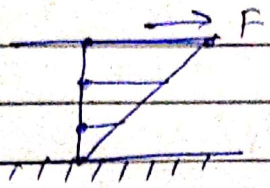
FLUID MECHANICS

FLUID: → A fluid is a substance which is capable of flowing, deforming or moving under the action of shear force. (However small the force may be)



Conversely a fluid is a substance which changes it's relative position under the action of shear force.

Example: → Liquid and gases.

FLUID PROPERTIES: →

(1) **Density (mass density), ρ:** → It is defined as the ratio of mass of the fluid to it's volume. It's unit is kg/m^3 and it's dimensional formula is ML^{-3}. Density of water for all calculation purpose is taken as $\boxed{\rho_{H_2O} = 1000 \frac{kg}{m^3}}$

→ Density depends of temperature and pressure.

$T\uparrow \; \rho\downarrow$

ρ

$P\uparrow \; \rho\uparrow$

(2) **Specific weight (weight density or w):** →

It is defined as the ratio of weight of the fluid to it's volume. It's unit is N/m^3

→ dimensional formula $ML^{-2}T^{-2}$

$$w = \frac{wt \text{ of fluid}}{\text{Volume}} = \frac{N}{m^3} \rightarrow \frac{MLT^{-2}}{L^3} = ML^{-2}T^{-2}$$

$$w = \left(\frac{m}{Vol}\right) g$$

$$\boxed{w = \rho g}$$

$w_{H_2O} = \rho_{H_2O}\, g$

$w_{H_2O} = 1000 \times 9.81 = 9810\ N/m^3 = 9.81\ kN/m^3$

$w \to \rho \to T, P$

$w \to g$ (location)

③ Relative density or specific gravity (S) :→

→ It is the ratio of density of fluid to the density of standard fluid. for liquids the standard fluid is water and for gases standard fluid is either Hydrogen (H_2) or air at a given temp. and pressure.

→ It is dimensionless ($M^0L^0T^0$)

→ It can also be defined as the ratio of specific wt. of the fluid to the sp. wt of standard fluid.

→ sp. gravity of water is one 1.

$$\boxed{\rho_{H_2O} = S \times 1000}$$

④ compressibility (β) :→ It is the reciprocal of bulk modulus K

$$\boxed{\beta = \frac{1}{K}}$$

$$K = \frac{dP}{\frac{-dV}{V}}$$

$$\boxed{\beta = \frac{1}{K}}$$

$$\rho = \frac{m}{V}$$

$$\rho V = m$$

$$\rho V = \text{const}$$

$$\rho\, dv + v\, d\rho = 0$$

$$\rho\, dv = -v\, d\rho$$

$$\frac{-dv}{V} = \frac{d\rho}{\rho}$$

$$k = \frac{dp}{\frac{-dv}{V}} \Rightarrow k = \frac{dp}{\frac{d\rho}{\rho}} \Rightarrow \boxed{k = \rho\left(\frac{dp}{d\rho}\right)}$$

$$k = \frac{\rho\, dp}{d\rho}$$

$$\beta = \frac{1}{k} = \frac{1}{\frac{\rho\, dp}{d\rho}}$$

$$\beta = \frac{d\rho}{\rho\, dp}$$

$\beta = 0 \Rightarrow$ incompressible

$\Rightarrow d\rho = 0 \Rightarrow \boxed{\rho = \text{const}}$

M → A fluid is said to be incompressible if its density remains constant. If the density varies than the fluid is compressible.

* Isothermal compressibility :-

$$pV = mRT$$

$$p = \frac{m}{V} RT$$

$$p = \rho RT$$

$$\frac{dp}{d\rho} = RT$$

$$k = \frac{\rho\, dP}{d\rho}$$

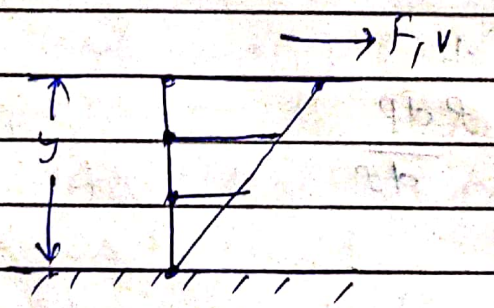
$$k_T = \rho R T$$

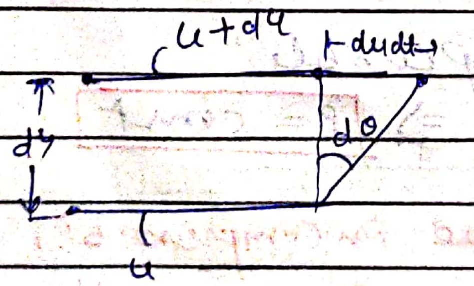
$$\boxed{k_T = P}$$

$$\boxed{\beta_T = \frac{1}{k_T} = \frac{1}{P}}$$

Note:→ Similarly it can be proved that adiabatic bulk modulus is equal to γ times P.

v. imp **Viscosity:→** It is the internal resistance offered by one layer of fluid to the other layer.





$V = \frac{d}{t}$

$d = Vt$

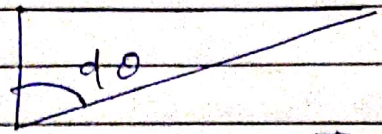
$du \times dt$

$$\tan d\theta = \frac{du\, dt}{dy}$$

$$d\theta = \frac{du\, dt}{dy}$$

$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

$\tau = \frac{F}{A}$

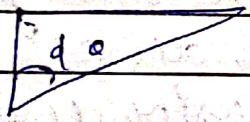
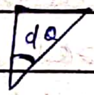


$\frac{d\theta}{dt}$ $\qquad$ $\tau \propto \frac{d\theta}{dt}$

$\tau = \mu \frac{d\theta}{dt}$

$$\tau = \mu \frac{d\theta}{dt}$$

$$\mu = \frac{\tau}{\frac{d\theta}{dt}}$$

A	B
	
$\frac{d\theta}{dt} \rightarrow$ large	$\frac{d\theta}{dt}$ is small
flow is easy	~~flow is not easy~~
μ is small	μ is large

→ μ is known as cofficient of viscocity or appsolute viscocity or dynamic viscocity and it represents resistance offered by one layer of fluid to the other layer.

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

$\frac{d\theta}{dt}$ ↓ Rate of angular deformation or Rate of shear strain

$\frac{du}{dy}$ → velocity gradient

$$\tau = \mu \frac{d\theta}{dt}$$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

* **Newtons law of viscosity :→** A fluid is said to be newtonian fluid if shear stress is directly proportional to rate of shear strain or rate of angular deformation or velocity gradient.

→ According to the newton's law of viscosity

$$\boxed{\tau = \mu \frac{du}{dy}}$$

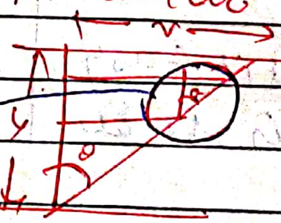
all the fluids which obey newton's law of viscosity are known as newtonian fluids.

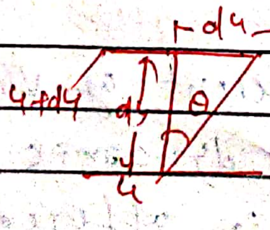
examples :→ Air, water, petrol, carocine, diesel, mercury.

