Syllabus

Mechanics of solids[3ME4-07]

UNIT-2

1	Theory of simple bending, bending moment and shear force diagrams for different types of static loading
2	Numericals based on Cantilever beam
3	Numericals based on Simply supported beam
4	Numericals based on Uniformly varying load
5	bending stresses, section modulus and transverse shear stress distribution in circular section,
6	section modulus and transverse shear stress distribution in hollow circular, I, Box, T, angle sections etc
7	Strain energy due to bending. Numericals based on above topics.

UNIT-2

Bending moments and shearing forces

Introduction

In Unit 1 we discussed the stresses set up in a bar due to axial forces of tension and compression. When a bar carries lateral forces, two important types of loading action are set up at any section: these are a bending moment and a shearing force.

Consider first the simple case of a beam which is fixed rigidly at one end B and is quite free at its remote end D, Figure ; such a beam is called a ***cantilever,*** a familiar example of which is a **fishing** rod held at one end. Imagine that the cantilever is horizontal, with one end B embedded in a wall, and that a lateral force W is applied at the remote end ***D.*** Suppose the cantilever is divided into two lengths by an imaginary section *C;* the lengths BC and CD must individually be in a state of statically equilibrium. If we neglect the mass of the cantilever itself, the loading actions over the section C of CD balance the actions of the force W at C. The length CD of the cantilever is in equilibrium if we apply an upwards vertical force F and an anti-clockwise couple ***A4*** at *C; F* is equal in magnitude to W, and M is equal to ***W(L*** - z), where z is measured from B. The force F at C is called a ***shearing force,*** and the couple M is a ***bending moment.***

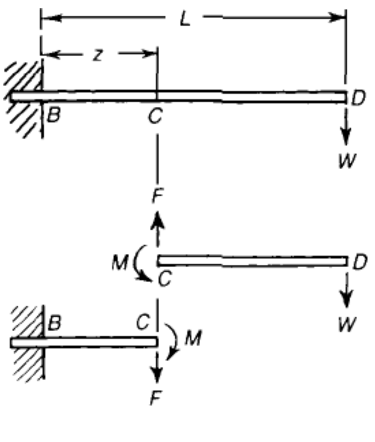


Figure Bending moment and shearing force in a simple cantilever beam.

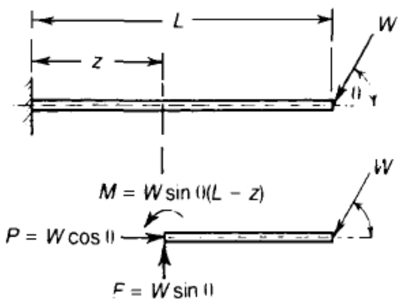


Figure Cantilever with and inclined end load.

But at the imaginary section C of the cantilever, the actions F and ***M*** on CD are provided by the length BC of the cantilever. In fact, equal and opposite actions F and ***M*** are applied by CD to BC. For the length BC, the actions at Care a downwards shearing force F, and a clockwise couple M.

When the cantilever carries external loads which are not applied normally to the axis of the beam, 2, axial forces are set up in the beam. If *W* is inclined at an angle **8** to the **axis** of the beam, 2, the axial thrust in the beam at any section is

$$P = W \cos \theta$$

The bending moment and shearing force at a section a distance *z* from the built-in end are

$$M = W(L - z) \sin \theta \qquad F = W \sin \theta$$

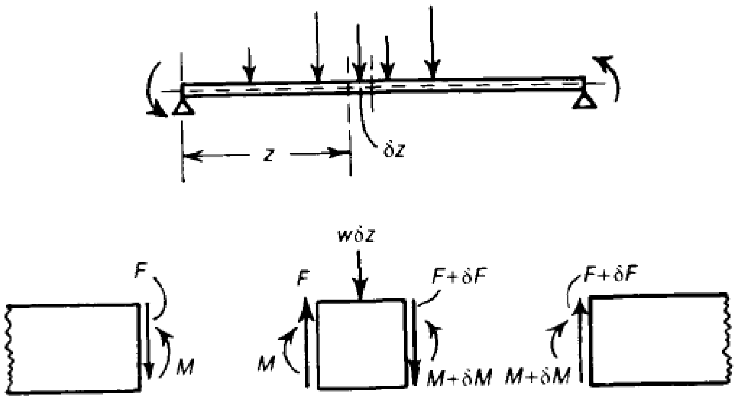
Concentrated and distributed loads

A concentrated load on a beam is one which can be regarded as acting wholly at one point of the beam. For the purposes of calculation such a load is localized at a point of the beam; in reality this would imply **an** infinitely large bearing pressure on the beam at the point of application of a concentrated load. All loads must be distributed in practice over perhaps only a small length of beam, thereby giving a finite bearing pressure. Concentrated loads arise frequently on a beam where the beam is connected to other transverse beams.

