Engineering Mechanics (3ME3-04)

DEPARTMENT

OF

MECHANICAL ENGINEERING

(Jaipur Engineering College and Research Centre, Jaipur)

UNIT: III

Friction: Types of Friction, Laws of friction, Angle of friction, Angle of repose, Ladder, Wedge, Belt Friction.

Belt and Rope drive: Types of belts, Types of belt drives, Velocity ratio, Effect of slip on Velocity ratio, Crowing of pulleys, Length of belt, Ratio of tensions in flat belt drive, Power transmission by belt drives, Advantage and disadvantages of V-Belt over Flat Belt.

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UNIT-3
FRICTION

Friction:— (1) The opposing force, which acts in the opposite direction of the motion is called the force of friction.

TYPES OF FRICTION:-

(I) Static friction

(II) Dynamic friction
- → Sliding friction
- → Rolling friction

→ Friction b/w unlubricated surface.
→ Friction b/w lubricated surface.

LIMITING FRICTION:— The maximum value of frictional force which comes into play, when a body just begins to slide over the surface of the other body is known as limiting force of friction.

LAWS OF STATIC FRICTION:—

→ Force of friction always acts in the opposite to the body tends to move.

→ Magnitude of the force of friction is exactly equal to the force, which tends to move the body.

→ $\frac{F}{R_N}$ = Constant

→ Force of friction is independent of the area of contact b/w the two surface.

→ Force of friction depends upon the roughness of the surface.

LAW'S OF SOLID FRICTION:—

→ Force of friction independent of the area of contact.

→ The force of friction always acts in the direction of opposite to the body movement.

→ Force of friction depends upon the material of the surface.

→ Force of friction is independent of the velocity of sliding.
→ Force of friction is directly proprotional to the normal load.

LAW'S OF FLUID FRICTION:-

→ Force of friction is independent of the load.
→ Force of friction is reduced with the increasing of the temp. of the lubricant.
→ Force of friction is different for different lubricants.
→ Force of friction is independent of the substance of the bearings surface.

LAW'S OF KINETIC OR DYNAMIC FRICTION:-

→ Force of friction always acts in opposite to the direction of the body movement.
→ In this case μ (co-efficient of friction) is less than the static friction.
→ For moderate speeds the force of friction remains constant but it decreases slightly with increase of speed

CO-EFFICIENT OF FRICTION:- It is defined as the ratio of force of friction to the normal reaction b/w the contact surfaces.
→ mathematically, the frictional force is directly proportional to the normal reaction b/w the contact surfaces. i.e.

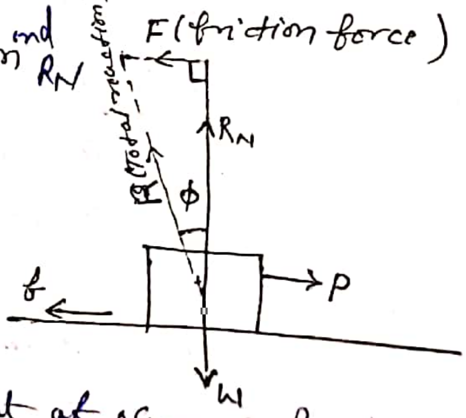
$$F \propto R_N$$

$$\Rightarrow F = \mu R_N$$

$$\Rightarrow \boxed{\mu = \frac{F}{R_N}} \leftarrow \text{co-efficient of friction.}$$

→ The co-efficient of friction would be high if the contact surfaces are rough.

ANGLE OF FRICTION:- It is defined as the angle which the resultant of normal reaction and R_N and limiting force of friction makes with the normal reaction.



$R_N \rightarrow$ Normal reaction
$F \rightarrow$ Limiting force of friction

$R = \sqrt{R_N^2 + F^2} \rightarrow$ Total or resultant reaction.

angle of friction is the angle b/w Normal Reaction and Resultant of Normal Reaction and friction force

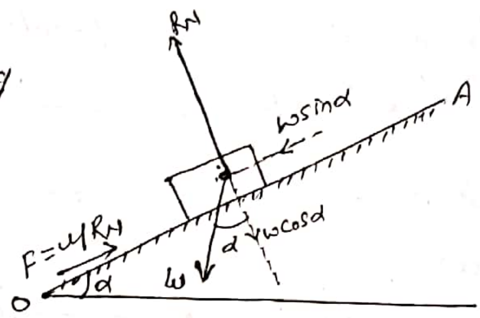
$\therefore \tan\phi = \frac{F}{R_N}$ $\Rightarrow$ $\boxed{\phi = \tan^{-1}\frac{F}{R_N}}$ ← Angle of friction.
or
Limiting angle of friction.

or $\mu = \frac{F}{R_N}$

$\therefore \boxed{\tan\phi = \mu}$

ANGLE OF REPOSE :—

consider a block of weight W resting on an inclined plane OA making an angle α with the horizontal.
Let the angle α be increased gradually till the block is just at the point of sliding.



Resolving these forces—

$F = w\sin\alpha$

$\Rightarrow \mu R_N = W\sin\alpha$ —— (I)

and

$R_N = w\cos\alpha$ —— (II)

Eqn (I) ÷ Eqn (II)

$$\frac{\mu \cancel{R_N}}{\cancel{R_N}} = \frac{\cancel{W}\sin\alpha}{\cancel{W}\cos\alpha}$$

$\Rightarrow \mu = \tan\alpha$ —— (III)

$\because$ we know that

$\mu = \tan\phi$ —— (IV)

From eqn (III) and (IV) we get

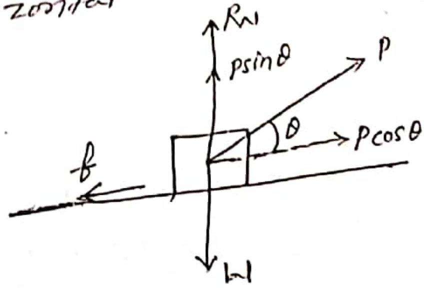
$\tan\alpha = \tan\phi$

$\Rightarrow \boxed{\alpha = \phi}$

The angle of α of the inclined plane at which a block resting on it is about to slide down to the plane is called the angle of repose and it is equal to angle of friction b/w block and Inclined plane.

MINIMUM FORCE REQUIRED TO SLIDE A BODY ON A ROUGH HORIZONTAL PL

P acts on the body inclined at θ to the horizontal surface.



Resolving the force P.

$P\cos\theta$ in horizontal direction

$P\sin\theta$ in vertical direction.

For equilibrium position-

$$R_N + P\sin\theta = W \quad \text{——— (I)}$$

$$\Rightarrow R_N = W - P\sin\theta \quad \text{——— (II)}$$

and

$$P\cos\theta = \mu R_N \quad \text{——— (III)}$$

Put the value of R_N from eqn (II) in eqn (III) we get

$$P\cos\theta = \mu(W - P\sin\theta)$$

$$\Rightarrow P\cos\theta = \mu(W - P\sin\theta)$$

$$\Rightarrow P\cos\theta = \tan\phi\,(W - P\sin\theta) \qquad \because \mu = \tan\phi$$

$$\Rightarrow P\cos\theta = \frac{\sin\phi}{\cos\phi}(W - P\sin\theta)$$

$$\Rightarrow P\cos\theta\cdot\cos\phi = W\sin\phi - P\sin\theta\cdot\sin\phi$$

$$\Rightarrow P\cos\theta\cdot\cos\phi + P\sin\theta\cdot\sin\phi = W\sin\phi$$

$$\Rightarrow P[\cos(\theta-\phi)] = W\sin\phi$$

$$\Rightarrow \boxed{P = \frac{W\sin\phi}{\cos(\theta-\phi)}}$$

For P to the minimum $\cos(\theta-\phi)$ should be maximum

$$\therefore \cos(\theta-\phi) = 1$$

$$\Rightarrow \cos(\theta-\phi) = \cos 0$$

$$\Rightarrow \theta - \phi = 0$$

$$\Rightarrow \theta = \phi$$

$$\therefore P = \frac{W\sin\phi}{\cos 0}.$$

$$\Rightarrow \boxed{P = W\sin\phi}$$ → Minimum force required to slide a body on rough horizontal plane.

TYPES OF FRICTION:-

Ⓐ **DRY FRICTION:-** Dry friction is said to occur when there is relative motion b/w the two completely unlubricated surfaces.

It is further divided into two parts:-

① **SLIDING FRICTION:-** When the two surfaces have a sliding motion relative to each other, it is called sliding friction.

Ⓘ **ROLLING FRICTION:-** Friction due to rolling of one surface over another is called rolling friction e.g. ball and roller bearings.

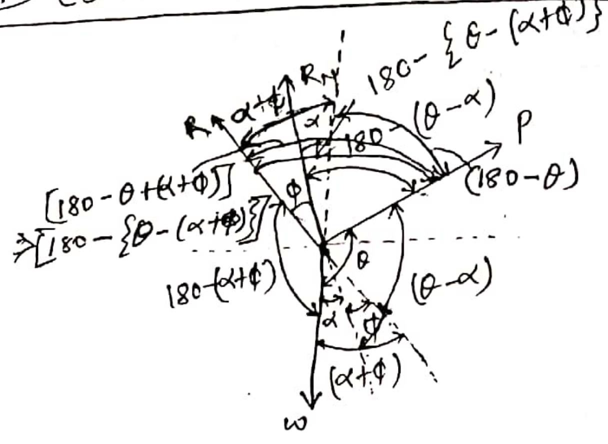
Ⓑ **SKIN OR GREASY FRICTION:-** When the two surfaces in contact have a minute thin layer of lubricant b/w them, It is known as skin friction or greasy friction. It is also known as boundry friction.

