Syllabus

Mechanics of solids[3ME04-07]

UNIT-1

Lecture	Topic to be discussed
1	Elementary definition of stress and strain, stress-strain relationship
2	Elastic, plastic and visco-elastic behavior of common materials in tension and compression test
3	Stress-strain curves, Hooke’s law, Poisson’s ratio
4	Elastic constants and their relations for an isotropic hookean material, anisotropic and orthotropic materials
5	Tension, compression, shearing stress and strain
6	Thermal stresses, composite bars, equations of static equilibrium
7	Concept of free body diagram. Strain energy due to axial loading.

MECHANICS
OF SOLID

UNIT-1

Tension and compression and direct stress

Introduction

The strength of a material, whatever its nature, is defined largely by the internal stresses, or intensities of force, in the material. A knowledge of these stresses **is** essential to the safe design of a machine, aircraft, or any type of structure. Most practical structures consist of complex arrangements of many component members; an aircraft fuselage, for example, usually consists of an elaborate system of interconnected sheeting, longitudinal stringers, and transverse rings. The detailed stress analysis of such a structure is a difficult task, even when the loading conditions are simple. The problem is complicated further because the loads experienced by a structure are variable and sometimes unpredictable. We shall be concerned mainly with stresses in materials under relatively simple loading conditions; we begin with a discussion of the behaviour of a stretched wire, and introduce the concepts of direct stress and strain.

Stretching of a steel wire

One of the simplest loading conditions of a material is that of ***tension,*** in which the fibres of the material are stretched. Consider, for example, a long steel wire held rigidly at its upper end, Figure 1.1, and loaded by a mass hung from the lower end. If vertical movements of the lower end are observed during loading it will be found that the wire is stretched by a small, but measurable, amount from its original unloaded length. The material of the wire is composed of a large number of small crystals which are only visible under a microscopic study; these crystals

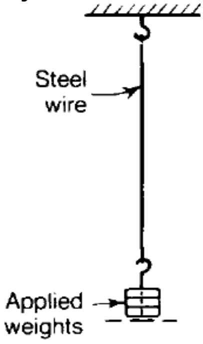


Figure 1.1 Stretching of a steel wire under end load.

have irregularly shaped boundaries, and largely random orientations with respect to each other; as loads are applied to the wire, the crystal structure of the metal is distorted.

For small loads it is found that the extension of the wire is roughly proportional to the applied load, Figure 1.2. **This**linear relationship between load and extension was discovered by Robert Hooke in 1678; a material showing this characteristic is said to obey ***Hooke's law.***

As the tensile load in the wire is increased, a stage is reached where the material ceases to show this linear characteristic; the corresponding point on the load-extension curve of Figure
1.2 is known **as** the ***limit of proportionality.*** If the wire is made from a hgh-strength steel then the load-extension curve up to the ***breakingpoint*** has the form shown in Figure 1.2. Beyond the limit of proportionality the extension of the wire increases non-linearly up to the elastic limit and, eventually, the breaking point.

The elastic h u t is important because it divides the load-extension curve into two regions. For loads up to the elastic limit, the wire returns to its original unstretched length on removal of the loads; **tlus**properly of a material to recover its original form on removal of the loads is known as ***elasticity;*** the steel wire behaves, in fact, as a still elastic spring. When loads are applied above the elastic limit, and are then removed, it is found that the wire recovers only part of its extension and is stretched permanently; in **tlus** condition the wire is said to have undergone an ***inelastic,*** or ***plastic,*** extension. For most materials, the limit of proportionality and the elastic limit are assumed to have the same value.

In the case of elastic extensions, work performed in stretching the wire is stored as ***strain energy*** in the material; this energy is recovered when the loads are removed. During inelastic extensions, work is performed in makmg permanent changes in the internal structure of the material; not all the work performed during an inelastic extension is recoverable on removal of the loads; this energy reappears in other forms, mainly as heat.

The load-extension curve of Figure is not typical of all materials; it is reasonably typical, however, of the behaviour of ***brittle*** materials, which are discussed more fully in Section 1.5. An important feature of most engineering materials is that they behave elastically up to the limit of proportionality, that is, all extensions are recoverable for loads up to this limit. The concepts of linearity and elasticity' form the basis of the theory of small deformations in stressed materials.

