Engineering Mechanics (3ME3-04)

DEPARTMENT

OF

MECHANICAL ENGINEERING

(Jaipur Engineering College and Research Centre, Jaipur)

UNIT: V

Work, Energy and power: Work of a force, weight, spring force and couple, Power, Efficiency, Energy, Kinetic energy of rigid body, Principle of work and energy, Conservative and Non-conservative Force, Conservation of energy.

Impulse and momentum: Linear and angular momentum, Linear and angular impulse, Principle of momentum for a particle and rigid body, Principle of linear impulse and momentum for a particle and rigid body, Principle of angular momentum and Impulse, Conservation of angular momentum, Angular momentum of rigid body, Principle of impulse and momentum for a rigid body, Central impact, Oblique impact, System of variable mass, Rocket.

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UNIT- 5

WORK ENERGY AND POWER

FORCE:- It is an external agent which tends to change the state of rest or of uniform motion of a system.

MOMENTUM:- It is the product of mass and velocity of a body. momentum represents the energy of motion stored in a moving body.

$$\boxed{\text{momentum} = mv}$$

LAW OF CONSERVATION OF MOMENTUM:-

Total momentum of any group of objects always remains the same if no external forces acts on it.

consider that a body A of mass m_1 moving with velocity u_1 collides with another body B of mass m_2 and moving with velocity u_2. Let v_1 and v_2 be their velocities after the collision. Then,-

momentum of masses before collision $= m_1u_1 + m_2u_2$

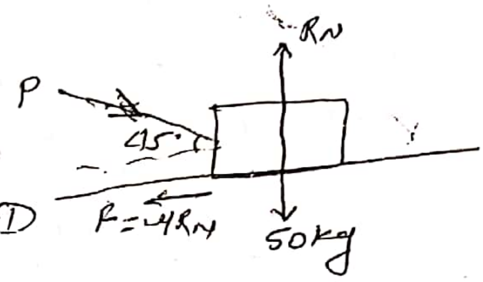
momentum of masses after collision $= m_1v_1 + m_2v_2$

According to law of conservation of momentum

$$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

Q. A body of mass 50 kg rests on a rough horizontal surface ($\mu = 0.4$) and is acted upon by a push applied at an angle of 45° to the horizontal. Determine the magnitude of push if it causes the body to move with an accn of $2\ m/s^2$.

Solⁿ:-



$R_N = P\sin 45 + (50 \times 9.81)$

$R_N = 0.707P + 490.5$ — (1)

$F = \mu R_N$

$= 0.4(0.707P + 490.5)$

$= 0.2828P + 196.2$

Unbalanced forces causing motion.

$f = P\cos 45 - F$

$f = P\cos 45 - (0.2828P + 196.2)$

$f = 0.4242P - 196.2$

Applying newton second law of motion

$f = ma$

$\Rightarrow 0.4242P - 196.2 = 50 \times 2$

$\Rightarrow 0.4242P = 100 + 196.2$

$\Rightarrow P = \dfrac{100 + 196.2}{0.4242} = 698.25\ N$

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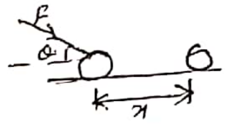
WORK:- work is said to be done when the point of application of force moves in the direction of force. The amt. of workdone equals the product of force and the resulting displacement in the direction of force.



$\boxed{\text{workdone} = \text{Force} \times \text{distance}}$ Joule (J)

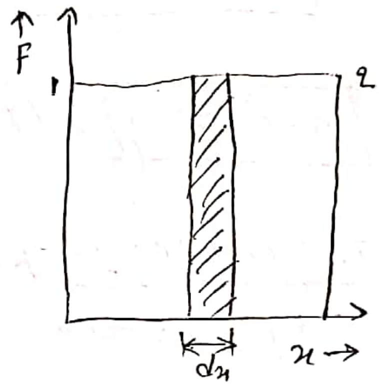
$\boxed{W = F\cos\theta \times x}$ J

$\boxed{W = F x}$ Joule (J).

→ If the force and displacement are at right angle then the workdone will be zero.

→ Workdone is taken to be +ve if the point of application of force moves in the direction of the force

$$\boxed{W = \int_1^2 F\, dx}$$



$\boxed{1\,J = 1\,Nm}$

POWER:- It is the rate of doing work and is obtained by dividing the workdone by time. The unit of power watt (W), kilowatt (kW) or megawatt (MW).

$1\,W = 1\,J/\text{sec.} = 1\,Nm/s$

$1\,kW = 10^3\,W$

$1\,MW = 10^6\,W$

ENERGY:- Capacity to do work is called energy. or The capacity to produce a change from existing conditions is called energy.

POTENTIAL ENERGY:- Potential energy is the energy possessed by a body due to its position or elevation. It represents the ability to do work against the body ~~due to its po~~ weight in lifting it from the surface of earth.

When a body of mass m kg is elevated through a distance h meters above the surface of Earth, then.

wt. of the body $= mg$

P.E. = workdone against gravity

$\boxed{PE = wt. \times height} \Rightarrow \boxed{PE = mgh}$

$\boxed{\Delta PE = mg \times (h_2 - h_1)}$

KINETIC ENERGY:- kinetic energy possessed by a body by virtue of its motion.

Consider a body of constant mass m which moves through a distance dx in the direction of force acting upon it.

Elementry workdone = Force x Elementry distance moved.