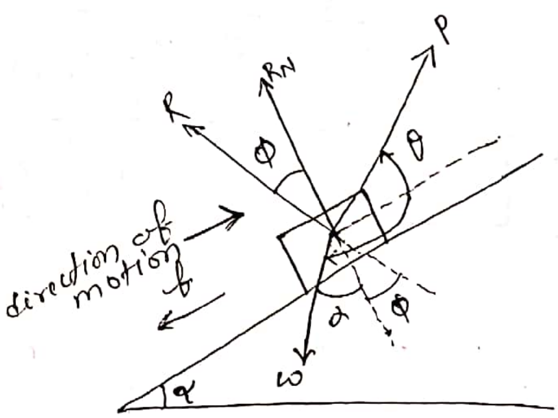
Ⓒ **FILM FRICTION:-** When the two surfaces in contact are completely seperated by a lubricant film, friction will occur due to shearing of different layers of the lubricants. This is known as film friction or viscous friction.

Ⓓ **FLUID FRICTION:-** It occurs when adjacent layers in fluid are moving at different velocities. This motion causes frictional forces b/w fluid element and depends upon the relative velocity b/w the layers.

FRICTION OF A BODY LAYING ON A ROUGH INCLINED PLANE:-

① CONSIDERING THE MOTION OF THE BODY UP TO THE PLANE:-





Applying sine rule:-

$$\frac{P}{\sin[180-(\alpha+\phi)]} = \frac{W}{\sin[180-(\theta-\alpha)+\phi]} = \frac{W}{\sin[180-\{\theta-(\alpha+\phi)\}]}$$

$$\frac{P}{\sin(\alpha+\phi)} = \frac{W}{\sin[\theta-(\alpha+\phi)]}$$

$$\boxed{P = \frac{W\sin(\alpha+\phi)}{\sin[\theta-(\alpha+\phi)]}} \quad \text{—— (I)}$$

CASE (I) When the force applied is horizontal then $\theta = 90°$

∴ Eqn (I) becomes as-

$$P_0 = \frac{W\sin(\alpha+\phi)}{\sin[90-(\alpha+\phi)]} = \frac{W\sin(\alpha+\phi)}{\cos(\alpha+\phi)}$$

$$\Rightarrow \boxed{P_0 = W\tan(\alpha+\phi)} \quad \text{—— (II)}$$

CASE (II) When the force applied is parallel to the plane then

$\theta = 90+\alpha$

∴ Equation (I) becomes as -

$$P = \frac{W\sin(\alpha+\phi)}{\sin[(90+\alpha)-(\alpha+\phi)]} = \frac{W\sin(\alpha+\phi)}{\sin(90-\phi)}$$

$$P = \frac{W(\sin\alpha\cdot\cos\phi+\cos\alpha\cdot\sin\phi)}{\cos\phi} = W(\sin\alpha+\cos\alpha\cdot\tan\phi)$$

$$\Rightarrow \boxed{P = W(\sin\alpha+\mu\cos\alpha)} \qquad \because \mu = \tan\phi$$

CASE-(III) P will be minimum if denominator is maximum.

$$P = \frac{W\sin(\alpha+\phi)}{\sin[\theta-(\alpha+\phi)]}$$

$$\sin[\theta-(\alpha+\phi)] = 1 = \sin 90$$

$$\theta-(\alpha+\phi) = 90$$

$$\Rightarrow \phi = \theta-\alpha-90$$

$$\Rightarrow \phi = \theta-(90+\alpha)$$

CASE-(IV) If friction force is neglected i.e. $(\phi=0)$

$$P_0 = \frac{W\sin(\alpha+\phi)}{\sin[\theta-(\alpha+\phi)]} = \frac{\sin\alpha}{\sin[\theta-\alpha]}$$

EFFICIENCY:- The efficiency of an inclined plane when a body slides up the plane is defined as the ratio of the forces required to move the body without consideration and with consideration of force of friction.

$$\eta = \frac{P_0}{P} = \frac{\dfrac{W\sin\alpha}{\sin(\theta-\alpha)}}{\dfrac{W\sin(\alpha+\phi)}{\sin[\theta-(\alpha+\phi)]}} = \frac{\cancel{W}\sin\alpha}{\sin(\theta-\alpha)} \times \frac{\sin[\theta-(\alpha+\phi)]}{\cancel{W}\sin(\alpha+\phi)}$$

$$= \frac{\sin\alpha}{\sin(\cancel{\theta+\alpha})} \times \frac{\sin\theta\cdot\cos(\alpha+\phi)-\cos\theta\cdot\sin(\alpha+\phi)}{\sin(\alpha+\phi)}$$

$$= \frac{\sin\alpha}{\sin(\alpha+\phi)} \times \frac{\sin\theta\cdot\cos(\alpha+\phi)-\cos\theta\cdot\sin(\alpha+\phi)}{\sin\theta\cdot\cos\alpha-\cos\theta\cdot\sin\alpha}$$

$$= \frac{\cancel{\sin\alpha}}{\cancel{\sin(\alpha+\phi)}} \times \frac{\cancel{\sin\theta}\,\cancel{\sin(\alpha+\phi)}\left[\dfrac{\cos(\alpha+\phi)}{\sin(\alpha+\phi)}-\dfrac{\cos\theta}{\sin\theta}\right]}{\cancel{\sin\theta}\cdot\cancel{\sin\alpha}\left[\dfrac{\cos\alpha}{\sin\alpha}-\dfrac{\cos\theta}{\sin\theta}\right]}$$

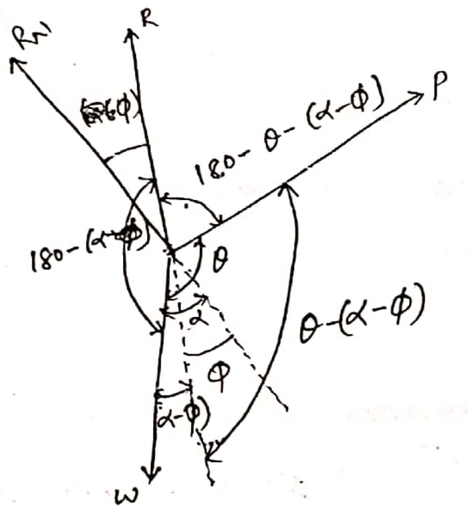
$$\boxed{\eta = \frac{\cot(\alpha+\phi)-\cot\theta}{\cot\alpha-\cot\theta}}$$

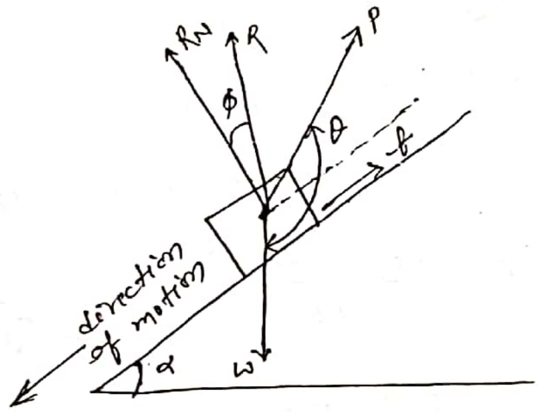
⇒ if the force is applied horizontally i.e. $\theta = 90$

$$\eta = \frac{\cot(\alpha+\phi) - \cot 90}{\cot\alpha - \cot 90} = \frac{\cot(\alpha+\phi)}{\cot\alpha}$$

$$\boxed{\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}}$$

CONSIDERING THE MOTION OF THE BODY DOWN THE PLANE:—





From sine rule

$$\frac{P}{\sin[180-(\alpha-\phi)]} = \frac{W}{\sin[180-\{\theta-(\alpha-\phi)\}]}$$

$$\Rightarrow \frac{P}{\sin(\alpha-\phi)} = \frac{W}{\sin[\theta-(\alpha-\phi)]}$$

$$\Rightarrow \boxed{P = \frac{W\sin(\alpha-\phi)}{\sin[\theta-(\alpha-\phi)]}} \quad \text{—— (1)}$$

CASE (I). When P is applied horizontally then $\theta = 90°$

∴ Equation (1) becomes as –

$$P = \frac{W\sin(\alpha-\phi)}{\sin[90-(\alpha-\phi)]} = \frac{W\sin(\alpha-\phi)}{\cos(\alpha-\phi)}$$

$$\Rightarrow \boxed{P = W\tan(\alpha-\phi)}$$

$\theta = 90 + \alpha$ (5)

CASE-(II) When P is applied parallel to the plane then,

$\therefore$ Equation (1) becomes as-

$$P = \frac{W\sin(\alpha-\phi)}{\sin[90+\alpha-\cancel{\alpha}+\phi]} = \frac{W\sin(\alpha-\phi)}{\sin(90-\phi)} = \frac{W(\sin\alpha\cdot\cos\phi - \cos\alpha\sin\phi)}{\cos\phi}$$

$$\Rightarrow P = W(\sin\alpha - \cos\alpha\cdot\tan\phi)$$

$$\Rightarrow \boxed{P = W(\sin\alpha - \mu\cos\alpha)} \qquad \because \mu = \tan\phi$$

CASE (III) If friction force is neglected then $(\phi = 0)$

$$P_0 = \frac{W\sin(\alpha-\phi)}{\sin[\theta-(\alpha-\phi)]}$$

$$\Rightarrow P_0 = \frac{W\sin\alpha}{\sin(\theta-\alpha)}$$

EFFICIENCY:- Efficiency of inclined plane when the body slides down the plane is defined as the ratio of the forces required to move the body with and without the consideration of force of friction i.e.

$$\eta = \frac{P}{P_0} = \frac{\cancel{W}\sin(\alpha-\phi)}{\sin[\theta-(\alpha-\phi)]}\times\frac{\sin(\theta-\alpha)}{\cancel{W}\sin\alpha} = \frac{\sin(\alpha-\phi)}{\sin\theta\cdot\cos(\alpha-\phi) - \cos\theta\cdot\sin(\alpha-\phi)}\times\frac{\sin\theta\cdot\cos\alpha - \cos\theta\cdot\sin\alpha}{\sin\alpha}$$

$$\Rightarrow \eta = \frac{\cancel{\sin(\alpha-\phi)}}{\sin\theta\cdot\cancel{\sin(\alpha-\phi)}\left[\frac{\cos(\alpha-\phi)}{\sin(\alpha-\phi)} - \frac{\cos\theta}{\sin\theta}\right]} \times \frac{\cancel{\sin\theta\cdot\sin\alpha}\left(\frac{\cos\alpha}{\sin\alpha} - \frac{\cos\theta}{\sin\theta}\right)}{\cancel{\sin\alpha}}$$