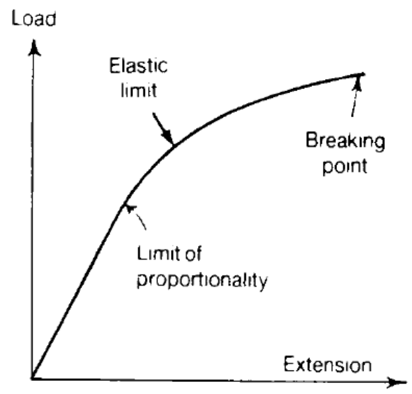


Figure 1.2 Load-extension curve for a steel wire, showing the limit of linear-elastic behavior (or limit of proportionality) **and** the breaking point.

Tensile and compressive stresses

The wire of Figure was pulled by the action of a mass attached to the lower end; in **this** condition the wire is in ***tension.*** Consider a cylindrical bar ***ab,*** Figure 1.3, which has a uniform cross-section throughout its length. Suppose that at each end of the bar the cross- section is dwided into small elements of equal area; the cross-sections are taken normal to the longitudinal axis of the bar. To each of these elemental areas an equal tensile load is applied normal to the cross-section and parallel to the longitudinal axis of the bar. The bar is then uniformly stressed in tension.

Suppose the total load on the end cross-sections is ***P;*** if **an** imaginary break is made perpendicular to the axis of the bar at the section **c,** Figure, then equal forces P are required at the section **c** to maintain equilibrium of the lengths ***ac*** and ***cb.*** This is equally true for any section across the bar, and hence on any imaginary section perpendicular to the axis of the bar there is a total force P.

When tensile tests are carried out on steel wires of the same material, but of different cross-sectional area, the breaking loads are found to be proportional approximately to the respective cross-sectional areas of the wires. This is ***so*** because the tensile strength is governed by the intensity of force on a normal cross-section of a wire, and not by the total force. **This** intensity of force is known as ***stress;*** in Figure the ***tensile stress*** (T at any normal cross-section of the bar is

$$\sigma = \frac{P}{A}$$

where P is the total force on a cross-section and A is the area of the cross-section.

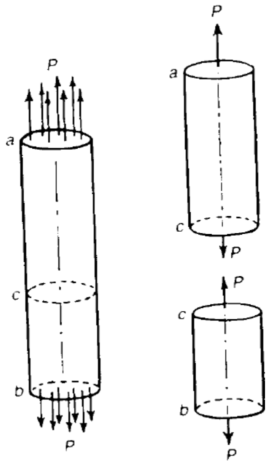


Figure 1.3 Cylindrical bar under uniform tensile stress; there is a similar state of tensile stress over any imaginary normal cross-section.

In Figure uniform stressing of the bar was ensured by applying equal loads to equal small areas at the ends of the bar. In general we are not dealing with equal force intensities of this type, and a more precise definition of stress is required. Suppose ***6A*** is **an** element of area of the cross- section of the bar, Figure **;** if the normal force acting on thls element is 6P, then the tensile stress at this point of the cross-section is defined as the limiting value of the ratio (6P/6A) as 6A becomes infinitesimally small. Thus

$$\sigma = \underset{\delta A \to 0}{\text{Limit}} \frac{\delta P}{\delta A} = \frac{dP}{dA}$$

Thls definition of stress is used in studying problems of non-uniform stress distribution in materials.

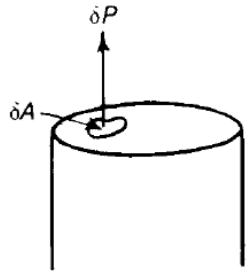


Figure 1.4 Normal load on **an** element of area of the cross-section.

When the forces ***P*** in Figure 1 are reversed in direction at each end of the bar they tend to ***compress*** the bar; the loads then give rise to ***compressive stresses.*** Tensile and compressive stresses are together referred to as ***direct*** (or *normal)* ***stresses,*** because they act perpendicularly to the surface.

