Engineering Mechanics (3ME3-04)

DEPARTMENT

OF

MECHANICAL ENGINEERING

(Jaipur Engineering College and Research Centre, Jaipur)

UNIT: II

Centroid & Moment of inertia: Location of centroid and center of gravity, Moment of inertia, Parallel axis and perpendicular axis theorem, Radius of gyration, M.I of composite section, Polar moment of inertia, M.I of solid bodies.

Lifting machines: Mechanical advantage, Velocity Ratio, Efficiency of machine, Ideal machine, Ideal effort and ideal load, Reversibility of machine, Law of machine, Lifting machines; System of pulleys, Simple wheel and axle, Wheel and differential axle, Weston's differential pulley block, Worm and worm wheel, Single purchase winch crab, Double purchase winch crab, Screw jack, Differential screw jack.

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UNIT-2

-: LIFTING MACHINE:-

MACHINE:- A machine may be defined as a device that receives energy in some available form and uses it for doing a particular useful work.

Example:- An I.C. engine which recieve the energy in the form of chemical and it converted into mechanical energy which can be used for doing some work.

SIMPLE M/C:- The devices which enables us to multiply force or to change direction of applied force so as to lift heavy loads are termed as simple machines: i.e. Lever, inclined plane, pulley, screw Jack, wedge, & wheel and axle.

COMPOUND M/C:- A compound m/c is a machine which is a combination of a number of simple machines.

DEFINITIONS RELATED TO LIFTING MACHINE:-

LOAD (W):- Any weight which is lifted by the machine.

EFFORT (P):- A force required to lift or displace the load

INPUT:- It is the work done on machine and is measured by the product of the effort (P) and distance (x) through which it has move. Its unit are N-m or (Joule).

$$\boxed{Input = P \times x} \text{ N-m or Joule}$$

OUTPUT:- It is useful work done by the simple machine and is measured as the product of the load (W) and the distance (d) through lifted by the machine and the distance through which it moves. Its units are N-m or Joule.

$$\boxed{output = W \times y} \text{ N-m or Joule}$$

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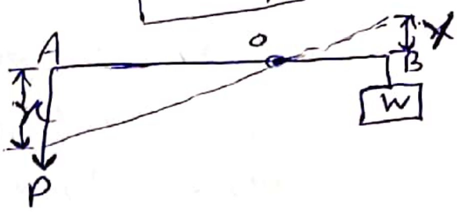
MECHANICAL ADVANTAGE :- (M.A.) :- This is the ratio of weight lifted (W) to the effort applied (P).

$P \times AO = W \times OB$

$\boxed{P = W \times \frac{OB}{AO}}$



$$M.A. = \frac{\text{weight lifted}}{\text{effort applied}}$$

$$\boxed{M.A. = \frac{W}{P}}$$

→ (M.A. > 1) → Mechanical advantage is always greater than one. because the effort applied is generally smaller than the load lifted.

VELOCITY RATIO :- It is the ratio of the distance (x) through which the effort is applied, to the distance (y) through which the weight is lifted in the same time.

$$\boxed{V.R. = \frac{\text{Distance moved by effort } (x)}{\text{Distance moved by load } (y)}}$$

EFFICIENCY OF MACHINE :- It is defined as the ratio of the useful work done by the machine (output), to the total work done upon (input) it. It is expressed as a percentage.

$$\eta = \frac{\text{output of machine}}{\text{Input of machine}}$$

$$\eta = \frac{W \times y}{P \times x} = \left(\frac{W}{P}\right) \times \left(\frac{y}{x}\right)$$

$$\boxed{\eta = M.A. \times \frac{1}{V.R.}}$$

→ No device is available in the universe whose η is 100%.

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IDEAL MACHINE :— If the friction losses are neglected then the machine efficiency is 100% (1.00) that means work input is equal to the work output.

$$\eta = \frac{\text{work output}}{\text{work input}}$$

$$\Rightarrow 1.00 = \frac{W \times y}{P \times x} = \frac{M.A.}{V.R.}$$

$$\Rightarrow \boxed{P \times x = W \times y} \Rightarrow \boxed{\frac{W}{P} = \frac{x}{y}}$$

$$\Rightarrow \boxed{M.A. = V.R}$$

If P_{ideal} is ideal effort then

$$V.R. = \frac{W}{P_i}$$

$$\Rightarrow \boxed{P_i = \frac{W}{V.R.}}$$

FRICTIONAL LOSSES IN MACHINE :— A large part of the workdone upon a machine is used up in overcoming friction b/w its various parts. Thus the useful workdone in lifting the load is reduced and the efficiency of machine is always less than 1 or 100%. Thus for actual machine :

$$\text{output} < \text{Input}$$

$$\text{output} = \text{Input} - \text{loss due to friction}$$

Let, P_{ideal} → Ideal effort required to overcome resistance W.

P_{actual} → Actual effort required to overcome same resistance W.

$P_{friction} \rightarrow$ Effort wasted in overcoming friction.

$$P_{friction} = P_{actual} - P_{ideal}$$

$$P_{actual} = P_{ideal} + P_{friction}$$

$$\eta = \frac{M.A.}{V.R.} = \left(\frac{W}{P_{actual}}\right) \times \frac{1}{V.R.}$$

$$\Rightarrow \boxed{P_{actual} = \frac{W}{\eta} \times \frac{1}{V.R.}} \quad — (1)$$

$$\Rightarrow \eta = \frac{\cancel{W}}{P_{actual}} \times \frac{P_{ideal}}{\cancel{W}}$$

$$\Rightarrow \boxed{\eta = \frac{P_{ideal}}{P_{actual}}} \quad — (2)$$

$\Rightarrow \boxed{M.A. = V.R} \leftarrow$ For ideal condition

$$\frac{W}{P} = V.R$$

$$\therefore \boxed{P_{ideal} = \frac{W}{V.R.}}$$

$$\Rightarrow V.R. = \frac{W}{P_{ideal}}$$

$\eta . P_{act} \times V.R = W.R$

$$P_{friction} = P_{actual} - P_{ideal}$$

$$= \frac{W}{\eta} \times \frac{1}{V.R.} - \frac{W}{V.R.}$$

$$\boxed{P_{friction} = \frac{W}{V.R.}\left(\frac{1}{\eta} - 1\right)} \quad — (3)$$

Similarly

$$W_{ideal} = P \times V.R.$$

and

$$W_{actual} = P\eta \times V.R.$$

$$\because W_{friction} = W_{ideal} - W_{actual}$$

$$= P \times V.R - P \times V.R. \times \eta$$

$$\boxed{W_{friction} = P \times V.R.(1 - \eta)}$$

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REVERSIBLE AND IRREVERSIBLE MACHINE :-

Let an effort P be applied through a distance x to lift a load W through a distance y. On removal of effort P, the following two conditions are likely to occur

(I) The workdone by the m/c is in reverse direction and the load falls. The machine is then called riversible m/c. Example:- A pulley used to draw water from a well with the help of bucket, is a riversible machine because the bucket falls down when the effort to pull it up is removed.

(II) The load does not fall i.e. The work is not done by the m/c in the riverse direction. The machine is then said to be irreversible or self locking. Example:- A screw jack used to lift the motor car is a self locking type lifting m/c. because it holds the car at the same position even when the application of effort is stopped.

In an irreversible machine some ~~useful~~ workdone is lost due to friction and is given by-

$$\text{Friction work} = \text{Input} - \text{Output}$$

$$= Px - Wy$$

On the removal of effort the load will not fall if the friction work is more than the output of machine.

i.e. $\text{Friction work} > Wy$

$$(Px - Wy) > Wy$$

$$\Rightarrow Px > 2Wy$$

$$\Rightarrow \frac{Wy}{Px} < \frac{1}{2}$$

$$\Rightarrow \eta < \frac{1}{2}$$

$$\Rightarrow \boxed{\eta < 50\%}$$

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Thus the condition for irreversibility or selflocking of a m/c is that the efficiency of m/c should be less than 50%. If the efficiency exceeds 50%. the m/c would be reversible.

✓ Q. A machine with velocity ratio 25 can lift a load of 200N on application of an effort of 20N. Comment on the riversibility of machine. Also make calculations for the friction loss of machine.

Soln:- given:-

$$V.R. = \frac{x}{y} = 25$$

$$W = 200 \text{ N}$$

$$P = 20 \text{ N}$$

$$M.A. = \frac{W}{P} = \frac{200}{20} = 10$$

$$\eta = \frac{M.A.}{V.R.} = \frac{10}{25} = 0.4 = 40\%.$$

Since the efficiency of the m/c is less than the 40%. The m/c is irreversible or self locking

Frictional loss in terms of load is—

$$W_{friction} = W_{ideal} - W_{actual}$$

$$= P \times V.R. - W_{actual}$$

$$= 20 \times 25 - 200$$

$$= 300 \text{ N}$$

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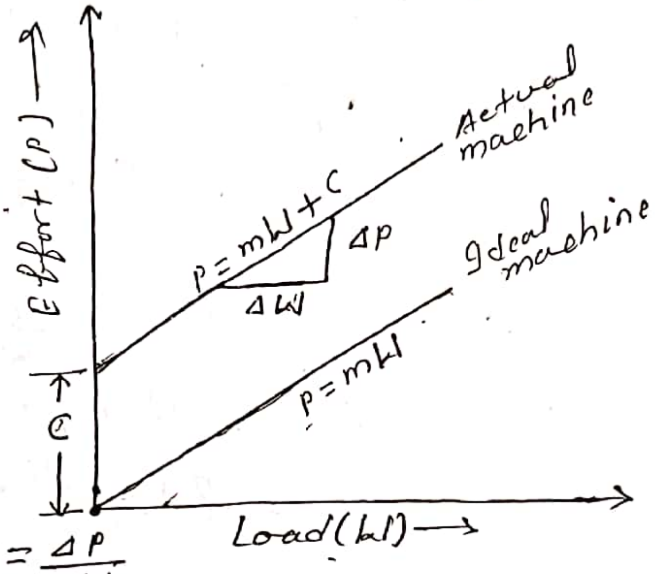
LAW OF MACHINE:-

The law of machine prescribes the relationship b/w the effort applied and the load lifted.

Let, P be the effort required to lift the load W. Then for a machine with constant velocity ratio the law of m/c is given as-

$$\boxed{P = mW + C}$$

straight line



where, m = slope of straight line $= \frac{\Delta P}{\Delta W}$

c = Intercept of the line on P-axis.