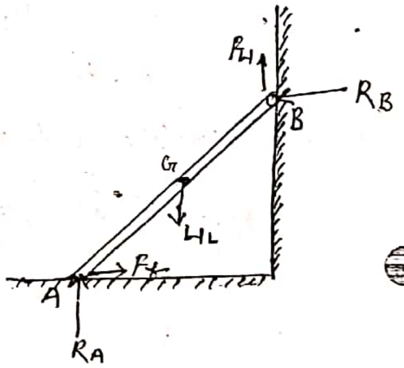
$$\Rightarrow \boxed{\eta = \frac{\cot\alpha - \cot\theta}{\cot(\alpha-\phi) - \cot\theta}}$$

→ If force is applied horizontally then $\theta = 90$

$$\eta = \frac{\cot\alpha - \cot 90}{\cot(\alpha-\phi) - \cot 90} \Rightarrow \boxed{\eta = \frac{\tan(\alpha-\phi)}{\tan\alpha}}$$

LADDER FRICTION :— A ladder is a device used for climbing or scaling higher location such as roofs or walls. It consists of two long uprights of woods or metal connected by a number of cross pieces called rungs. These rungs serves as steps. It is generally placed in such a way that it rests on the floor and leans against a wall. The ladder has a tendency to slip away from the wall at its lower end to the floor. Thus the upper end tends to slip downwards so that the force of friction b/w the ladder and the wall F_W acts upwards. at the lower ends which tends to move away from the wall the force of friction F_f b/w ladder and the floor will be towards the wall. Both ends are also subjected to normal reactions R_A and R_B which acts perpendicular to the floor and wall respectively. As the man climb the ladder it has a tendency to slip away and even topple about the lower end.

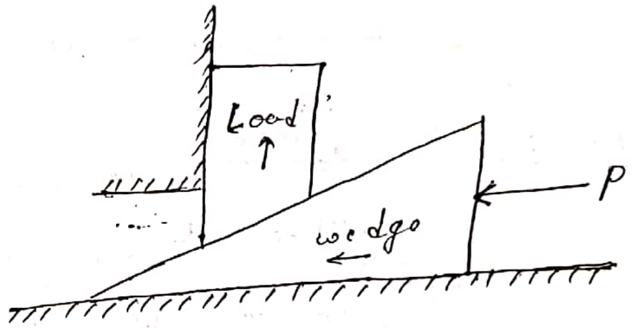
⇒ In the state of equilibrium impending any such motion.



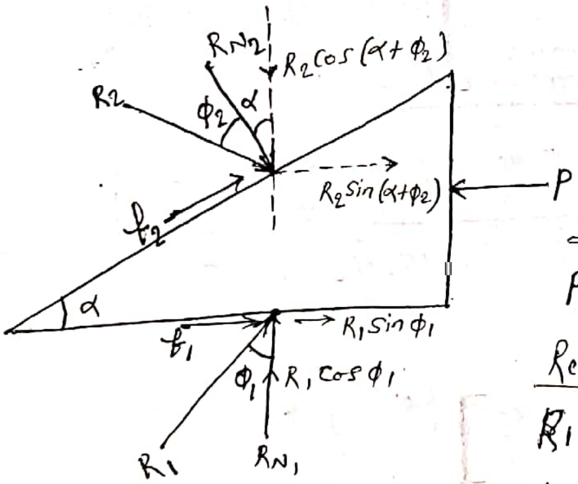
(I) The algebraic sum of horizontal and vertical components of the forces acting on it must be zero.

(II) The clockwise and anticlockwise moments about any fixed end must be equal and opposite i.e. The algebric sum of moments must also becomes zero.

WEDGE FRICTION:- A wedge is a piece of metal or wood whose section is usually in the shape of a prism trapezoidal or triangular. It is used for lifting loads. When lifting loads the wedge is placed below the load and a horizontal force P is applied. The wedge moves towards left and the loads moves upwords.



considering a wedge:-



where,

$R_1 \to$ Total reaction which is the resultant of R_{N_1} and f_1.

$R_2 \to$ Total reaction force which is the resultant of R_{N_2} and f_2.

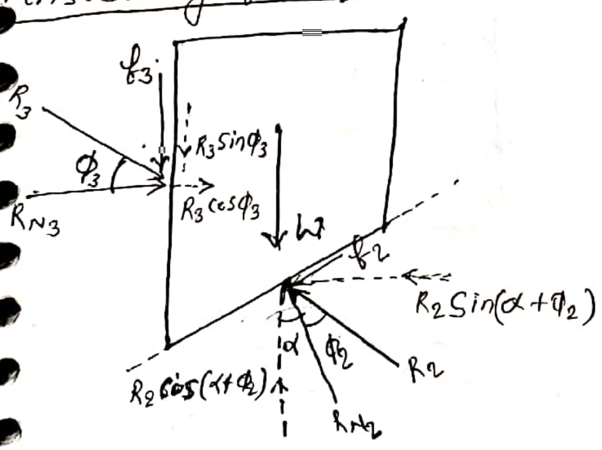
Resolving all the forces horizontally -

$$P - R_1 \sin\phi_1 - R_2 \sin(\alpha+\phi_2) = 0 \quad \text{—(I)}$$

Resolving all the forces vertically

$$R_1 \cos\phi_1 - R_2 \cos(\alpha+\phi_2) = 0 \quad \text{—(II)}$$

considering forces act on the load:-



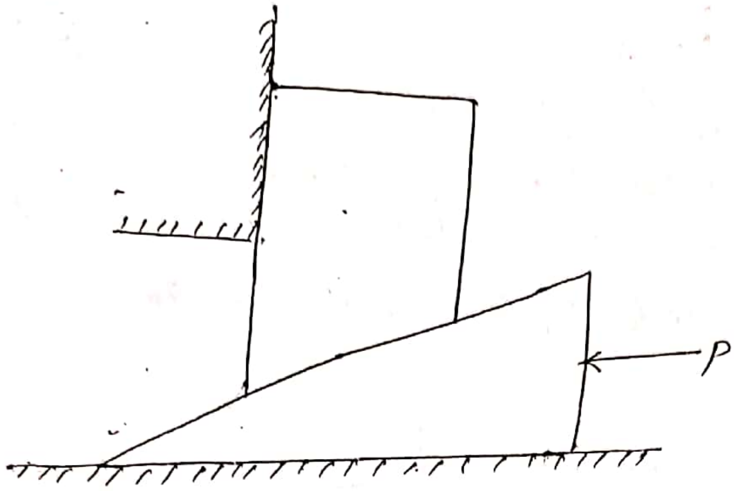
Resolving all the horizontall forces:-

$$R_3 \cos\phi_3 - R_2 \sin(\alpha+\phi_2) = 0 \quad \text{—(III)}$$

Resolving all the vertical forces:-

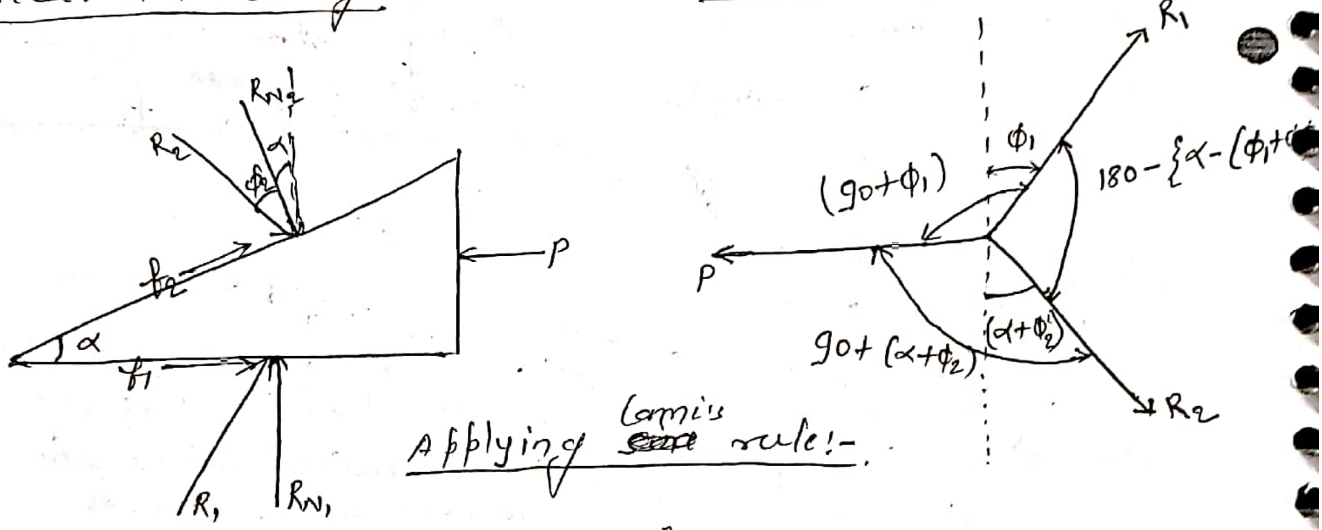
$$R_3 \sin\phi_3 + W - R_2 \cos(\alpha+\phi_2) = 0 \quad \text{—(IV)}$$