→ for a Newtonian fluid viscosity μ is constant

Note :→ when the velocity profile is linear

$$\boxed{\tau = \frac{\mu v}{y}}$$





$\tan\theta = \frac{v}{y}$ $\quad\quad$ $\tan\theta = \frac{du}{dy}$

$\frac{v}{y} = \frac{du}{dy}$

→ $\tau = \frac{\mu v}{y}$

$\frac{F}{A} = \frac{\mu v}{y} \Rightarrow \boxed{F = \frac{\mu A v}{y}}$

Unit of viscosity (μ):→

$$F = \frac{\mu A V}{y}$$

$$\mu = \frac{Fy}{AV} = \frac{Nm}{m^2 \times m/s} = \frac{N\text{-}s}{m^2} = Pa\text{-}s$$

$N = mg$

$N = Kg \frac{m}{s^2}$

$$\frac{NS}{m^2} \rightarrow \frac{Kg\,\cancel{m}}{s^{\cancel{2}}} \times \frac{\cancel{s}}{m^{\cancel{2}}} = \frac{Kg}{m\text{-}s}$$

$$\boxed{\frac{N\text{-}s}{m^2} = Pa\text{-}s = \frac{Kg}{m\text{-}s}}$$

$$\downarrow$$

$$\frac{M}{LT} = \boxed{ML^{-1}T^{-1}}$$

Kinematic viscosity (ν):→

In fluid mechanics the term $\frac{\mu}{\rho}$ appears frequently and for convenience $\frac{\mu}{\rho}$ is known as kinematic viscosity.

$$\boxed{\nu = \frac{\mu}{\rho}}$$

Unit of kinematic viscosity:→

$$\nu = \frac{\frac{Kg}{m\text{-}s}}{\frac{Kg}{m^3}} = \frac{m^3}{m\text{-}s} = \frac{m^2}{sec}$$

$$\boxed{\nu = \frac{m^2}{sec}}$$

$\mu \rightarrow$ Dynamic viscosity $\rightarrow$ F

$\nu \rightarrow$ Kinematic vis $\rightarrow$ F(

→ Unit of absolute viscosity (μ) kinematic viscosity in (ν) in C.G.S systems

$$\mu = \frac{kg}{m\text{-}s}$$

$$\mu = \frac{gm}{cm\text{-}s} = \text{poise}$$

$$\frac{1\,kg}{m\text{-}s} = \frac{10^3\,gm}{10^2\,cm\text{-}s}$$

$$\frac{1\,kg}{m\text{-}s} = \frac{10\,gm}{cm\text{-}s}$$

$$\frac{1\,kg}{m\text{-}s} = 10\ \text{poise}$$

$$\boxed{1\ \text{poise} = 0.1\,\frac{kg}{m\text{-}s}}$$

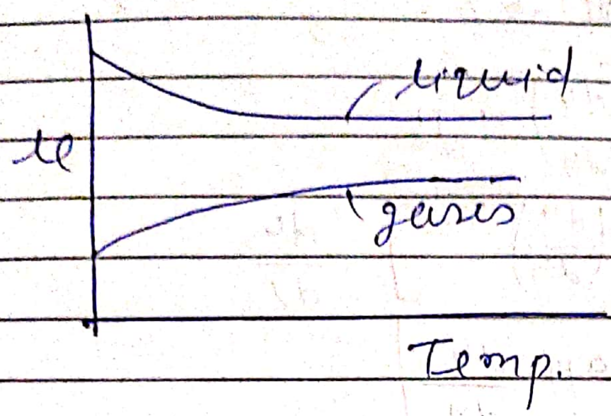
→ The unit of kinematic viscosity in C.G.S is

$$\boxed{\frac{cm^2}{sec} = \text{stoke}}$$

Variation of viscosity with Temperature :→

In case of liquids the intermolecular distance is small and hence cohesive forces are large. with rise in temp. cohesive forces decrease and hence the resistance to the flow is also decreasing therefore for a liquid with rise in temp. viscosity decreases

In case of gases the inter molecular distance is large and hence cohesive forces are negligible. with rise in temp. the molecular agitation (disturbance) increases and hence the resistance to the flow also increases. increases therefore viscosity of a gas increases with increase in temperature.



Non newtonian fluids :→ All the fluids which do not obey newtons' law of viscosity are known as non-newtonian fluids and the study of non-newtonian fluids is known as Rheology.

→ The general relationship b/w shear stress (τ) and velocity gradient $\left(\frac{du}{dy}\right)$ is :—

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

Case I :→ Dilatant fluid ($B = 0$; $n > 1$)

$$\tau = A\left(\frac{du}{dy}\right)^n + 0$$

$$\tau = A\left(\frac{du}{dy}\right)^n$$

$$\tau = \left[A\left(\frac{du}{dy}\right)^{n-1}\right]\left(\frac{du}{dy}\right)$$

$$\tau = \mu_{app.}\frac{du}{dy}$$

Shear thickening fluid → rice starch

for a dilatant fluid the apparent viscosity increases with rate of deformation.

and hence dilatant fluid is known as shear thickening fluid

case - II Pseudoplastic fluid :→ [B = 0 ; n < 1]

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

$$\tau = \left[A\left(\frac{du}{dy}\right)^{n-1}\right]\frac{du}{dy}$$

$$\boxed{\tau = \mu_{app.}\frac{du}{dy}}$$

A fluid is said to be pseudo plastic fluid if its apparent viscosity decreases with rate of deformation. and hence these fluids are known as shear thinning fluids

Examples :→ Blood; milk, (colloidal solⁿ).

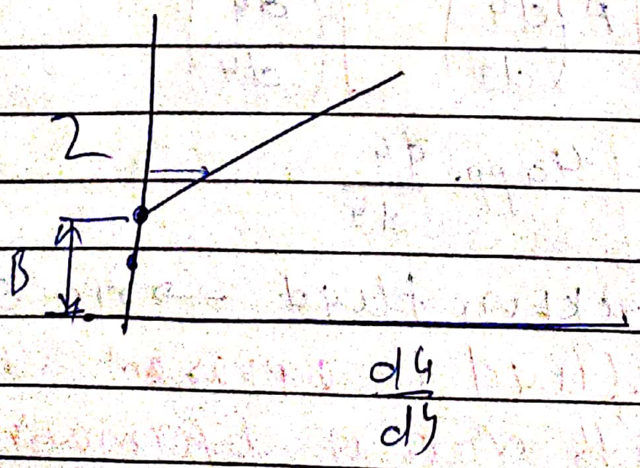
case - III :→ Bingham plastic fluid :→ [B ≠ 0 ; n = 1]

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

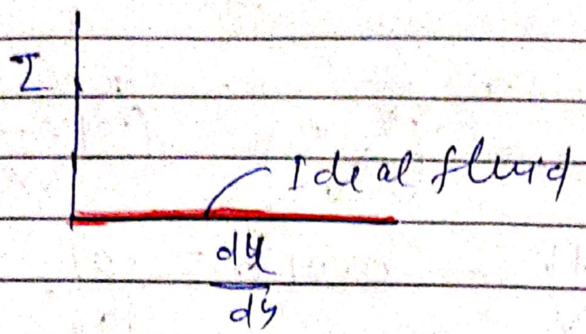
$$\tau = A\left(\frac{du}{dy}\right) + B$$

$$y = mx + c$$

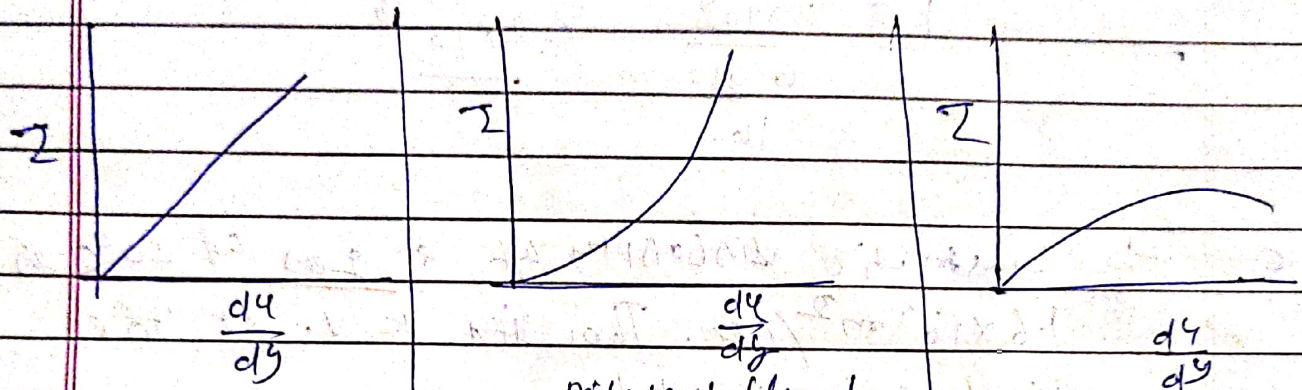
Ex :→ Tooth paste



Ideal fluid :→ A fluid is said to an Ideal fluid if it is noviscus and incompressible fluid.