Tensile and compressive strains

In the steel wire experiment of Figure we discussed the extension of the whole wire. If we measure the extension of, say, the lowest quarter-length of the wire we find that for a given load it is equal **to** a quarter of the extension of the whole wire. In general we find that, at a given load, the ratio of the extension of any length to that length is constant for all parts of the wire; this ratio is **knownas**the ***tensile strain.***

Suppose the initial unstrained length of the wire is *Lo,* and the ***e*** is the extension due to straining; the tensile strain **E** is defined as

$$\varepsilon = \frac{e}{L_0}$$

Thls definition of strain is useful **only** for small distortions, in which the extension ***e*** is small compared with the original length *Lo;* this definition is adequate for the study of most engineering problems, where we are concerned with values of **E** of the order 0.001, or so.

If a material is compressed the resulting strain is defined in a similar way, except that ***e*** is the contraction of a length.

We note that strain is a ***Ron-dimensional*** quantity, being the ratio of the extension, or contraction, of a bar **to** its original length.

Stress-strain curves for brittle materials

Many of the characteristics of a material can be deduced from the tensile test. **In** the experiment of 1 we measured the extensions of the wire for increasing loads; it is more convenient to compare materials in terms of stresses and strains, rather than loads and extensions of a particular specimen of a material.

The tensile ***stress-struin*** curve for a hgh-strength steel has the form shown in Figure 1**.5.** The stress at any stage is the ratio of the load of the ***original*** cross-sectional area of the test specimen; the strain is the elongation of a unit length of the test specimen. For stresses up to about **750** MNlm2the stress-strain curve is linear, showing that the material obeys Hooke's law in this range; the material is also elastic in this range, and no permanent extensions remain after removal of the stresses. The ratio of stress to strain for this linear region is usually about 200 GN/m2for steels; this ratio is known as ***Young's modulus*** and is denoted by E. The strain at the limit of proportionality is of the order 0.003, and is small compared with strains of the order 0.100 at fracture.

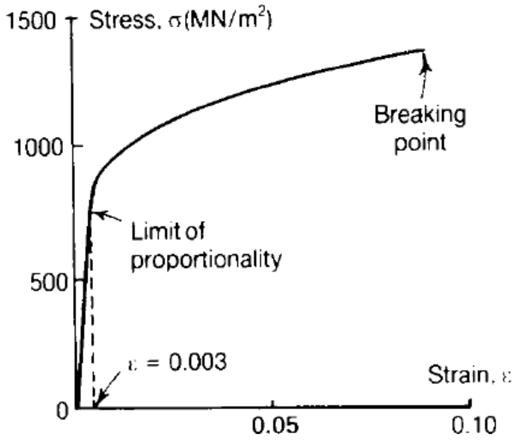


Figure 1.5:Tensile stress-strain curve for a high-strength steel.

for the linear-elastic range. If ***P*** is the total tensile load in a bar, A its cross-sectional area, and
Lo its length, then

$$E = \frac{\sigma}{\varepsilon} = \frac{P / A}{e / L_0}$$

where ***e*** is the extension of the length *Lo*. Thus the expansion is given by

$$e = \frac{PL_0}{EA}$$

If the material is stressed beyond the linear-elastic range the limit of proportionality is exceeded, and the strains increase non-linearly with the stresses. Moreover, removal of the stress leaves the material with some permanent extension; hrange is then bothnon-linear and inelastic. The maximum stress attained may be of the order of 1500 MNlm’, and the total extension, or ***elongation,*** at this stage may be of the order of 10%.

The curve of 5 is typical of the behaviour of ***brittle*** materials-as, for example, area characterized by small permanent elongation at the breaking point; in the case of metals this is usually lo%, or less.

When a material is stressed beyond the limit of proportionality and is then unloaded, permanent deformations of the material take place. Suppose the tensile test-specimen of 5 is stressed beyond the limit of proportionality, (point ***a*** in Figure l.5), to a point ***b*** on the stress- strain diagram. If the stress is now removed, the stress-strain relation follows the curve ***bc;*** when the stress is completely removed there is a residual strain given by the intercept Oc on the &- axis. If the stress is applied again, the stress-strain relation follows the curve ***cd*** initially, and finally the curve ***df*** to the breaking point. Both the unloading curve ***bc*** and the reloading curve ***cd*** are approximately parallel to the elastic line ***Oa;***they are curved slightly in opposite directions. The process of unloading and reloading, ***bcd,*** had little or no effect on the stress at the breaking point, the stress-strain curve being interrupted by only a small amount ***bd,*** 6.