The relationship has been depicted graphically as shown in figure. Both for the ideal and actual machine.

(I) For an ideal machine the straight line passes through the origin and the intercept $\boxed{C = 0}$

(II) For an actual m/c the straight line has an intercept c on the P-axis. The intercept represents the effort required to overcome friction. If the effort applied is less than C then the load will not be lifted.

$$\because M.A. = \frac{W}{P} = \frac{W}{mW+C} = \frac{1}{m + \frac{C}{W}}$$

→ By increasing W the value of factor $\frac{C}{W}$ decreases, and that in turn increases the mechanical advantage.

→ The maximum or limiting value of the mechanical advantage will be $\frac{1}{m}$ when $\frac{C}{W}$ approaches zero.

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$$\because \eta = \frac{M.A.}{V.R} = \frac{W/P}{VR} = \frac{W}{P \times V.R}$$

$$= \frac{W}{(mW+C)\,V.R}$$

$$\boxed{\eta = \frac{1}{\left(m + \frac{C}{W}\right) V.R}}$$

Here the value of $\frac{C}{W}$ decreased and that in turn increasing the (η) efficiency. The maximum value of the (η) efficiency will be.

$$\boxed{\eta_{max} = \frac{1}{m \times VR}}$$ when $\frac{C}{W}$ approaches Zero

Q. In an experimental test conducted on a hoasting m/c, it was observed that an effort of 20 kN was applied to lift a load of 90 kN, where as an effort of 16 kN was required to lift a load of 70 kN. Determine the following:—
(I) Law of m/c (II) The limiting Mechanical advantage (III) The limiting efficiency (IV) The effort required to lift a load of 15 kN.

What would be the mechanical advantage and η of the m/c at this moment. Take velocity ratio = 25.

Soln:- $P = mW + C$

(I) $20 = m90 + C$ ——— (I)
$16 = m70 + C$ ——— (II)

From eqn (I) and (II)

$m = 0.2$ and $C = 2$

$\therefore$ The law of m/c is

$P = mW + C$

$\Rightarrow P = 0.2W + 2$

(II) $(MA)_{max} = \frac{1}{m} = \frac{1}{0.2} = 5$

(III) $\eta_{max} = \frac{1}{m \times VR} = \frac{1}{0.2 \times 25} = 0.2$ or 20%

(IV) $P = 0.2 \times 15 + 2 = 5$ kN

$MA = \frac{W}{P} = \frac{15}{5} = 3$

$\eta = \frac{MA}{VR} = \frac{3}{25} = 0.12$ or 12%

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Q. An effort of 50N is required by a machine to lift a load of 500N. The distance moved by the effort is 63 cm and the corresponding load movement is 6 cm. Make calculations for the mechanical advantage, velocity ratio and efficiency of the m/c.

Sol^n:- given:-

$P = 50N$

$W = 500N$

distance moved by the effort $= x = 63$ cm

distance moved by the load $= y = 6$ cm

∵ we know that

$$M.A. = \frac{W}{P} = \frac{500}{50} = 10$$

$$V.R. = \frac{\text{distance moved by the effort}}{\text{distance moved by the load}} = \frac{x}{y}$$

$$\Rightarrow V.R = \frac{63}{6} = 10.5$$

$$\eta = \frac{M.A.}{V.R.} = \frac{10}{10.5} = 0.952 = 95.2\%$$

Q. The velocity ratio of the m/c is 15 and its η is 65%. Determine the load which can be raised on application of an effort of 50N.

Sol^n:- given:- $V.R. = 15$, $\eta = 65\% = 0.65$, $P = 50N$

$$\because \eta = \frac{M.A.}{V.R}$$

$$\Rightarrow M.A. = \eta \times V.R.$$

$$\Rightarrow \frac{W}{P} = 0.65 \times 15$$

$$\Rightarrow \frac{W}{50} = 9.75$$

$$\Rightarrow W = 9.75 \times 50$$

$$\Rightarrow \boxed{W = 487.5\ N}$$

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Q. An effort of 60 N is applied to a m/c to lift a load of 900 N. If the velocity ratio of the m/c is 20. Determine:-

(a) Efficiency of the m/c.

(b) Frictional force in terms of effort.

(c) Frictional force in terms of load.

soln:- given:-

$P = 60 N$

$W = 900 N$

$V.R. = 20$

(a) $\therefore \eta = \frac{M.A.}{V.R.} = \frac{W/P}{V.R.} = \frac{900/60}{20} = \frac{\cancel{900}^{3}}{\cancel{60}_{2} \times \cancel{20}} = \frac{3}{4} = 0.75$ or 75%

(b) $P_{friction} = \frac{W}{V.R.}\left(\frac{1}{\eta} - 1\right)$

$= \frac{900}{20}\left(\frac{1}{0.75} - 1\right)$

$= 15 N$

(c) $W_{friction} = P \times V.R. (1 - \eta)$

$= 60 \times 20 (1 - 0.75)$

$= 300 N$

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Q. When an effort of 280N is applied to lifting m/c It was found that the 25% effort applied is lost in friction. The velocity ratio is 12. Find the load which can be lifted and the efficiency of the m/c at this load.

Soln.- given:- $P = 280N$

$F_p = 25\%$ of $P = 0.25P$

$V.R. = 12$

$\because$ $\boxed{F_p = P - P_i}$

Effort lost in friction = Actual effort − Ideal effort

$$F_p = P - P_i$$

$$\Rightarrow 0.25P = P - \frac{W}{V.R.} \qquad \boxed{\because P_i = \frac{W}{V.R.}}$$

$$\Rightarrow 0.25P = P - \frac{W}{12}$$

$$\Rightarrow \frac{W}{12} = P - 0.25P$$

$$\Rightarrow \frac{W}{12} = 0.75P$$

$$\Rightarrow W = 0.75P \times 12$$

$$\Rightarrow W = 0.75 \times 280 \times 12 = 2520N$$

$$\eta = \frac{M.A.}{V.R.} = \frac{W/P}{12}$$

$$= \frac{W}{P} \times \frac{1}{V.R} = \frac{2520}{280} \times \frac{1}{12}$$

$= 0.75$ or 75%.

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[illegible]

Q.(I) In a m/c it was found that the effort had to be moved through a distance of 250 mm to lift the load by 5 mm. Using this m/c a load of 40000 N was raised by an effort of 1000 N. Determine:

(I) Velocity ratio of the m/c
(II) Mechanical advantage (III) Efficiency
(IV) Effort required to lift the load under ideal condition.
(V) Effort lost in friction.
(VI) The load which ~~can~~ could have been lifted with the given effort under ideal conditions.
(VII) Friction of the m/c.

Soln:- Given:- $W = 40{,}000\,N$

$P = 1000\,N$

distance moved by effort $(x) = 250\,mm$

distance moved by load $(y) = 5\,mm$

(I) Velocity ratio = $\dfrac{\text{distance moved by effort}}{\text{distance moved by load}}$

$$\Rightarrow V.R. = \frac{x}{y} = \frac{250}{5} = 50$$

(II) Mechanical Advantage = $\dfrac{\text{load lifted}}{\text{Effort applied}}$

$$\Rightarrow M.A. = \frac{W}{P} = \frac{40000}{1000} = 40$$

(III) $$\eta = \frac{M.A.}{V.R.} = \frac{40}{50} = 0.8 \text{ or } 80\%$$

(IV) Ideal effort (P_i) = $\dfrac{\text{load lifted}}{\text{Velocity ratio}} = \dfrac{W}{V.R.}$

$$= \frac{40000}{50} = 800\,N$$

(V) Effort lost in friction = Actual effort − Ideal effort

$$= 1000 - 800 = 200\,N$$

(VI) Ideal load which can be lifted with an effort of 1000

$$W_i = P(V.R.) = 1000 \times 50 = 50000\,N$$

(VII) Friction of the m/c = Ideal load − Actual load = $W_i - W$

$$F_W = 50000 - 40000 = 10000\,N$$

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Q. In a lifting m/c whose velocity ratio is 40, a load of 2000N was lifted with an effort of 160N. Suppose the effort is removed, will there be a reversal of the machine? Also find the frictional load of the m/c.

Sol^n:- given:- $W = 2000\,N$
$P = 160\,N$
$V.R. = 40$

M/c to be reversible if the $\boxed{\eta > 50\%}$

$$\therefore \quad \eta = \frac{M.A}{V.R.} = \frac{W/P}{V.R.} = \frac{2000}{160\times 40} = 0.3125 = 31.25\%$$

Since the η is less than 50%. the m/c is non-reversible.

Ideal load $(W_i) = P\,(V.R.)$

$W_i = 160\times 40 = 6400\,N$

Actual load $(W) = 2000\,N$

Frictional load $(F_W) = W_i - W$

$= 6400 - 2000 = 4400\,N$

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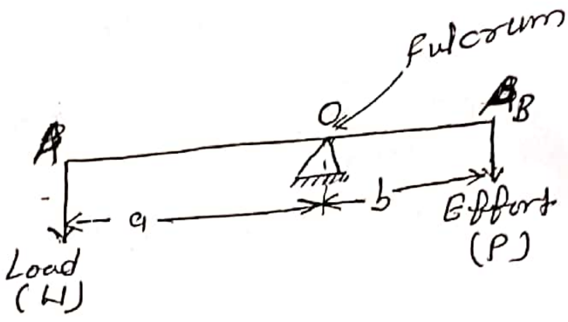
LEVERS



A lever is essentially a rigid straight bar which rests on and can turn about a point called fulcrum. It enables a small effort to overcome a large load

- The perpendicular distance of point A, at which load is applied from the fulcrum (O) is called load arm (OA = a).

The perpendicular distance of point B, at which effort is applied from the fulcrum (O) is called effort arm (OB = b).

When the lever is in equilibrium $\Sigma M = 0$

Taking moments about the fulcrum point O

$$W \times a = P \times b$$

$$\Rightarrow \frac{W}{P} = \frac{b}{a}$$

$$\Rightarrow \boxed{\frac{W}{P} = \frac{\text{length of effort arm}}{\text{Length of load arm}} = \frac{b}{a}}$$

← This is known as law of arm

This relation has been setup with the assumptions:-

(I) The lever is weightless.

(II) The friction is neglected.

"The mechanical advantage of a lever is equal to the ratio of the length of effort arm to the length of load arm." $\boxed{\frac{W}{P} = MA = \frac{b}{a}}$

→ For greater mechanical advantage that is to lift a greater load with less effort, the effort arm should be as larger as possible. The ratio of length of effort arm to the length of load arm (b/a) is called leverage.