$\tau = \mu \frac{du}{dy}$



Newtonia fluid
$\tau = \mu \frac{du}{dy}$
const
$y = mx$

Dilatent fluid
$\tau = \mu_{app} \frac{du}{dy}$

Pseudoplastic
$\tau = \mu_{app} \frac{du}{dy}$

Q Match the following

	List - I		List II
(A)	Sp. wt	(1)	L/T^2
(B)	density	(2)	F/L^3
(C)	Shear stress	(3)	F/L^2
(D)	Viscosity.	(4)	FT/L^2
		(5)	FT^2/L^4

where $F \to$ Force

(A) $w \to \frac{N}{m^3} - \frac{F}{L^3}$

(B) $w = \rho g$

$\rho = \frac{w}{g} = \frac{\frac{F}{L^3}}{\frac{m}{T^2}} = \frac{FT^2}{L^4}$

(C) $\tau = \frac{P}{A} = \frac{F}{L^2}$

(D) $\mu \to \frac{NS}{m^2} \quad \frac{Ft}{L^2}$

Q An increase in pressure of 2 bar decreases the Volume of a liquid by 0.01% than bulk modulus is

A $K = \frac{dP}{-\frac{dV}{V}} = \frac{2}{-\left(\frac{V_2-V_1}{V}\right)} = \frac{2}{\left(\cancel{.01}\,\cancel{\frac{V-V_1}{V_1}}\right)}$

$-\frac{dV}{V} = 0.01\% = \frac{.01}{100}$

$K = \frac{2\times10^5}{\frac{0.01}{100}} = 2\times10^9$

Q the kinematic viscosity of a gas at $20°C$ is $1.6\times10^{-5}\ m^2/sec$. Than K.V. at $70°C$ can be approximately.

(a) $1\times10^{-5}\ m^3/\mathrm{s}$

(b) $7.6\times10^{-5}\ m^2/s$

(c) 1.2×10^{-P}

(d) 2.2×10^{-5}

[∵ temp increase viscosity increase]

Q The shear stress in a fluid may be express as $\tau = \mu\left(\frac{du}{dy}\right)^n$, than the values of the n for newtonian and non-newtonian will be respectively.

(a) $n=1$ & $n>1$

(b) $n<1$ & $n<1$

(c) $n=1$ & $n<1$

(d) $n=1$ & $n\neq1$

Q. the shear stress developed in a lubricating oil of viscosity 0.98 poise filled b/w two parallel plates 1 cm apart and moving with a velocity of 2 m/s is

Ans

$\mu = 0.98$ poise $= 0.0981$ kg/m-s

$y = 1$ cm

$v = 2$ m/s

[assume linear velocity]

$$\tau = \frac{\mu V}{y} = \frac{0.0981 \times 2}{10^{-2}}$$

$$\tau = 19.62 \text{ N/m}^2.$$

Q. The velocity distribution over a flat plate is given by $u = \frac{3}{4}y - y^2$, the shear stress at a location 0.3 m above the plate is K times the shear at a location 0.2 m above the plate than the value of K is

(a) 7/3

(b) 3/7

(c) 3/2

(d) 9/4.

Ans

$$\frac{du}{dy} = \frac{3}{4}(1) - 2y$$

$$\frac{du}{dy} = \frac{3}{4} - 2y$$

$$\tau_{0.3} = K\tau_{0.2}$$

$$\cancel{\mu}\frac{du}{dy}\Big|_{0.3} = K\cancel{\mu}\frac{du}{dy}\Big|_{0.2}$$

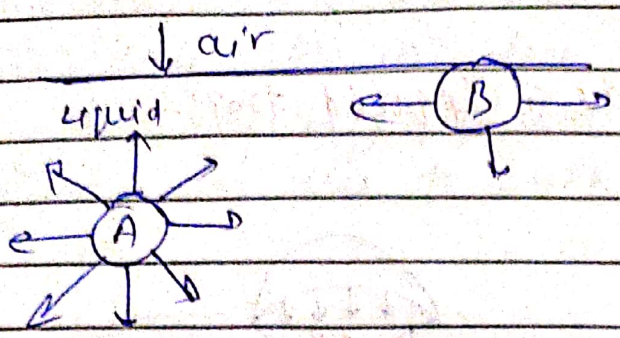
$$\frac{du}{dy}\Big|_{0.3} = K\frac{du}{dy}\Big|_{0.2}$$

$$\frac{du}{dy}\Big|_{0.3} = \frac{3}{4} - 2(0.3) = 0.15$$

$$\frac{du}{dy}\Big|_{0.2} = \frac{3}{4} - 2(0.2) = 0.35$$

$$K = \frac{0.15}{0.35} = 3/7$$

Surface Tension! →



Consider the molecule A which is below the free surface of the liquid this molecule surrounded by various corresponding molecule and because of various cohesive forces it will be under equilibrium.

Now consider the molecule B which is at the free surface of the liquid. This molecules is under the influence of net downward force and because of this they seems to be a layer form because of this at the surface which can resist small tensile loads. This phenomenon is known as surface tension.

→ surface tension is a line force and it is expressed as force per unit length drawn on the surface and acts normal to the line in the plane of surface

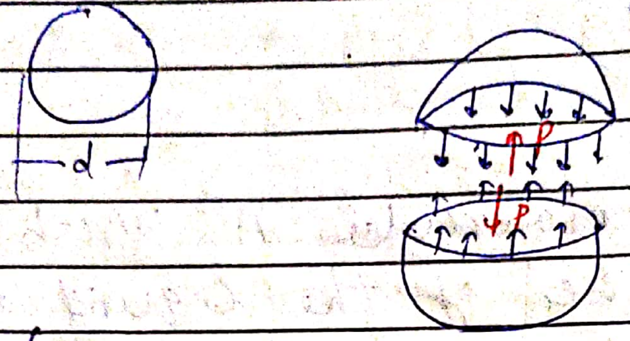
Its unit is $\frac{N}{m}$ and its dimensional formula is $\frac{MLT^{-2}}{L} = MT^{-2}$

→ with rise in temp. cohesive forces decrease and hence surface tension also decreases

$$\boxed{\sigma = \frac{F}{l}}$$

→ Surface tension for water – air interface at 20°c is 0.0736 N/m

pressure inside a liquid drop! →



for equilibrium, surface tension force F_s = pressure force

$p = \frac{F_p}{A}$

$F_p = P \times A \Rightarrow F_p = P \times \frac{\pi}{4} d^2$

$\sigma = \frac{F}{l} \Rightarrow F = \sigma l = \sigma \pi d$

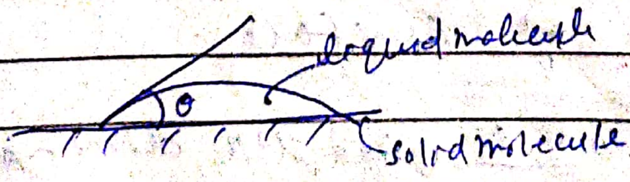
$P \times \frac{\pi}{4} d^2 = \sigma \pi d$

$\boxed{P = \frac{4\sigma}{d}}$

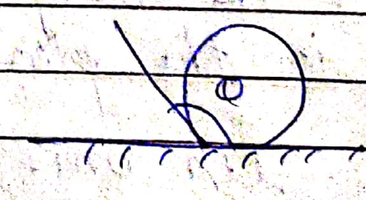
Note → In case of Bubble there are two surfaces and here, $\boxed{P = \frac{8\sigma}{d}}$

24/11/09

capilarity →

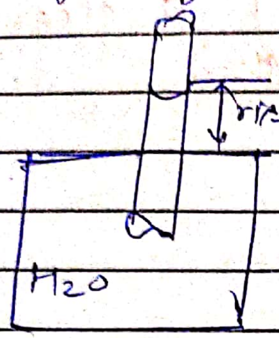


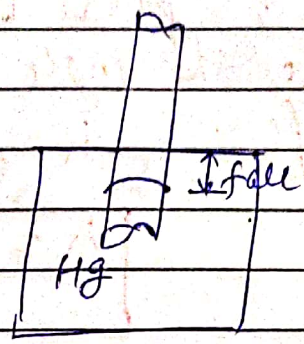
Adhesion is more
$\theta < 90$
wetting liquid



Cohesion is more
$\theta > 90$
Non-wetting liquid

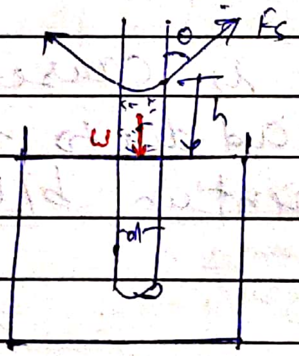
The rise or fall of a liquid when a glass tube is immersed in it is known as capillarity.
→ the capillary rise is due to adhesion (water) and the capillary fall due to cohesion (Hg).
→ capillarity is due to both adhesion and cohesion.