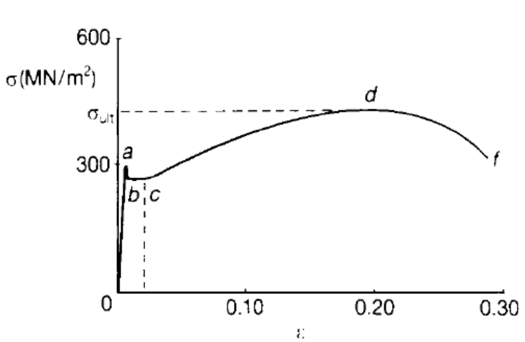
The stress-strain curves of brittle materials for tension and compression are usually similar in form, although the stresses at the limit of proportionality and at fracture may be very different for the **two**loading conditions. Typical tensile and compressive stress-strain curves for concrete are shown in 7; the maximum stress attainable in tension is only about one-tenth of that in compression, although the slopes of the stress-strain curves in the region of zero stress are nearly equal.

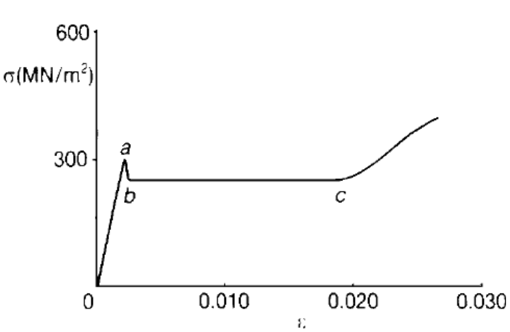
Ductile materials

A brittle material is one showing relatively little elongation at fracture in the tensile test; by contrast some materials, such as mild steel, copper, and synthetic polymers, may be stretched appreciably before breaking. These latter materials are ductile in character.

If tensile and compressive tests are made on a mild steel, the resulting stress-strain curves are different in form from those of a brittle material, such as a high-strength steel. If a tensile test

Specimen of mild steel is loaded axially, the stress-strain curve is linear and elastic up to a point ***a,*** 8; the small strain region of 8. is reproduced to a larger scale in 3. The ratio of stress to strain, or Young's modulus, for the linear portion ***Oa*** is usually about 200 GN/m2, ie, 200 x109 N/m2. The tensile stress at the point ***a*** is of order 300 MN/m2, i.e. 300 **x** lo6 N/m2. If the test specimen is strained beyond the point ***a,*** Figures 1.8 and 1.9, the stress must be reduced almost immediately to maintain equilibrium; the reduction of stress, ***ab,*** takes place rapidly, and the form of the curve ***ab*** is difficult to define precisely. Continued straining proceeds at a roughly constant stress along ***bc.*** In the range of strains from ***a*** to **c** the material is said to ***yield; a*** is the ***upper yield point,*** and ***b*** the ***lower yield point.*** Yielding at constant stress along ***bc*** proceeds usually to a strain about **40** times greater than that at ***a;*** beyond the point **c** the material ***strain-hardens,*** and stress again increases with strain where the slope from **c** to ***d*** is about 1150th that from 0 to ***a* .** The stress for a tensile specimen attains a maximum value at ***d*** if the stress is evaluated on the basis of the original cross-sectional area of the bar; the stress corresponding to the point ***d*** is **known** as the ***ultimate stress,*** (T, of the material. From ***d*** to f there is a reduction in the nominal stress until fracture occurs at The ultimate stress in tension is attained at a stage when ***necking*** begins; this is a reduction of area at a relatively weak cross-section of the test specimen. It is usual to measure the diameter of the neck after fracture, and to evaluate a true stress at fracture, based on the breaking load and the reduced cross-sectional area at the neck. Necking and considerable elongation before fracture are characteristics of ductile materials; there is little or no necking at fracture for brittle materials.





