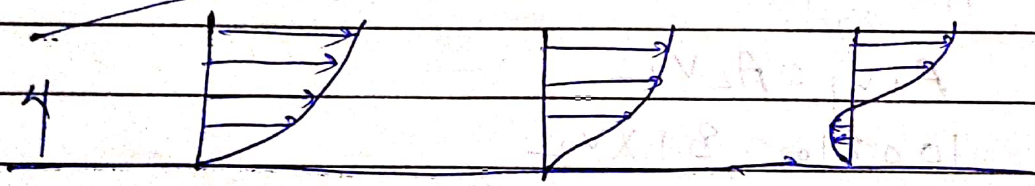
Boundry Layer Separation



In case of convergent flow (accelerating flow) the velocity increases and pressure decreases therefore fluid slides down the pressure hill. When the fluid flows through a divergent passage the velocity decreases and pressure increases therefore the fluid moves under the positive pressure gradient and hence the fluid has to climb a pressure hill. If the velocity reduction is more the momentum of the fluid particles may not support to climb a pressure hill and hence the flow may reverse it's direction from it's boundary & this is known as boundary layer separation. & this occurs under positive pressure gradients & these gradients are also known as adverse pressure gradients.



$\left.\frac{du}{dy}\right|_{y=0} > 0$ — No separation

$\left.\frac{du}{dy}\right|_{y=0} = 0$ — About to separate (separation point)

$\left.\frac{du}{dy}\right|_{y=0} < 0$ — Separated flow

Flow through pipes

When fluid flows through pipes it encounters various losses

Losses
- Major loss
  - (1) loss of head due to friction.
- Minor loss
  - (1) losses due to sudden expansion
  - (2) sudden contraction
  - (3) Bend loss etc.

Major loss:→

(I) Darcy - Weisbach eqn

$$h_L = \frac{fLV^2}{2gD}$$

$$Q = AV$$

$$Q = \frac{\pi}{4} D^2 V$$

$$V = \frac{4Q}{\pi D^2}$$

$$h_L = \frac{fL}{2gD}\left(\frac{4Q}{\pi D^2}\right)^2$$

$$h_L = \frac{16 fLQ^2}{\pi^2 \times 2g D^5}$$

$$h_L = \frac{fLQ^2}{\left(\frac{2g\pi^2}{16}\right) D^5}$$

$$h_L = \frac{fLQ^2}{12 D^5}$$

$$\boxed{h_L = \frac{fLV^2}{2gD} = \frac{fLQ^2}{12 D^5}}$$

chazi's formula:→

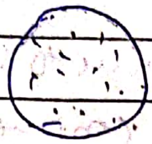
$$V = C\sqrt{mi}$$

C = Chezys constant

m = hydraulic mean depth

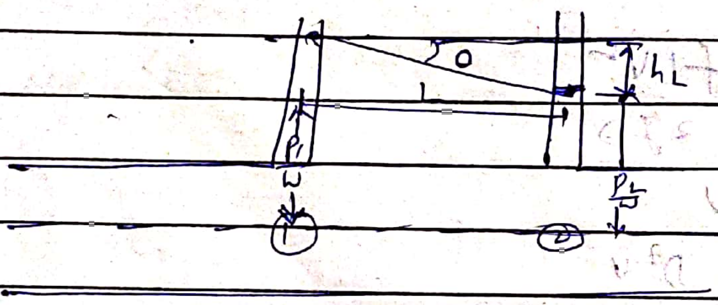
i = hydraulic slope.

v = velocity of flow.



$$m = \frac{A}{P}$$

$$m = \frac{\frac{\pi}{4}D^2}{\pi D} = D/4$$



$$i = \tan\theta = \frac{h_L}{L}$$

$$\frac{P_1}{w} + \cancel{\frac{V_1^2}{2g}} + \cancel{Z_1} = \frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}} + \cancel{Z_2} + h_L$$

