Engineering Mechanics (3ME3-04)

DEPARTMENT

OF

MECHANICAL ENGINEERING

(Jaipur Engineering College and Research Centre, Jaipur)

UNIT: IV

Kinematics of particles and rigid bodies: Velocity, Acceleration, Types of Motion, Equations of Motion, Rectangular components of velocity and acceleration, Angular velocity and Angular acceleration, Radial and transverse velocities and accelerations, Projectiles motion on plane and Inclined Plane, Relative Motion.

Kinetics of particles and rigid bodies: Newton's second law, Equation of motion in rectangular coordinate, Equation of motion in radial and transverse components, Equation of motion in plane for a rigid body, D'Alembert principle.

Faculty: AKHILESH PALIWAL
(Assistant Professor)

UNIT-4

9.10, 11 → 17 → 105 → 50% = 65

KINEMATICS OF PARTICLES AND RIGID BODY

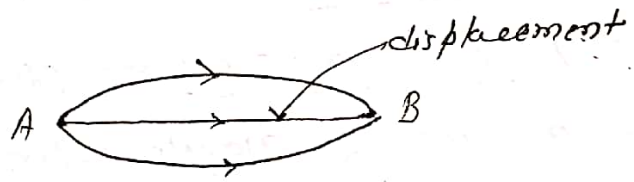
Dynamic (motion) → kinematics, kinetics

KINEMATICS:- kinematics is concerned with the description of motion of objects independent of causes of motion.

KINETICS:- kinetics relates to action of forces and the resulting motion.

SOME BASIC DEFINITIONS:-

DISPLACEMENT:- The displacement of a particle is defined as the ~~distance~~ change in its position of particle in a definite direction. It is measured by a straight distance b/w the initial and final position of particles.



VELOCITY:- The rate of change of displacement with respect to time in a specific direction is called velocity.

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}} \ m/s$$

→ velocity is a vector quantity because it has ~~both~~ both magnitude and direction. The magnitude of velocity is called speed.

AVERAGE VELOCITY:- The total displacement covered in total time is called average velocity.

$$\boxed{V_{avg} = \frac{\Delta x}{\Delta t}}$$

$\Delta x \rightarrow$ Total displacement
$\Delta t \rightarrow$ Time interval

INSTANTANEOUS VELOCITY:- The velocity at a particular instant of time is called Instantaneous velocity.

$$\boxed{V_{instan} = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta t}} = \frac{dx}{dt}$$

ACCELERATION:- Acceleration is defined as the rate of change of velocity with respect to time is called acceleration.

$$\boxed{\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}} \; m/s^2$$

$$\boxed{a = \frac{v-u}{t}} \; m/s^2$$

TYPE'S OF MOTION:-

(I) **RECTILINEAR MOTION:-** Rectilinear motion occurs when a particle move's along a straight line path

(II) **CURVILINEAR MOTION:-** Curvilinear motion occurs when a particle moves along a curved line path.

(III) **CIRCULAR MOTION OR ROTARY MOTION:-** Circular or rotary motion occurs when a particle move along a circular line path.

EQUATIONS OF RECTILINEAR MOTION

When a body moves in a straight line with uniform acceleration the equations of motions are —

(I) $v = u + at$

(II) $v^2 = u^2 + 2as$

(III) $s = ut + \frac{1}{2}at^2$

(IV) $S_n = u + \frac{a}{2}(2n-1)$ distance travelled in n^{th} second.

(I) $v = u + at$

a = Acceleration = Rate of change of velocity

let, $v \rightarrow$ final velocity

$u \rightarrow$ initial velocity

$$a = \frac{v-u}{t}$$

$$\Rightarrow at = v - u$$

$$\Rightarrow \boxed{v = u + at} \text{ ——— (I)}$$

(III) $s = ut + \frac{1}{2}at^2$

distance travelled = Average velocity × time.

$$s = \frac{u+v}{2} \times t$$

$$\Rightarrow s = \frac{u + u + at}{2} \times t$$

From eq^n (I)

$v = u + at$

$$\Rightarrow s = \left(\frac{2u + at}{2}\right)t$$

$$\Rightarrow s = \left(u + \frac{at}{2}\right)t$$

$$\Rightarrow \boxed{s = ut + \frac{1}{2}at^2} \text{ ——— (II)}$$

(III) $v^2 = u^2 + 2as$

distance = Average velocity × time

$$s = \frac{u+v}{2} \times t$$

From eqn (1)
$v = u + at$
$\Rightarrow v - u = at$
$\Rightarrow t = \frac{v-u}{a}$

$$\Rightarrow s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$\Rightarrow s = \frac{v^2 - u^2}{2a}$$

$$\Rightarrow v^2 - u^2 = 2as$$

$$\Rightarrow \boxed{v^2 = u^2 + 2as} \quad \text{(III)}$$

(IV) $S_{nth} = u + \frac{a}{2}(2n-1)$

distance covered in n seconds

$$S_n = un + \frac{1}{2}an^2 \qquad \because s = ut + \tfrac{1}{2}at^2$$

distance covered in (n−1) seconds

$$S_{(n-1)} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$= u(n-1) + \frac{1}{2}a(n^2 + 1 - 2n)$$

distance covered in nth second

$$S_{nth} = S_n - S_{n-1}$$

$$= un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n^2+1-2n)$$

$$= \cancel{un} + \cancel{\frac{1}{2}an^2} - \cancel{un} - u - \cancel{\frac{1}{2}an^2} - \frac{1}{2}a + an$$

$$= u - \frac{1}{2}a + an = u + an - \frac{1}{2}a$$

$$\boxed{S_{nth} = u + \frac{a}{2}(2n-1)} \quad \text{(IV)}$$

Q. A car travels from one station to another along a straight ~~Line~~ road. First half of the distance is covered with velocity of 60 km/hr and the second half is covered with velocity 90 km/hr. Determine the average speed of the motor.