Adhesion is more   Cohesion is more

Expression for capillary rise or fall! →



Under equilibrium conditions

Vertical comp of surface tension force = wt of liquid

w = wt of liquid / Vol

wt of fluid = $w \times$ vol

wt of fluid = $w \times \frac{\pi}{4} d^2 h$

$\sigma = \frac{F_s}{l}$

$F_s = \sigma l$

$F_s = \sigma \pi d$

Vertical comp = $F_s \cos\theta = \sigma \pi d \cos\theta$

$\sigma \pi d \cos\theta = w \times \frac{\pi}{4} d^2 h$

$$\boxed{h = \frac{4\sigma \cos\theta}{w d}}$$

Note :→ In case of ordinary water w.r.to glass tube the angle of contact θ is equal to 21°

→ In case of mercury the angle of contact θ is 130°

→ In case of pure water w.r.t a clean glass tube the angle of contact θ=0.

therefore $h = \frac{4\sigma \cos\theta}{wd}$

$$\boxed{h = \frac{4\sigma}{wd}}$$

Q Assertion :- ~~[illegible]~~ viscosity in liquid increase and in gases it decrease with rise in temp.

Reason :- viscosity is caused by intermolecular forces of cohesion and due to transfer of momolecular momentum b/w fluid layer

A= Assertion ✓

Reason ✗

Q math the following

Ans 1. List I — List II

(A) Ideal fluid → (1) viscosity does not change with rate of deformation

(B) Newtonian fluid (2) fluid of zero viscosity

(C) μ / ρ (3) Dynamic viscosity

(d) mercury in glass → (4) capillary depression

(5) capillary rise

(6) kinematic viscosity

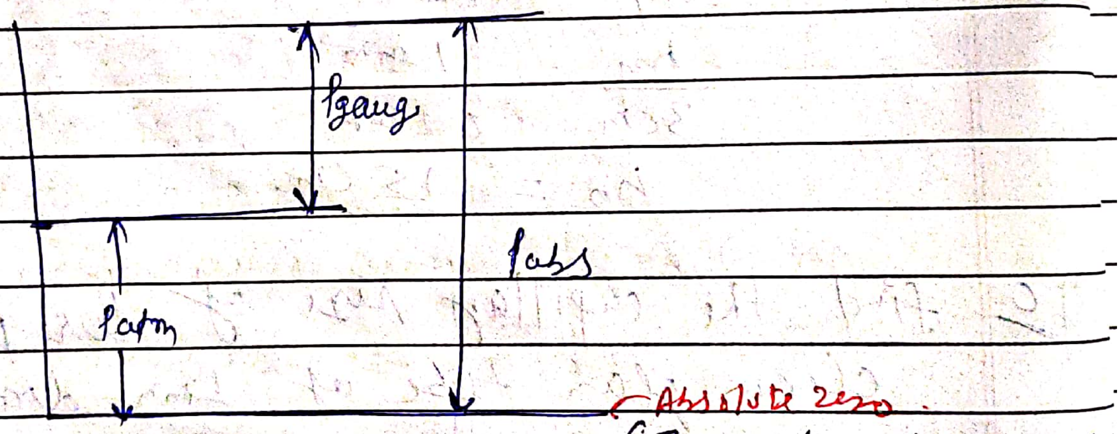
Fluid pressure measurement (Manometry):→

pressure → It is defined as normal force per unit area

Pascal's law:→ In a static fluid the pressure at a point is equal in all the directions or if pressure is applied at a point in a static fluid it is transmitted equally in all directions.

Gauge pressure (P_g):→ The pressure measured relative to atmospheric pressure is known as gauge pressure

Absolute pressure:→ The pressure measured w.r.t absolute zero is known as Absolute pressure.

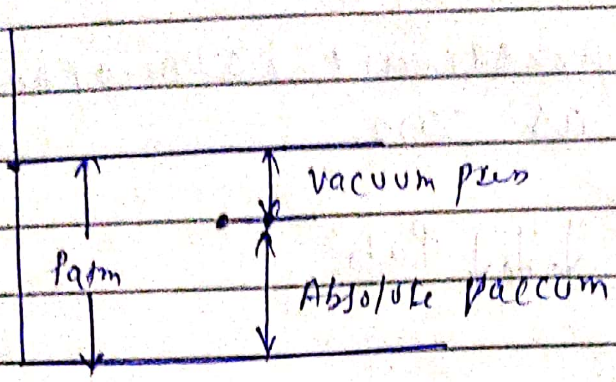


$$\boxed{P_{abs} = P_g + P_{atm}}$$

Note:→ In finding absolute pressure local atm. pressure is taken into consideration

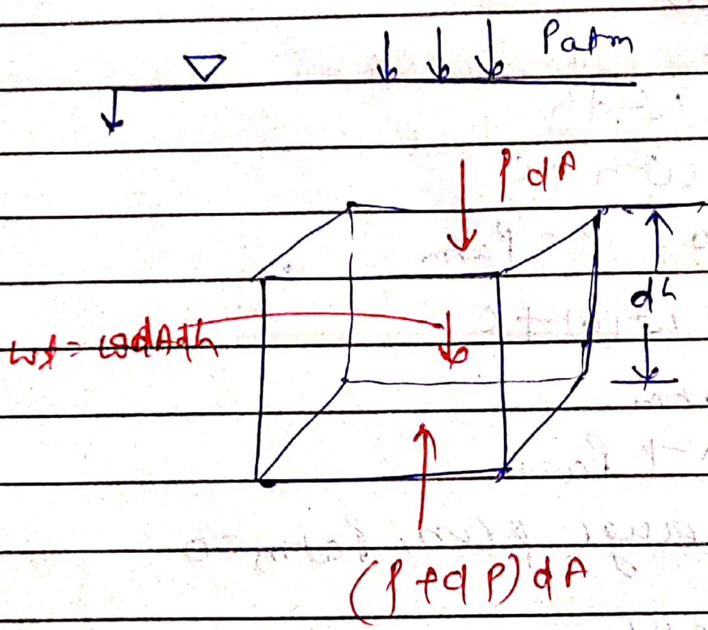
Vaccum pressure:→

the pressure less than atm. pressure is known as vaccum pressure.



$$\boxed{\text{Absolute vacuum} = P_{atm} - \text{vacuum pres.}}$$

Hydrostatic law:→



$w = \frac{wt}{vol}$

$wt = w \times vol$

$wt = w \times dA\,dh$

$p\,dA + w\,dA\,dh = (p+dp)\,dA$

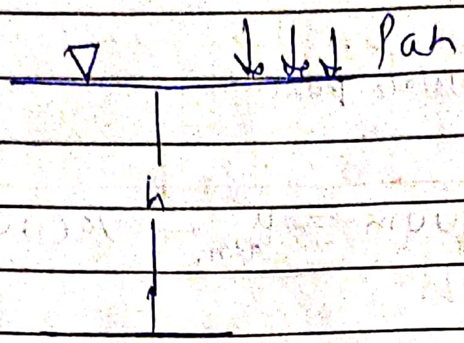
$p + w\,dh = p + dp$

$$\boxed{\frac{dp}{dh} = w}$$

The variation of pressure in vertical direction is proportional to specific weight.