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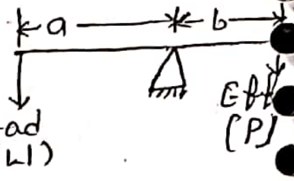
CLASSIFICATION OF LEVERS

① LEVER OF FIRST KIND :-

→ fulcrum is b/w the load and the effort



→ $M.A. = \frac{\text{Effort arm}}{\text{load arm}} = \frac{b}{a}$

M.A. can be more than 1, equal to 1 or less than 1

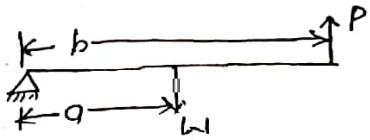
→ M.A. is increases with movement of fulcrum towoards the load.

→ When M.A. is greater than one less effort will be required to lift a heavy load the lever is then reffered to as effort multiplier lever.

→ Handle of water pump, plier, Sea-saw, scissor, ~~extractor~~

② LEVER OF SECOND KIND :-

→ load is b/w the effort and fulcrum.



→ $M.A. = \frac{b}{a}$, M.A. is always greater than one.

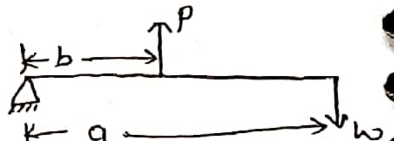
→ M.A. increases by moving the load towards fulcrum.

→ Since M.A. is always greater than 1, lever is known as multiplexer lever.

→ Example :- wheel-barrow (Thela), nut-cracker, lemon crusher etc.

③ LEVER OF THIRD KIND :-

→ Effort is b/w the fulcrum and the load.



→ $M.A. = \frac{b}{a}$, M.A. is always less than 1.

→ M.A. can not be made greater than 1 by any movement of load point.

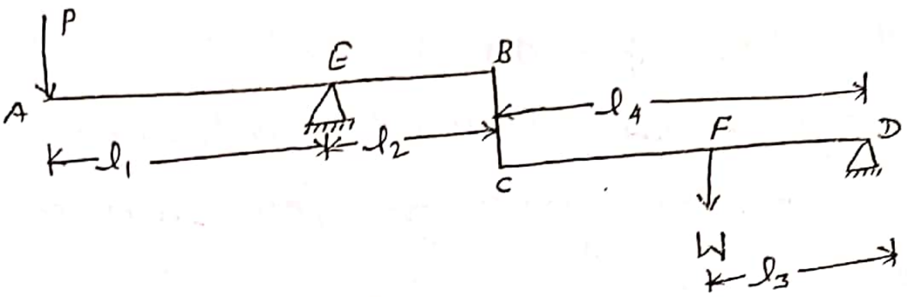
→ Since M.A is always less than 1, lever of this kind is only as speed multiplier lever. This type of levers cannot lift heavy loads but provide an increase in the speed of lifting.

→ Examples :- Fire tongs (चिमटा), ~~knife~~, human arm etc.

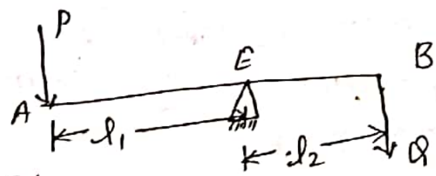
Fishing Rod

COMPOUND LEVER:-

A compound lever is ~~used~~ combination of simple levers linked with one another. Such levers are used to obtain higher mechanical advantage,



With refrence to figure:-
AB is a simple lever connected to another simple lever CD with the help of a link BC. P is the effort applied at end A to lift a load W acting at point F

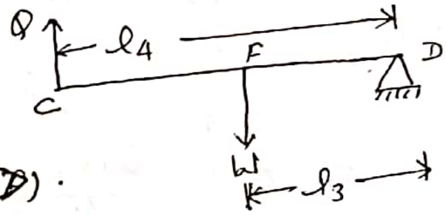


consider the F.B.D of lever AB, we have:-
Taking moments about point (E).

$$P \times l_1 = Q \times l_2$$

$$\Rightarrow \boxed{Q = \frac{P \times l_1}{l_2}} \quad \text{(I)}$$

Q is the force in link BC.



consider the F.B.D of lever CD we have:-
Taking moments about Point (D).

$$Q \times l_4 = W \times l_3$$

$$\Rightarrow \boxed{Q = \frac{W \times l_3}{l_4}} \quad \text{(II)}$$

From eqn (I) and (II) we get

$$\frac{P \times l_1}{l_2} = \frac{W \times l_3}{l_4}$$

$$\Rightarrow \frac{W}{P} = \frac{l_1 \times l_4}{l_2 \times l_3}$$

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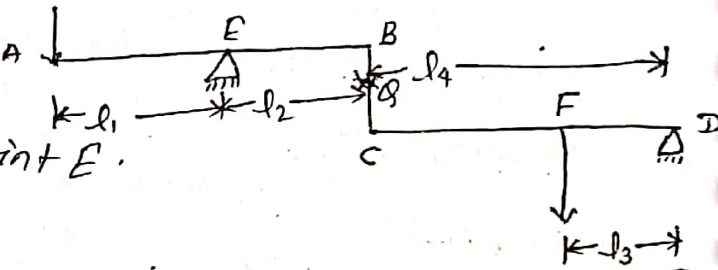
$$\boxed{M.A. = \frac{W}{P} = \frac{l_1}{l_2} \times \frac{l_4}{l_3}}$$

Let, $\frac{l_1}{l_2} = \frac{l_4}{l_3} = 10$

If only the lever AB is used the mechanical advantage would be 10. By combining two levers the mechanical advantage gets increased to $10 \times 10 = 100$.

Q. It is desired to lift 20kN load acting at point F with the help of a system of levers as shown in fig. What effort should be applied at end A of the lever so that the load just gets lifted. Also determine the mechanical advantage of the composite lever. Take – $l_1 = 150\,mm$, $l_2 = 30\,mm$, $l_3 = 60\,mm$, and $l_4 = 300\,mm$.



Sol^n:- For lever AB taking moments about Point E.

$P \times l_1 = Q \times l_2$

$Q = \frac{P \times l_1}{l_2}$ ——— (I)

For lever CD taking moments about Point D:

$Q \times l_4 = W \times l_3$

$\Rightarrow Q = \frac{W \times l_3}{l_4}$ ——— (II)

From eq^n (I) and (II) we get

$\frac{P \times l_1}{l_2} = \frac{W \times l_3}{l_4}$

$\Rightarrow P = \frac{W \times l_2 \times l_3}{l_1 \times l_4} = \frac{20 \times 30 \times 60}{150 \times 300} = \frac{4}{5} = 0.8\,kN$

$M.A. = \frac{W}{P} = \frac{20}{0.8} = 25$

PULLEYS :- FIXED AND MOVABLE

A pulley is essentially a metalic or wooden wheel which is capable of rotation about an axis. The wheel has groove cut along its periphery and a rope is made to rest in the groove. When a chain is used instead of rope, sprocket teeth are cut on the periphery of the wheel.

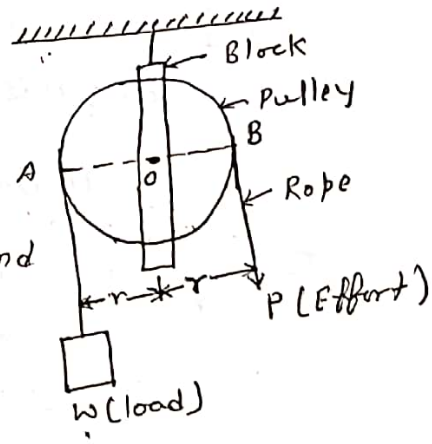
Pulleys are of two type's (I) fixed pulley and (II) movable pulley.

Assumptions for pulley arrangement :-

→ The wt. of pulley is small compared to the wt. to be lifted and hence is neglected.

→ The pulley is smooth i.e. the tension of the string or rope passing ~~through~~ arround the pulley is same throughout.

SINGLE FIXED PULLEY :-



The block or axle supporting the pulley is fixed, its position does not change when the chain or rope passing around its periphery is moved. The wt. W is attached to one end of the rope and the effort P is applied at the other end.

For the equilibrium condition

$\Sigma M = 0$ (Taking moments about point O)

$P \times r - W \times r = 0$

$\Rightarrow P \times \cancel{r} = W \times \cancel{r}$

$\Rightarrow P = W$

$\therefore M.A. = \frac{W}{P} = 1$

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In the absence of friction

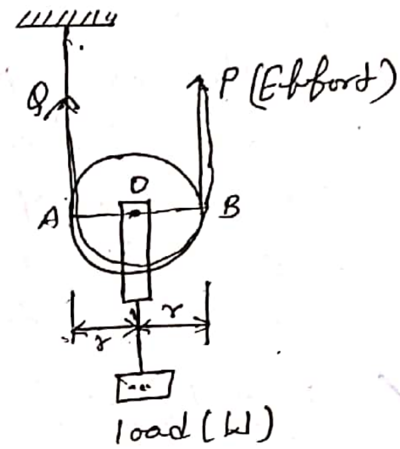
$$M.A = V.R = 1$$

$$\therefore \eta = \frac{M.A}{V.R} = 1 \text{ or } 100\%$$

→ To change the direction of applied force which is always easier to apply in the ~~direction~~ downward direction.

→ To raise a load in upward direction by applying effort in downward direction.

SINGLE MOVABLE PULLEY:-



A movable pulley changes its position when the work is being done. Load to be raised is attached to the pulley itself and the axle of the pulley rises and decends with the load.

Under equilibrium conditions $\Sigma m = 0$

Taking moments of all forces about the axle.

$$Q \times r - P \times r + \cancel{W \times 0} = 0$$

$$\Rightarrow Q \times r = P \times r$$

$$\Rightarrow Q = P \quad \text{—— (I)}$$

Taking moments about point A,

$$W \times r - P \times 2r = 0$$

$$\Rightarrow W \times \cancel{r} = P \times 2\cancel{r}$$

$$\Rightarrow P = \frac{W}{2} \quad \text{—— (II)}$$

From eqn (I) and

$$\boxed{P = Q = \frac{W}{2}}$$

M.A $\Rightarrow \boxed{\frac{W}{P} = 2}$

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SYSTEM OF PULLEY :—

A number of pulleys are so arranged that the composite system results into gain in mechanical advantage. There are essentially three systems of pulleys i.e. The first, second and third system.