Tensile stress-strain curve for an annealed mild steel, showing the drop in stress at Upper and lower yield points of a yielding from the upper yield point *a* to the lower mild steel yield point *b.*

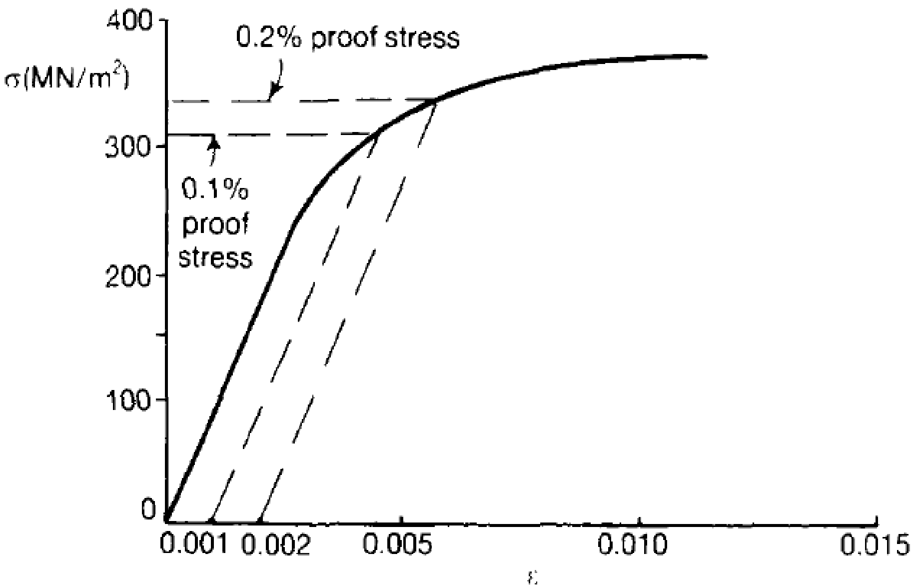
Compressive tests of mild steel give stress-strain curves similar to those for tension. If we consider tensile stresses and strains as positive, and compressive stresses and strains as negative, we can plot the tensile and compressive stress-strain curves on the same diagram; 10 shows the stress- strain curves for an annealed mild steel. In determining the stress-strain curves experimentally, it is important to ensure that the bar is loaded axially; with even small eccentricities of loading the stress distribution over any cross-section of the bar is non uniform, and the upper yield point stress is not attained in all fibres of the material simultaneously. For this reason the lower yield point stress is taken usually as a more realistic definition of yielding of the material. Some ductile materials show no clearly defined upper yield stress; for these materials the limit of proportionality may be lower than the stress for continuous yielding. The term yieldstress refers to the stress for continuous yielding of a material; this implies the lower yield stress for a material in which an upper yield point exists; the yield stress is denoted by **oy.**

Tensile failures of some steel bars are shown in fig; specimen (ii) is a brittle material, showing little or no necking at the fractured section; specimens (i) and (iii) are ductile steels showing a characteristic necking at the fractured sections. The tensile specimens of 12 show the forms of failure in a ductile steel and a ductile light-alloy material; the steel specimen (i) fails at a necked section in the form of a 'cup and cone'; in the case of the light-alloy bar, two 'cups' are formed. The compressive failure of a brittle cast iron is shown in 13. In the case of a mild steel, failure in compression occurs in a 'barrel-lke' fashion, as shown in fig.

The stress-strain curves discussed in the preceding paragraph refer to static tests carried out at negligible speed. When stresses are applied rapidly the yield stress and ultimate stresses ofmetallic materials are usually raised. At a strain rate of 100 per second the yield stress of a mild steel may be twice that at negligible speed.

Proof stresses

Many materials show no well-defmed yield stresses when tested in tension or compression. **A**
typical stress-strain curve for an alum inium alloy is shown in fig.



Proof stresses of **an**aluminium-alloy material; the proof stress is **found**by drawing the line parallel to the linear-elastic line at the appropriate proof strain.

The limit of proportionality is in the region of 300 MN/m^2,but the exact position of this limit is difficult to determine experimentally. To overcome this problem a ***proof stress*** is defined; the 0.1% proof stress required to produce a permanent strain of 0.001 **(or** 0.1%) on removal of the stress. Suppose we draw a line from the point 0.001 on the strain axis, 15, parallel to the elastic line of the material; the point where this line cuts the stress-strain curve defines the proof stress. The **0.2%**proof stress is defined in a similar way.