$= ma \times dx$

$= m\left(\frac{dv}{dt}\right) \cdot dx$

$= mv\,dv$ $\qquad \frac{dx}{dt} = v$

K.E. gained by the body = Total work done $= \int mv\,dv$

Change in K.E. $= \Delta KE = m\int_1^2 v\,dv = \frac{1}{2}m(v_2^2 - v_1^2)$

$\boxed{K.E. = \frac{1}{2}mv^2}$

Q. A box of mass 80 kg is moving at a speed of 10 m/s on a ropeway. If the box is 40 m above the ground level. Estimate the potential energy and kinetic energy of the box.

Soln:-

$P.E. = mgh = 80 \times 9.81 \times 40 = 31392\, J$

$= 31.392\, kJ$

$K.E. = \frac{1}{2}mv^2 = \frac{1}{2} \times 80 \times 10^2 = 4000\, J$

$= 4\, kJ$

Q. A vehicle accelerates a glider of 125 kg mass from rest to a speed of 50 km/hr. Make calculation for the workdone on the glider by the vehicle. What change would occur in the KE of the glider if subsequently its velocity reduces to 20 km/hr on the application of brakes.

Soln:-

$V_1 = 0\, m/s$

$V_2 = 50\, km/hr = \frac{50 \times 1000}{3600} = 13.89\, m/s$

workdone on the glider in accelerating it from V_1 to V_2 equals to change in KE of the glider

$\therefore$ change in KE = workdone on the glider $= \frac{1}{2}m(V_2^2 - V_1^2)$

$W = \frac{1}{2} \times 125\,(13.89)^2 = 12058\, J$

when application of brake

velocity of glider $(V_3) = 20\, km/hr = 5.56\, m/s$

change in $KE = \frac{1}{2}m(V_3^2 - V_2^2)$

$= \frac{1}{2} \times 125\,(5.56^2 - 13.89^2)$

$= 10126\, J.$

Q. What will be the KE in kWh of an aeroplane which has a mass of 30 tones and is travelling at 1000 km/h speed ? If this plane is made to nose vertically upwards at this speed with power off, calculate the vertical distance through which the plane will move. Take ($g = 9.80$ m/s^2)

Sol^n:- speed of the plane $v = 1000\ km/hr = 277.78\ m/s$

KE of the plane $= \frac{1}{2}mv^2 = \frac{1}{2} \times (30 \times 10^3) \times (277.78)^2$

$= 7716 1743\ J = 77162\ KJ$.

$1\ kWh = 3600\ KJ$

$\therefore$ KE of the plane in kWh $= \frac{77162}{3600} = 21.43\ kWh$

When the power is off and the plane is made to move vertically upwards, the KE gets converted to potential energy. thus

$$77161743 = mgH$$

$$\Rightarrow H = \frac{77161743}{mg}$$

$$= \frac{77161743}{(30 \times 10^3) \times 9.80}$$

$$= \frac{77161743}{294000}$$

$$\boxed{H = 262.45\ m}$$

IMPULSE:- The product of the force and the time during which it acts ($F \times t$) is called the impulse of force. Thus the impulse equals the change in momentum.

change in momentum $= m(v-u)$ kg m/s or NS

Impulse $= F \times t$ NS

$\therefore$ $\boxed{\text{Impulse} = \text{change in momentum}}$

$\Rightarrow \boxed{F \times t = m(v-u)}$ NS

Q. A cricket ball of mass 175 gm is moving with a speed of 36 km/hr. What average force will be required to stop the ball in 0.02 seconds.

Solⁿ:-

$$u = 36\,km/hr = \frac{36 \times 1000}{3600} = 10\,m/s$$

From the impulse momentum relation

$$F \times t = m(v-u)$$

$$\Rightarrow F \times 0.2 = m(0-10)$$

$$\Rightarrow F = \frac{0.175 \times -10}{0.2}$$

$$\Rightarrow \boxed{F = -8.75\,N}$$

Q: A pile hammer of 250 kg mass is made to fall freely on a pile from a height of 6 m. If the hammer comes to rest in 0.012 sec., determine the change in momentum, impulse and average force.

Soln:-

$\because u = 0$

$\because v^2 = u^2 + 2gH$

$v^2 = 0 + 2gH$

$\Rightarrow v = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 6}$

$\Rightarrow v = 10.85 \text{ m/s}$

change in momentum $= m(v-u)$

$= 250(10.85 - 0)$

$= 2712.5 \text{ Ns}$

$\because$ change in momentum = Impulse

$2712.5 = F \times t$

$\Rightarrow F = \dfrac{2712.5}{t}$

$\Rightarrow F = \dfrac{2712.5}{0.012}$

$\Rightarrow \boxed{F = 226042 \text{ N}}$

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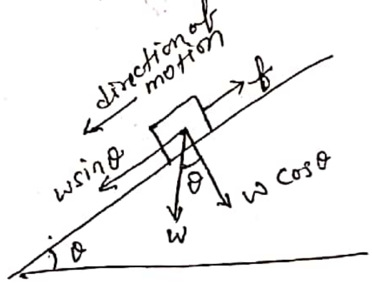
Q. A Truck weighing 5kN just moves freely (engine is not running) at 30 km/hr down a slope of 1 in 50, and the road resistance at this speed is just sufficient to prevent any acceleration. Estimate the road resistance per kN weight of truck.

What power will the engine have to exert to run up the same slope at double the speed when the road resistance remains the same? Take efficiency of engine as 40%.