Soln:- Let total distance covered by car $= S$.

Now, Let $\frac{S}{2}$ distance is covered by the car in t_1 hour with the velocity of 60 km/hr $= \frac{S/2}{60}$

$$\Rightarrow t_1 = \frac{S}{120} \text{ hr}$$

Next $S/2$ distance is covered by the car in t_2 hour with the velocity of 90 km/hr $= \frac{S/2}{90}$

$$\Rightarrow t_2 = \frac{S}{180} \text{ hr.}$$

$$\text{Total time taken } (t) = t_1 + t_2 = \frac{S}{120} + \frac{S}{180}$$

$$= \frac{3S + 2S}{360} = \frac{5S}{\cancel{360}\,72} = \frac{S}{72}$$

Now

$$V_{avg} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$V_{avg} = \frac{S}{t}$$

$$= \frac{S}{S/72}$$

$$= \frac{\cancel{S}}{\cancel{S}} \times 72$$

$$= 72 \text{ km/hr}$$

Imp Q. A ~~train~~ toy car accelerates from rest at a constant rate of $2\,m/s^2$ for some time. Then it retards at a constant rate of $4\,m/s^2$ and comes to rest. If the car remain in motion for 3 seconds. Determine the maximum speed attained at the total distance travelled by the car.

Soln:- ~~given~~

$$\boxed{v = u + a.t}$$

Let v be the maximum speed attained by the car

$$v = 0 + a_1 t_1$$

$\Rightarrow v = a_1 t_1 \rightarrow$ when car accelerates

$\Rightarrow 0 = v - a_2 t_2$

$\Rightarrow v = a_2 t_2 \rightarrow$ when car retards

$$v = a_1 t_1 = a_2 t_2$$

$$\Rightarrow t_1 = \frac{a_2}{a_1} t_2 = \frac{4}{2} t_2 = 2t_2$$

Total time take for ~~rest~~ come into rest

$\therefore t = t_1 + t_2$

$\Rightarrow 3 = 2t_2 + t_2$

$\Rightarrow 3 = 3t_2$

$\Rightarrow t_2 = 1\,sec$

$\therefore t_1 = 2t_2 = 2\,Sec.$

$$\boxed{v = a_1 t_1 = 2\times 2 = 4\,m/s}$$

$\because s = ut + \frac{1}{2}at^2$

$s_1 = 0 * 2 + \frac{1}{2} \times 2 \times 2^2$

$\Rightarrow s_1 = 4\,m$ when accelerated

$s_2 = 4\times 1 + \frac{1}{2}(-4)\times 1^2$

$= 4 - 2 = 2\,m$ when car deaccelerated

Total distance travelled by the car to come to rest

$s = s_1 + s_2 = 4 + 2$

$= 6\,m$

MOTION UNDER GRAVITY:—

When a body falls freely, then its velocity increases as it approaches the earth. The increase in the velocity of falling body is due to gravitational acceleration (g) whose value is normally taken as- $9.81\ m/s^2$.

For rectilinear motion under gravity:-

(I) $v = u + gt$

(II) $v^2 = u^2 + 2gh$

(III) $S_{nth} = u + \frac{g}{2}(2n-1)$

(IV) $S = ut + \frac{1}{2}gt^2$

→ At the point of maximum height attained by a body thrown vertically upwards, the velocity of v of body becomes zero.

$\therefore\ v = u - gt$

$\Rightarrow 0 = u - gt$

$\Rightarrow \boxed{t = \frac{u}{g}}$ ← Time taken in attaining maximum height.

$\because\ v^2 = u^2 - 2gh$

$\Rightarrow 0 = u^2 - 2gh$

$\Rightarrow \boxed{h = \frac{u^2}{2g}}$ ← Maximum height attained.

→ When the body falls down freely then the striking velocity to the earth of the particle

$v^2 = u^2 + 2gh$

$\Rightarrow v^2 = 0 + 2gh$ $\Rightarrow \boxed{v = \sqrt{2gh}}$

Q. A stone is dropped into a well and the sound of splash is heared after 4 sec. Assuming velocity of sounds to be 350 m/s, make calculations for the depth of well.

Soln:- Let,

$h \rightarrow$ depth of well

$t_1 \rightarrow$ Time taken by the stone to strike water

$t_2 \rightarrow$ Time taken by sound to reach from surface of water to top of well.

$$t_1 + t_2 = 4 \quad \text{——— (I)}$$

$$\because \quad h = ut_1 + \tfrac{1}{2}gt_1^2$$

$$h = 0 \times t_1 + \tfrac{1}{2} \times 9.81 \times t_1^2$$

$$\Rightarrow h = 4.905\, t_1^2 \quad \text{——— (II)}$$

$$t_2 = \frac{(h)\text{ depth of well}}{\text{Velocity of sound}} = \frac{4.905\, t_1^2}{350} \quad \text{——— (III)}$$