Ductility measurement

The Ductility value of a material can be described as the ability of the material to suffer plastic deformation whle still being able to resist applied loading. The more ductile a material is the more it is said to have the ability to deform under applied loading.

The ductility of a metal is usually measured by its percentage reduction in cross-sectional area or by its percentage increase in length, i.e.

$$\text{percentage reduction in area} = \frac{(A_I - A_F)}{A_I} \times 100\%$$

and

$$\text{percentage increase in length} = \frac{(L_I - L_F)}{L_I} \times 100\%$$

where

A_I = initial cross-sectional area of the tensile specimen

A_F = final cross-sectional area of the tensile specimen

L_I = initial gauge length of the tensile specimen

L_F = final gauge length of the tensile specimen

It should be emphasised that the shape of the tensile specimen plays a major role **on** the measurement of the ductility and some typical relationships between length and character for tensile specimens i.e. given in Table 1.1

Materials such **as**copper and mild steel have high ductility and brittle materials such as bronze and cast **iron**have low ductility.

Place	L_I	L_I/D_I*
UK	4√area	3.54
USA	4.51√area	4.0
Europe	5.65√area	5.0

area = cross-sectional area

* D_I = initial diameter of the tensile specimen

Working stresses

In many engineering problems the loads sustained by a component of a machine or structure are reasonably well-defined; for example, the lower stanchions of a tall buildmg support the weight of material forming the upper storeys. The stresses which are present in a component, under normal working conditions, are called the ***working stresses;*** the ratio of the yield stress, ***oy,***of a material to the largest working stress, ***ow,***in the component is the ***stress factor*** against yielding. The stress factor on yielding is then

$$\frac{\sigma_Y}{\sigma_W}$$

If the material has **no**well-defined yield point, it is more convenient to use the ***proof stress,op;*** the stress factor **on**proof stress is then

$$\frac{\sigma_P}{\sigma_W}$$

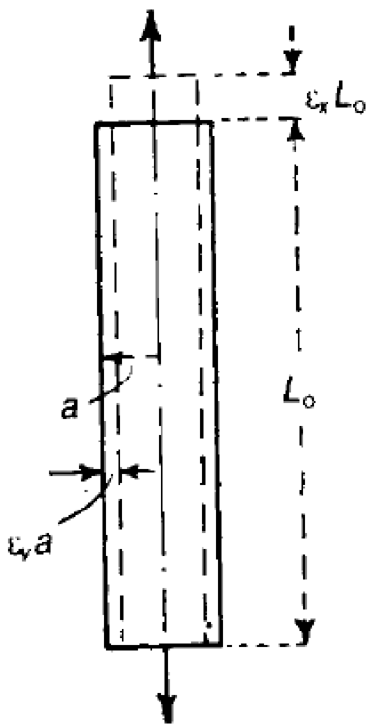
Some writers refer to the stress factor defined above as a ‘safety factor’. It is preferable, however, to avoid any reference to ‘safe’ stresses, as the degree of safety in any practical problem is difficult to define. The present writers prefer the term ‘stress factor’ as this defines more precisely that the worlung stress is compared with the yield, or proof stress of the material. Another reason for using ‘stress factor’ will become more evident after the reader has studied Section fig.

Lateral strains due to direct stresses

When a bar of a material is stretched longitudinally-as in a tensile test-the bar extends in the direction of the applied load. This longitudinal extension is accompanied by a lateral contraction of the bar, as shown in 17. In the linear-elastic range of a material the lateral strain is proportional to the longitudinal strain; if **E,** is the longitudinal strain of the bar, then the lateral strain is

$$\varepsilon_y = \nu\varepsilon_x$$

The constant v in this relationship is known **as *Poisson* 's *ratio,*** and for most metals it has a value of about 0.3 in the linear-elastic range; it cannot exceed a value of 0.5. For concrete it has a value of about 0.1. If the longitudinal strain is tensile, the lateral strain is a contraction; for a compressed bar there is a lateral expansion.