Soln:- ① Truck moves freely down the plane

$u = 30 \text{ km/hr} = \frac{30 \times 1000}{3600} = \cancel{10 \text{ m/s}}$

$= 8.33 \text{ m/s}$



Slope of the truck = 1 in 50

$\sin\theta = \tan\theta = \frac{1}{50}$

Net force on the truck in the direction of motion

$F = W\sin\theta - f$

As the truck moves freely with uniform velocity, its accn is zero and so will be the net force $F = ma$

$F = w\sin\theta - f$

$\Rightarrow ma = w\sin\theta - f$

$\Rightarrow 0 = w\sin\theta - f$

$\Rightarrow f = w\sin\theta$

$\Rightarrow f = 5 \times 10^3 \sin \frac{1}{50}$

$\Rightarrow \boxed{f = 100 N}$ ← Total road resistance

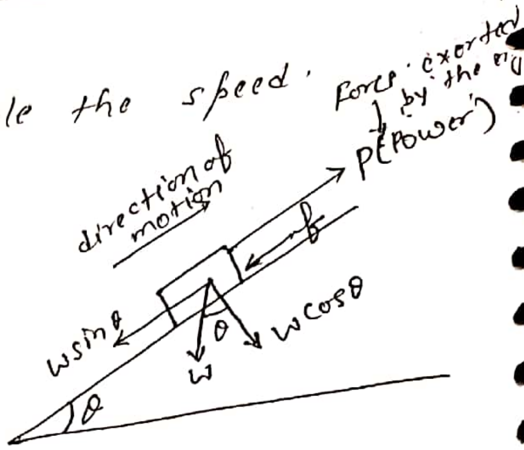
Road resistance per kN wt. of truck $= \frac{100}{5} = 20 N$

(ii) Truck moves upward with double the speed.

speed of truck $(v) = 2u$

$= 2 \times 8.33 = 16.66\ m/s$

Road resistance $(f) = 100\ N$ (same as before)



slope $\sin\theta = \tan\theta = \frac{1}{50}$

Net force on the truck in the direction of motion

$F = P - f - w\sin\theta$

Again the truck moves upward with uniform speed

i.e. no acceleration

$\therefore\ ma = P - f - w\sin\theta$

$\Rightarrow\ 0 = P - f - w\sin\theta$

$\Rightarrow\ P = w\sin\theta + f$

$\Rightarrow\ P = \left(5\times10^3 \times \frac{1}{50}\right) + 100$

$\Rightarrow\ P = 200\ N$

Output power required $= P \times v$

$= 200 \times 16.66$

$= 3332\ W = 3.332\ kW$

$\because\ \eta = \frac{\text{output}}{\text{Input}}$

$\Rightarrow$ Input power of engine $= \frac{\text{output power}}{\eta \text{ of engine}}$

$= \frac{3.332}{0.4} = 8.33\ kW$

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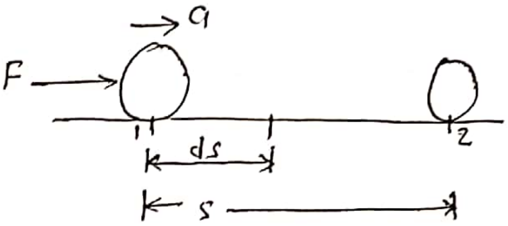
WORK - ENERGY PRINCIPLE:-



consider a force F acting on an object which may be displaced from position 1 to position 2 over the course of action covering distance S as shown in fig. When the object is subjected to more than one force, then F represents the net force or the resultant force acting on the object.

For an elementary distance ds travelled by the object in time dt, the work done by the force would be

$$dW = F \cdot ds \quad \text{——— (I)}$$

from Newton second law of motion

$$F = ma$$

$$\Rightarrow F = m\frac{dv}{dt}$$

$$\Rightarrow F = m\frac{dv}{ds} \times \frac{ds}{dt}$$

$$\Rightarrow F = m \cdot v \cdot \frac{dv}{ds}$$

put the value of F in eqn (I) we get

$$dW = mv\frac{dv}{\cancel{ds}} \cdot \cancel{ds}$$

$$\Rightarrow dW = mv\,dv \quad \text{——— (II)}$$

Integrating the above equations

$$\int_1^2 dW = m\int_1^2 v\,dv$$

$$W_{1\text{-}2} = m \times \frac{1}{2}\left[v^2\right]_1^2 = m \times \frac{1}{2}\left[v_2^2 - v_1^2\right]$$

$$\boxed{W_{1\text{-}2} = (K \cdot E)_2 - (K E)_1}$$

CONSERVATION OF MECHANICAL ENERGY:—

OR

CONSERVATION OF ENERGY:—

The total energy possessed by an object remains constant provided no energy is added to or substracted from it.

or

The energy can neither be created nor be destroyed through it can be transfered from one form to another.

In the conservative force field the workdone ~~with that~~ in moving an object from position 1 to 2 is independent of the path followed and depends on a change in the potential energy of the end states only.

$$W_{1-2} = (PE)_1 - (PE)_2 \quad \text{——— (I)}$$

$$\because W_{1-2} = (KE)_2 - (KE)_1 \quad \text{——— (II)}$$

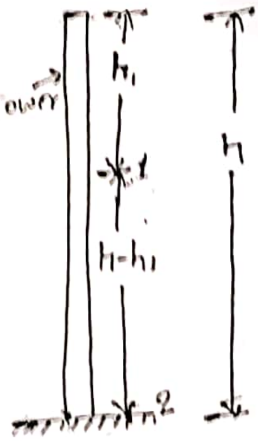
from eqn (I) and (II) we get

$$(PE)_1 - (PE)_2 = (KE)_2 - (KE)_1$$

$$\Rightarrow \boxed{(P.E + K.E)_1 = (PE + KE)_2 = \text{constant}}$$

The function (PE+KE) i.e. the sum of potential energy and kinetic energy of an object is called mechanical energy.

consider a body of mass m resting on the top of a tower of height 'h'. at this position



K.E. of body $= \frac{1}{2}mv^2$

$= 0$

P.E. of the body $= mgh$

$\therefore$ Total energy of the body ~~with respect to ground~~ at the top of tower is $= KE + PE$

$= 0 + mgh = mgh$ ——— (I)

Energy at position 1

$\because u = 0$
$v = v_1$
$h = h_1$ from the top.