Note:→ if h is taken in vertically upward direction then $\boxed{\frac{dp}{dh} = -w}$

→ for gauge pressure atmospheric pressure is treated as zero.



$$\frac{dp}{dh} = w$$

$$dp = w\,dh$$

$$p = wh + C$$

At $h = 0$; $p = P_{atm}$

$$P_{atm} = w(0) + C$$

$$C = P_{atm}$$

$$p = wh + P_{atm}$$

If p is gauge pres, $P_{atm} = 0$

$$p = wh$$

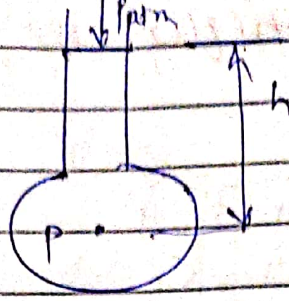
$$\boxed{p = \rho g h}$$

Note:→ We know that $p = \rho g h$, ρ & g are constants and pressure depends on h. Because of this reason pressure is sometimes expressed in height (column), therefore

$h = \frac{p}{\rho g}$ i.e. $\boxed{h = \frac{p}{w}}$ and

this is known as pressure head.

Piezometer! → It is the simplest device use for measuring pressure



$0 + \rho g h = P$

$\boxed{P = \rho g h}$

→ piezometer are used for finding out moderate liquid pressures

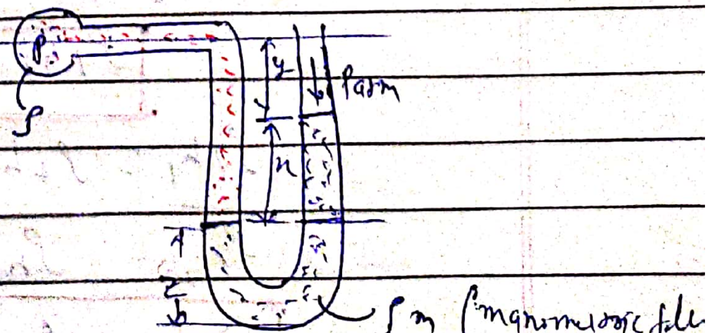
→ To nullify the effect of capillarity the dia. of the tube is generally taken as greater than 1 cm.

Manometer! →

Manometers
- Simple ↓ measure press at a point.
- Differential ↓ measure press diff b/w two point.

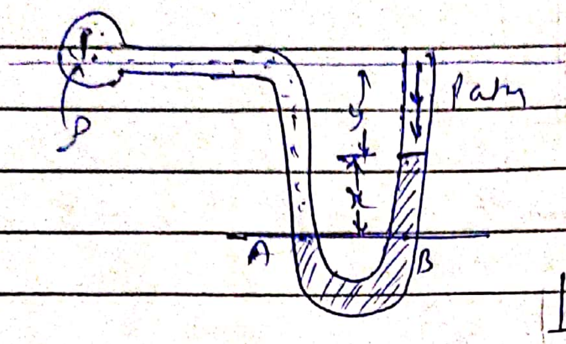
U-tube manometer! →

1) Jumping of fluid Technique :-



$$P + \rho g(x+y) - \rho_m g x \Rightarrow$$
$$P = \rho_m g x - \rho g(x+y)$$

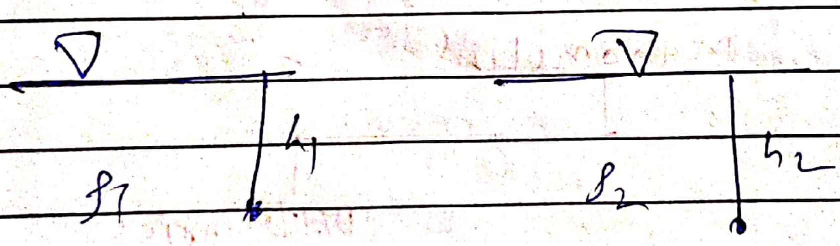
(2) Datum line technique :-



$$\boxed{P_A = P_B}$$

$$P + \rho g(x+y) = P_A$$
$$0 + \rho_m g x = P_B$$
$$P + \rho g(x+y) = \rho_m g x$$
$$\boxed{P = \rho_m g x - \rho g(x+y)}$$

conversion of one fluid column into the other column :-



$$P = \rho_1 g h_1 \qquad P = \rho_2 g h_2$$

$$\rho_1 g h_1 = \rho_2 g h_2$$
$$\boxed{\rho_1 h_1 = \rho_2 h_2}$$

$$\frac{\rho_1 h_1}{\rho_{H_2O}} = \frac{\rho_2 h_2}{\rho_{H_2O}}$$

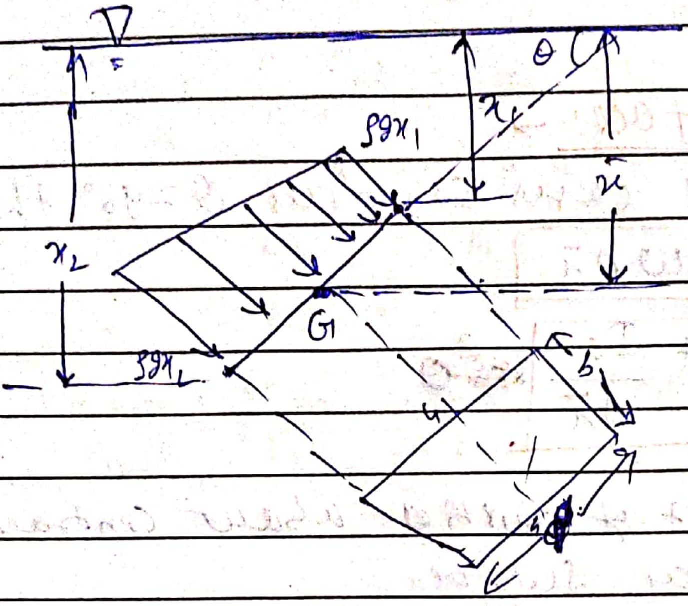
$$\boxed{S_1 h_1 = S_2 h_2}$$

$$h_m S_m = h_w S_w$$

* Hydrostatic forces:→

Hydrostatic forces on plane surfaces:→

ex(1) inclined surfaces:→



G = Centre of gravity

$\bar{x}$ is always measured from free surface

$$\boxed{F = w A \bar{x}}$$

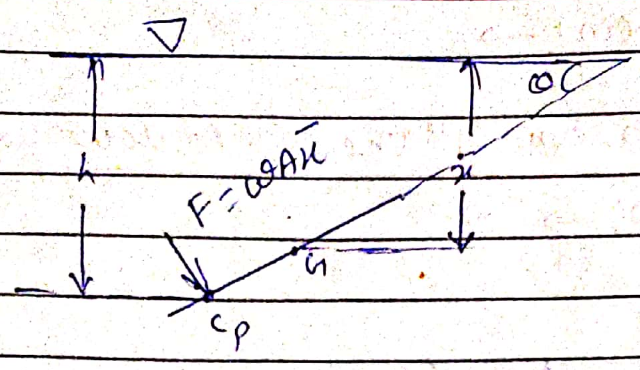
where F → Hydrostatic force

Centre of pressure C_p :→ It is the point from which the Hydrostatic force is acting

from the principle of moments we have centre of pressure,

$$\boxed{h = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}}$$

where I_G = moment of inertia of the surface about its centroidal axis passing through G-G



case-2 :- Vertical surface :→

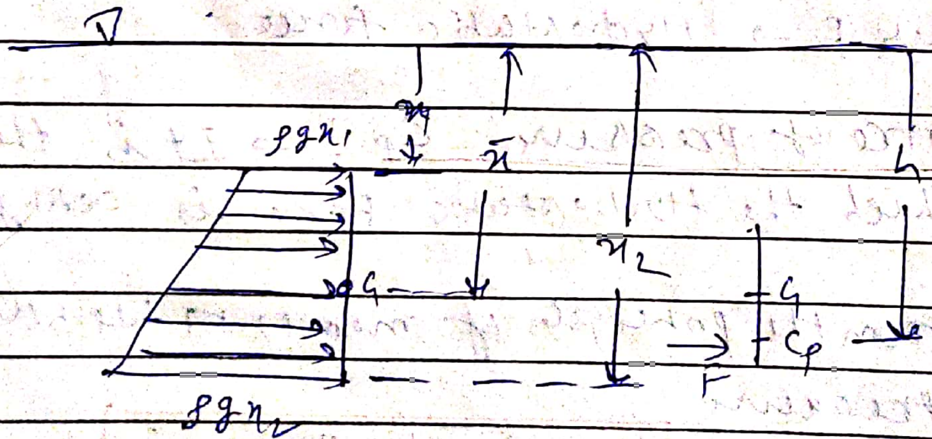
for vertical cases substitute $\theta = 90^\circ$, therefore