$$\cancel{A_1}V_1 = \cancel{A_2}V$$

$$\frac{P_1}{w} = \frac{P_2}{w} + h_L$$

$$V = C\sqrt{mi}$$

$$V = C\sqrt{\frac{D}{4}\times\frac{h_L}{L}}$$

$$V^2 = C^2 \cdot \frac{D h_L}{4L}$$

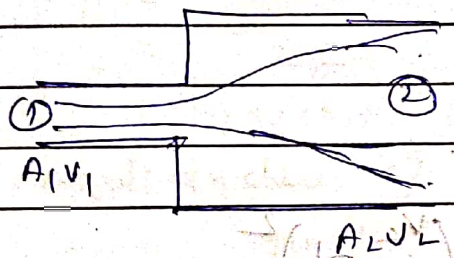
$$h_L = \frac{4LV^2}{C^2 D}$$

$$\frac{4LV^2}{C^2 D} = \frac{fLV^2}{2gD}$$

$$\frac{4}{C^2} = \frac{f}{2g} \Rightarrow \boxed{C = \sqrt{\frac{8g}{f}}}$$

MINOR LOSSES:→

I) Losses due to sudden expansion:→



$$h_{L\,exp} = \frac{(V_1 - V_2)^2}{2g}$$

$$h_{L\,exp} = \frac{V_1^2}{2g}\left[1 - \frac{V_2}{V_1}\right]^2$$

$$A_1 V_1 = A_2 V_2$$

$$\frac{V_2}{V_1} = \frac{A_1}{A_2}$$

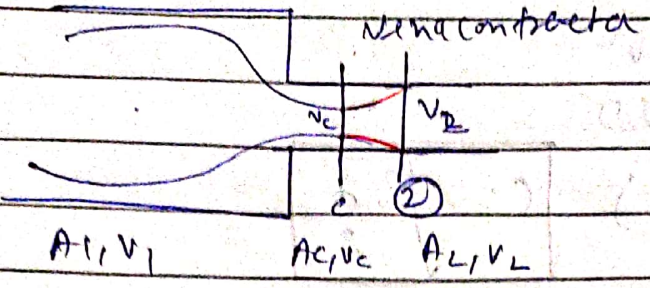
$$\boxed{h_{L\,exp} = \frac{V_1^2}{2g}\left[1 - \frac{A_1}{A_2}\right]^2}$$

II) Losses at the exit of pipe:→

this is similar to sudden expansion with $A_2 = \infty$ therefore exit loss is equal to

$$\boxed{h_{L\,exp} = \frac{V_1^2}{2g}}$$

III) Losses due to sudden contraction :-



$$C_c = \frac{A_c}{A_2}$$

$$A_c V_c = A_2 V_2$$

$$\frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{C_c}$$

$$h_{Lc} = \frac{(V_c - V_2)^2}{2g}$$

$$h_{L\,contraction} = \frac{V_2^2}{2g}\left(\frac{V_c}{V_2} - 1\right)^2$$

$$\boxed{h_{L\,contra} = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2}$$

if the C_c (coefficient of contraction) is not given than the losses due to sudden contraction is taken as $\boxed{h_c = 0.5\frac{V_2^2}{2g}}$ where V_2 = velocity in the smaller pipe.

VI) Losses at the entrance of pipe :-

It is similar to sudden contraction and hence ~~sudden contraction~~ entrance loss is equal to $0.5\frac{V^2}{2g}$, where V is velocity in the pipe.

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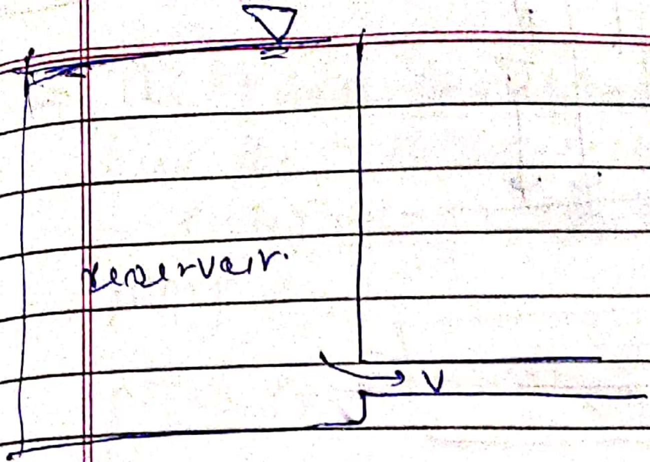
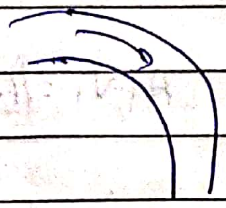