Put the value of t_1 and t_2 in eqn (I) we get

$$t_1 + \frac{4.905\, t_1^2}{350} = 4$$

$$\Rightarrow 350\, t_1 + 4.905\, t_1^2 = 1400$$

$$\Rightarrow 4.905\, t_1^2 + 350\, t_1 - 1400 = 0$$

$$t_1 = \frac{-350 \pm \sqrt{350^2 + 4 \times 4.905 \times 1400}}{2 \times 4.905} = \frac{-350 \pm 387.26}{9.81}$$

$$\Rightarrow t_1 = 3.798 \text{ Sec.}$$

$$\therefore \quad h = 4.905\, t_1^2 = 4.905 \times (3.798)^2$$

$$\Rightarrow h = 70.75 \text{ m.}$$

CURVILINEAR MOTION :-

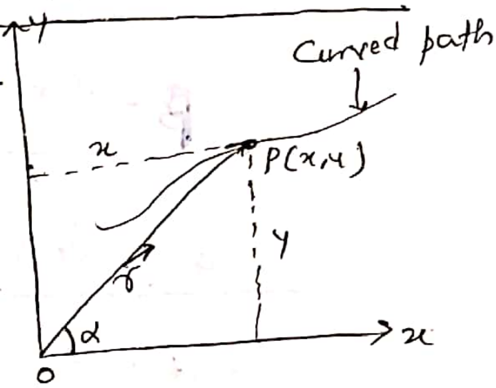
- Examples of curvilinear motion are:-

→ An automobile a turn on the road.
→ A projectile motion of bullet fired from a gun.
→ Motion of bob of pendulum oscillating in vertical plane.
→ motion of satellite around the earth.

Rectangular component of velocity and acceleration:-

- The position of particle on curved path at any instant is defined by position vector ($\vec{r}$)

$\vec{r}$ → position vector



$$\boxed{\vec{r} = xi + yj}$$ where i and j are unit vectors.

magnitude $|\vec{r}| = r = \sqrt{x^2+y^2}$

velocity vector ($\vec{v}$) $= \dfrac{d\vec{r}}{dt} = \dfrac{d(xi+yj)}{dt}$

$$\Rightarrow \vec{v} = \frac{dx}{dt}i + \frac{dy}{dt}j$$

$$\Rightarrow \boxed{\vec{v} = V_x i + V_y j}$$

magnitude of velocity

$$\boxed{V = |\vec{V}| = \sqrt{(V_x)^2 + (V_y)^2}}$$

direction of velocity

$$\tan\alpha = \frac{V_y}{V_x} \Rightarrow \boxed{\alpha = \tan^{-1}\frac{V_y}{V_x}}$$

Acceleration ($\vec{a}$) : $= \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}\left(\dfrac{dx}{dt}i + \dfrac{dy}{dt}j\right)$

$$\Rightarrow \vec{a} = \frac{d^2x}{dt}i + \frac{d^2y}{dt}j$$

$$\Rightarrow \boxed{\vec{a} = a_x i + a_y j}$$

magnitude of acceleration

$$\boxed{a = |\vec{a}| = \sqrt{(a_x)^2 + (a_y)^2}}$$

direction of acceleration

Let β is the angle made by resultant acc. with x- axis then

$$\tan\beta = \frac{a_y}{a_x}$$

$$\Rightarrow \boxed{\beta = \tan^{-1}\frac{a_y}{a_x}}$$

For a motion in space we may write:-

$$\vec{r} = xi + yj + zk$$

$$\vec{v} = V_x i + V_y j + V_z k$$

$$\vec{a} = a_x i + a_y j + a_z k$$

RELATIVE VELOCITY:-

train A $\longrightarrow V_a = 60 km/hr$

train B $\longrightarrow V_b = 45 km/hr$

Consider two trains A and B moving in parallel tracks and in the same direction with velocity 60km/h and 45 km/hr respectively. To a passenger sitting in train B, the train A will appear to be moving with speed of $(60-45) = 15 km/hr$.

This implies that the relative velocity of A with respect to B is

$$\boxed{V_{ab} = V_a - V_b} = 60 - 45 = 15 \ km/hr$$

train A $\longrightarrow V_a = 60 km/hr$

train B $\longleftarrow V_b = 45 km/hr$

When the motion of the trains are along parallel tracks but opposite in direction then relative velocity of A with respect to B is

$$\boxed{V_{ab} = V_a - (-V_b)} = 60 - (-45) = 105 \ km/hr.$$

Imp

Q. A 200 m long passenger ~~car~~ train running with a velocity of 72km/hr is to overtake a 150 m long goods train that is moving on a parallel track in the same direction. If the speed of the goods train is 36 km/hr, how much time will be taken for its complete overtake.

Soln:- Velocity of passenger train

$$V_a = 72 km/hr = \frac{72 \times 1000}{3600} = 20 \ m/s$$

Scanned by CamScanner

velocity of goods train $(V_b) = 36\,km/hr = \frac{36\times1000}{3600} = 10\,m/s$

Relative velocity of passenger train w.r.to goods train

$V_{ab} = V_a - V_b$

$V_{ab} = 20 - 10 = 10\,m/s$

Total distance to be covered $= 200 + 150 = 350\,m$

$\because$ distance $= V \times$ time

$350 = 10 \times t$

$\Rightarrow \boxed{t = 35\ sec.}$

Q: Two train A (length 125 m) and B (length 150 m) move in opposite direction along parallel tracks. At the instant of complete pass over the train A is moving at 10 m/s with a constant accⁿ of 0.1 m/s^2, and the train B has a uniform speed. If the train take 12 sec. to pass one another, determine the uniform speed of train B.