The Poisson ratio effect leading to lateral contraction of a **bar** in tension.

With a knowledge of the lateral contraction of a stretched bar it is possible to calculate the change in volume due to straining. The bar of 17 is assumed to have a square cross-section of side *a;* Lo is the unstrained length of the bar. When strained longitudinally an amount **E,** the corresponding lateral strain of contractions is **E ~ .**The **bar** extends therefore an amount *& Ao,*and each side of the cross- section contracts an amount **E, Q.** The volume of the bar before stretching is

$$V_0 = a^2 L_0$$

After straining the volume is

$$V = (a - \varepsilon_y a)^2 \ (L_0 + \varepsilon_x L_0)$$

which may be written

$$V = a^2 L_0 (1 - \varepsilon_y)^2 (1 + \varepsilon_x) = V_0 (1 - \varepsilon_y)^2 (1 + \varepsilon_y)$$

If ε_x and ε_y are small quantities compared to unit, we may write

$$(1 - \varepsilon_y)^2 (1 + \varepsilon_x) = (1 - 2\varepsilon_y)(1 + \varepsilon_x) = 1 + \varepsilon_x - 2\varepsilon_y$$

ignoring squares and products of ε_x and ε_y. The volume after straining is then

$$V = V_0 (1 + \varepsilon_x - 2\varepsilon_y)$$

The *volumetric strain* is defined as the ratio of the change of volume to the original volume, and is therefore

$$\frac{V - V_0}{V_0} = \varepsilon_x - 2\varepsilon_y$$

If **E,** = ***v*** **E~** then the volumetric strain is **E,** (1 ***2v).*** Equation shows why ***v*** cannot be greater- than 0.5; if it were, then under hydrostatic stress a ***positive*** volumetric strain will ***compressive*** result, whch is impossible.

Weight and stiffness economy of materials

In some machine components and structures it is important that the weight of material should be as small as possible. This is particularly true of aircraft, submarines and rockets, for example, in which less structural weight leads to a

$$\frac{\sigma_{ult}}{\rho}$$

larger pay-load. If **odt** is the ultimate stress of a material in tension and **p** is its density, then a measure of the strength economy is the ratio

The materials shown in Table 1.2 are compared on the basis of strength economy in Table
1.3 from which it is clear that the modern fiber-reinforced composites offer distinct savings in weight over the more common materials in engineering use.

In some engineering applications, stiffness rather thanstrength is required of materials; this is ***so*** in structures likely to buckle and components governed by

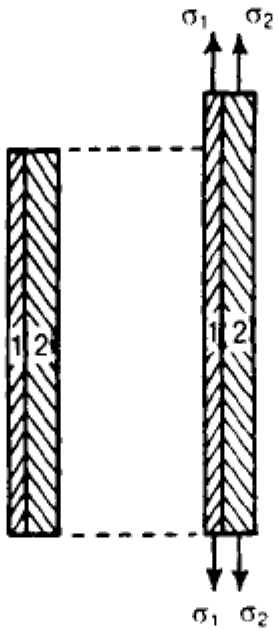
$$\frac{E}{\rho},$$

deflection limitations. A measure of the stiffness economy of a material is the ratio some values of which are shown in Table. Boron composites and carbon-fibre composites show outstanding stiffness properties, whereas glass-fibre composites fall more into line with the best materials already in common use.

Composite bars in tension or compression

A composite bar is one made of two materials, such as steel rods embedded in concrete. The construction of the bar is such that constituent components extend or contract equally under load. To illustrate the behaviour of such bars consider a rod made of two materials, **1** and 2, **20;** A,, A, are the cross-sectional areas of the bars, and E,, E, are the values of Young's modulus. We imagine the bars to be rigidly connected together at the ends; then for ***compatibility,*** the longitudinal strains to be the same when the composite bar is stretched we must have

$$\varepsilon = \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$



Composite bar in tension; if the bars are connected rigidly
at **their ends, they suffer the same extensions.**

where σ_1 and σ_2 are the stresses in the two bars. But from *equilibrium* considerations

$$P = \sigma_1 A_1 + \sigma_2 A_2$$

By equations

$$\sigma_1 = \frac{PE_1}{A_1 E_1 + A_2 E_2}, \quad \sigma_2 = \frac{PE_2}{A_1 E_1 + A_2 E_2}$$