FIRST SYSTEM OF PULLEY :-

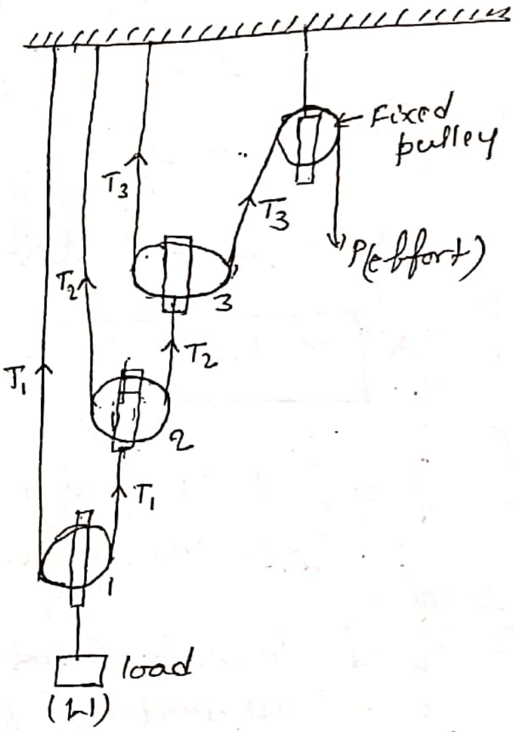

Figure shows first system of pulley using three pulleys.

→ All the pulleys 1, 2 and 3 are movable pulleys.

→ A separate rope passes around the periphery of each pulley. One end of the rope is fastend to a fixed support and the other end is connected to the axle of the next upper pulley.

→ The load is attached to the bottom most pulley where as the effort is applied to the effort end of rope which passes round the upper most pulley.

For equilibrium conditions:-

$$W = 2T_1$$

$$T_1 = 2T_2$$

$$T_2 = 2T_3$$

$$T_1 = 2T_2 = 2 \times 2T_3 = 4T_3$$

$$W = 2T_1 = 2 \times 4T_3 = 8T_3$$

$$P = T_3$$

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$\because MA = \frac{W}{P} = \frac{8\cancel{T_3}}{\cancel{T_3}} = 8$

$\Rightarrow MA = 2^3$

$\boxed{Input = P \times x}$

$\boxed{output = W \times y}$

For ideal condition there is no friction losses.

Input = output

$Px = W \times y$

$\Rightarrow \frac{W}{P} = \frac{x}{y}$

$\Rightarrow M.A. = V.R = 2^3$

$\Rightarrow \boxed{M.A. = V.R. = 2^n}$ where n = No. of movable pulleys

Q. An effort of 100N is required to lift a load of 2500N by the first system of pulleys which has 5 movable pulleys, Determine:-

(I) η of the m/c. (II) Effort wasted in friction.

(III) Load wasted in friction:-

Soln:- For a first system of pulleys

$VR = 2^n$

$VR = 2^5 = 32$

$MA = \frac{W}{P} = \frac{2500}{100} = 25$

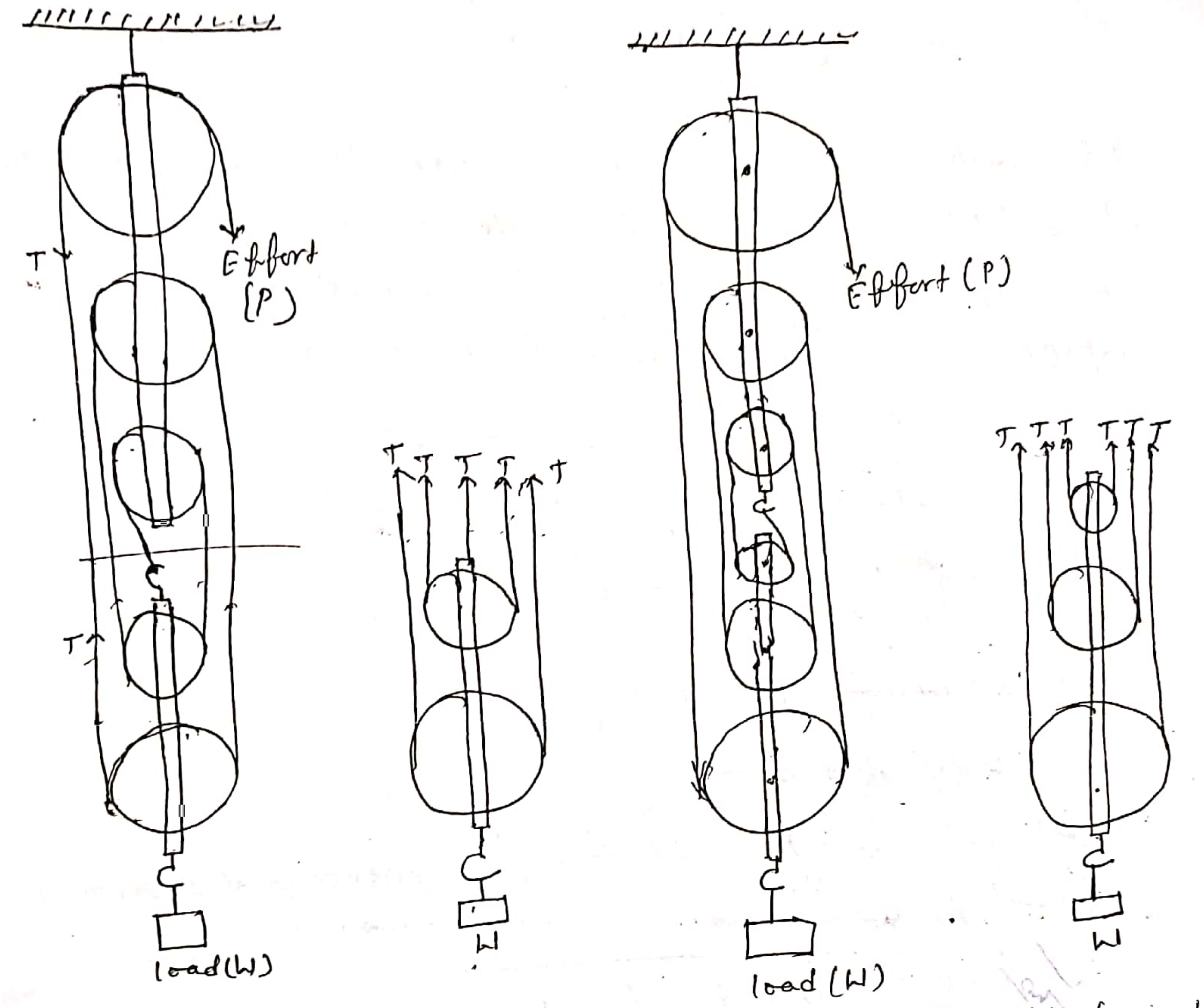
(I) $\eta = \frac{MA}{VR} = \frac{25}{32} = 0.781$ or 78.1%.

(II) $P_{friction} = \frac{W}{VR}\left(\frac{1}{\eta} - 1\right) = \frac{2500}{32}\left(\frac{1}{0.781} - 1\right) = 21.9$ N

(III) $W_{friction} = P \times VR(1-\eta) = 100 \times 32(1 - 0.781)$

$= 700.8$ N

SECOND SYSTEM OF PULLEYS:- Figure shows an arrangement for the second system of pulleys



→ The system has three pulleys in the upper block and two pulleys in the lower block

→ The upper block is fixed to a support and lower block is movable.

→ The weight W is attached to the lower block and the effort is applied at the free end of the rope.

Therefore equilibrium of the lower block.

$W = 5T$ $\qquad \boxed{T = P}$

$\Rightarrow W = 5P$

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$\therefore MA = \frac{W}{P} = \frac{5P}{P} = 5$

For an ideal condition

$$MA = VR = 5$$

If both the upper and lower block are same number of pulleys then start is made from one end of the rope fixed to the lower ~~end~~ most pulley in the upper block. Equilibrium of the lower block then gives

$$W = 6T \qquad T = P$$

$$\therefore W = 6P$$

$$\therefore MA = \frac{W}{P} = \frac{6P}{P} = 6$$

For an ideal condition

$$VR = MA = 6$$

From the result obtained, In the second system of pulley

$\boxed{VR = \text{Number of pulley}}$

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THIRD SYSTEM OF PULLEYS:-

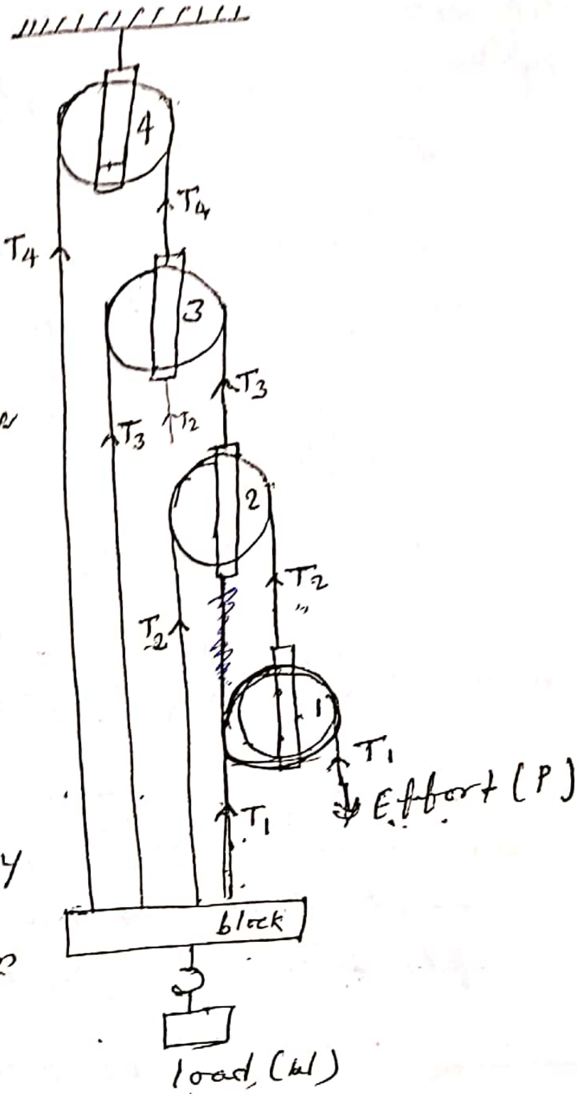

Figure shows the arrangement of third system of pulleys.

→ Several moveable pulleys are used and the topmost pulley is kept fixed.