solⁿ:- Total distance to be passed $= 125 + 150 = 225\,m$

$s = ut + \frac{1}{2}at^2$

$V_{ab} \rightarrow$ Relative velocity of train A w.r.to train B

$225 = V_{ab} \times 12 + \frac{1}{2}\,0.1 \times 12^2$

$V_{ab} = \frac{225 - 7.2}{12} = 18.15\,m/s$

$V_{ab} = V_a - (-V_b)$

$18.15 = 10 + V_b$

$\Rightarrow \boxed{V_b = 8.15\,m/s}$

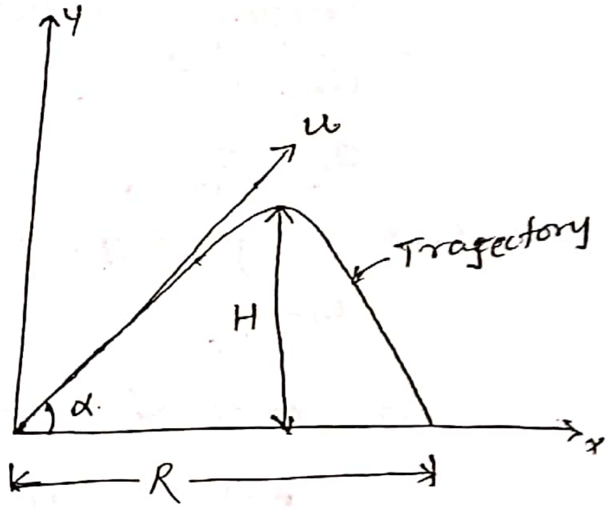
A Particle, moving under the combined effect of vertical and horizontal component is called a Projectile.

PROJECTILE MOTION:- If a particle is thrown in any direction inclined to the vertical with a certain velocity then it moves along a curved path instead of straight line. This freely projected particle which is having combined effect of a vertical and horizontal is called projectile.

→ The motion of a projectile has two components namely vertical and horizontal.

→ The vertical component is subjected to gravitational acceleration or retardation, while the horizontal component remains constant, if the resistence due to air is neglected.

→ The point from where it is projected is called point of projection.



(I) VELOCITY OF PROJECTION:- The initial velocity with ~~respect~~ which a particle is projected is called the velocity of projection.

(II) ANGLE OF PROJECTION:-

The angle b/w the direction of projection and the horizontal is called the angle of projection.

(III) TRAJECTORY:- The path followed by the projectile is called its trajectory.

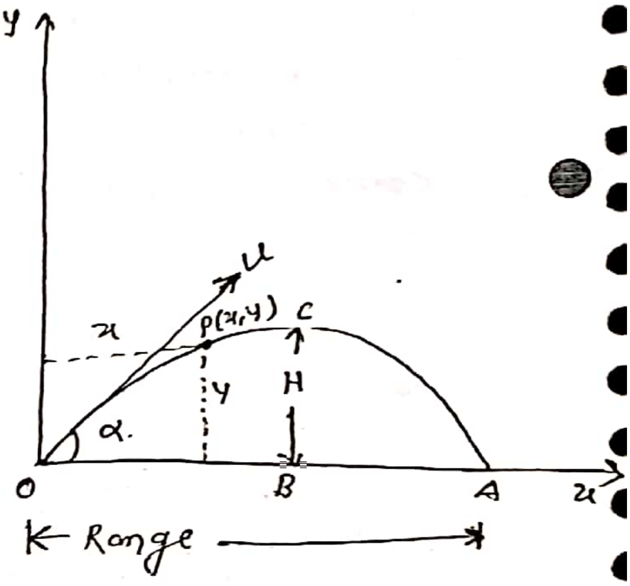
(IV) HORIZONTAL RANGE:- The distance b/w the point of projection and the point where the projectile strikes the horizontal planes at the end of its journey is called the horizontal range or range.

Scanned by CamScanner

(V) TIME OF FLIGHT:- The time interval during which the projectile is in motion, is called the time of flight.

(VI) MAXIMUM HEIGHT:- The maximum vertical distance of projectile from the horizontal plane is called ~~the~~ its maximum height.

MOTION OF PROJECTILE AND ITS TRAJECTORY:-



$u_x = u\cos\alpha$ ——— (I)

$u_y = u\sin\alpha$ ——— (II)

For vertical motion

$u_y = u\sin\alpha$ (upward)

$a_y = -g$

For horizontal motion

$u_x = u\cos\alpha$ (Constant)

$a_x = 0$

Let, $P(x,y)$ be the position of the particle after time t.

For vertical motion

$$y = u_y t + \frac{1}{2}at^2$$

$$\Rightarrow y = u\sin\alpha t - \frac{1}{2}gt^2 \quad \text{——— (III)}$$

For horizontal motion

$$x = u_x t + \frac{1}{2}at^2$$

$$\Rightarrow x = u\cos\alpha\, t + 0$$

$$\Rightarrow x = u\cos\alpha\, t \quad \text{——— (IV)}$$

$$\Rightarrow t = \frac{x}{u\cos\alpha} \quad \text{——— (V)}$$

Put the value of t in eqn (III) we get

$$Y = u\sin\alpha t - \frac{1}{2}gt^2$$

$$= u\sin\alpha \frac{x}{u\cos\alpha} - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$$

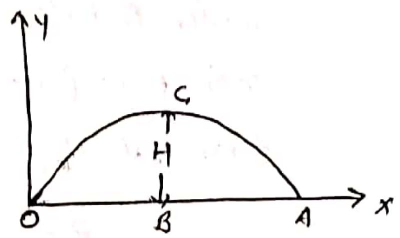
$$\boxed{Y = x\tan\alpha - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2}$$

Equation of projectile motion is a Parabola equation.

MAXIMUM HEIGHT:- At point C where the particle attains the maximum height, the vertical component of its velocity will be zero.