(VI) Bend losses:—



$$h_L = \frac{kV^2}{2g}$$

$K = 1.2$ for $90°$

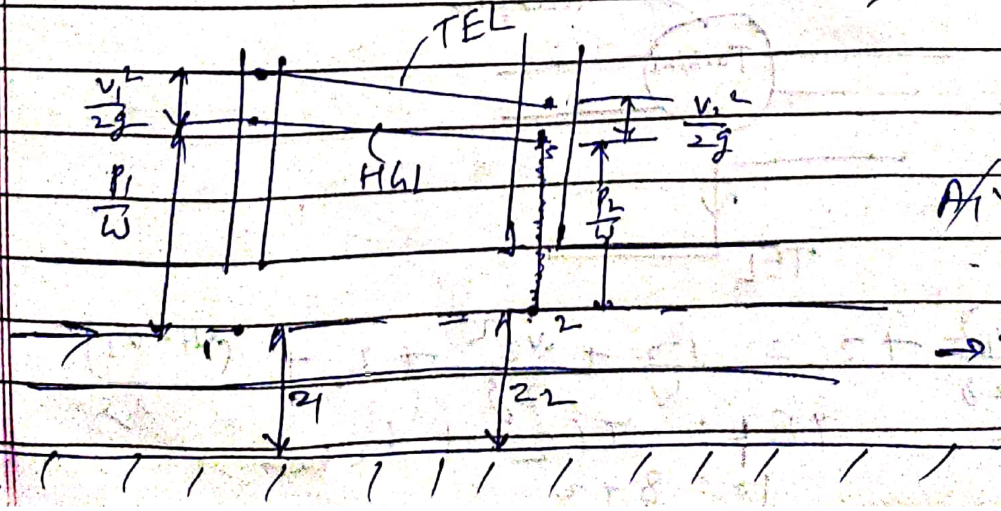
$K = .4$ for $45°$

k = constant which depends on angle of Bend and radius of curvature of bend

Hydraulic gradient line and total energy line! → (HGL) (TEL)

HGL! → It is the line which joints piezometric head $\left(\frac{p}{w} + z\right)$ at various points is known as Hydraulic gradient line.

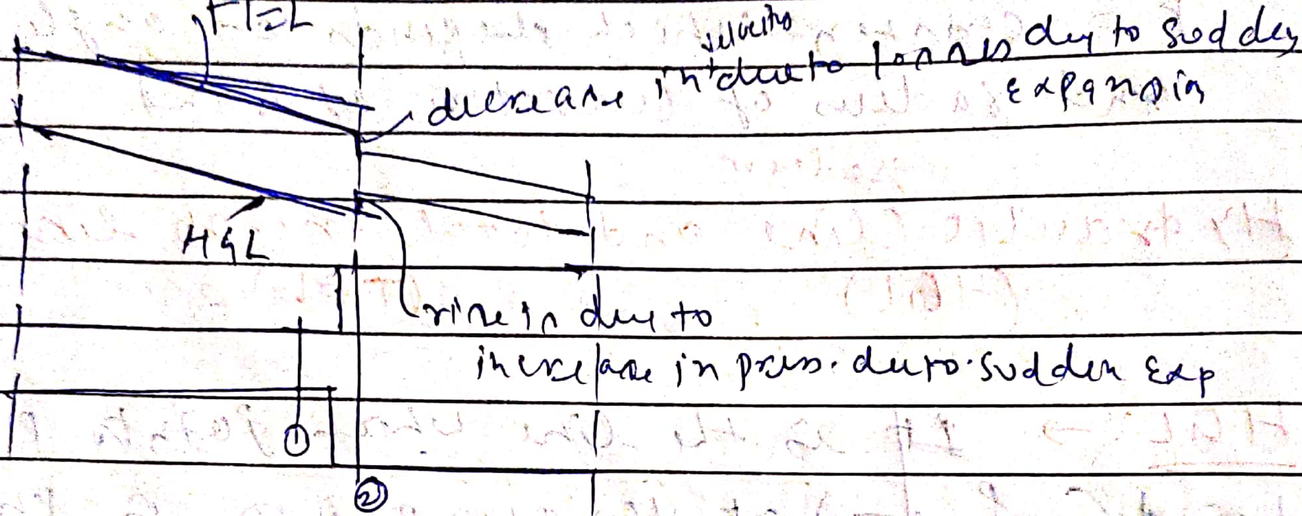
TEL: → The line which joints the total energy $\left(\frac{p}{w} + z + \frac{v^2}{2g}\right)$ at various points is know as Total energy line. The distance b/w TEL & HGL = kinetic energy head (or) velocity head.