Temperature stresses

When the temperature of a body is raised, or lowered, the material expands, or contracts. If this expansion or contraction is wholly or partially resisted, stresses are set up in the body. Consider a long bar of a material; suppose *Lo* is the length of the bar at a temperature **e,** and that a is the coefficient of linear expansion of the material. The bar is now subjected to an increase 8 in temperature. If the bar is completely free to expand, its length increases,and the length becomes *Lo* (1 + ***a8)*** were compressed to a length *Lo;* in this case the compressive strain is

$$\varepsilon = \frac{\alpha L_0 \theta}{L_0 (1 + \alpha\theta)} = \alpha\theta$$

since a8 is small compared with **unity;**the corresponding stress is

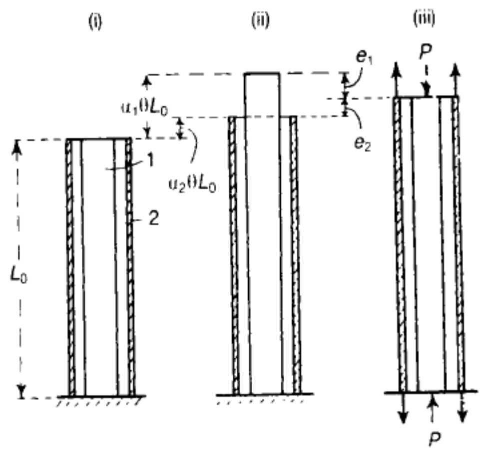
$$\sigma = E\varepsilon = \alpha\theta E$$

By a similar argument the tensile stress set up in a constrained bar by a fall **8**in temperature is a8
E. It is assumed that the material remains elastic.
In the case of steel a = 1.3 **x**per "C; the product *aE* is approximately 2.6 MN/m^2per "C,
so that a change in temperature of **4°C** produces a stress of approximately 10 h4N/m2if the bar is completely restrained.

Temperature stresses in composite bars

In a component or structure made wholly of one material, temperature stresses arise only if external restraints prevent thermal expansion or contraction. In composite bars made of materials with different rates of thermal expansion, internal stresses can be set up by temperature changes; these stresses occur independently of those due to external restraints.

Consider, for example, a simple composite bar consisting of two members-a solid circular bar, 1, contained inside a circular tube, 2, 2 1. The materials of the bar and tube have different coefficients of linear expansion, a, and **q,** respectively. If the ends of the bar and tube are attached rigidly to each other, longitudinal stresses are set up by a change of temperature. Suppose firstly, however, that the bar and tube are quite free of each other; if ***Lo*** is the original length of each bar, 21, the extensions due to a temperature increase **0** are a, ***015,*** and a, ***OLo,*** 21(ii). The difference in lengths of the two members is (a, - **q)** ***L0,;*** this is now eliminated by compressing the inner bar with a force P, and pul l(iii).



(i) **(ii)** **(iii)**

Temperature stress in a composite **bar.**

If A , and E, are the cross-sectional area and Young's modulus, respectively, of the inner bar, and A, and E, refer to the outer tube, then the contraction of the

$$e_1 = \frac{PL_0}{E_1 \; A_1}$$

inner bar to P is

and the extension of the outer tube due to P is

$$e_2 = \frac{PL_0}{E_2 \; A_2}$$

Then from compatibility considerations, the difference in lengths (a, - %) ***OL,*** is eliminated completely when

On substituting for e, + ***e2,***we have

$$(\alpha_1 - \alpha_2)\,\theta L_0 = e_1 + e_2$$

$$(\alpha_1 - \alpha_2)\theta L_0 = PL\left(\frac{1}{E_1 A_1} + \frac{1}{E_2 A_2}\right)$$

The force P is induced by the temperature change ***8*** if the ends of the two members are attached rigidly to each other; from equation (1.22), P has the value

$$P = \frac{(\alpha_1 - \alpha_2)\,\theta}{\left(\frac{1}{E_1 A_1} + \frac{1}{E_2 A_2}\right)}$$

An internal load is **only** set up if a, is different from **q.**