→ There is same number of ropes as the numbers of pulleys

→ One end of each rope is connected to a block which carries the load and other end is fixed to the next lower pulley.

→ The effort is applied to the free end of the lowest pulley

For equilibrium of these system

$W = T_1 + T_2 + T_3 + T_4$ —— ①

$T_1 = P$

$T_2 = 2T_1 = 2 \times P = 2P$

$T_3 = 2T_2 = 2 \times 2P = 4P = 2^2 P$

$T_4 = 2T_3 = 2 \times 4P = 8P = 2^3 P$

$\therefore W = (P + 2P + 2^2 P + 2^3 P + \text{-----}) = P(2^4 - 1)$

$\therefore M.A. = \frac{W}{P} = \frac{P(2^4-1)}{P} = 2^4 - 1$

In general if there are n = no. of pulley

$\therefore \boxed{M.A. = 2^n - 1}$

For ideal codition $MA = VR$

$\therefore \boxed{VR = 2^n - 1}$.

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Q. In a system of pulleys with one string there are five segments of the string at the lower block. What is the velocity ratio of the pulley arrangement? If a force of 200N is req. to lift a load of 600N, calculate the η of the system.

Sol^n:- Since there is only one string the arrangement corresponds to second system of pulleys.

$$VR = \text{No. of pulleys.}$$

$$VR = 5$$

$$M.A. = \frac{W}{P} = \frac{600}{200} = 3$$

$$\therefore \eta = \frac{MA}{VR} = \frac{3}{5} = 0.6 \text{ or } 60\%$$

Q. For a third order pulley system having three pulleys an effort of 200N is required to lift a load of 1000N. Calculate the η of the system and the effort lost in friction.

Sol^n:- For a third order pulley system

$$VR = 2^n - 1 = 2^3 - 1 = 7$$

$$MA = \frac{W}{P} = \frac{1000}{200} = 5$$

$$\eta = \frac{MA}{VR} = \frac{5}{7} = 0.714 \text{ or } 71.4\%$$

$$P_{friction} = \frac{W}{VR}\left(\frac{1}{\eta} - 1\right)$$

$$= \frac{1000}{7}\left(\frac{1}{0.714} - 1\right)$$

$$= 57.22 \text{ N}$$

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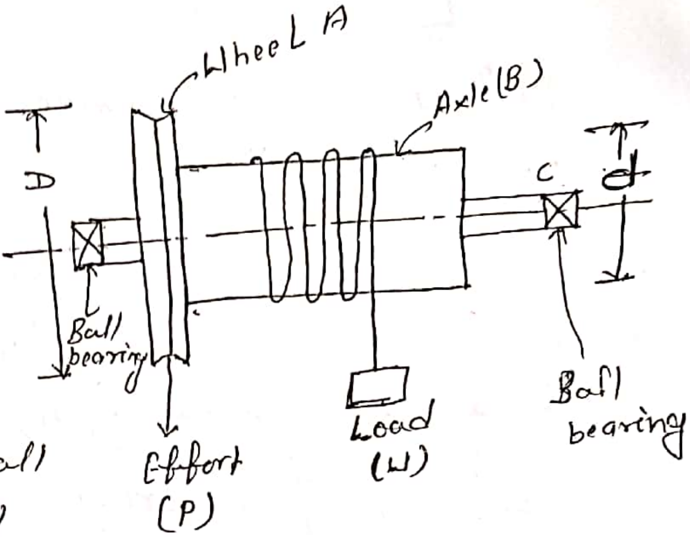
SIMPLE WHEEL AND AXLE:-

A simple and axle unit consists of a wheel A of larger diameter and an axle B of small dia. Both are keyed to the same spindle C. The entire assembly is mounted on ball bearing so that the wheel



and axle can be rotated. The load W to be lifted is attached to a string which is wound round the axle. Another string is wound round the wheel and the effort P is applied to it. These two strings are wound in opposite direction. which is makes the load move upward when the effort is applied downward.

The wheel and axle are keyed to the same spindle and therefore when the wheel makes one revolution, the axle would also turn one revolution.

Let,
D = Diameter of the wheel.
d = Diameter of the axle.

In one revolution of wheel the distance traveled by the effort $= \pi D$

In one revolution of exle the distance travelled. by the load $= \pi d$

$$\therefore VR = \frac{\text{distance moved by effort}}{\text{Distance moved by load}} = \frac{\pi D}{\pi d}$$

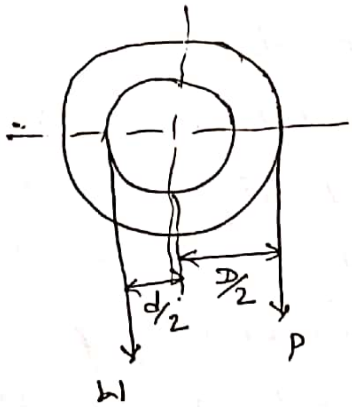
$$\Rightarrow \boxed{VR = \frac{D}{d}}$$

If t_1 and t_2 represents the thickness of string on the wheel and axle, then

$$\boxed{V.R. = \frac{D+t_1}{d+t_2}}$$

<u>If friction force is neglected</u>

For an equilibrium condition $\Sigma m = 0$



$$W \times \frac{d}{2} - P \times \frac{D}{2} = 0$$

$$\Rightarrow \frac{W \times d}{2} = \frac{P \times D}{2}$$

$$\Rightarrow \boxed{\frac{W}{P} = \frac{D}{d}}$$

$$\Rightarrow \boxed{MA = VR}$$

<u>Effect of friction:-</u>

Let P' be the effort required to lift the load W.

work input $= P' \times \pi D$

work output $= W \times \pi d$

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{W \times \cancel{\pi} d}{P' \times \cancel{\pi} D}$$

$$\Rightarrow \boxed{P' = \frac{W}{\eta} \times \frac{d}{D}}$$

M.A (actual)

$$\Rightarrow \boxed{MA = \frac{D}{d} \times \eta}$$

MA (Actual) $= \frac{W}{P'} = \frac{W}{\frac{W}{\eta} \times \frac{d}{D}} = \frac{D}{d} \times \eta$.

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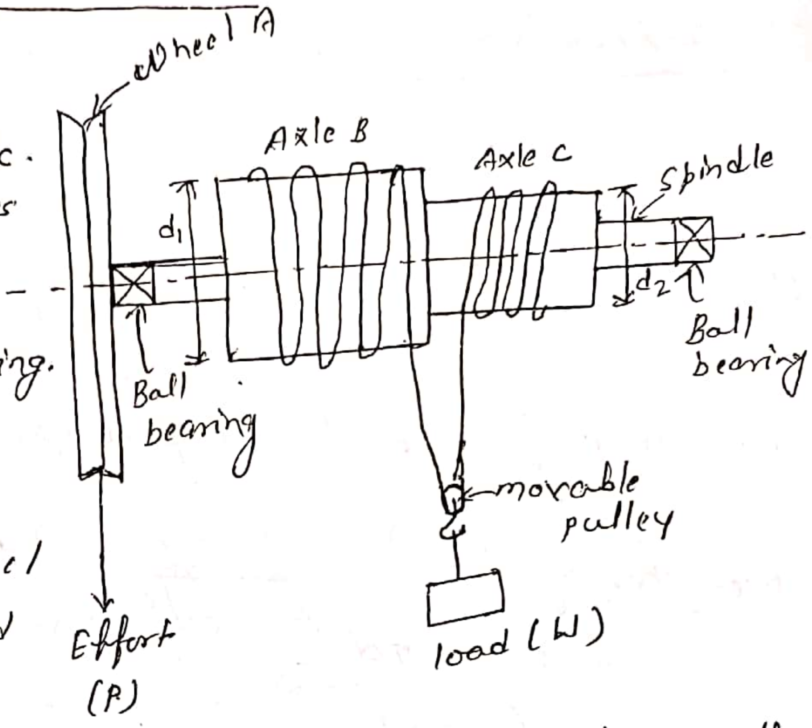
DIFFERENTIAL WHEEL AND AXLE:—



The unit consist of a wheel A and two axle B & C. The wheel and the two axles are keyed to the same shaft (spindle) which is supported in ball bearing. The effort is applied to the string which is wound round the wheel another string is wound on the two axle and it carries the load throug a pulley. The string ~~on the~~ on the wheel and smaller axle are wound in the same direction where as winding of string on the bigger axle is in the opposite direction.

When the effort P is applied in the downward direction there is unwinding of the string on the wheel and smaller axle. The string winds on the bigger axle at the same times and the load W is lifted upward.

Distance moved by the effort $= \pi D$

Length of string that winds on bigger axle $= \pi d_1$

Length of string that unwinds on smaller axle $= \pi d_2$

Net length of string which will get wound on bigger axle

$= \pi d_1 - \pi d_2$

Distance moved by the load $= \dfrac{\pi d_1 - \pi d_2}{2} = \dfrac{\pi}{2}(d_1 - d_2)$

$$\therefore VR = \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{\pi D}{\frac{\pi}{2}(d_1 - d_2)} = \frac{2D}{(d_1 - d_2)}$$

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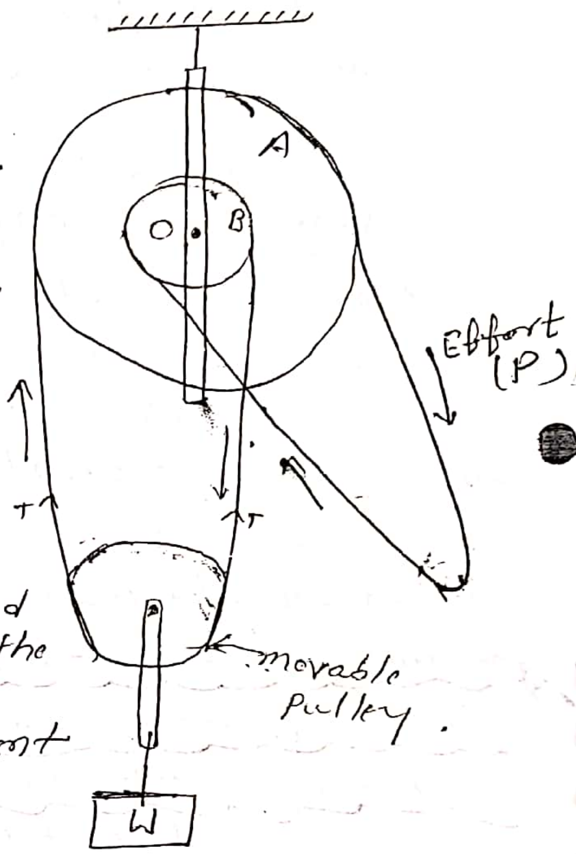
for a greater VR the d_1 made nearly equal to d_2.