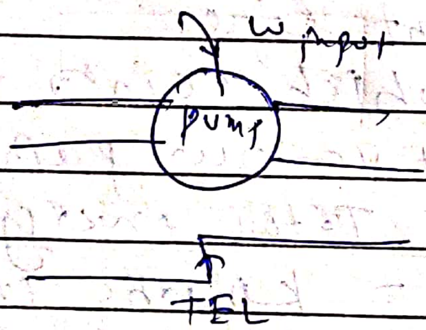


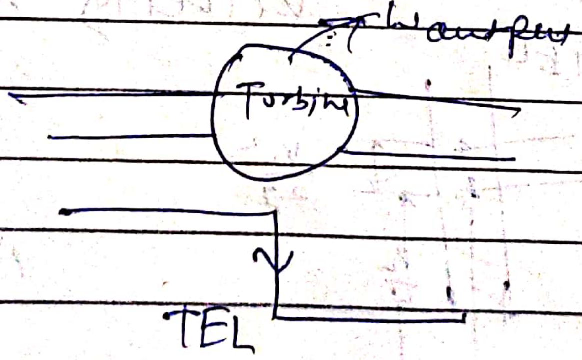
$\cancel{A_1}V_1 = \cancel{A_2}V_2$

→ for sudden contraction

for sudden expansion:-







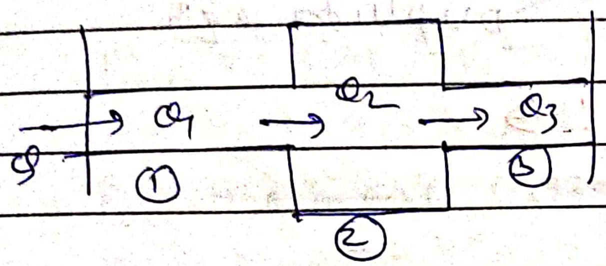
$$\frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2 + \frac{(v_1 - v_2)^2}{2g}$$

$$TE_1 = TE_2 + h_{L\,exp}$$

$$10 = 8 + 2$$

→ The hydraulic gradient line can rise or fall but the total energy line will rise only when there is external energy input as in the case of a pump

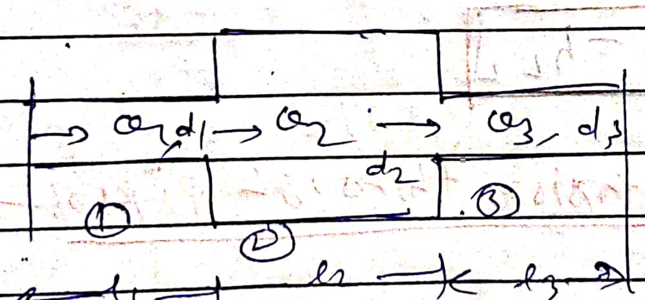
pipes in series! →

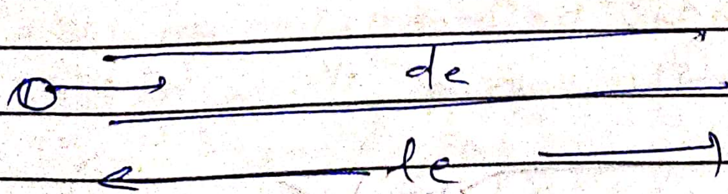


$$Q = Q_1 = Q_2 = Q_3$$

$$h_L = h_1 + h_2 + h_3$$

Equivalent pipe:→ A pipe is said to be equivalent to a compound pipe when the losses in both the pipes are same and discharge in compound pipe is equal to discharge in equivalent pipe