In practice there are many examples of distributed loads: they arise when a wall is built on a girder; they occur also in many problems of fluid pressure, such as wind pressure on a tall building, and aerodynamic forces on an aircraft wing.

Relation between the intensity of loading, the shearing force, and bending moment in a straight beam

Consider a straight beam under any system of lateral loads and external couples, 3; an element length ***6z*** of the beam at a distance *z* from one end is acted upon by an external lateral load, and internal bending moments and shearing forces. Suppose external lateral loads are distributed ***so*** that the intensity of loading on the elemental length 6z is ***w.***



Shearing and bending actions on an elemental length of a straight beam.

Then the external vertical force on the element is *W ~ Z* Figure,; this is reacted by **an** internal bending moment *M* and shearing force *F* on one face of the element, and *M + 6M* and *F +6F* on the other face of the element. For vertical equilibrium

$$(F + \delta F)\,\delta z - (M + \delta M) + M + wdz\left(\frac{1}{2}\,\delta z\right) = 0$$

Then, to the first order of small quantities,

$$F\delta z - \delta M = 0$$

Then, in the limit as δz approaches zero,

$$\frac{dM}{dz} = F$$

On integrating between the limits $z = z_1$ and $z = z_2$, we have

$$\int_{z=z_1}^{z=z_2} dM = \int_{z_1}^{z_2} Fdz$$

of the element we have

Then, the decrease of shearing force from z_1, to z_2 is equal to the area below the load distribution curve over this length of the beam, or the difference between *F*, and ***F2*** is the net lateral load over this length of the beam.

Furthermore, for rotational equilibrium of the elemental length

$$(F + \delta F)\,\delta z - (M + \delta M) + M + wdz\left(\frac{1}{2}\,\delta z\right) = 0$$

Then, to the first order of small quantities,

$$F\delta z - \delta M = 0$$

Then, in the limit as δz approaches zero,

$$\frac{dM}{dz} = F$$

On integrating between the limits $z = z_1$ and $z = z_2$, we have

$$\int_{z=z_1}^{z=z_2} dM = \int_{z_1}^{z_2} Fdz$$

where M_1, and $M_2$are the values M at $z = z_1$ and $z = z_2$,respectively. Then the increase of bending moment from z_l to z, is the area below the shearing force curve for that length of the beam.

Equations and are extremely useful for finding the bending moments and shearing forces in beams with irregularly distributed loads. From equation the shearing force F a t a section distance *z* from one end of the beam is

On substituting **this** value of F into equation ,

Thus

$$M_2 - M_1 = \int_{z_1}^{z_2} F dz$$

From equation we have that the bending moment M has a stationary value when the shearing force F is zero. Equations give

$$\frac{d^2M}{dz^2} = \frac{dF}{dz} = -w$$

For the directions of ***M,*** F and ***w*** considered in 3, M is ***mathematically*** a maximum, since ***&M/d?*** is negative; the significance of the word mathematically will be made clearer in Section .

All the relations developed in **this** section are merely statements of statical equilibrium, and are therefore true independently of the state of the material of the beam.

Cantilevers

A cantilever is a beam supported at one end only; for example, the beam already discussed in Section **7.1,** and shown in 1, is held rigidly at B. Consider first the cantilever shown in , which carries a concentrated lateral load W at the free end. The bending moment at a section a distance z from B is

$$M = -W(L - z)$$

the negative sign occurring since the moment is hogging, **as**shown . The variation of bending moment is linear, as shown in. The shearing force at any section is