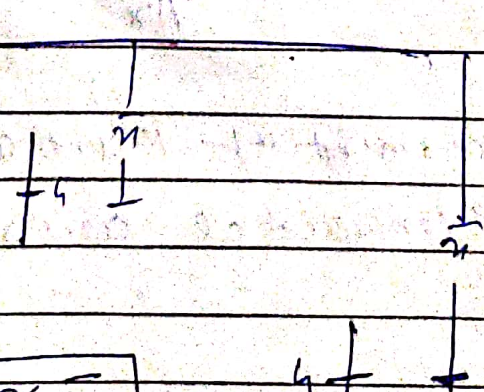
$$\boxed{F = wA\bar{x}}$$

$$\boxed{h = \bar{x} + \frac{I_G}{A\bar{x}}}$$

→ I_G is moment of inertia about centroidal axis parallel to free surface

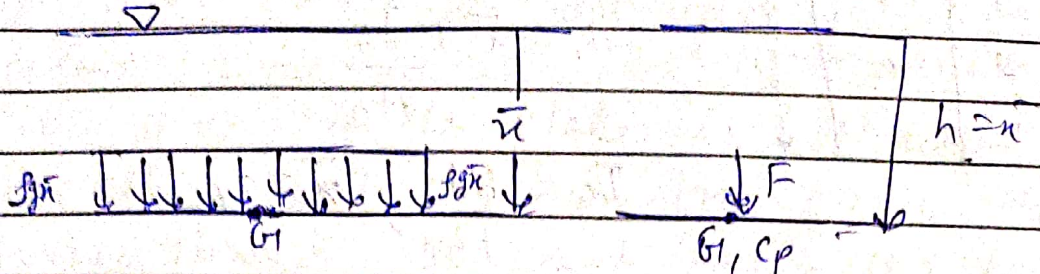
→ the centre of pressure is almost equal to centre of gravity when the depth of immersion ($\bar{x}$) is very large



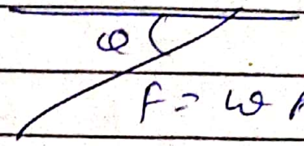


$$\boxed{h \approx \bar{x}}$$

Case 3:- Horizontal surface:→



$p = F/A$
$F = pA$
$F = \rho g \bar{x} \times A$
$F = w A \bar{x}$
$h = \bar{x}$



$F = w A \bar{x}$
$h = \bar{x} + \frac{I_G}{A\bar{x}} \sin^2\theta$

in this case $\theta = 0$, therefore $\boxed{F = w A \bar{x}}$
and $\boxed{h = \bar{x}}$

case	force	C.P.
Horizontal	$wA\bar{x}$	$\bar{x}$
vertical	$wA\bar{x}$	$\bar{x} + \frac{I_G}{A\bar{x}}$
Inclined	$wA\bar{x}$	$\bar{x} + \frac{I_G}{A\bar{x}} \sin^2\theta$

→ All the distances are measured from the free surface

Q. Match the following

List I (Surfaces) — List II (C.P)

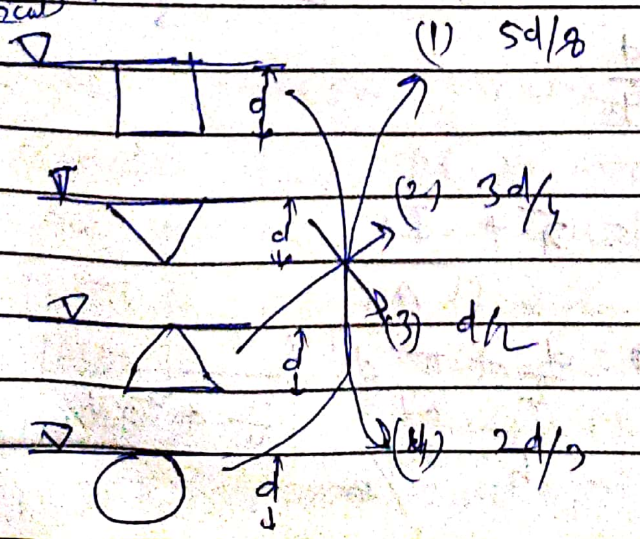
(all are vertical)

(A)
(1) $5d/8$

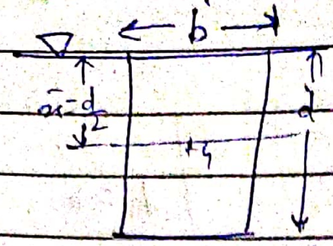
(B)
(2) $3d/4$

(C)
(3) $d/2$

(D)
(4) $2d/3$



Que (1)



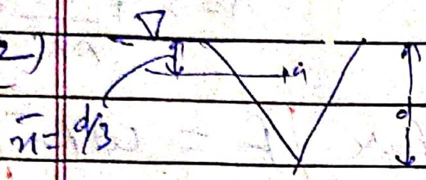
$$h=\bar{x}+\frac{I_G}{A\bar{x}}$$

$$I_G=\frac{bd^3}{12},\quad \bar{x}=d/2$$

$$h=\frac{d}{2}+\frac{bd^3}{12}\times\frac{1}{bd\times\frac{d}{2}}$$

$$h=\frac{d}{2}+\frac{d}{6}=2d/3$$

(2)



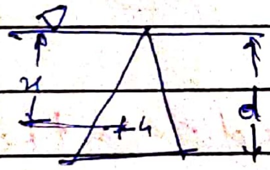
$$I_G=\frac{bd^3}{36},\quad \bar{x}=\frac{d}{3}$$

$$h=\bar{x}+\frac{I_G}{A\bar{x}}$$

$$h=\frac{d}{3}+\frac{bd^3}{36}\times\frac{1}{\frac{1}{2}bd\times d/3}$$

$$h=\frac{d}{3}+\frac{d}{6}=d/2.$$

(3)



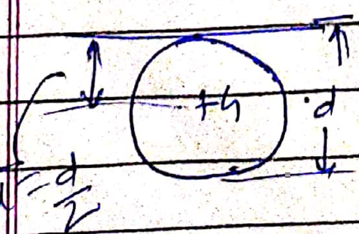
$$x=\frac{2d}{3}$$

$$h=\bar{x}+\frac{I_G}{A\bar{x}}$$

$$h=\frac{2d}{3}+\frac{bd^3}{36}\times\frac{1}{\frac{1}{2}bd\times\frac{2d}{3}}$$

$$h=\frac{3d}{4}.$$

(4)

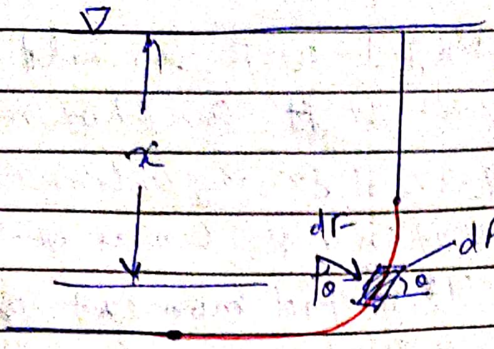


$$h=\bar{x}+\frac{I_G}{A\bar{x}}$$

$$h=\frac{d}{2}+\frac{\pi}{64}d^4\times\frac{1}{\frac{\pi}{4}d^2\times d/2}$$

$$h=5d/8.$$

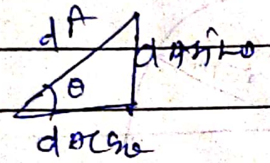
* Hydrostatic forces on curved surfaces :→