ANOTHER METHOD To solve the wedge problem:-



consider the wedge

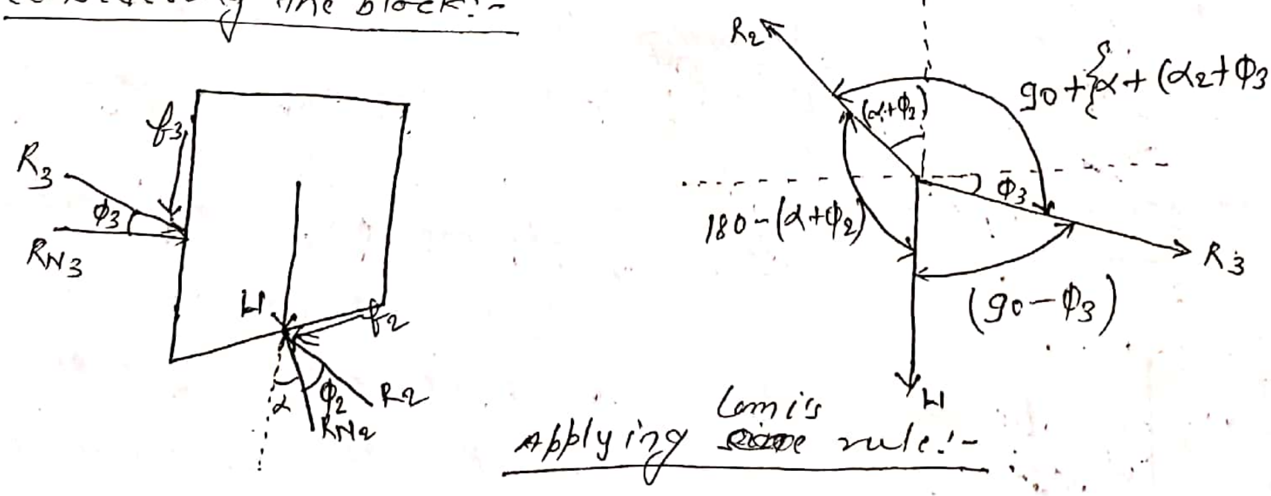
F.B.D



Applying ~~same~~ Lami's rule:-

$$\frac{P}{\sin[180-\{\alpha-(\phi_1+\phi_2)\}]} = \frac{R_1}{\sin[90+(\alpha+\phi_2)]} = \frac{R_2}{\sin(90+\phi_1)}$$

considering the block:-



Applying ~~same~~ Lami's rule:-

$$\frac{W}{\sin[90+\{\alpha+(\phi_2+\phi_3)\}]} = \frac{R_2}{\sin(90-\phi_3)} = \frac{R_3}{\sin[180-(\alpha+\phi_2)]}$$

Using Lami's theorem:-

For block:-

If co-efficient of friction is same i.e. friction angle is same for all surfaces.

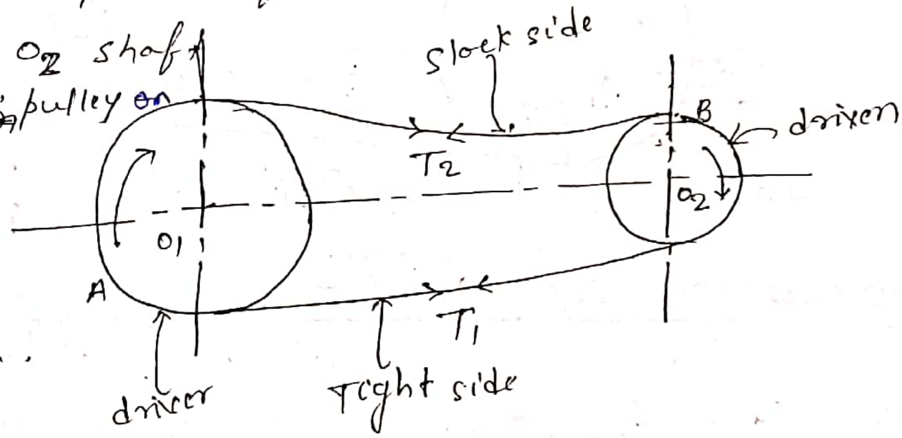
$$\frac{W}{\sin[90+(\alpha+2\phi)]} = \frac{R_2}{\sin(90-\phi)} = \frac{R_1}{\sin[180-(\alpha+\phi)]}$$

For wedge:-

$$\frac{P}{\sin[180-(\alpha+2\phi)]} = \frac{R_2}{\sin(90+\phi)} = \frac{R_3}{\sin[90+(\alpha+\phi)]}$$

BELT FRICTION:- The power is transmitted from one rotating shaft to another by means of belts which is work on the principle of friction.

If we transmit the power from O_1 shaft to the O_2 shaft then we mounted the pulley on the both shaft. after that we connected the both pulley by the belt.



if the pulley A rotates in clockwise direction then the belt is also move along with the pulley because of the friction force applied b/w the pulley surface and the belt surface. This friction force is called the belt friction.

→ The belts or ropes are used to transmit power from one shaft to another shaft.

→ The amount of power transmitted depends upon the following factors:-

(I) The velocity of the belt. (II) The tension of the belt.
(III) The arc of contact b/w the belt and the smaller pulley.

TYPES OF BELT DRIVE:-

(I) Light drives → Belt speed up to 10 m/s.

(II) Medium drives :- Belt speeds over 10 m/s.

(III) Heavy drives :- Belt speeds above 22 m/s.

TYPES OF BELT:-

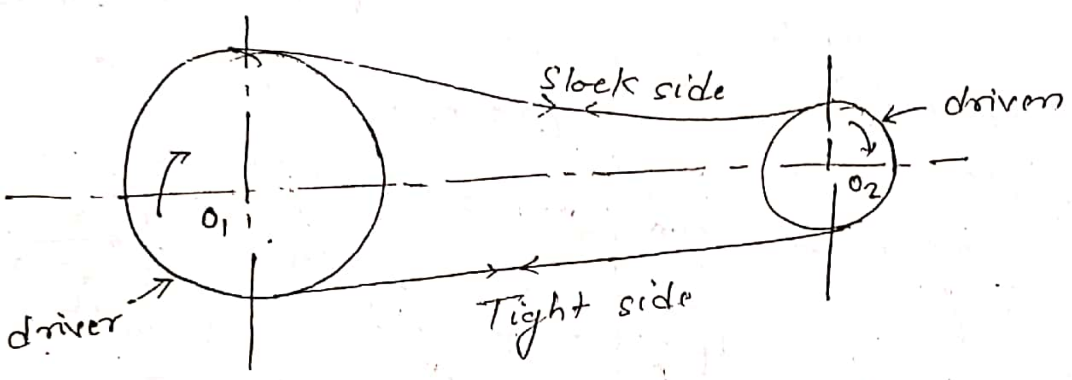
(I) Flat Belt (II) V- Belt (III) Circular belt or rope.

MATERIAL USED FOR BELT:-

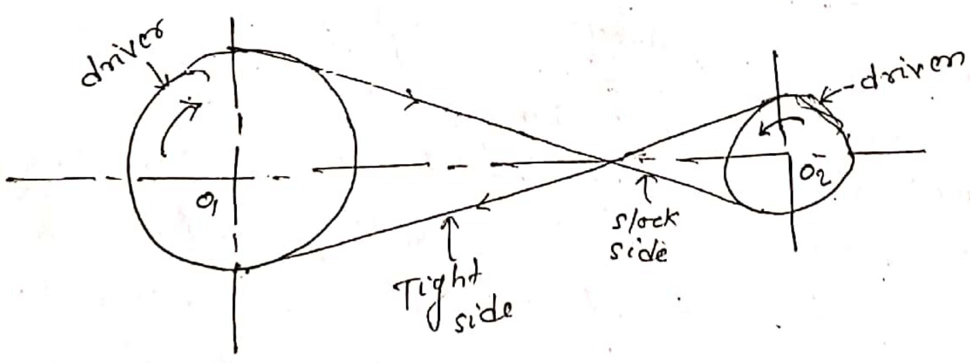
(I) Leather (II) Cotton or fabrics (III) Rubber

TYPES OF FLAT BELT DRIVE:-

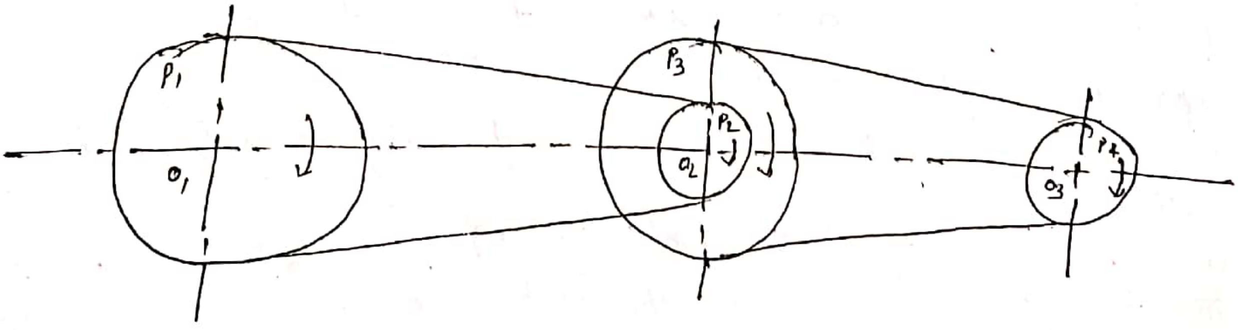
(I) OPEN BELT DRIVE:-



(II) CROSSED BELT DRIVE:-



(III) COMPOUND BELT DRIVE:-



VELOCITY RATIO OF BELT DRIVE:—

Ratio b/w the velocities of the driver and the follower or driven.

Let, $d_1 \rightarrow$ Diameter of the driver
$d_2 \rightarrow$ Diameter of the follower
$N_1 \rightarrow$ speed of the driver in r.p.m
$N_2 \rightarrow$ speed of the follower in r.p.m

∵ Circumference of the pulley $= \pi d$ ~~($2\pi r$)~~

$\boxed{\because \text{Arc} = r\theta}$

∴ Length of the belt passes over the driver in one minute $= \pi d_1 N_1$

∴ Length of the belt that passes over the driven in one minute $= \pi d_2 N_2$

∵ Length of the belt passes over the driver in one minute is equal to the length of the belt passes over the driven in one minute.