$\because$ We know that

$v^2 = u^2 + 2gh$

$\Rightarrow v_1^2 = 0 + 2gh_1$

$\Rightarrow v_1^2 = 2gh_1$ ——— (a)

K.E. of the body at position 1 $= \frac{1}{2}mv_1^2$

$= \frac{1}{2}m\,2gh_1 = mgh_1$ ——— (b)

P.E. of the body at position 1 $= mg(h-h_1)$ ——— (c)

$\therefore$ Total energy of the body at the position 1

$= KE + PE$

$= mgh_1 + mgh - mgh_1$

$= mgh$ ——— (II)

Energy at position 2

$\because$ $v = v_2$
$u = 0$

$\because$ we know that

$v^2 = u^2 + 2gh$

$\Rightarrow v_2^2 = 0 + 2gh$

$\Rightarrow v_2^2 = 2gh$

$\therefore$ K.E. of the body at ground level $= \frac{1}{2} m v_2^2$

$= \frac{1}{2} \times m \times 2gh$

$= mgh$ ——— (A)

P.E. of the body at ground level $= mgh$

$= 0$ ——— (B)

$\therefore$ Total energy of the body at ground level

$= KE + PE$

$= mgh + 0$

$= mgh$ ——— (III)

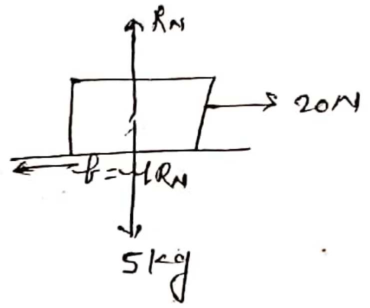
Thus the total energy of an object is always same or remain constat during its motion.

Q. A body of 5 kg mass is initially at rest on a rough horizontal surface ($\mu = 0.2$) and is acted upon by a 20N pull applied horizontally. calculate :-

(a) The workdone by the net force on the body in 5sec.

(b) change in KE of the body in 5 sec. Comments on the result.

Soln:-



$R_N = 5 \times 9.81 = 49.05N$

$f = \mu R_N = 0.2 \times 49.05 = 9.81$

Net force causing motion

$F = 20 - f$

$\Rightarrow ma = 20 - 9.81 = 10.19N$ Net force causing motion

$\Rightarrow 5 \times a = 10.19 N$

$\Rightarrow a = \frac{10.19}{5} = 2.038 \, m/s^2$

the distance moved by the body in 5 sec.

$s = ut + \frac{1}{2}at^2$

$\Rightarrow s = 0 \times 5 + \frac{1}{2} \times 2.038 \times 5^2$

$\Rightarrow s = 25.475 \, m$

(a) workdone by the net force $= F \cdot s$

$= 10.19 \times 25.475$

$= 259.59 \, Nm$

(b) The body starts from rest ($u=0$) according to this the initial KE of the body is zero. $\boxed{(K.E.)_i = 0}$

initial K.E. $= \frac{1}{2}mu^2 = \frac{1}{2}m \times 0 = 0$

Final K.E $= \frac{1}{2}mv^2$

$\because v = u + at$

$\Rightarrow v = 0 + 2.038 \times 5$

$\Rightarrow v = 10.19\ m/s$

$\therefore (K.E.)_2 = \frac{1}{2} m v^2$

$= \frac{1}{2} \times 5 \times (10.19)^2$

$= 259.59\ Nm$

$\therefore$ change in $K.E. = (KE)_2 - (KE)_1$

$= 259.59 - 0$

$= 259.59\ Nm$

Comment :- workdone by net force on the body is equal to the change in kinetic energy of the body. This is according with the work-energy principle.

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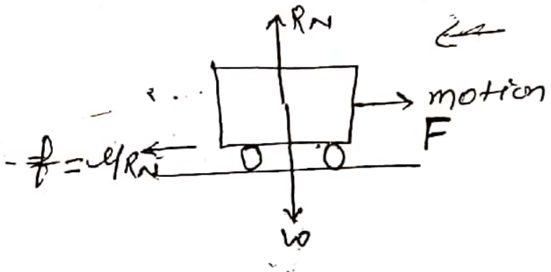
Q: An Automobile is travelling along a straight level highway when the brakes are applied, the vehicle slides for 2 sec. and covers a distance of 10m before coming to rest. Assuming that the automobile moves with constant deacceleration during this period, Determine the co-efficient of friction b/w the tyres and the road.

Soln:-



$\because v = u + at$

$\Rightarrow 0 = u + a \times 2$

$\Rightarrow u = -2a$

$s = ut + \frac{1}{2}at^2$

$10 = -2a \times 2 + \frac{1}{2} a \times 4$

$\Rightarrow 10 = -2a$

$\Rightarrow \boxed{a = -5 \ m/s^2}$

$R_N = W$

$R_N = mg$

Retarding force = F

$\Rightarrow f = F$

$\Rightarrow \mu R_N = ma$

$\Rightarrow \mu \times W = ma$

$\Rightarrow \mu \times \cancel{m}g = \cancel{m}a$

$\Rightarrow \mu = \frac{a}{g}$

$\Rightarrow \mu = \frac{5}{9.81}$

$\Rightarrow \boxed{\mu = 0.51}$

$f + F = 0$

$\Rightarrow f = -F$

$\Rightarrow \mu W = -(+ma)$

$\mu W = -(m.-5)$

$\mu W = +5m$

Q: A bullet of mass 0.1 kg and travelling at a speed of 180 m/s penetrated 10 cm when fired into a wooden log. Determine the velocity with which this bullet would emerge when fired with the same velocity into a similar 5 cm thick wooden plank. Also determine the force of resistance assuming it to be uniform.