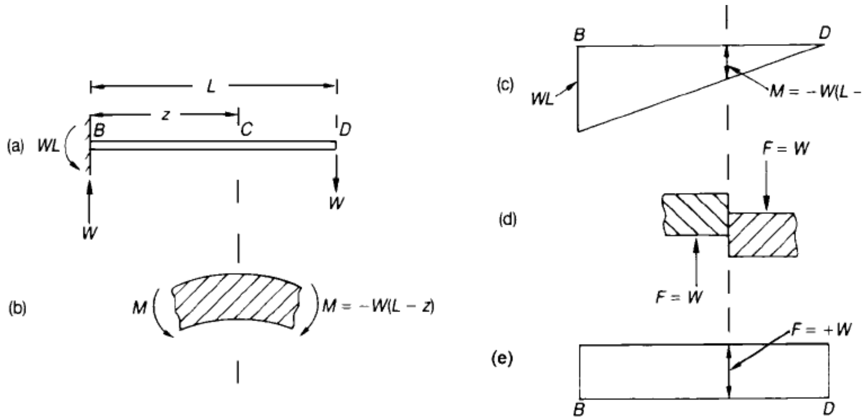
$$F = +W$$

the shearing force being positive as it is clockwise, as shown . The shearing force is constant throughout the length of the cantilever. We note that

$$\frac{dM}{dz} = W = F$$

Further $dF/dz = 0$, as there are no lateral loads between B and ***D.***

The bending moment diagram is shown and the shearing force diagram is shown



Bending-moment and shearing-force diagrams for a cantilever with a concentrated load at the free end.

Now consider a cantilever carrying a uniformly distributed downwards vertical load of intensity ***w*,**. The shearing force at a distance z from B is

$$F = +w(L - z)$$

as shown in . The bending moment at a distance ***z*** from B is

$$M = -\frac{1}{2} w(L - z)^2$$

as shown. The shearing force varies linearly and the bending moment parabolically along the length of the beam, as show. We see that

$$\frac{dM}{dz} = w(L - z) = +F$$

Simply-supported beams

By simply-supported we mean that the supports are of such a nature that they do not apply any resistance to bending of a beam; for instance, knife-edges or functionless pins perpendicular to the plane of bending cannot transmit couples to a beam. The remarks concerning bending moments and shearing forces, which were made in Section in relation to cantilevers, apply equally to beams simply-supported at each end, or with any conditions of end support.

As an example, consider the beam shown in (a), which is simply-supported at B and ***C,*** and cames a vertical load ***W*** a distance ***a*** from B. If the ends are simply-supported no bending moments are applied to the beam at B and C. By taking moments about B and C we find that the reactions at these supports are

$$\frac{W}{L}(L - a) \text{ and } \frac{Wa}{L}$$

respectively. Now consider a section of the beam a distance z from B; if $z < a$, the bending moment and shearing force are

$$M = +\frac{Wz}{L}(L - a), \quad F = +\frac{W}{L}(L - a),$$ as shown by Figures 7.7(b) and 7.7(d)

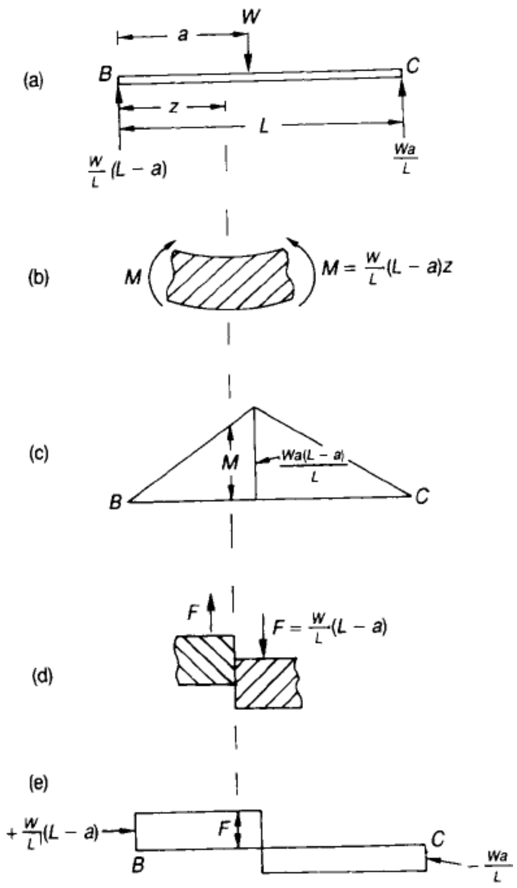
If $z > a$,

$$M = +\frac{Wz}{L}(L - a) - W(z - a) = +\frac{Wa}{L}(L - z)$$

$$F = -\frac{Wa}{L}$$

The bending moment and shearing force diagrams show discontinuities at $z = a$; the maximum bending moment occurs under the load W, and has the value

$$M_{\max} = \frac{Wa}{L}(L - a)$$



Bending-moment and shearing-force diagrams
for a simply-supported beam with a single
concentrated lateral load.