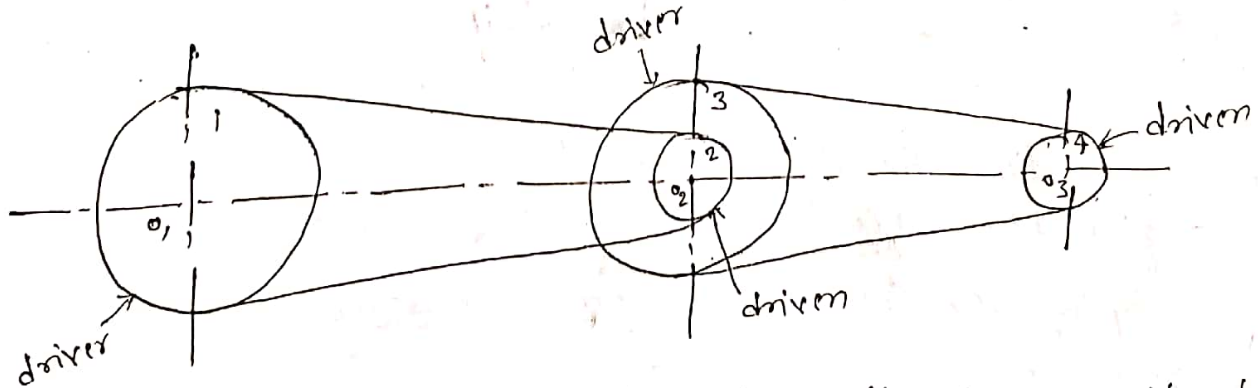
$$\therefore \cancel{\pi} d_1 N_1 = \cancel{\pi} d_2 N_2$$

$$\Rightarrow \boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$ ← Velocity ratio of the belt.

→ when thickness of the belt is considered.

$$\boxed{\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}}$$

VELOCITY RATIO OF COMPOUND BELT DRIVE:—



Consider a pulley one driving the pulley 2. Since the pulley 2 and 3 are keyed to the same shaft.
∴ The pulley 1 also drives the pulley 3 which in turn drives the pulley 4.

Let, d_1 → Diameter of pulley 1.
N_1 → Speed of pulley 1 in mm.
d_2, d_3, d_4 and N_2, N_3, N_4 → corresponding value of pulley 2, 3 and 4.

∵ we know that velocity ratio of pulley 1 and 2.

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{———} (I)$$

Similarly, The velocity ratio of pulley 3 and 4

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \quad \text{———} (II)$$

Multiplying eqn (I) and Eqn (II) we get

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\Rightarrow \boxed{\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}} \qquad ∵ N_2 = N_3$$

∴

$$\boxed{\frac{\text{speed of last driven}}{\text{speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}}$$

SLIP OF BELT:– When the frictional grip b/w the belt and pulley becomes insufficient, there occurs some foreward motion of the drivers without carrying the belt with it. The relative motion b/w the pulley and belt is called slip.

Let,
S_1% → Slip b/w the driver and belt.
S_2% → Slip b/w the driven and belt.

∴ Velocity of the belt passing over the driver per second

$$V = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{S_1}{100}$$

$$\Rightarrow V_{belt} = \pi d_1 N_1 \left(1 - \frac{S}{100}\right) \quad \text{———} (I)$$

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~~velocity of the belt passing over the follower per second~~ (9)

$$\frac{\pi d_2 N_2}{60} = V - V\frac{S_2}{100}$$

$$\Rightarrow \frac{\pi d_2 N_2}{60} = V\left(1 - \frac{S_2}{100}\right)$$

$$\Rightarrow V = \frac{\pi d_2 N_2 / 60}{\left(1 - S_2/100\right)}$$

Put the value of V ~~from eqn (i) in eqn (ii)~~ we get

$$V = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)$$

$$\frac{\frac{\pi d_2 N_2}{60}}{\left(1 - \frac{S_2}{100}\right)} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)$$

$$\Rightarrow \frac{\pi d_2 N_2}{\cancel{60}} = \frac{\pi d_1 N_1}{\cancel{60}}\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

$$V = \omega R \qquad \omega = 2\pi N$$

$$= 2\pi N \times \frac{d}{2}$$

$$V = \pi d N$$

$$V_1 = \frac{\pi d_1 N_1}{60} \qquad V_2 = \frac{\pi d_2 N_2}{60}$$

Velocity of the belt after S_1 slip

$$V_{belt} = V_1 - V_1 \times \frac{S_1}{100} = V_1\left(1 - \frac{S_1}{100}\right)$$

Velocity of the driven after S_1% and S_2% slip.

$$V_2 = V_{belt} - V_{belt} \times \frac{S_2}{100}$$

$$\Rightarrow V_2 = V_{belt}\left(1 - \frac{S_2}{100}\right)$$

$$\Rightarrow V_2 = V_1\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

$$\Rightarrow \frac{V_2}{V_1} = \left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_2}{100} - \frac{S_1}{100} + \frac{S_1 \times S_2}{100 \times 100}\right)$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_2}{100} - \frac{S_1}{100}\right) \qquad \because \frac{S_1 \times S_2}{100 \times 100} \rightarrow \text{Neglected}$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_1 + S_2}{100}\right)$$

$$\Rightarrow \boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S}{100}\right)} \qquad \because S = S_1 + S_2$$

If thickness of the belt is considered.

$$\boxed{\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{S}{100}\right)}$$

CREEP OF BELT :—

When the belt passes from the slack side to the tight side a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length there is a relative motion b/w the belt and the pulley surfaces. This relative motion is termed as creep of belt.

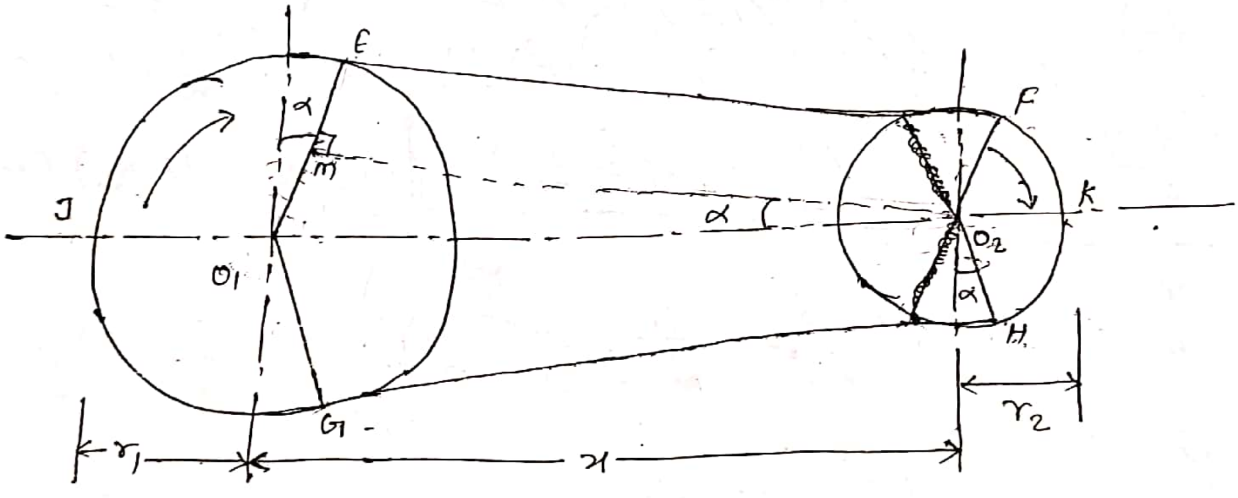
when creep is considerd then velocity ratio is -

$$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E+\sqrt{\sigma_2}}{E+\sqrt{\sigma_1}}}$$

$\sigma_1, \sigma_2 \rightarrow$ stress in the belt

$E \rightarrow$ young modulus.

LENGTH OF AN OPEN BELT DRIVE :-



In the open belt drive both the pulleys rotates in the same direction.

Let, r_1 and $r_2 \rightarrow$ Radius of the longer and smaller pulleys.

$x \rightarrow$ Distance b/w the centres of two pulleys.

$L \rightarrow$ Total length of the belt.

Let, the belt leaves the larger pulley at E and G and the smaller pulley at F and H.

→ Draw O_2m parallel to EF.

→ Let the angle $mO_2O_1 = \alpha$ radians

We know that length of the belt:-

$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$

$= \text{Arc } GJ + \text{Arc } JE + EF + \text{Arc } FK + \text{Arc } KH + HG$

$= 2(\text{Arc } JE + EF + \text{Arc } FK)$ —— (A)

From ΔO_1O_2M

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x} \quad \text{—— (i)}$$

$\because$ α is very small

$\therefore$ $\sin\alpha = \alpha$

$$\therefore \alpha = \frac{r_1 - r_2}{x} \quad \text{—— (ii)}$$

$$\text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \quad \text{—— (iii)}$$

$\boxed{\text{Arc} = r\theta}$

$$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right) \quad \text{—— (iv)}$$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2}$$

$$= \sqrt{x^2 - (r_1 - r_2)^2} = x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2} = x\left[1 - \left(\frac{r_1 - r_2}{x}\right)^2\right]^{1/2}$$

Expanding this equation by binomial theorem

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \text{-----}\right]$$

$$= x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2\right]$$

$$EF = x - \frac{(r_1 - r_2)^2}{2x} \quad \text{—— (v)}$$

Put these value in eqn (A) we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + \left[x - \frac{(r_1 - r_2)^2}{2x}\right] + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2\frac{\pi}{2} - r_2\alpha\right]$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2 + \text{---}$$

$$L = 2\left[\frac{\pi}{2}(r_1+r_2) + \alpha(r_1-r_2) + x - \frac{(r_1-r_2)^2}{2x}\right]$$

$$L = \pi(r_1+r_2) + 2\alpha(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

Put the value of α from eq^n (II) we get

$$L = \pi(r_1+r_2) + 2\frac{(r_1-r_2)}{x}(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

$$= \pi(r_1+r_2) + \frac{2(r_1-r_2)^2}{x} + 2x - \frac{(r_1-r_2)^2}{x}$$

$$\boxed{L = \pi(r_1+r_2) + 2x + \frac{(r_1-r_2)^2}{x}}$$

$$\boxed{L = \frac{\pi}{2}(d_1+d_2) + 2x + \frac{(d_1-d_2)^2}{4x}}$$

~~LENGTH~~

INITIAL TENSIONS IN THE BELT :- (T_0)