DIFFERENTIAL PULLEY BLOCK :-



$W = 2T$, $T = \frac{W}{2}$

in one revolution.

Unwinding of rope from pulley A.

$= \pi D$

Unwinding of rope from pulley B.

$= \pi d$.

Net shorting of the ~~chain~~ rope

$= \pi D - \pi d$

$= \pi (D - d)$

The shortening of the rope is divided equally b/w two segments of the rope supporting the pulley in the lower block. Hence displacement of the load is

$= \frac{\pi (D - d)}{2}$

$\therefore$ V.R. $= \frac{\text{distance moved by effort}}{\text{" " " load}} = \frac{\pi D}{\frac{\pi (D-d)}{2}}$

$$\boxed{V.R. = \frac{2D}{(D-d)}}$$

if friction force is neglected.

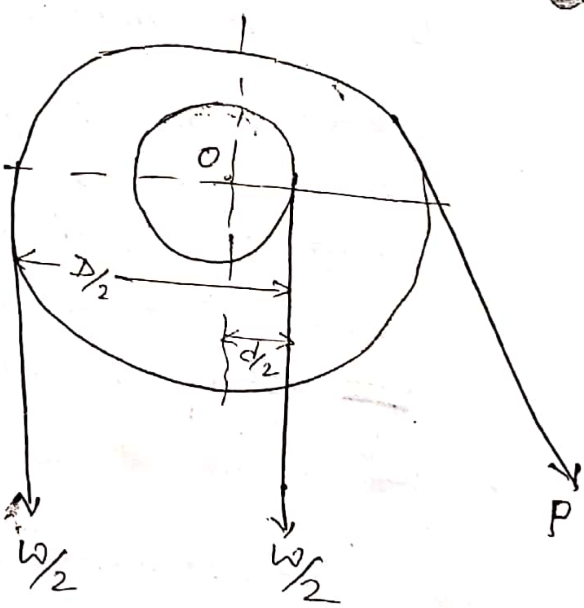
Taking moment about point O.

$\therefore P \times \frac{D}{2} + \frac{W}{2} \times \frac{d}{2} = \frac{W}{2} \times \frac{D}{2}$

$\Rightarrow \frac{PD}{\cancel{2}} = \frac{W}{\cancel{4}2} (D - d)$

$\because MA = V.R$

$= \frac{W}{P} = \frac{2D}{D - d}$



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UNIT – 2

CENTROID AND MOMENT OF INERTIA.

CENTRE OF GRAVITY:– Centre of gravity of a body is the point through which the resultant of the distributed gravitational parallel forces passes, irrespective to the position of the body.

OR

Centre of gravity is the point where whole weight of the body is assumed to be concentrated.

$$\boxed{\bar{x} = \frac{\Sigma w x}{\Sigma w}} \quad \text{or} \quad \boxed{\bar{x} = \frac{\Sigma m x}{\Sigma m}}$$

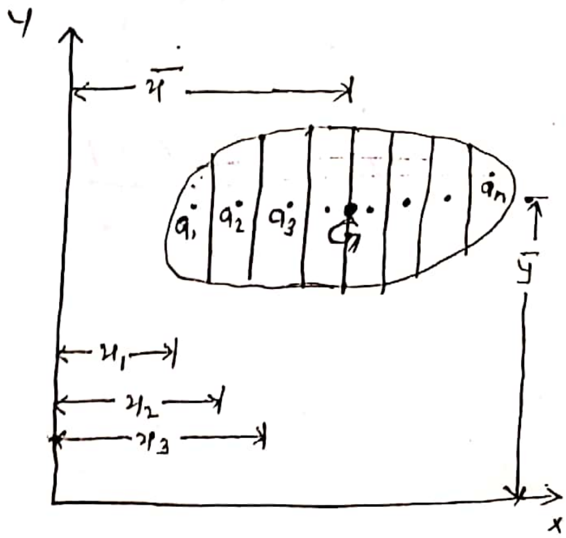
$$\boxed{\bar{y} = \frac{\Sigma w y}{\Sigma w}} \quad \text{or} \quad \boxed{\bar{y} = \frac{\Sigma m y}{\Sigma m}}$$

CENTROID:– The entire area of the body assumed to be concentrated at a point is known as centroid.

From the "varignon theorem"

moment of areas of all strips about y-axis.

$= a_1 x_1 + a_2 x_2 + a_3 x_3 + \cdots\cdots = \Sigma a x$



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Moment of total Area A about the y-axis

$$= A\bar{x}$$

$$\therefore \quad A\bar{x} = \Sigma a x$$

$$\Rightarrow \boxed{\bar{x} = \frac{\Sigma a x}{A}} \qquad \boxed{\bar{x} = \frac{\Sigma a x}{\Sigma a}}$$

Similarly when the moments are taken about x-axis, we get

$$A\bar{y} = \Sigma a y$$

$$\boxed{\bar{y} = \frac{\Sigma a y}{A}} \qquad \boxed{\bar{y} = \frac{\Sigma a y}{\Sigma a}}$$

$$\because \quad m = \rho . V$$

$$\therefore \quad m_1 = \rho V_1, \quad m_2 = \rho V_2, \quad m_3 = \rho V_3 \text{ etc.}$$

$$V = V_1 + V_2 + V_3 + \text{-----}$$

$$\therefore \quad \bar{x} = \frac{\Sigma \rho V x}{\Sigma \rho V} = \frac{\Sigma V x}{\Sigma V}$$

$$\bar{y} = \frac{\Sigma \rho V y}{\Sigma \rho V} = \frac{\Sigma V y}{\Sigma V}$$

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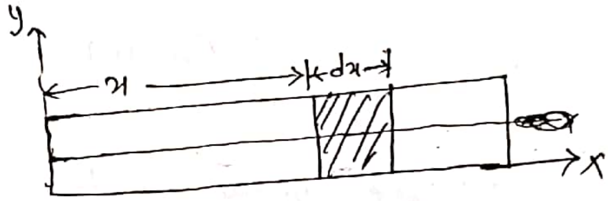
→ A body has only one centre of gravity.

→ Its location does not change even with a change in the orientation of the body.

→ It is an imaginary point which may occur inside or outside the body.

CENTROID OF A UNIFORM WIRE OF LENGTH(L)



we know that

$$\therefore \bar{x} = \frac{\Sigma x\,dl}{\Sigma dl}$$

$$\bar{y} = \frac{\Sigma y\,dl}{\Sigma dl}$$

when the x- axis is so choosen that it passes through the centre of the wire and along its length $\bar{y} = 0$

$$\Sigma \bar{x}\,dl = \int_0^l x\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^L = L^2/2$$

$$\Sigma dl = \int_0^L dx = [x]_0^L$$

$$= L$$

$$\therefore \bar{x} = \frac{L^2/2}{L} = L/2$$

$$\boxed{\bar{x} = L/2}$$

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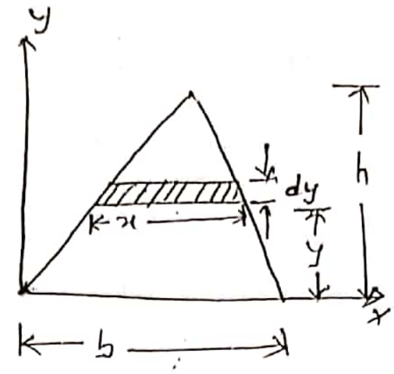
CENTROID OF THE TRIANGLE:-



Consider a rectangular lamina :

$$dA = x.dy$$

By Similar triangle

$$\frac{x}{h-y} = \frac{b}{h}$$

$$\Rightarrow x = \frac{b}{h}(h-y)$$

$\because$ we know that

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{\int y.dA}{\int dA}$$

$$= \frac{\int_0^h y\frac{b}{h}(h-y)\,dy}{\int_0^h \frac{b}{h}(h-y)\,dy}$$

$$= \frac{\int_0^h \left(yb - \frac{y^2 b}{h}\right) dy}{\int_0^h \left(b - \frac{by}{h}\right) dy}$$

$$= \frac{\left[\frac{y^2 b}{2} - \frac{y^3 b}{3h}\right]_0^h}{\left[yb - \frac{y^2 b}{2h}\right]_0^h}$$

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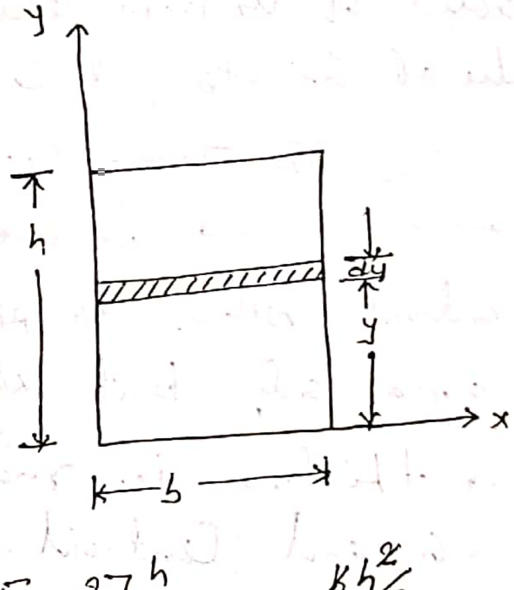
$$= \frac{\left(\frac{bh^2}{2} - \frac{bh^{\cancel{3}2}}{3K}\right)}{\left(bh - \frac{bh^{\cancel{2}}}{2\cancel{K}}\right)}$$

$$= \frac{\frac{3bh^2 - 2bh^2}{6}}{\frac{2bh - bh}{2}}$$

$$= \frac{\cancel{b}h^{\cancel{2}}}{\cancel{6}3} \times \frac{\cancel{2}}{\cancel{bK}}$$

$$= \frac{h}{3}$$

CENTROID OF THE RECTANGLE:-



Area of strip

$dA = b \times dy$

Area of ~~triangle~~ rectangle $= (b \times h)$

$$\therefore \bar{y} = \frac{\int y \times dA}{\int dA}$$

$$= \frac{\int_0^h y \cdot b\,dy}{b \times h} = \frac{\left[b \cdot \frac{y^2}{2}\right]_0^h}{bh} = \frac{\cancel{b}h^{\cancel{2}}/2}{\cancel{bK}}$$

$$\Rightarrow \boxed{\bar{y} = h/2}$$

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Similarly

$$\boxed{\bar{x} = b/2}$$

So, the centroid is at $(b/2, h/2)$

A solid of Uniform density throughout, then centroid, Centre of Gravity and Centre of mass are Coinside.