By the eqn of motion

$$v^2 = u^2 + 2as$$



$v = 0$
$u = u\sin\alpha$
$a = -g$, $s = H$

$$0 = u^2\sin^2\alpha - 2gh$$

$$\Rightarrow \boxed{H = \frac{u^2\sin^2\alpha}{2g}}$$

TIME TAKEN TO REACH THE MAXIMUM HEIGHT:-

By the equation of motion

$$v = u + at$$

$v = 0$
$u = u\sin\alpha$
$a = -g$

$$\therefore 0 = u\sin\alpha - gt$$

$$\Rightarrow \boxed{t = \frac{u\sin\alpha}{g}}$$

TIME OF FLIGHT:- Since after the coverage of trajectory on the horizontal plane for time t, the vertical distance moved by the partcle is zero i.e. $y = 0$

$\because$ $y = u\sin\alpha t - \frac{1}{2}gt^2$ from eq^n (III).

$0 = u\sin\alpha t - \frac{1}{2}gt^2$

$\Rightarrow \frac{1}{2}gt^2 = u\sin\alpha t$

$\Rightarrow \boxed{t = \frac{2u\sin\alpha}{g}}$

HORIZONTAL RANGE:- During the time of flight (t), the particle has been moving horizontally with uniform velocity $(u\cos\alpha)$, so that the horizontal distance traced by the projectile in this time.

$R = u\cos\alpha \times t = u\cos\alpha \times \frac{2u\sin\alpha}{g}$

$\Rightarrow R = \frac{2u^2\sin\alpha \cdot \cos\alpha}{g}$

$\Rightarrow \boxed{R = \frac{u^2\sin 2\alpha}{g}}$

MAXIMUM RANGE:-

R is maximum i.e.

$\frac{u^2\sin 2\alpha}{g}$ is maximum when $\sin 2\alpha$ is maximum

$\sin 2\alpha = 1$

$\sin 2\alpha = \sin 90$

$\boxed{\alpha = 45^\circ}$ $\therefore \boxed{R_{max} = \frac{u^2}{g}}$

Hence the horizontal range is maximum when the angle of projection is 45° and the maximum range $\boxed{R_{max} = \frac{u^2}{g}}$

Scanned by CamScanner

VELOCITY AND DIRECTION OF PROJECTILE AT GIVEN TIME:-

component of velocity u and v along x-directions are -

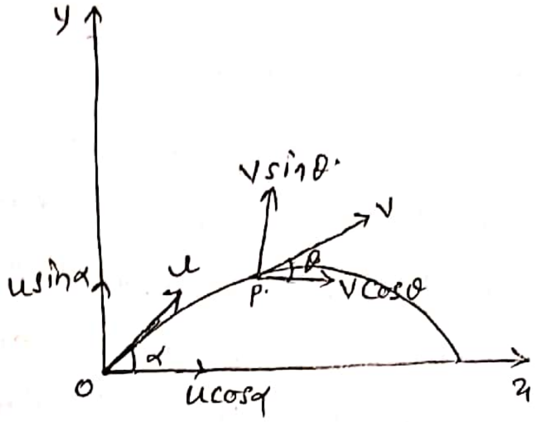
$u_x = u\cos\alpha$

$V_x = V\cos\theta$

along y-direction are -

$u_y = u\sin\alpha$

$V_y = v\sin\theta$



since horizontal motion (along x-direction) is with uniform velocity so it is equal to the initial component of horizontal velocity.

$$v\cos\theta = u\cos\alpha \quad \text{——— (I)}$$

and vertical component

$$v\sin\theta = u\sin\alpha - gt \quad \text{——— (II)} \quad v = u - gt$$

Squaring the both equations and adding.

$$v^2\cos^2\theta + v^2\sin^2\theta = u^2\cos^2\alpha + (u\sin\alpha - gt)^2$$

$$\Rightarrow v^2(\cos^2\theta + \sin^2\theta) = u^2\cos^2\alpha + u^2\sin^2\alpha + g^2t^2 - 2ugt\sin\alpha$$

$$\Rightarrow v^2(\cos^2\theta + \sin^2\theta) = u^2(\cos^2\alpha + \sin^2\alpha) + g^2t^2 - 2ugt\sin\alpha.$$