The simply-supported beam of carries a uniformly-distributed load of intensity ***w*.** The vertical reactions at *B* and Care ***%wL*.** Consider a section at a distance *z* from *B*. The bending moment at this section is

$$M = \frac{1}{2}wLz - \frac{1}{2}wz^2$$

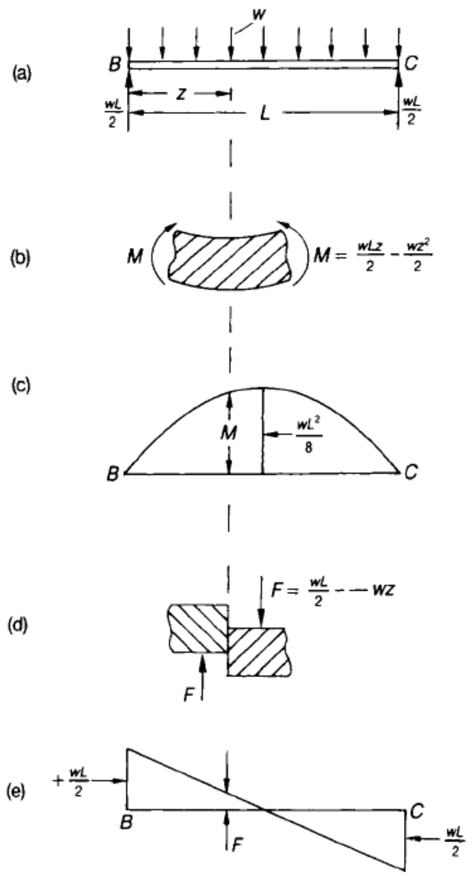
$$= \frac{1}{2}wz\,(L - z)$$

as **shown** in fig and the shearing force is

$$F = +\frac{1}{2}wL - wz$$

$$= w\left(\frac{1}{2}L - z\right)$$

as shown in



Bending-moment and shearing-force diagrams for a simply- supportedbeam with a uniformly distributed lateral load.

The bending moment is a maximum at $z = ½L$, where

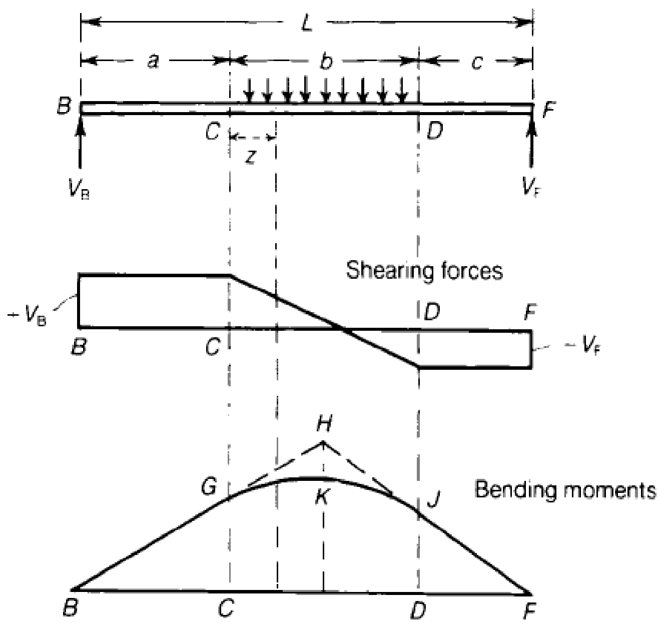
At $$M_{max} = \frac{wL^2}{8}$$

$$\frac{dM}{dz} = + F = 0$$

Simply-supported beam with a uniformly distributed load over part of a span

The beam *BCDF*, shown , carries a uniformly distributed vertical load ***w***per unit length over the portion *CD*. On takmg moments about ***B***and *F*,

$$V_B = \frac{bw}{2L}(b + 2c), \quad V_F = \frac{bw}{2L}(b + 2a)$$



Shearing-force and bending-moment diagrams for simply-supported beam with distributed load over part of the span.