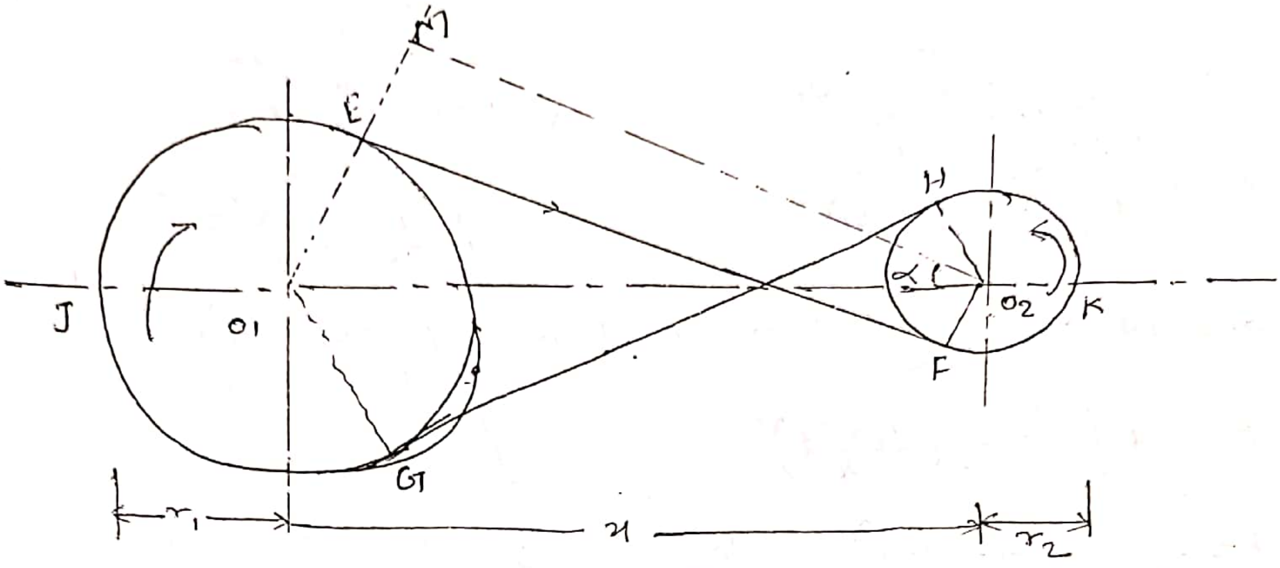
$$\boxed{T_0 = \frac{T_1+T_2}{2}}$$

If centrifugal tension is considerd then

$$T_0 = \frac{(T_1+T_c)+(T_2+T_c)}{2} = \frac{T_1+T_2+2T_c}{2}$$

$$\Rightarrow \boxed{T_0 = \frac{T_1+T_2+2T_c}{2}}$$

LENGTH OF CROSS BELT DRIVE:-



→ In a cross belt drive both the pulley rotates in opposite directions

Let, r_1 & r_2 → Radius of the larger and smaller pulley

x → distance b/w the centre of two pulleys

L → Total length of the belt.

→ Let the belt leaves the larger pulley at G and E and the smaller pulley at F and H.

→ Draw O_2M parallel to EF.

→ Let the angle $MO_2O_1 = \alpha$ radians.

we know that length of the belt:-

$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$

$L = \text{Arc } GJ + \text{Arc } JE + EF + \text{Arc } FK + \text{Arc } KH + HG$

$L = 2(\text{Arc } JE + EF + \text{Arc } FK)$ ——— (A)

From ΔO_1O_2M

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

∵ α is very small

∴ $\sin\alpha = \alpha$

$$\alpha = \frac{r_1 + r_2}{x} \quad \text{——— (I)}$$

Now

Arc $JE = r_1 \times \left(\frac{\pi}{2}+\alpha\right)$ ——— (II)

Arc $FK = r_2\left(\frac{\pi}{2}+\alpha\right)$ ——— (III)

$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{(x)^2 - (r_1+r_2)^2}$

$= x\sqrt{1-\left[\frac{r_1+r_2}{x}\right]^2} = x\left[1-\left(\frac{r_1+r_2}{x}\right)^2\right]^{1/2}$

Expanding this equation by binomial theorem

$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1+r_2}{x}\right)^2 + \text{-------}\right]$

$EF = x - \frac{(r_1+r_2)^2}{2x}$ ——— (IV)

Put these values in equation (A)

$L = 2\left[r_1\left(\frac{\pi}{2}+\alpha\right) + \left[x - \frac{(r_1+r_2)^2}{2x}\right] + r_2\left(\frac{\pi}{2}+\alpha\right)\right]$

$= 2\left[r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1+r_2)^2}{2x} + r_2\pi/2 + r_2\alpha\right]$

$= 2\left[\frac{\pi}{2}(r_1+r_2) + \alpha(r_1+r_2) + x - \frac{(r_1+r_2)^2}{2x}\right]$

$= \pi(r_1+r_2) + 2\alpha(r_1+r_2) + 2x - \frac{(r_1+r_2)^2}{x}$

put the value of α from eqn (I)

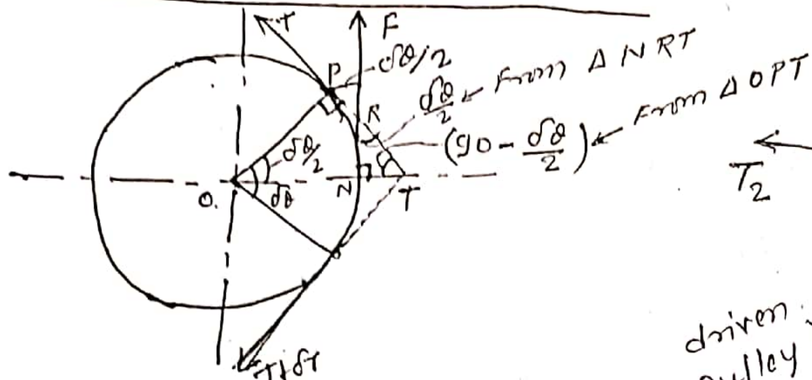
$L = \pi(r_1+r_2) + 2\frac{(r_1+r_2)}{x}(r_1+r_2) + 2x - \frac{(r_1+r_2)^2}{x}$

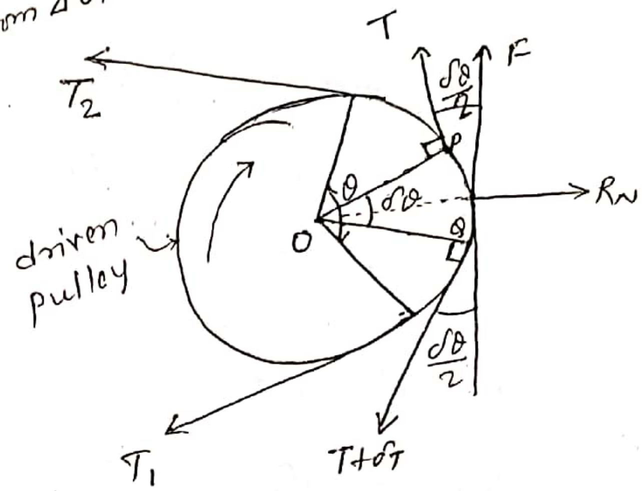
$\Rightarrow L = \pi(r_1+r_2) + 2\frac{(r_1+r_2)^2}{x} + 2x - \frac{(r_1+r_2)^2}{x}$

$\Rightarrow \boxed{L = \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{2x}}$

$\boxed{L = \frac{\pi}{2}(d_1+d_2) + 2x + \frac{(d_1+d_2)^2}{4x}}$

RATIO OF DRIVING TENSIONS:-





driven pulley rotates in clockwise direction.

Let,

$T_1 \rightarrow$ Tension in the belt on the tight side.

$T_2 \rightarrow$ Tension in the belt on the slack side.

$\theta \rightarrow$ Angle of contact in radians.

$T \rightarrow$ Tension in the belt at P.

$T+\delta T \rightarrow$ Tension in the belt at Q.

$R_N \rightarrow$ Normal reaction

$F = \mu R_N \rightarrow$ Frictional force b/w the belt and pulley.

Resolving all the forces horizontally.

$$R_N = (T+\delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \quad \text{——— (I)}$$

$\because \frac{\delta\theta}{2}$ is very small

$$\therefore \sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$$

$$\Rightarrow \therefore R_N = (T+\delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2}$$

$$= 2T\frac{\delta\theta}{\cancel{2}} + \delta T\frac{\delta\theta}{2}$$

$$= T\delta\theta + \delta T.\frac{\delta\theta}{2}$$

$\because \delta T\frac{\delta\theta}{2}$ is neglected due to very small term

$$\Rightarrow R_N = T.\delta\theta \quad \text{——— (II)}$$

Now resolving the forces vertically :-

$$F = (T+\delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

$$\Rightarrow \mu R_N = (T+\delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

$\because \frac{\delta\theta}{2}$ is very small

$\therefore \cos\frac{\delta\theta}{2} = 1$

$$\therefore \mu R_N = \cancel{T} + \delta T - \cancel{T}$$

$$\Rightarrow \mu R_N = \delta T$$

$$\Rightarrow R_N = \frac{\delta T}{\mu} \text{ ——— (III)}$$

From eqn (II) and eqn (III) we get

$$T \cdot \delta\theta = \frac{\delta T}{\mu}$$

$$\Rightarrow \mu\,\delta\theta = \frac{\delta T}{T}$$

$$\Rightarrow \frac{\delta T}{T} = \mu\,\delta\theta$$

Integrating both sides we get

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta$$

$$\Rightarrow \log [T]_{T_2}^{T_1} = \mu[\theta]_0^{\theta}$$

$$\Rightarrow \log(T_1 - T_2) = \mu(\theta - 0)$$

$$\Rightarrow \log T_1 - \log T_2 = \mu\theta$$

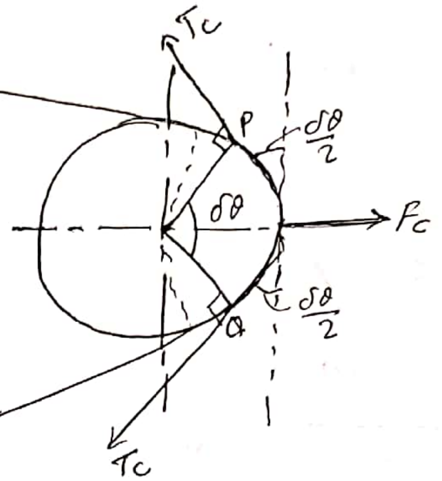
$$\Rightarrow \log_e\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\Rightarrow \boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$

$$\Rightarrow \boxed{2.3 \log \frac{T_1}{T_2} = \mu\theta}$$

CENTRIFUGAL TENSION IN THE BELT.—

since the belt continiously runs over the pulleys therefore some centrifugal force is caused whose effect is to increase the tension on both sides tight as well as slack sides. The tension caused by centrifugal force is called centrifugal tension.