$$\Rightarrow \boxed{v^2 = u^2 + g^2t^2 - 2gtu\sin\alpha}$$

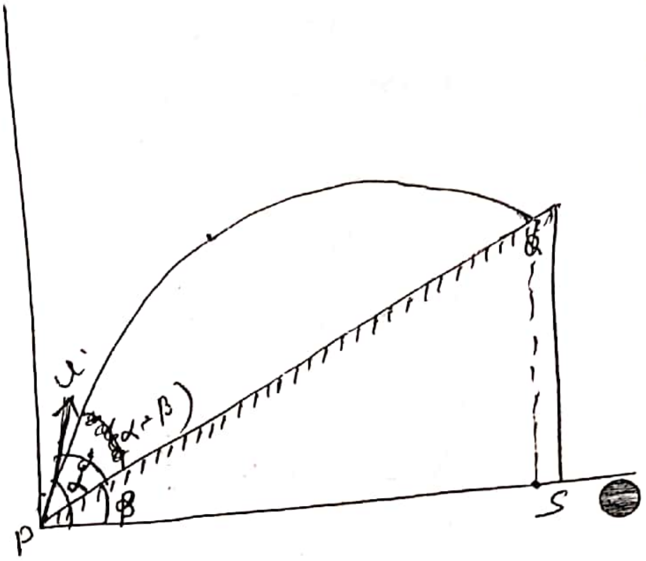
Eq^n (II) ÷ Eq^n (I) we get

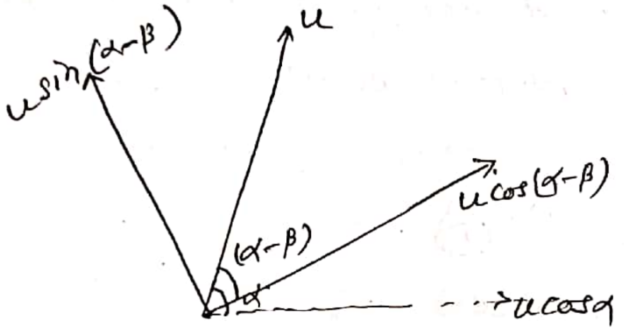
$$\frac{v\sin\theta}{v\cos\theta} = \frac{u\sin\alpha - gt}{u\cos\alpha}$$

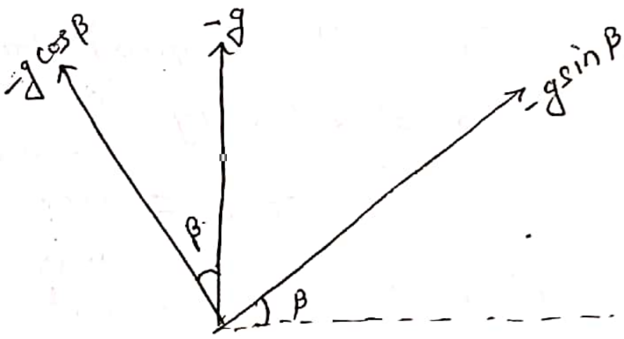
$$\Rightarrow \boxed{\tan\theta = \frac{u\sin\alpha - gt}{u\cos\alpha}}$$

PROJECTILE MOTION ON AN INCLINED PLANE:-

Let, u = Velocity of projection. This velocity of projection can be resolved into two components, one along the inclined plane and other along to normal to inclined plane.







Velocity along (PQ) = $u\cos(\alpha-\beta)$

Velocity normal to th plane $= u\sin(\alpha-\beta)$

Accn due to gravity along $PQ = -g\sin\beta$

Accn due to gravity along normal to the plane $= -g\cos\beta$

TIME OF FLIGHT:-

initial velocity normal to inclined plane $= u\sin(\alpha-\beta)$

Accn due to gravity " " " " $= -g\cos\beta$

$$\because \; s = ut + \tfrac{1}{2}gt^2$$

$$s = u\sin(\alpha-\beta)\cdot T + \tfrac{1}{2}(-g\cos\beta)T^2$$

$s=0$, Normal to inclined Plane PQ

$$0 = u\sin(\alpha-\beta)T - \tfrac{1}{2}g\cos\beta\, T^2$$

$$\boxed{T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}}$$

RANGE ON THE INCLINED PLANE:-

Range of inclined plane is found by distance PQ.

Horizontal distance PS = component of velocity of projection in horizontal direction × time of flight

$$\Rightarrow PS = u\cos\alpha \times T$$

$$= u\cos\alpha \times \frac{2u\sin(\alpha-\beta)}{g\cos\beta} = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos\beta}$$

From the right angle Δ PQS.

$$\cos\beta = \frac{PS}{PQ}$$

$$\Rightarrow PQ = PS/\cos\beta$$

$$\Rightarrow PQ = \frac{2u^2\sin(\alpha-\beta)\cdot\cos\alpha/g\cos\beta}{\cos\beta} = \frac{2u^2\cos\alpha\cdot\sin(\alpha-\beta)}{g\cos^2\beta}.$$

$$\Rightarrow PQ = \frac{u^2}{g\cos^2\beta}\sin(\alpha+\alpha-\beta) - \sin[\alpha-(\alpha-\beta)]$$

$$\Rightarrow PQ = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha-\beta) - \sin\beta]$$

$$\Rightarrow \boxed{\text{Range} = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha-\beta) - \sin\beta]}$$

$\because [2\cos\alpha\cdot\sin\beta = (\sin\alpha+\beta) - \sin(\alpha-\beta)]$

Q. A body is projected at an angle such that its horizontal range is 3 times the maximum height. Find the angle of projection.

Solⁿ:- we know that

$$H = \frac{u^2 \sin^2\alpha}{2g}$$

$$R = \frac{u^2 \sin 2\alpha}{g}$$

From question

$$R = 3H$$

$$\frac{u^2 \sin 2\alpha}{\cancel{g}} = 3 \times \frac{u^2 \sin^2\alpha}{2\cancel{g}}$$

$$\Rightarrow 2\cancel{u^2} \sin\alpha \cdot \cos\alpha = 3 \times \frac{\cancel{u^2} \sin^2\alpha}{2}$$

$$\Rightarrow \sin\alpha \cdot \cos\alpha = \frac{3}{4} \sin^2\alpha$$

dividing both side by $\sin^2\alpha$, we get

$$\frac{\cancel{\sin\alpha} \cdot \cos\alpha}{\sin^{\cancel{2}}\alpha} = \frac{3}{4} \frac{\cancel{\sin^2\alpha}}{\cancel{\sin^2\alpha}}$$

$$\Rightarrow \cot\alpha = \frac{3}{4}$$

$$\Rightarrow \tan\alpha = \frac{4}{3}$$

$$\Rightarrow \tan\alpha = 1.333$$

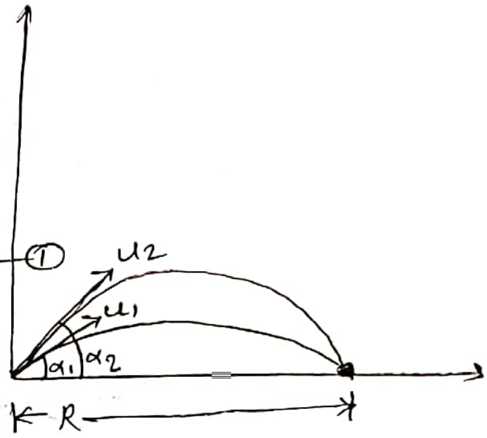
$$\Rightarrow \alpha = \tan^{-1}(1.333) = 53.13^\circ$$