Soln:- $s = 10\,cm = 0.1\,m$

$\because v^2 = u^2 + 2as$

$\Rightarrow v^2 - u^2 = 2as$

$\Rightarrow 0 - (180)^2 = 2a \times 0.1$

$\Rightarrow a = \frac{-(180 \times 180)}{0.2}$

$\Rightarrow a = -162000\ m/s^2$

Let the bullet emerge with velocity v from the 5 mm thick plank of wood. $s = 5\,cm = 0.05$

$v^2 = u^2 + 2as$

$\Rightarrow v^2 = (180)^2 - 2 \times 162000 \times 0.05$

$\Rightarrow v^2 = 32400 - 16200 = 16200$

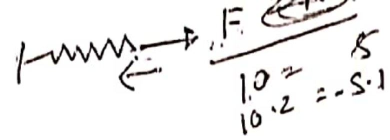
$\Rightarrow \boxed{v = 127.28\ m/s}$

Resistance force $= F$.

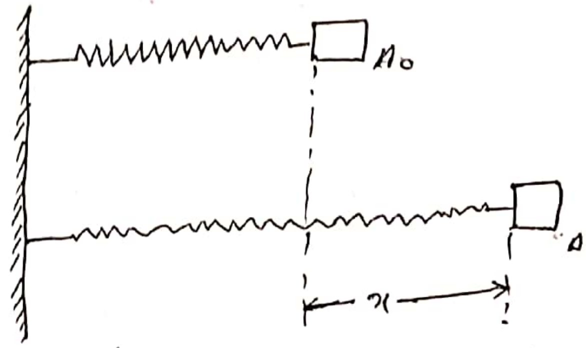
$f = ma$

$\Rightarrow f = 0.1 \times 162000$

$\Rightarrow \boxed{f = 16200 N}$

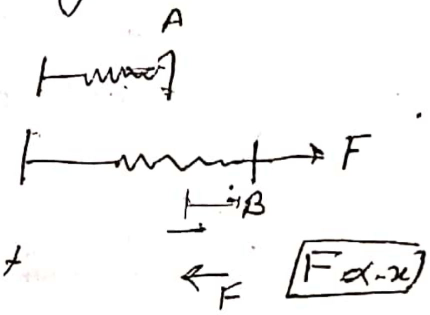


WORKDONE BY A SPRING:-



consider a body attached to a fixed support by a spring. Initially when the body is at position A_0, the spring is underformed. When the body is acted upon by a force, the spring extends and the body shifted to a new position which depends upon the magnitude of applied force.

When a spring is deformed (compressed or elongated) a corresponding force is developed in the spring. The magnitude of force developed is directly proportional to the displacement of spring from the undeformed position.



$F \propto x$

$\Rightarrow \boxed{F = kx}$

$\Rightarrow \boxed{k = \frac{F}{x}}$ N/m ← spring constent

$\boxed{F \propto -x}$

Elementry workdone by the spring force

$dw = -F\,dx$

$\Rightarrow dw = -kx\,dx$

-ve siagn shows the force induced in the spring acts in a direction opposite to that of displacement.

$\Rightarrow \int dw = -\int_0^x kx\,dx$

$\Rightarrow \boxed{W = -\frac{1}{2}kx^2}$

If displacement of body from x_1 to x_2 then

$\int dw = -\int_{x_1}^{x_2} kx\,dx$

$\boxed{W = -\frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}k(x_1^2 - x_2^2)}$

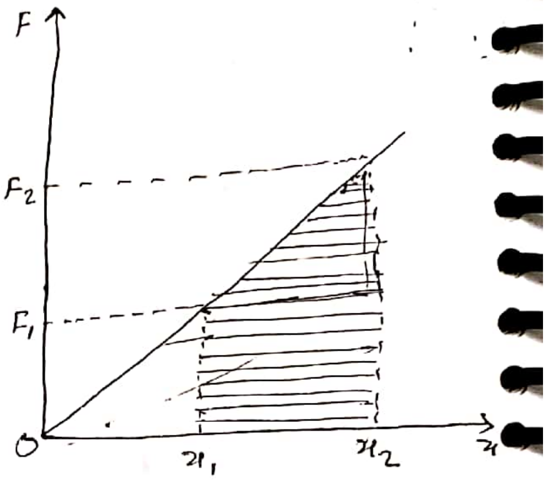
The workdone by the spring force is represented by the shaded area

$$W = -\frac{F_2 + F_1}{2} \times (x_2 - x_1)$$

$$W = -\frac{kx_2 + kx_1}{2} \times (x_2 - x_1)$$

$$W = \frac{1}{2} k (x_1 + x_2)(x_1 - x_2)$$

$$\boxed{W = \frac{1}{2} k (x_1^2 - x_2^2)}$$



$= \frac{1}{2}(F_2 - F_1)(x_2 - x_1) + (F_1 - 0)($

Q. The spring in a gun is held in vertical position and compressed by 0.25 m. A ball of 40N wt. is placed on the compressed spring which is subsequently released. Calculate the height ~~to~~ which the ball rise and the velocity it will attain at a height of 2 m. It may be presumed that the spring has a stiffness of 5000 N/m.