Consider the small portion PQ of the belt subtending an angle $\delta\theta$ the centre of the pulley.

Let,
m = mass of the belt per unit length in kg.
v = linear velocity of the belt in m/s.
r = Radius of the pulley over which the belt runs in meters.
T_c = Centrifugal tension acting tangentially at P and Q in newton's.

Length of the belt (PQ) = $r\,d\theta$

mass of the belt = $m\,r\,d\theta$

∴ Centrifugal force acting on the belt PQ.

$$F_c = (m\cancel{r}\,d\theta)\frac{v^2}{\cancel{r}}$$

$$\Rightarrow F_c = m\,d\theta\,v^2$$

Now centrifugal tension T_c acting tangentially at P and Q keeps the belt in equilibrium.

Now resolving the forces:-

$$T_c \sin\frac{\delta\theta}{2} + T_c \sin\frac{\delta\theta}{2} = F_c$$

$$\Rightarrow 2T_c \sin\frac{\delta\theta}{2} = F_c$$

$\because \frac{d\theta}{2}$ is very small

$\therefore \sin\frac{d\theta}{2} = \frac{d\theta}{2}$

$\therefore \cancel{2} T_c \frac{d\theta}{\cancel{2}} = F_c$

$\Rightarrow T_c\, d\theta = F_c$

$\Rightarrow T_c\, \cancel{d\theta} = mv^2 \cancel{d\theta}$

$\Rightarrow \boxed{T_c = mv^2}$

$\therefore$ Tension in the tight side

$$T_{t_1} = T_1 + T_c$$

Tension in the slack side

$$T_{t_2} = T_2 + T_c$$

Power transmitted (P).

$$P = (T_{t_1} - T_{t_2})\, v$$

$$= (T_1 + \cancel{T_c} - T_2 - \cancel{T_c})\, v$$

$$\boxed{P = (T_1 - T_2)\, v}$$

$\therefore$ The centrifugal tensions does not effect the Power transmission

<u>Condition for transmit the maximum power :-</u>

$$P = (T_1 - T_2)\, v$$

$$= T_1\left(1 - \frac{T_2}{T_1}\right)v$$

$$= T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)v$$

$$= T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)v$$

Let $\left(1 - \frac{1}{e^{\mu\theta}}\right) = K$

$\because T = T_1 + T_c$

$\Rightarrow T_1 = T - T_c$

$\therefore P = T_1 K V$

$P = (T - T_c) K V$

$P = (T - mv^2) K V$

$P = (TV - mv^3) K$

$\frac{dP}{dv} = T - 3mv^2 = 0$

$\Rightarrow T = 3mv^2$

$\Rightarrow \boxed{T = 3T_c}$

$\Rightarrow \boxed{V = \sqrt{\frac{T}{3m}}}$

$\Rightarrow \boxed{T_c = \frac{T}{3}}$

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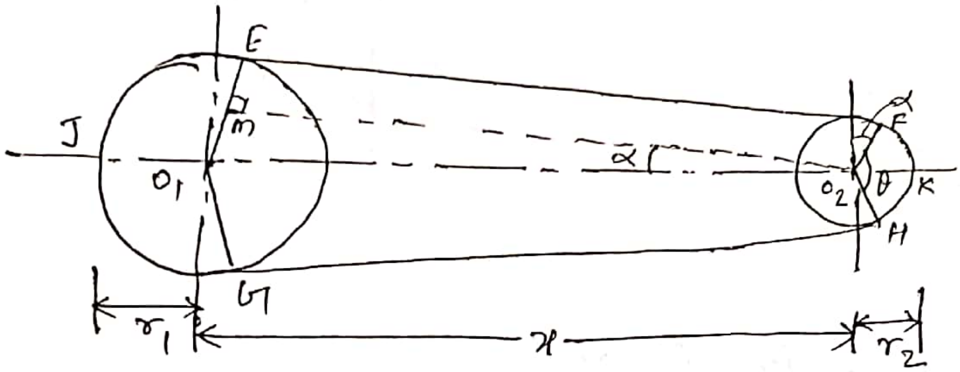
ANGLE OF CONTACT:-

FOR AN OPEN BELT:-

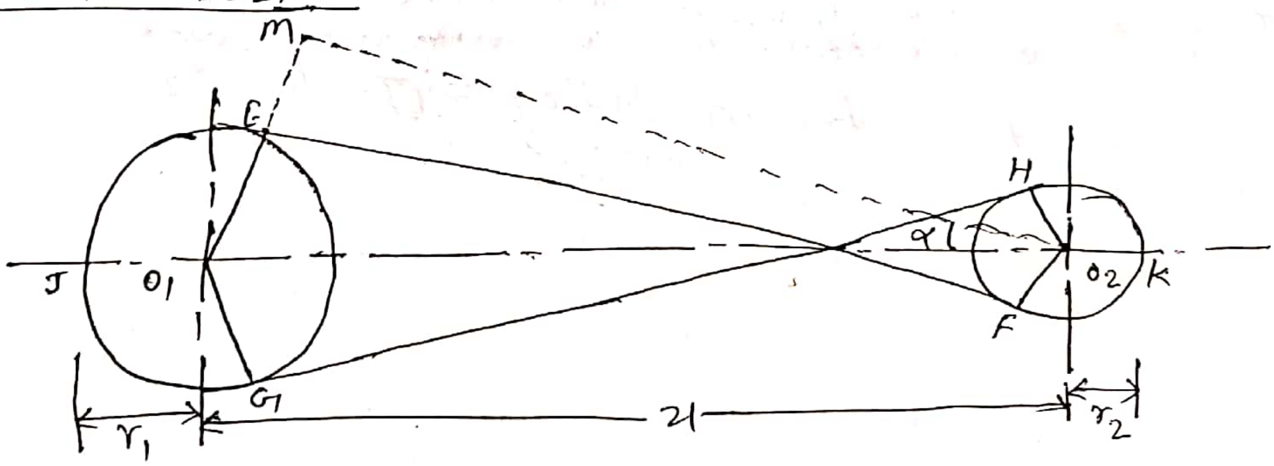


$$\sin\alpha = \frac{O_1 m}{O_1 O_2} = \frac{O_1 E - EM}{O_1 O_2}$$

$$\Rightarrow \boxed{\sin\alpha = \frac{r_1 - r_2}{x}}$$

$\therefore$ angle of contact $\boxed{(\theta) = (180 - 2\alpha) \times \frac{\pi}{180}}$ radians

FOR CROSS BELT:-

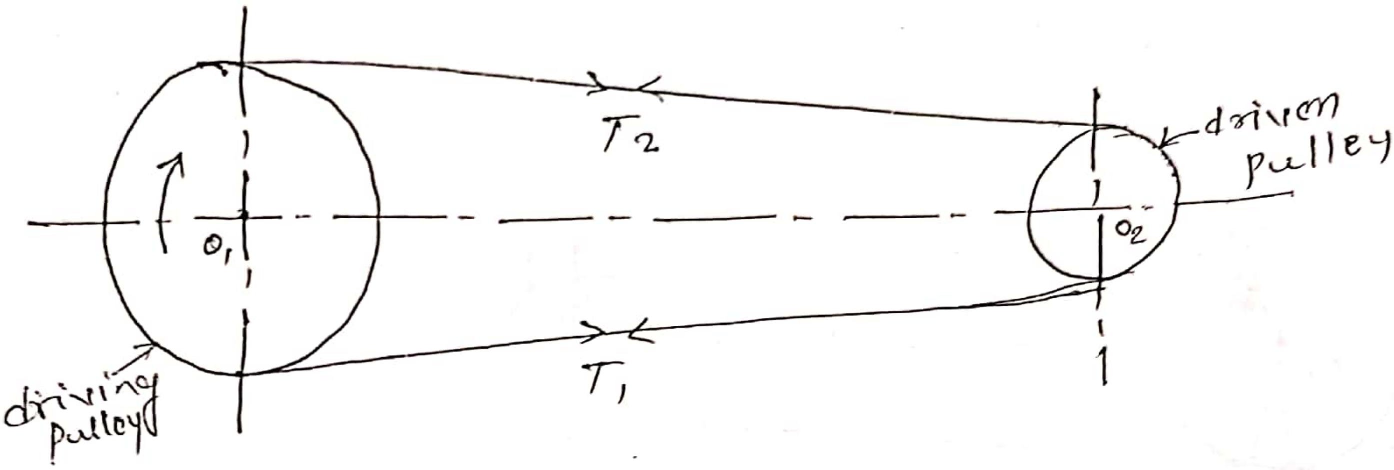


$$\sin\alpha = \frac{O_1 m}{O_1 O_2} = \frac{O_1 E + EM}{O_1 O_2}$$

$$\Rightarrow \boxed{\sin\alpha = \frac{r_1 + r_2}{x}}$$

$\therefore$ Angle of contact $\boxed{(\theta) = (180 + 2\alpha) \frac{\pi}{180}}$ radians

POWER TRANSMITTED BY A BELT:-



work done/sec. $= (T_1 - T_2) v$

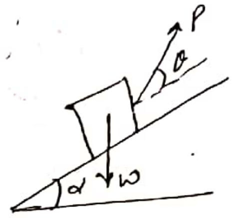
$\therefore$ $\boxed{\text{power } (P) = (T_1 - T_2) v}$ watt

$T_1 \rightarrow$ Tension in the tight side.
$T_2 \rightarrow$ Tension in the slack side.
$v \rightarrow$ Velocity of the belt.