Q. Two adjacent guns having shot the bullets at a velocity of 300 m/s simultaneously at angles α_1 and α_2 for the same target at range of 5.6 km. Calculate the time difference b/w the hits.

Soln:-

$\because R = \frac{u^2 \sin 2\alpha}{g}$

$\therefore R = \frac{u_1^2 \sin 2\alpha_1}{g} = \frac{u_2^2 \sin 2\alpha_2}{g} = 5600\,m$ — ①



$\because u_1 = u_2$

$\frac{\cancel{u_1^2} \sin 2\alpha_1}{\cancel{g}} = \frac{\cancel{u_1^2} \sin 2\alpha_2}{\cancel{g}}$

$\sin 2\alpha_1 = \sin 2\alpha_2$

$\sin 2\alpha_1 = \sin(\pi - 2\alpha_2)$

$2\alpha_1 = \pi - 2\alpha_2$

$\Rightarrow \boxed{\alpha_1 = \frac{\pi}{2} - \alpha_2}$

From eqn ①

$\frac{u^2 \sin 2\alpha_1}{g} = 5600$

$\Rightarrow \frac{300^2 \sin 2\alpha_1}{9.81} = 5600$

$\Rightarrow \sin 2\alpha_1 = \frac{5600 \times 9.81}{300^2} = 0.6104$

$\Rightarrow 2\alpha_1 = \sin^{-1}(0.6104)$

$\Rightarrow 2\alpha_1 = 37.62°$

$\Rightarrow \alpha_1 = 18.8°$

$\therefore \alpha_2 = \frac{\pi}{2} - \alpha_1$

$\alpha_2 = \frac{\pi}{2} - 18.8 = 71.2°$

$t_1 = \frac{2u \sin \alpha_1}{g}, \quad t_2 = \frac{2u \sin \alpha_2}{g}$

$t_2 - t_1 = \frac{2u}{g}(\sin \alpha_2 - \sin \alpha_1)$

$= \frac{2 \times 300}{9.81}[\sin 71.2 - \sin 18.8]$

$= \frac{600}{9.81}[0.946 - 0.332]$

$\boxed{t_2 - t_1 = 37.55 \text{ sec.}}$

NEWTON'S LAW OF MOTION :-

NEWTON'S FIRST LAW OF MOTION :- or Law of inertia:-

A body continues in its state of rest or uniform rectilinear motion, unless an external force is applied to it to change the state.

Examples:-

→ when bus starts suddenly the passanger falls backword.

→ On shaking of the branch of tree, the fruits fall down.

NEWTON'S SECOND LAW OF MOTION :-

The rate of change of momentum of a moving body is ~~selected in such a way that constant of proportionality reduces to unity.~~ proportional to the applied forces on its and change takes place in same direction, in which force acts.

$$\boxed{\text{momentum} = \text{mass} \times \text{velocity}}$$

initial momentum = mu

final momentum = mv

$$\text{Rate of change of momentum} = \frac{mv - mu}{t}$$

$$= \frac{m(v-u)}{t}$$

$$= ma \qquad \frac{v-u}{t} = a$$

According to second law of motion

$$\boxed{F \propto ma}$$

$$\Rightarrow \boxed{F = kma} \qquad k = 1$$

$$\Rightarrow \boxed{F = ma}$$

Example:-

→ Pushing a car that if two people push a car on a flat road it will accelerate faster than if one person was pushing it.

NEWTON'S THIRD LAW OF MOTION:-

For every action there is an equal and opposite reaction.

Example:-

→ The rocket action is to push down on the ground with the force of its powerful engines and the reaction is that the ground pushes the rocket upward with an equal force.

NEWOTON'S LAW OF GRAVITATION:-

The gravitational force b/w two particles varies directly with the product of their masses and inversally proportional with the square of the distance b/w them

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

F → Gravitational attraction of force.

G → The universal constant $= 6{\cdot}67 \times 10^{-11}\ m^3/kg\text{-}s^2$

m_1, m_2 → The masses of the two particles.

r → The distance b/w them.

Scanned by CamScanner

D' ALEMBERT'S PRINCIPLE:-

It states if a rigid body is acted upon by a system of forces this system may be reduced to single resultant force whose magnitude direction and the line of action may be found out by the method of graphic statics.

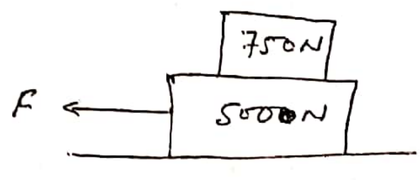
$$F = ma \quad \text{———} \quad (I)$$

$$F - ma = 0 \quad \text{———} \quad (II)$$

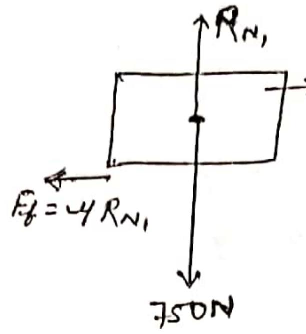
It may be noted that equation (I) is the eqn of dynamics whereas the equation (II) is the equation of statics. The equation (II) is also known as the equation of dynamic equilibrium under the action of the real force F. This principle is known as D' Alembert's principle.