Soln:- When the spring is compressed, it has a potential energy stored in it $= \frac{1}{2}kx^2 = \frac{1}{2} \times 5000 \times (0.25)^2 = 156.25$ Nm

The potential energy of the spring gets transformed into potential energy and kinetictic energy of the ball

(a) Let the ball rise to a maximum height of h m. At maximum height the velocity of the ball will be ze

$$156.25 = PE + KE$$

$$\Rightarrow 156.25 = mgh + \frac{1}{2}mv^2$$

$$\Rightarrow 156.25 = 40h + 0$$

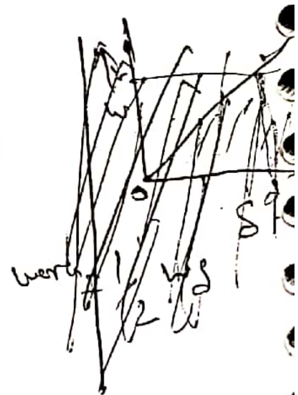
$$\Rightarrow h = \frac{156.25}{40} = 3.9 \text{ m}$$

(b) At height of 2 m velocity ($v = ?$)

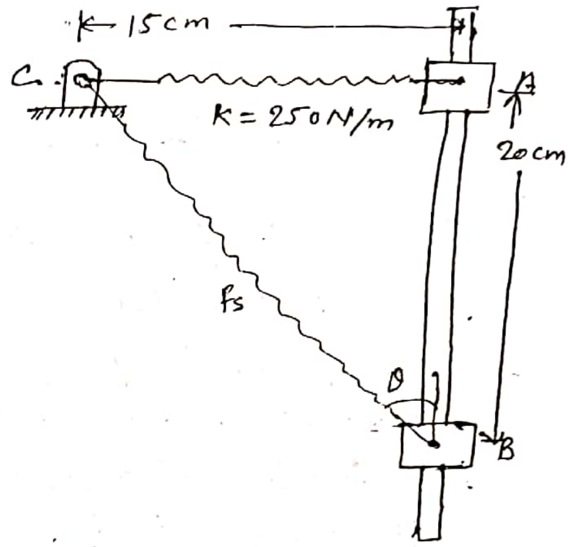
$$156.25 = mgh + \frac{1}{2}mv^2$$

$$\Rightarrow 156.25 = (40 \times 2) + \frac{1}{2} \times \left(\frac{40}{9.81}\right) \times v^2$$

$$\Rightarrow \boxed{v = 6.11} \text{ m/s}$$



Q: A collar of 5 kg mass slides without friction along a vertical rod as shown in fig. The spring attached to the collar has a spring constant of 250 N/m and its undeformed length is 15 cm. The collar is released from rest at position A and slides 20 cm downward to position B. What will be the velocity of collar at position B?



Soln:-

Extension of spring

$x = BC - AC$

$= \sqrt{(AC)^2 + (AB)^2} - AC$

$= \sqrt{15^2 + 20^2} - 15$

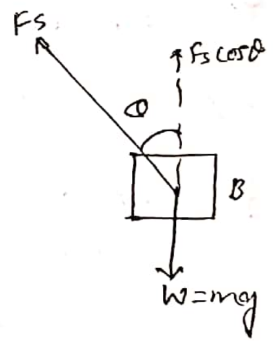
$= 10 \text{ cm} = 0.1 \text{ m}$

Force exerted by spring

$F_s = kx$

$= 250 \times 0.1 = 25 \text{ N}$

F.B.D at collar



$\tan\theta = \frac{AC}{AB} = \frac{15}{20} = 0.667$

$\Rightarrow \theta = \tan^{-1} 0.667$

$\Rightarrow \theta = \cancel{33.70^\circ}\ 36.86$

From the eqn of equilibrium

$\Sigma F_y = ma$

$\Rightarrow mg - F_s \cos\theta = ma$

$\Rightarrow (5 \times 9.81) - 25 \cos 33.70 = 5 \times a$

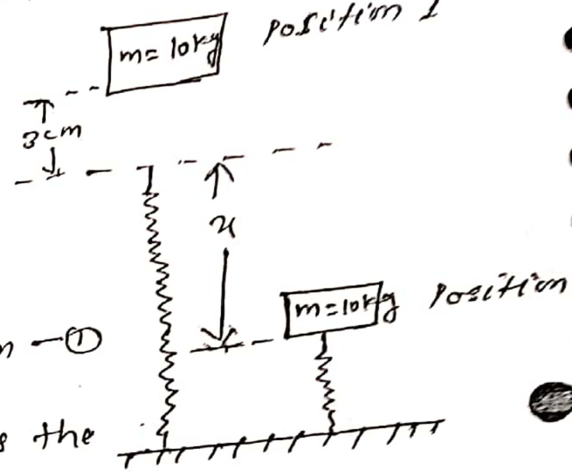
$\Rightarrow a = \frac{5 \times 9.81 - 25 \times 0.832}{5} = 5.65 \text{ m/s}^2$

∵ we know that

$v^2 = u^2 + 2as$ $\quad s = 20 \text{ cm} = 0.2 \text{ m}$

$\Rightarrow v^2 = 0 + 2 \times 5.65 \times 0.2$ $\quad \Rightarrow v = 1.50 \text{ m/s.}$

Q. A body of mass 10 kg is made to fall 3 cm height on a spring of stiffness 120 N/cm. Find the displacement of spring. Use the concept of that ~~total~~ energy of the mass spring system remains constant.