The bending moments at *C* and *D* are

$$M_C = aV_B = \frac{baw}{2L}(b + 2c)$$

$$M_D = cV_F = \frac{bcw}{2L}(b + 2a)$$

The bending moments in *BC* and *FD* vary linearly. The bending moment in *CD,* at a distance ***z***
From c is

$$M = \left(1 - \frac{z}{b}\right) M_C + \frac{z}{b} M_D + \frac{1}{2} wz\,(b - z)$$

Then

$$\frac{dM}{dz} = \frac{1}{b}\left(M_D - M_C\right) + \frac{1}{2} w\,(b - 2z)$$

On substituting for M_C and M_D from equations (7.20)

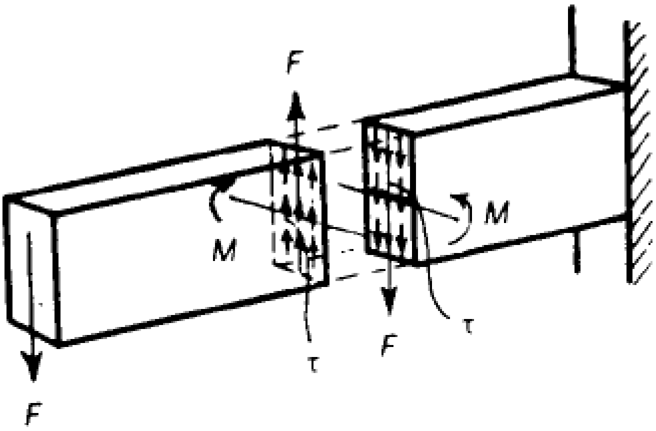
$$\frac{dM}{dz} = \frac{bw}{2L}(c - a) + \frac{1}{2} w\,(b - 2z)$$

At C, $z = 0$, and

$$\frac{dM}{dz} = \frac{bw}{2L}(b + 2c) = V_B$$

Shearing stresses in beams

We referred earlier to the existence of longitudinal direct stresses in a cantilever with a lateral load at the free end; on a closer study we found that these stresses are distributed linearly over the cross-section of a beam carrying a uniform bending moment. In general we are dealing with bending problems in which there are shearing forces present at any cross-section, as well as bending moments. Impractice we find that the longitudinal direct stresses in the beam are almost unaffected by the shearing force at any section, and are governed largely by the magnitude of the bending moment at that section. Consider again the bending of a cantilever with a concentrated lateral load *F,* at the free end, Figure 10.1; Suppose the beam **is** of rectangular cross-section. If we cut the beam at any transverse cross-section, we must apply bending moments *M* and shearing forces *F* at the section to maintain equilibrium. The bending moment Mis distributed over the cross-section in the form of longitudinal direct stresses, as already discussed.



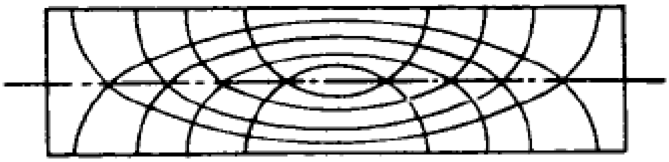
Shearing actions in a cantilever carrying an end load.

The shearing force *F* is distributed in the form of shearing stresses **T,** acting tangentially to the cross-section of the beam; the form of the distribution of ***T*** is dependent on the shape of the cross- section of the beam, and on the direction of application of the shearing force *F.* An interesting feature of these shearing stresses is that, as they give rise to complementary shearing stresses, we find that shearing stresses are also set up in longitudinal planes parallel to the axis of the beam.

Principal stresses in beams

We have shown how to find separately the longitudinal stress at any point in a beam due to bending moment, and the mean horizontal and vertical shearing stresses, but it does not follow that these are the greatest direct or shearing stresses. Within the limits of our present theory we can employ the formulae of Sections **5.7** and 5.8 to find the principal stresses and the maximum shearing stress.

We can draw, on a side elevation of the beam, lines showing the direction of the principal stresses. Such lines are called the ***lines ofprincipal stress;*** they are such that the tangent at any point gives the direction of principal stress. As an example, the lines of principal stress have been drawn in Figure for a simply-supported beam of uniform rectangular cross-section, carrying a uniformly distributed load. The stresses are a maximum where the tangents to the curves are parallel to the axis of the beam, and diminish to zero when the curves cut the faces of the beam at right angles. On the neutral axis, where the stress is one of shear, the principal stress curves cut the axis at **45".**



Principal Mess lines in a simply-supported rectangular beam canying a uniformiy distributed load.