The Term Centre of Gravity applies to bodies with mass & weight.

and Centroid applies to ~~plane~~ plane figures which have area only but no mass.

When thickness i.e. mass of body is not considered, the CG and Centroid are ~~Synonymous~~ Synonymous and pass through same Point

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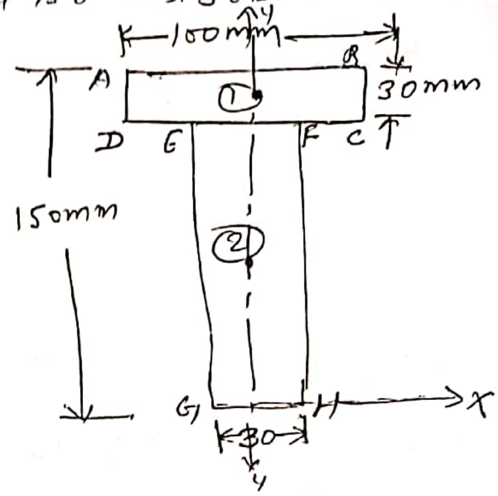
Q. Find the centroid of a 100mm × 150mm × 30mm T-section



Sol^n:- The section is symmetrical about y-y axis, so Centre of gravity lies on this axis.

So that only calculate the $\bar{y}$ in this question.

Let GH be the axis of refrence from the bottom.

(I) Rectangle ABCD

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2.$$

$$y_1 = \left(150 - \frac{30}{2}\right) = 135 \text{ mm}$$

(II) Rectangle EFGH

$$a_1 = (150-30) \times 30 = 3600 \text{ mm}^2$$

$$y_1 = (150-30)/2 = 60 \text{ mm}$$

∵ we know that the distance of C.G. from bottom

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$\Rightarrow \bar{y} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} =$$

$$\Rightarrow \boxed{\bar{y} = 94.1 \text{ mm}}$$

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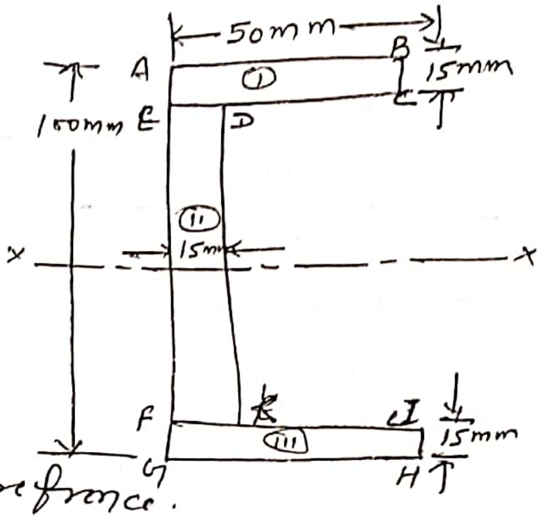
Q. Find the centroid of a channel section 100 mm × 50 mm × 15

The section is symmetrical about x-x-axis. so the centroid is lies on this axis.

$\therefore \bar{y} = 0$

only calculate the $\bar{x}$.



Let, Face AG be the axis of reference.

(I) Rectangle ABCE

$a_1 = 50 \times 15 = 750 \text{ mm}^2$

$x_1 = \frac{50}{2} = 25 \text{mm}$

(II) Rectangle EDFK

$a_2 = (100-30) \times 15 = 1050 \text{mm}^2$

$x_2 = \frac{15}{2} = 7.5 \text{mm}$

(III) Rectangle FGHJ

$a_3 = 50 \times 15 = 750 \text{ mm}^2$

$x_3 = \frac{50}{2} = 25 \text{ mm}$

$\because$ we know that the distance of centroid from face AG.

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750}$$

$$\Rightarrow \boxed{\bar{x} = 17.8 \text{ mm}}$$

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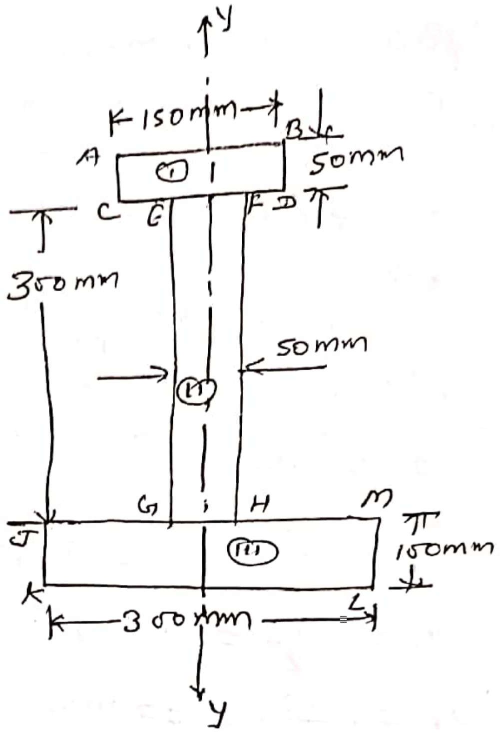
Q. An I-section has the following dimensions in mm

Bottom flange = 300 × 100

Top flange = 150 × 50

web = 300 × 50



Soln.- The section is symmetrical about y-y-axis, so its centroid is lies on this axis.

$\therefore \bar{x} = 0$

only determine $\bar{y} = ?$

(I) Rectangle ABCD

$a_1 = 150 \times 50 = 7500\ mm^2$

$y_1 = \left(100 + 300 + \frac{50}{2}\right) = 425\ mm$

(II) Rectangle EFGH

$a_2 = 300 \times 50 = 15000\ mm^2.$

$y_2 = \left(100 + \frac{300}{2}\right) = 250\ mm$

(III) Rectangle JKLM

$a_3 = 300 \times 100 = 30000\ mm^2$

$y_3 = \frac{100}{2} = 50\ mm$

∵ we know that from bottom.

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(7500 \times 425) + (15000 \times 250) + (30000 \times 50)}{7500 + 15000 + 30000}$$

$\Rightarrow \boxed{\bar{y} = 160.7\ mm.}$

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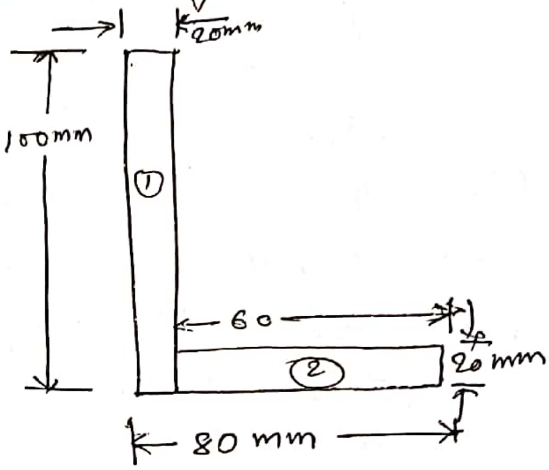
CENTROID OF UNSYMMETRICAL SECTION :-

Q. Find the centroid of an unequal angle section 100 mm X 80 mm X 20 mm.



Sol^n :- This section is not symmetrical about any axis. ∴ we have to find out the value of $\bar{x}$ and $\bar{y}$.

(I) Rectangle ①

$a_1 = 100 \times 20 = 2000 \ mm^2$

$x_1 = \frac{20}{2} = 10 \ mm$

$y_1 = \frac{100}{2} = 50 \ mm$

(II) Rectangle - II

$a_2 = (80-20) \times 20 = 1200 \ mm^2$

$x_2 = 20 + \frac{60}{2} = 50 \ mm$

$y_2 = \frac{20}{2} = 10 \ mm$

∵ we know that

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \ mm$$

$\Rightarrow \boxed{\bar{x} = 25 \ mm}$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35 \ mm$$

$\Rightarrow \boxed{\bar{y} = 35 \ mm}$

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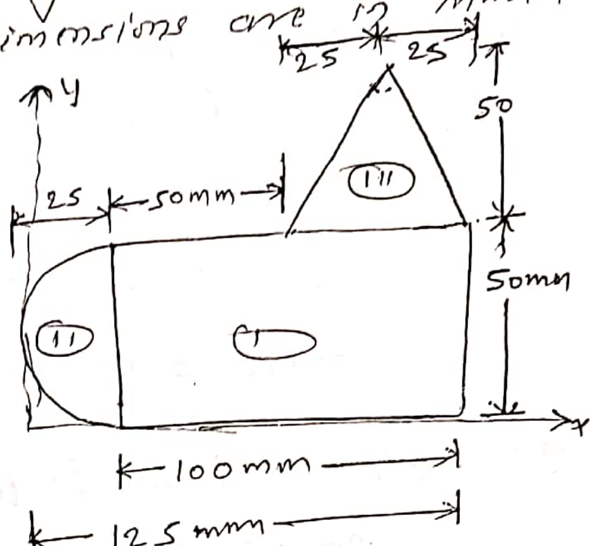
Q. A uniform lamina shown in fig. consists of a rectangle, a circle and a triangle. Determine the centroid of the lamina. All dimensions are in mm.



Sol:- This section is not symmetrical about any axis therefore we have to find out the both the values $\bar{x}$ and $\bar{y}$.