Soln.:-

At position 1:- The body has only potential energy and the spring has no energy as it is not deformed.

$$E_1 = mgh = 10 \times 9.81 \times 3 = 294.3 \text{ N-cm} \quad \text{(I)}$$

At position 2:- The body as well as the spring have potential energy.

$$E_2 = -mgx + \tfrac{1}{2}Kx^2 \qquad x \rightarrow \text{compression of spring}$$

$$\Rightarrow E_2 = -10 \times 9.81 \times x + \tfrac{1}{2} \times 120 \times x^2$$

$$\Rightarrow E_2 = -98.1x + 60x^2 \quad \text{(II)}$$

From law of conservation of Energy

$$E_1 = E_2$$

$$294.3 = -98.1x + 60x^2$$

$$\Rightarrow 60x^2 - 98.1x - 294.3 = 0$$

$$\Rightarrow x^2 - 1.635x - 4.905 = 0$$

$$\Rightarrow x = \frac{-(-1.635) \pm \sqrt{\cancel{4 \times 1 \times 4}\ (-1.635)^2 - 4 \times 1 \times (-4.905)}}{2 \times 1}$$

$$\Rightarrow x = \frac{1.635 \pm \sqrt{2.673 + 19.62}}{2} = \frac{1.635 \pm 4.721}{2}$$

$$\Rightarrow x = \frac{1.635 + 4.721}{2} = 3.178$$

$$\Rightarrow \boxed{x = 3.178 \text{ cm}}$$

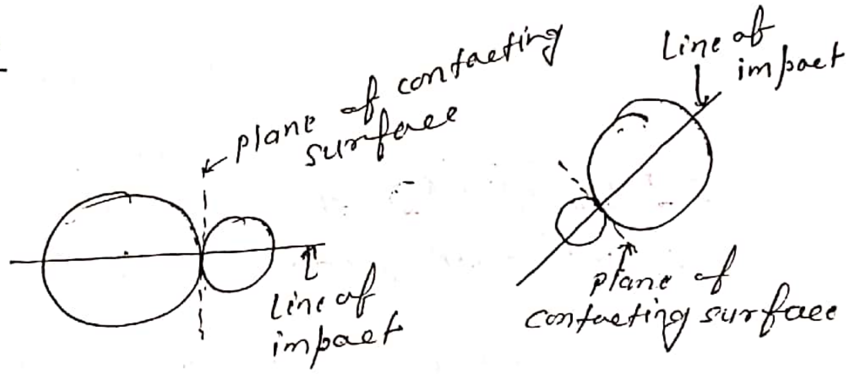
IMPACT:- collision means contact b/w two bodies for a short duration. During collision the bodies produce impulsive forces on each other and this impulsive force is much larger than any other finite force that may be acting.

The phenomenon of collision b/w two bodies which occurs in a very short duration of time and during which the bodies exert relatively larger forces on each other is called an impact.

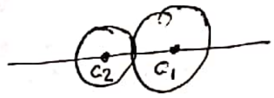
TYPES OF IMPACT:-

Line of impact:-

→ The line joining the centres of colliding bodies and passing through the point of contact is called the line of impact.

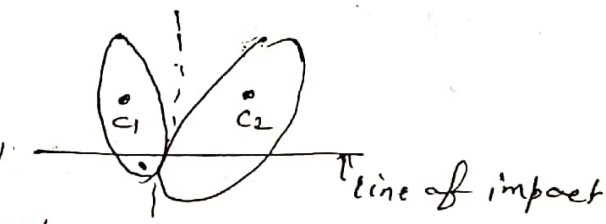


CENTRAL IMPACT:- The impact is called central when the mass centres of the colliding bodies are located on the line of impact.

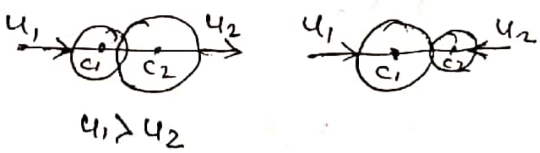


ECCENTRIC IMPACT:-

When the centre of mass of colliding bodies are not located on the line of impact, this type of impact is called eccentric impact.

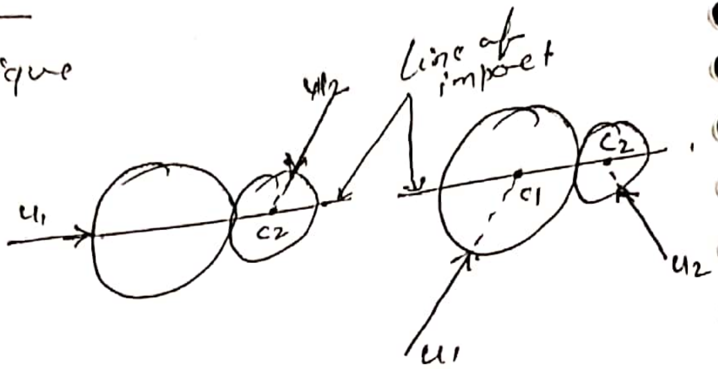


DIRECT IMPACT:- The impact is said to be direct if before impact the bodies are moving along the line of impact i.e the motion of the colliding bodies is directed along the line of impact.



INDIRECT OR OBLIQUE IMPACT:-

The impact is indirect or oblique if the motion of one or both of colliding bodies before impact is not directed along the line of impact.



NOTE:-

→ The property of bodies which leads to rebound after impact is called elasticity.

→ The impact is elastic if the body rebonds after impact.