$P = \frac{F}{A}$

$dF = P\,dA$ $\qquad P = \rho g x$

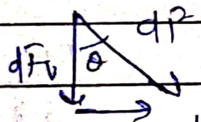
$dF = \rho g x\,dA$

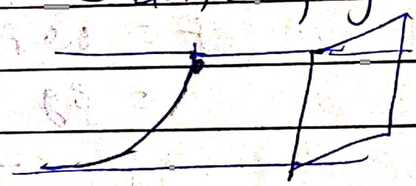


$dF_H = dF \sin\theta$

$dF_H = \rho g x \underbrace{dA \sin\theta}$

↳ vertical projection area

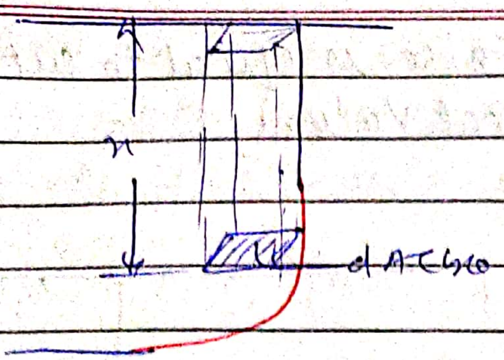




$dF_H = dF \sin\theta$

$dF_V = dF \cos\theta$

→ The Horizontal ~~force~~ component of force on curved surface is equal to the Hydrostatic force on vertical projection area and it acts at the centre of pressure



$dF_V = dF\cos\theta$

$dF_V = \rho g \underbrace{x\, dA\cos\theta}_{vol.}$

$dF_V = \rho g\, vol.$

dF_V = wt of the fluid

$w = \dfrac{\text{wt of fluid}}{vol.}$

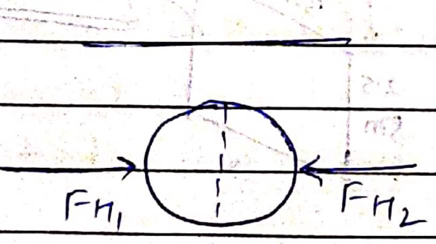
wt of fluid = w × vol

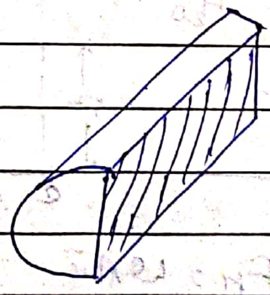
wt of fluid = ρg vol.

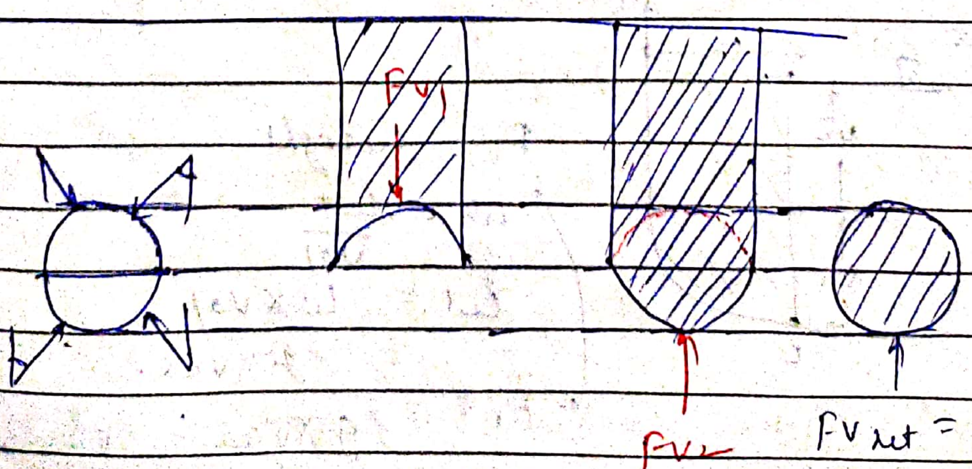
→ Vertical component of the force on the curved surface is equal to wt of the liquid contained by the curved surface upto free surface and this force will act at the centre of gravity of corresponding weight.

Special cases :→

Case (1)



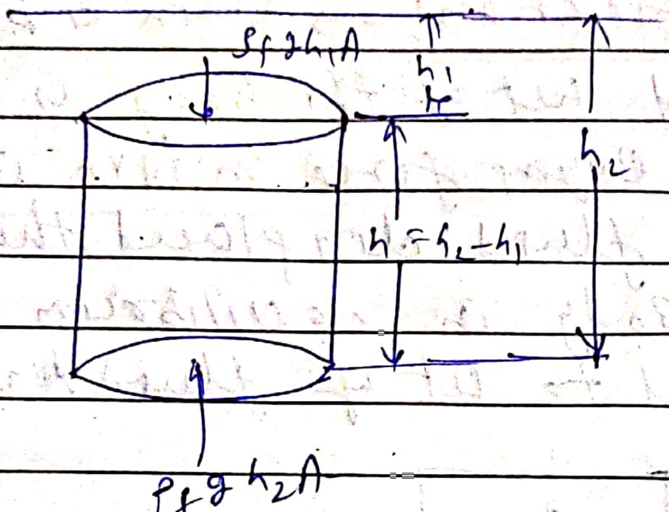




$F_{V\,net} = F_{V_2} - F_{V_1}$

Buyoncy and Floatation

When a Body is immirsed either partially or completly the net vertical force enterted by the fluid on the body is known as Buyonce force and Buyont force is equal to wt of the fluid displaced this is known as archimedies principle. Buyoncy force is basically due to pressure difference

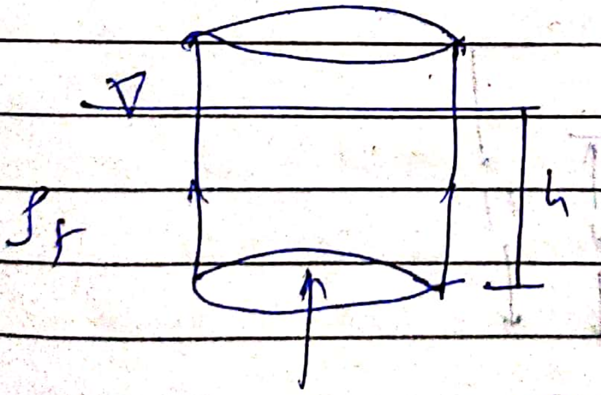


$$F_{net} = \rho_f g h_2 A - \rho_f g h_1 A$$

$$F_{V\,net} = \rho_f g (h_2 - h_1) A \Rightarrow \rho_f g h A \Rightarrow$$

Volfd

$$F_{V\,net} = \rho_f g V_{fd}$$

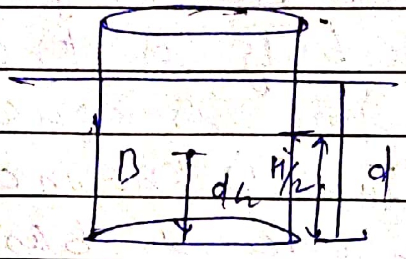


$F_{V\,net} = \rho_f g h A$ ~~vol of submerged or~~ ~~vol of fluid displaced~~ (V_f

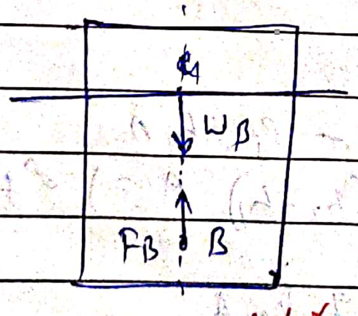
$F_{Vnet} = \rho_f g \cdot V_{fd}$

F_{Vnet} = wt of fluid displaced.

Centre of Buoyancy :→ (B) It is the point from which the Buoyancy force is acting. this force will act at the centroid of displaced volume



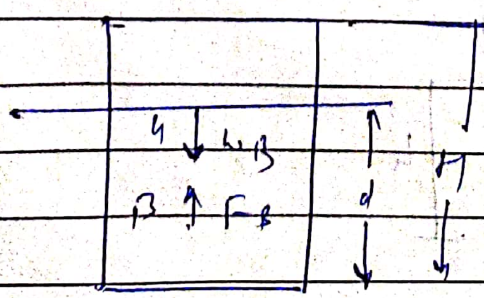
principle of Floatation :→ When a body is floating in a fluid wt of the body is equal to Buoyant force. But Buoyant force in turn is equal to wt of the fluid displaced, therefore for a floating Body in equilibrium wt of Body is equal to wt of fluid displaced.