$$h_{Le} = \frac{f L_e Q^2}{12 d_e^5}$$

$$h_L = h_{L1} + h_{L2} + h_{L3}$$

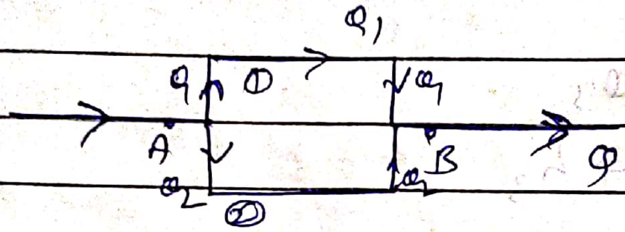
$$12d_1^5 \quad 12d_2^5 \quad 12d_3^5$$

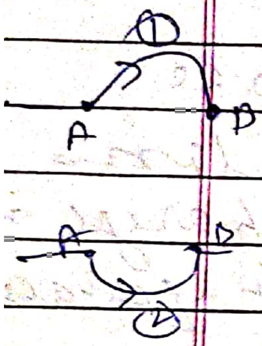
$$\frac{fL_eQ^2}{12d_e^5} = \frac{fL_1Q^2}{12d_1^5} + \frac{fL_2Q^2}{12d_2^5} + \frac{fL_3Q^2}{12d_3^5}$$

$$\boxed{\frac{L_e}{d_e^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}}$$

Dupuit eq^n

Pipes in parallel :→





$$Q = Q_1 + Q_2$$

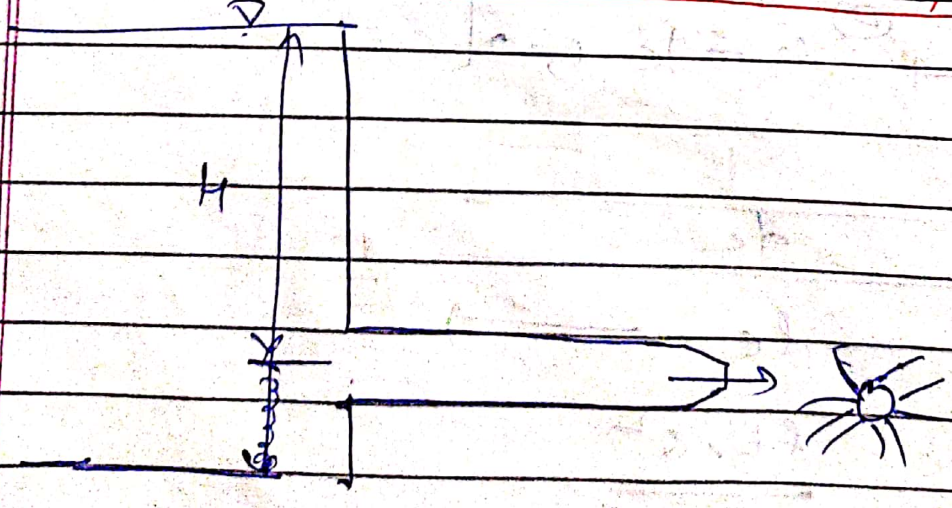
$$\frac{P_A}{w} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{w} + \frac{V_B^2}{2g} + Z_B + h_{L_1}$$

$$\frac{P_A}{w} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{w} + \frac{V_B^2}{2g} + Z_B + h_{L_2}$$

$\frac{P_A}{w}$

$$\boxed{h_{L_1} = h_{L_2}}$$

Power transmission through pipes →



$$P_{th} = wQH$$

$$P_{act} = wQ(H - h_L)$$

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$$\eta = \frac{P_{act}}{P_{th}} = \frac{wQ(H-h_L)}{wQH}$$

$$\boxed{\eta = \frac{H-h_L}{H}}$$

$$P_{act} = wQ(H-h_L)$$

$$P_{act} = wQ\left(H - \frac{fLQ^2}{12d^5}\right)$$

$$P_{act} = w\left(HQ - \frac{fLQ^3}{12d^5}\right)$$

P_{act} is max then $\frac{dP_{act}}{dQ} = 0$

$$\frac{dP_{act}}{dQ} = w\left(H \times 1 - \frac{3fLQ^2}{12d^5}\right) = 0$$

$$H = \frac{3fLQ^2}{12d^5}$$

$$\boxed{H = 3h_L}$$

$$\eta_{max} = \frac{3h_L - h_L}{3h_L}$$

$$\eta_{max} = \frac{2}{3} = 0.6667$$

$$\boxed{\eta_{max} = 66.67\%}$$