Q. A 750 N crate rest on a 500 N cart the co-efficien of friction b/w the crate and cart is 0.3 and b/w the cart and the road is 0.2. If the cart is to be pulled by a force F such that the crate does not slip determine (a) The maximum allowable magnitude of F and (b) The corresponding accn of the cart.

Soln:-



FBD at upper block



inertia force

$F_i = ma = \frac{w}{g} a = \frac{750}{9.81} \times a$

<u>vertical forces:-</u>

$R_{N_1} - W = 0$

$R_{N_1} = W = 750\,N$

Horizontal forces

$F_f = \mu R_{N_1}$
$= 0.3 \times 750 = 225\,N$

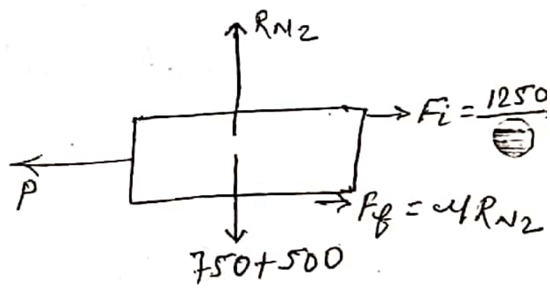
$F_f = F_i$

$\Rightarrow F_f = \frac{750}{9.81} \times a =$

$\Rightarrow 225 = \frac{750}{9.81} a$

$\Rightarrow \boxed{a = 2.943\ m/s^2}$

<u>F.B.D at lower block</u>



<u>vertical forces</u>

$R_{N_2} = 750 + 500 = 1250\,N$

$F_{f_0} = \mu R_{N_2} = 0.2 \times 1250 = 250\,N$

<u>Horizontal forces</u>

$P = F_i + F_f$?

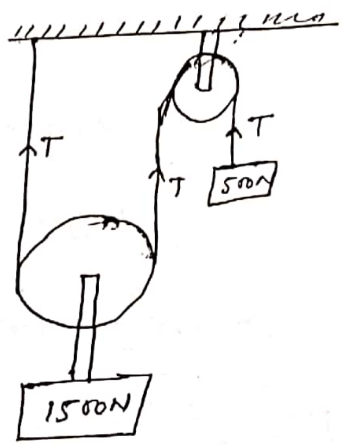
$\Rightarrow P = \left(\frac{1250}{9.81} \times 2.943\right) + 250$

$\Rightarrow P =$

$\Rightarrow \boxed{P = 625\,N}$

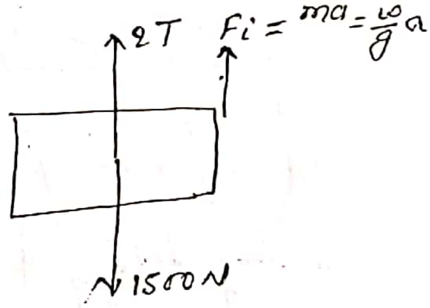
Scanned by CamScanner

Q. Determine the tension in the string and acceleration of blocks A and B weighing 1500N and 500N connected by an extensible string as shown in fig. Assume pulleys as frictionless and weightless.



Soln:- In this system it may be observed that if 1500 N block moves downward by distance x, 500N block moves up by $2x$. Hence accn of 1500N block is 'a' then for 500N block is '2a'.

F.B.D of 1500 N block



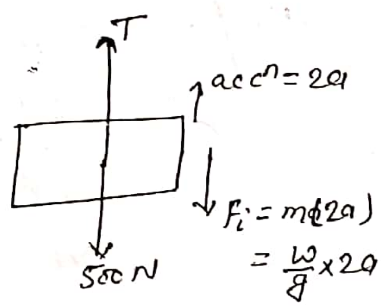
$$\Sigma V = 0$$

$$2T + F_i = 1500$$

$$2T + \frac{w}{g}a = 1500$$

$$2T + \frac{1500}{9.81} \times a = 1500 \quad \text{——— (I)}$$

F.B.D of 500N block



$$\Sigma V = 0$$

$$T = F_i + 500$$

$$\Rightarrow T - F_i = 500$$

$$\Rightarrow T - \frac{w}{g} \times 2a = 500 \quad \text{——— (II)}$$

$$\Rightarrow T - \frac{500}{9.81} \times 2a = 500$$

Eqn (I) − Eqn (II) × 2

$$\cancel{2T} + \frac{1500a}{9.81} = 1500$$

$$\cancel{2T} - \frac{500}{9.81} \times 4a = 1000$$

$$- \quad + \quad -$$

$$\frac{1500\,a}{9.81} + \frac{2000\,a}{9.81} = 500$$

$$\Rightarrow \frac{3500\,a}{9.81} = 500$$

$$\Rightarrow a = \frac{\cancel{500} \times 9.81}{\cancel{3500}_{7}} = \frac{9.81}{7} = 1.401 \text{ m/sec}^2$$

put the value of a in eq^n (ii) we get

$$T = \frac{w}{g} * 2a + 500$$

$$= \frac{500}{9.81} \times 2 \times 1.401 + 500$$

$$\cancel{= 71.406 + 500} =$$

$$= 142.81 + 500$$

$$\boxed{T = 642.81 \text{ N}}$$