$W_B = F_B$

$F_B = W_{fd}$

*** $\boxed{W_B = W_{fd}}$ ***



W_B

$$\rho_B = \frac{m_B}{V_B}$$

$$m_B = \rho_B V_B$$

$$W_B = m_B g$$

$$W_B = \rho_B V_B g$$

$$W_B = \rho_B g V_B$$

$$W_B = \rho_B g A H \longleftarrow (1)$$

$$W_{fd} = \rho_f g V_{fd}$$

$$W_{fd} = \rho_f g A d$$

$$W_{fd} = \rho_f g A d \longrightarrow (2)$$

$$W_B = W_{fd}$$

$$\rho_B \cancel{g} \cancel{A} H = \rho_f \cancel{g} \cancel{A} d$$

$$\boxed{\rho_B H = \rho_f d}$$

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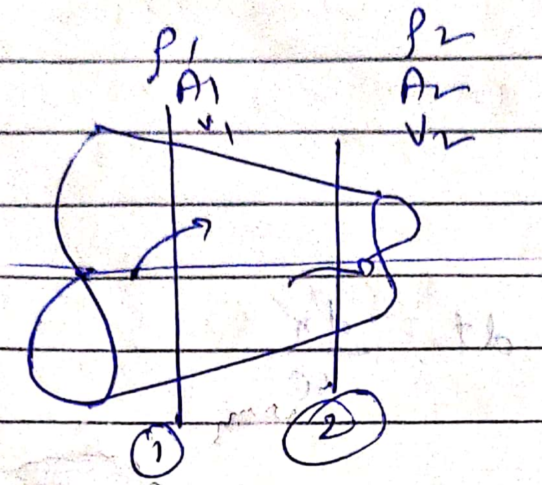
* continuity equation! → (conservation of mass

I Steady 1 - dimensional flow! →



$$\rho = \frac{m}{Vol.}$$

$$m = \rho \times Vol$$

$m = \rho \times A \times L$

$\dot{m} = \frac{m}{t} = \frac{\rho A L}{t}$

$\dot{m} = \rho A V$

$\dot{m}_1 = \dot{m}_2$

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$\boxed{\rho A V = \text{const}}$

for incompressible cases

$\rho_1 = \rho_2$

$\cancel{\rho_1} A_1 V_1 = \cancel{\rho_2} A_2 V_2$

$\boxed{A_1 V_1 = A_2 V_2} \rightarrow$ steady, 1-d & incompressible

Note:- any type of fluid flow must satisfied the continuity equation and if the continuity eqn is vailated than that flow is not possible.

Discharge (Q):→ volume flow rate is known as discharge

$Q = \frac{\text{vol}}{\text{time}} = \frac{A \times L}{\text{time}}$

$\boxed{Q = AV}$ $\quad \frac{m^3}{sec} \quad \frac{L}{sec} \quad L/\text{min} \quad \left[\frac{L^3}{T} = L^3 T^{-1}\right]$

$A_1 V_1 = A_2 V_2 \rightarrow$ steady, 1-d & incomp.

$\boxed{Q_1 = Q_2 = \text{const}}$

* Discharge remains const. for steady one-dimensional incompressible flow

II) Generalised continuity eqn:→

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0.$$

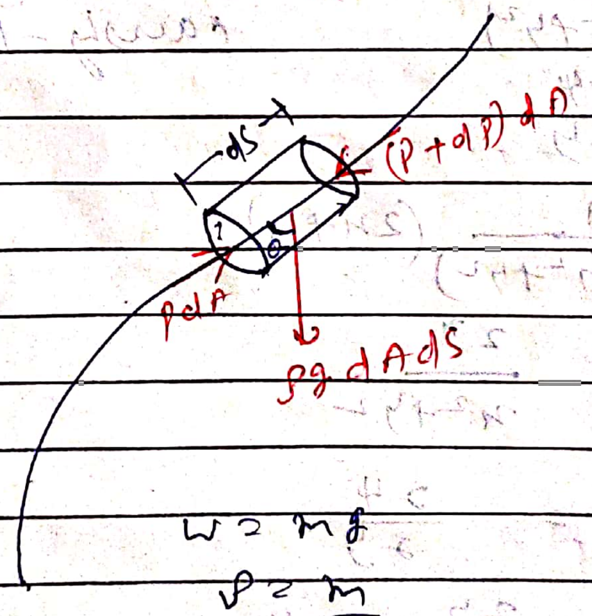
FLUID DYNAMICS

generally the forces acting on the fluid particle are pressure force (F_p), gravity force (F_g) and viscous force (F_v). In navier stokes analysis all these three forces are taken into account. In euler's analysis only pressure and gravity forces are taken into account i.e. viscous forces are neglected

Bernoulli's equation: → (Law of conservation of energy)

Assumptions: →

1. non viscous flow
2. steady flow
3. incompressible flow



$w = mg$

$\rho = \frac{m}{V}$

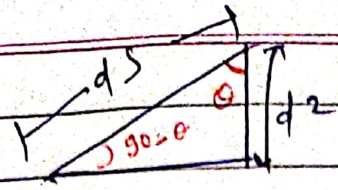
$m = \rho \times vol.$

$m = \rho \times dA\,ds$

$wt = mg$

$wt = \rho dA\,ds\,g$

$wt = \rho g\,dA\,ds$



$$\cos\theta = \frac{dz}{ds}$$

$$dz = ds\cos\theta$$

$$p\,dA - (p+dp)\,dA - \rho g\,dA\,ds\cos\theta = ma$$ (steady flow)

$$p\,dA - (p+dp)\,dA - \rho g\,dA\,ds\cos\theta = \rho\,dA\,ds\left[v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right]$$

$$p - (p+dp) - \rho g\,ds\cos\theta = \rho\,ds\left[v\frac{\partial v}{\partial s}\right] = \rho\,ds\left[v\frac{dv}{ds}\right]$$

$$-dp - \rho g\,dz = \rho\,\cancel{ds}\,v\frac{dv}{\cancel{ds}}$$

$$-dp - \rho g\,dz = \rho v\,dv$$

$$dp + \rho g\,dz + \rho v\,dv = 0$$

$$\boxed{\frac{dp}{\rho} + g\,dz + v\,dv = 0}$$ ← Euler's eqn of motion

integrating

$$\boxed{\int\frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{const.}}$$ ~~Euler's eqn~~

$$\boxed{\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{const}}$$

This equation is known as Bernoulli's equation.

→ In the above equation each term represents energy of the fluid per unit mass

Bernoulli's Theorem: → In a steady incompressible & non-viscous flow along a stream line the sum of pressure energy, potential energy & kinetic energy is constant and hence Bernoulli's theorem is also known as law of conservation of energy.

$$\frac{E}{mass} = \frac{P}{\rho} + gz + \frac{V^2}{2} = const$$

$$\frac{E}{mass} \quad \frac{P}{\rho g} + z + \frac{V^2}{2g} = const$$

$$\boxed{\frac{P}{w} + \frac{V^2}{2g} + z = const}$$

in the above eqn. each term represents energy of the fluid per unit weight

$$\frac{P}{w} + \frac{V^2}{2g} + z = const$$

$\frac{P}{w}$ → press. head

$\frac{V^2}{2g}$ → K.E. head or Velocity head

z → Datum head or P.E. head

$$\text{piezometric head} = \frac{P}{w} + z$$

* Relationship b/w first Law of thermodynamics and Bernoulli's ~~Equation~~ Theorem :-

$$h_1 + \frac{V_1^2}{2} + z_1 g + q = h_2 + \frac{V_2^2}{2} + z_2 g + w$$

$$u_1 + p_1 v_1 + \frac{V_1^2}{2} + z_1 g + q = u_2 + p_2 v_2 + \frac{V_2^2}{2} + z_2 g + w$$

$$u_1 + \frac{p_1}{\rho_1} + \frac{V_1^2}{2} + z_1 g + q = u_2 + \frac{p_2}{\rho_2} + \frac{V_2^2}{2} + z_2 g + w$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + z_1 g = \frac{p_2}{\rho} + \frac{V_2^2}{2} + z_2 g$$

1. $u_1 = u_2$
2. $q = 0$
3. $w = 0$
4. $\rho_1 = \rho_2$ (incompressible)

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