$\therefore$ Torque on the driving pulley $= (T_1 - T_2) r_1$

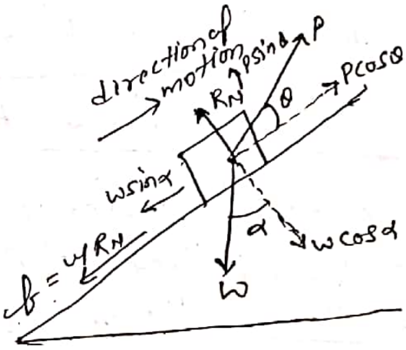
Torque on the driven pulley $= (T_1 - T_2) r_2$

Q. Find the least force required to drag a block of wt. W placed on a rough inclined plane having inclination α with the horizontal. The force ~~required~~ applied to the block makes an angle θ to the inclined plane. Consider the following the cases:- (a) The block is to move up the plane, (ii) The block is to move down the plane.



Soln:-

motion up to the plane:-



Perpendicular to the plane

$$R_N + P\sin\theta = W\cos\alpha$$

$$\Rightarrow R_N = W\cos\alpha - P\sin\theta \quad \text{(a)}$$

Parallel to the plane:-

$$P\cos\theta - f - W\sin\alpha = 0$$

$$\Rightarrow P\cos\theta = \mu R_N + W\sin\alpha$$

Put the value of R_N from eqn (a) we get

$$\Rightarrow P\cos\theta = W\sin\alpha + \mu(W\cos\alpha - P\sin\theta)$$

$$\Rightarrow P\cos\theta = W\sin\alpha + \mu W\cos\alpha - \mu P\sin\theta$$

$$\Rightarrow P\cos\theta + \mu P\sin\theta = W\sin\alpha + \mu W\cos\alpha$$

$$\Rightarrow P(\cos\theta + \mu\sin\theta) = W(\sin\alpha + \mu\cos\alpha)$$

$$\Rightarrow P = W\frac{\sin\alpha + \mu\cos\alpha}{\cos\theta + \mu\sin\theta}$$

$$\because \mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$$

$$\Rightarrow P = W\frac{\sin\alpha + \frac{\sin\phi}{\cos\phi}\cdot\cos\alpha}{\cos\theta + \frac{\sin\phi}{\cos\phi}\cdot\sin\theta} = W\frac{\sin\alpha\cdot\cos\phi + \sin\phi\cdot\cos\alpha}{\cos\theta\cdot\cos\phi + \sin\theta\cdot\sin\phi}$$

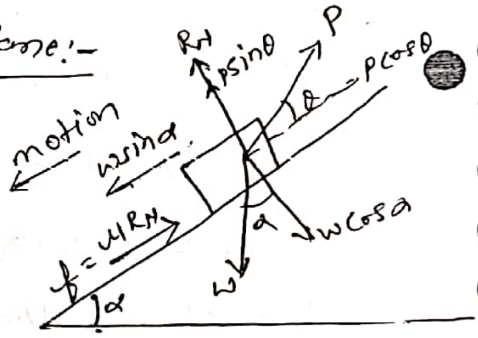
$$\Rightarrow \boxed{P = W\frac{\sin(\alpha+\phi)}{\cos(\theta-\phi)}}$$

P will be minimum when $\cos(\theta-\phi)$ is maximum

$$\cos(\theta-\phi) = 1$$
$$\Rightarrow \cos(\theta-\phi) = \cos 0$$
$$\Rightarrow (\theta-\phi) = 0$$
$$\Rightarrow \boxed{\theta = \phi}$$

$$\therefore \quad P = W \frac{\sin(\alpha+\phi)}{1}$$

$$\Rightarrow \boxed{P = W\sin(\alpha+\phi)}$$



when the block move down to the plane:-

Perpendicular to the plane:-

$$R_N + P\sin\theta = W\cos\alpha$$
$$\Rightarrow R_N = W\cos\alpha - P\sin\theta \quad \text{---} \quad (1)$$

Parallel to the plane:-

$$P\cos\theta + \mu R_N = W\sin\alpha$$
$$\Rightarrow P\cos\theta = W\sin\alpha - \mu R_N$$

Put the value of R_N from eq^n (1) we get

$$P\cos\theta = W\sin\alpha - \mu(W\cos\alpha - P\sin\theta)$$
$$\Rightarrow P\cos\theta + \mu P\sin\theta = W\sin\alpha - \mu W\cos\alpha$$
$$\Rightarrow P(\cos\theta + \mu\sin\theta) = W(\sin\alpha - \mu\cos\alpha)$$

$$\mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$$

$$\Rightarrow P\left(\cos\theta + \frac{\sin\phi}{\cos\phi}\cdot\sin\theta\right) = W\left(\sin\alpha - \frac{\sin\phi}{\cos\phi}\cdot\cos\alpha\right)$$

$$\Rightarrow P[\cos\theta\cdot\cos\phi + \sin\theta\cdot\sin\phi] = W(\sin\alpha\cdot\cos\phi - \cos\alpha\cdot\sin\phi)$$

$$\Rightarrow P[\cos(\theta+\phi)] = W\sin(\alpha-\phi)$$

$$\Rightarrow \boxed{P = W\frac{\sin(\alpha-\phi)}{\cos(\theta+\phi)}}$$

for minimum value of P $\cos(\theta+\phi)$ will be maximum

$$\boxed{P = W\sin(\alpha-\phi)}$$

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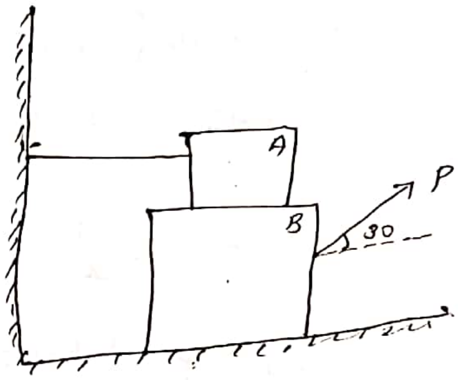
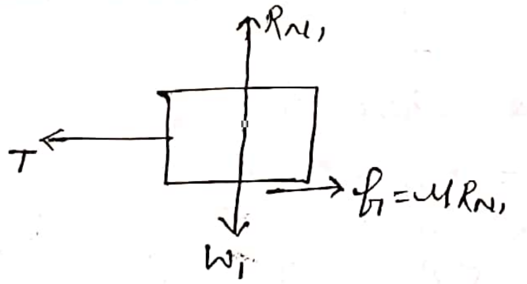
Q. Two body A and B weighing 50 kg and 80 kg respectively are in equilibrium in the position shown in fig. Calculate the force P required to move the lower block B and tension in the cable. Take coefficient of friction at all contact surfaces to be 0.3. (16)



Soln:-

F.B.D at Block A



For equilibrium of block A.

$$\Sigma F_x = 0$$

$$T - f_1 = 0$$

$$\Rightarrow T = \mu R_{N_1}$$

$$\Rightarrow T = 0.3 R_{N_1} \text{ ——— (a)}$$

$$\Sigma F_y = 0$$

$$R_{N_1} = W_1$$

$$\Rightarrow R_{N_1} = mg = 50 \times 9.81$$

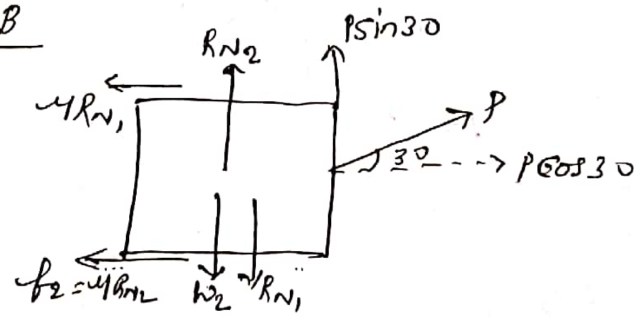
$$\Rightarrow R_{N_1} = 490.5 \text{ N}$$

Put the value of R_{N_1} in eqn (a) we get

$$T = 0.3 \times 490.5$$

$$\Rightarrow \boxed{T = 147.15 \text{ N}}$$

F.B.D. at block B



$\boxed{\Sigma F_x = 0}$

$f_1 + f_2 - P\cos 30 = 0$

$\Rightarrow \mu R_{N_1} + \mu R_{N_2} - P\cos 30 = 0$

$\Rightarrow (0.3 \times 490.5) + 0.3 R_{N_2} = 0.866 P$

$\Rightarrow 147.15 + 0.3 R_{N_2} = 0.866 P$ —— (b)

$\boxed{\Sigma F_y = 0}$

$W_2 + R_{N_1} - R_{N_2} - P\sin 30 = 0$

$\Rightarrow (80 \times 9.81) + (490.5) = R_{N_2} + 0.5P$

$\Rightarrow R_{N_2} = 1275.3 - 0.5P$

Put the value of R_{N_2} in eqn (b)

$147.15 + 0.3(1275.3 - 0.5P) = 0.866P$

$\Rightarrow 147.15 + 382.6 - 0.15P = 0.866P$

$\Rightarrow 529.75 = 1.016P$

$\Rightarrow P = \frac{529.75}{1.016} = 521.40N$

$\Rightarrow \boxed{P = 521.40N}$