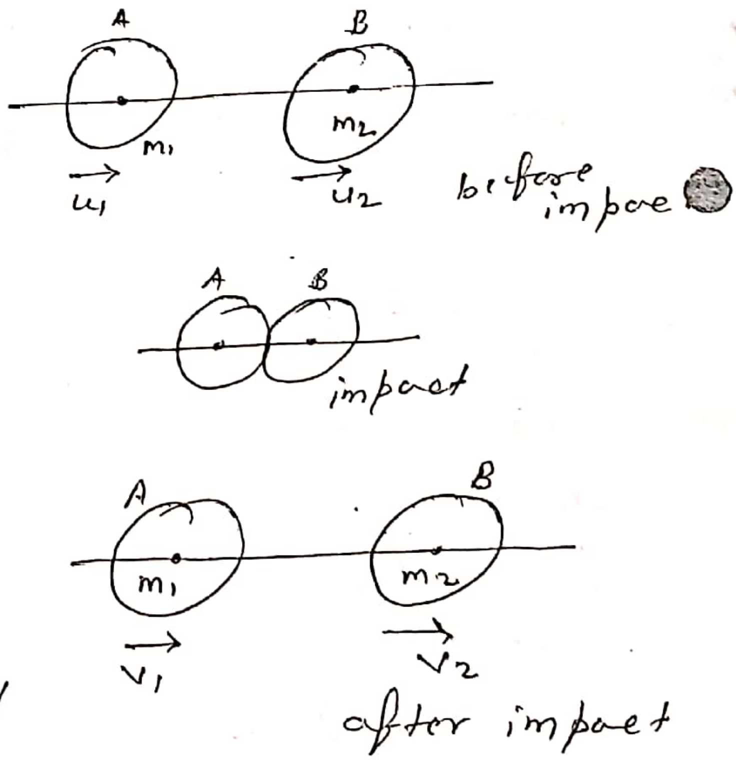
→ Greater the elasticity of the body, greater will be the rebound.

→ The impact is inelastic if the body does not rebound at all.

CONSERVATION OF MOMENTUM:-

consider two bodies A and B of mass m_1 and m_2 respectively.

Let these bodies be moving with respective velocity of u_1 and u_2 before impact and v_1 and v_2 after impact.



During collision, there is an impulse (F×t) exerted by body 'A' on body 'B'.

This impulse on body B is measured by the change in its momentum.

Impulse on body B = change in momentum of body B.

$$\Rightarrow \quad F \times t = m_2 V_2 - m_2 u_2 \quad \text{——— (I)}$$

According to Newton's third law of motion, Action and reaction b/w the colliding bodies is equal in magnitude and opposite in direction, and it acts for the same time. then impulse on body A will be -

Impulse on body A = change in momentum of body A.

$$\Rightarrow -F \times t = m_1 V_1 - m_1 u_1$$

$$\Rightarrow F \times t = m_1 u_1 - m_1 V_1 \quad \text{——— (II)}$$

From eq^n (I) and (II) we get

$$m_1 u_1 - m_1 V_1 = m_2 V_2 - m_2 u_2$$

$$\Rightarrow \boxed{m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2}$$

Thus the momentum before collison is equal to the momentum after collision.

NEWTON'S LAW OF COLLISION :-

or

COEFFICIENT OF RESTITUTION :-

consider two bodies A and B of mass m_1 and m_2 respectively. ~~Th~~ Let these bodies be moving with respective velocities u_1 and u_2 before impact. The impact will take place only if $u_1 > u_2$

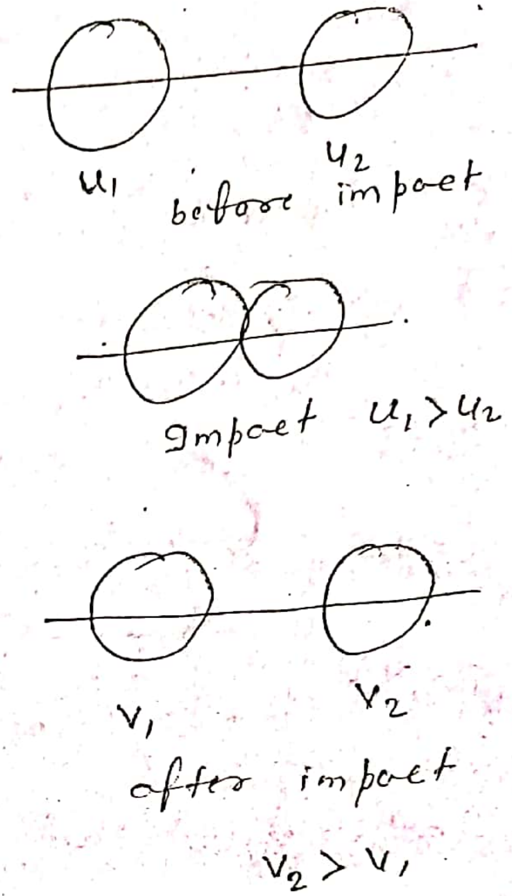
∴ Velocity of approach = $(u_1 - u_2)$

After a short period of contact, the bodies will separate and will start moving with velocity V_1 and V_2 The seperation will occur only if $V_2 > V_1$

∴ Velocity of separation $= V_2 - V_1$

Newton's law of collision :-

When two ~~bodies~~ moving bodies collides with each other their velocity of separation bears constant ratio to their velocity of approach.



$$V_2 - V_1 = e\,(u_1 - u_2)$$

$$\Rightarrow \boxed{e = \frac{V_2 - V_1}{u_1 - u_2}} \leftarrow \text{coefficient of restitution.}$$

$$\Rightarrow \boxed{e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}}$$

↑ co-efficient of restitution

→ The value of coefficient of restitution (e) lies between 0 and 1.

→ If $e = 0$ the body are inelastic

→ If $e = 1$ the bodies are perfectly elastic.

→ The value of coefficient of restitution depends not only on the material but it also depends on the shape and size of the body.