(I) Rectangular portion

$a_1 = 100 \times 50 = 5000 \text{ mm}^2$

$x_1 = 25 + \frac{100}{2} = 75 \text{ mm}$

$y_1 = \frac{50}{2} = 25 \text{ mm}$

(II) semicircular portion

$a_2 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2} \times (25)^2 = 982 \text{ mm}^2$

$\boxed{x_2 = 25 - \frac{4r}{3\pi}} = 25 - \frac{4 \times 25}{3\pi} = 14.4 \text{ mm}$

$y_2 = \frac{d}{2} = \frac{50}{2} = 25 \text{ mm}$

(III) Triangular portion

$a_3 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 50 \times 50 = 1250 \text{ mm}^2$

$y_3 = 50 + \frac{h}{3} = 50 + \frac{50}{3} = 66.7 \text{ mm}$

$x_3 = 25 + 50 + \frac{50}{2} = 100 \text{ mm}$

$\because$ we know that

$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 100)}{5000 + 982 + 1250} = 71.7 \text{ mm}$

$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 25) + (982 \times 25) + (1250 \times 66.7)}{5000 + 982 + 1250} = 32.2 \text{ mm}$

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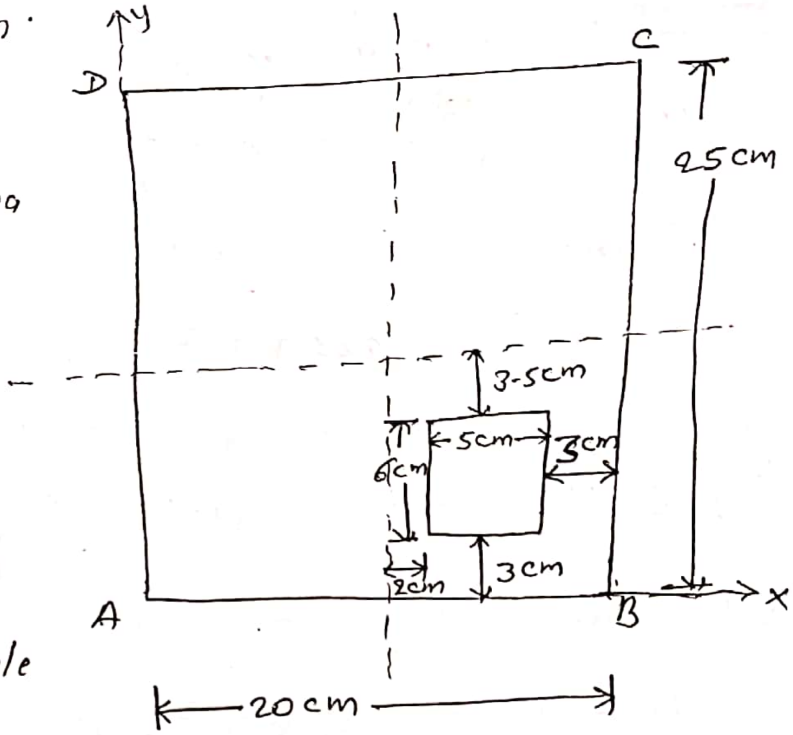
Q: A rectangular lamina ABCD 20cm x 25cm has a rectangular hole of 5cm x 6cm as shown in fig. Locate the centroid of the section.



Soln.-

(I) For the rectangular lamina ABCD

$A_1 = 20 \times 25 = 500\, cm^2$

$x_1 = \frac{20}{2} = 10\, cm$

$y_1 = \frac{25}{2} = 12.5\, cm$

(II) For the cut rectangular hole

$A_2 = 5 \times 6 = 30\, cm^2$

$x_2 = 10 + 2 + \frac{5}{2} = 14.5\, cm$

$y_2 = 3 + \frac{6}{2} = 6\, cm$

∵ we know that

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{(500 \times 10) - (30 \times 14.5)}{500 - 30} = 9.71\, cm$$

$\Rightarrow \boxed{\bar{x} = 9.71\, cm}$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{(500 \times 12.5) - (30 \times 6)}{500 - 30} = 12.91\, cm$$

$\Rightarrow \boxed{\bar{y} = 12.91\, cm}$

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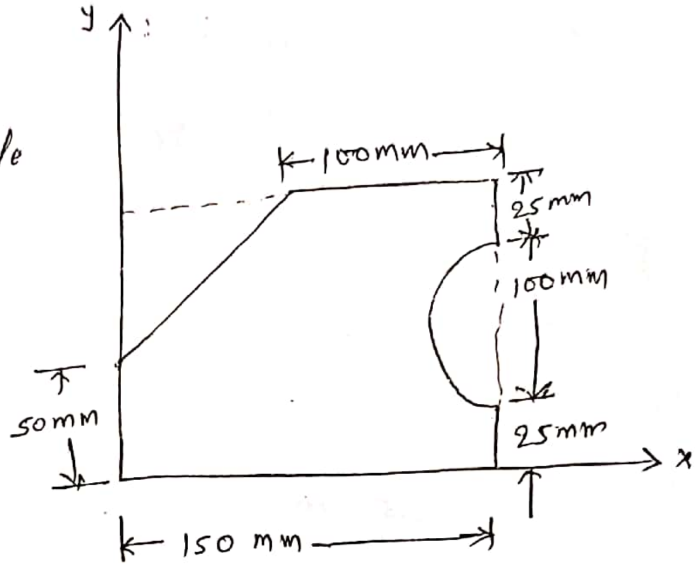
Q. Locate the centroid of the area shown in fig.

soln:- Total area can be considered as a rectangle



∴ (I) Rectangle

$a_1 = 150 \times 150 = 22500 mm^2$

$x_1 = \frac{150}{2} = 75 mm$

$y_1 = \frac{150}{2} = 75 mm$

(II) Semicircle

$a_2 = \frac{\pi}{2} r^2 = \frac{\pi}{2} \times (50)^2 = 3925 mm^2$

$x_2 = 150 - \frac{4r}{3\pi} = 150 - \frac{4 \times 50}{3\pi} = 128.77 mm$

$y_2 = 25 + \frac{d}{2} = 25 + 50 = 75 mm$

(III) Triangle

$a_3 = \frac{1}{2} b h = \frac{1}{2} \times 50 \times 100 = 2500 mm^2$

$x_3 = \frac{b}{3} = \frac{50}{3} = 16.67 mm$

$y_3 = 150 - \frac{h}{3} = 150 - \frac{100}{3} = 116.67 mm$

∵ we know that

$$\bar{x} = \frac{a_1x_1 - a_2x_2 - a_3x_3}{a_1 - a_2 - a_3} = \frac{(22500 \times 75) - (3925 \times 128.77) - (2500 \times 16.67)}{22500 - 3925 - 2500}$$

$\Rightarrow \boxed{\bar{x} = 70.94 mm}$

$$\bar{y} = \frac{a_1y_1 - a_2y_2 - a_3y_3}{a_1 - a_2 - a_3} = \frac{(22500 \times 75) - (3925 \times 75) - (2500 \times 116.67)}{22500 - 3925 - 2500}$$

$\Rightarrow \boxed{\bar{y} = 68.52 mm}$

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MOMENT OF INERTIA (MOI) :-

→ Moment of force about a point is the product of force "F" and the perpendicular distance "x" b/w the point and the line of action of force.

moment of force = Fx

If this moment Fx is further multiplied by the distance x, then a quantity Fx^2 is known as moment of moment or the second moment of force.

moment of moment = $Fx \times x = Fx^2$

If the term F is replaced by (Area) or (mass) of the body the resulting parameter is called the moment of inertia (MOI).

moment of inertia of a plane area = Ax^2 (mm^4) form

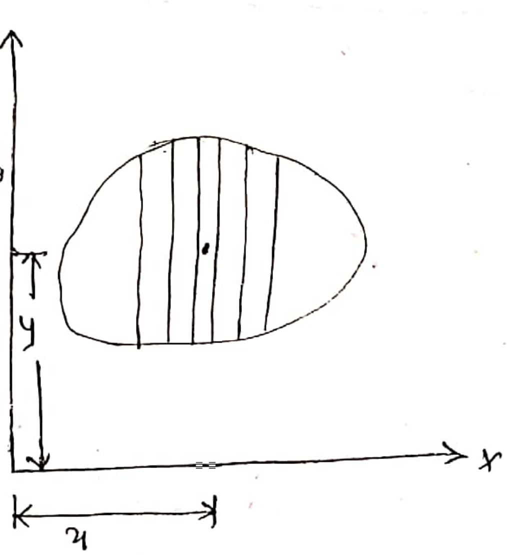
mass moment of inertia of a body = mx^2 ($kg\ m^2$)

moment of inertia by integration:-

I_{xx} = moment of inertia about x-axis = $\Sigma(y\,da)y = \Sigma y^2 da = \int y^2 da$

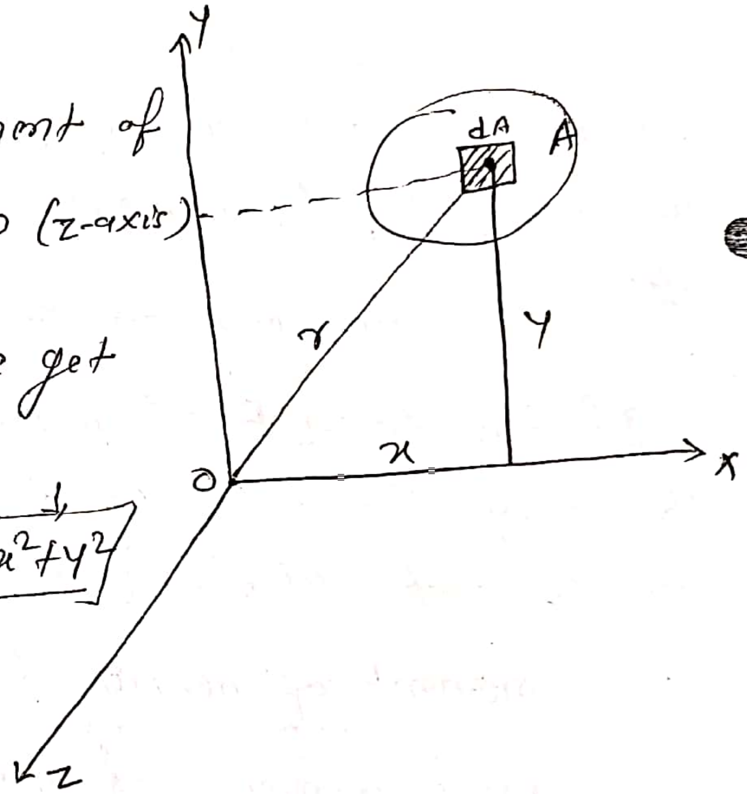
da → Area of strip.

I_{yy} → Moment of inertia about y-axis = $\Sigma x^2 da = \int x^2 da$



~~PARALLEL AXIS THEOREM~~

POLAR MOMENT OF INERTIA:- The moment of inertia of an area of passing figure with respect to an axis perpendicular to the (x-y) plane and passing through a pole (z-axis) is called polar moment of inertia and is denoted by I_{zz} or J_0.



From figure polar moment of inertia of dA about Pole O (z-axis)

$$dI_{zz} = r^2 dA$$

Integrating both sides we get

$$\int dI_{zz} = \int r^2 dA$$

$$\boxed{r^2 = x^2 + y^2}$$

$$\Rightarrow I_{zz} = \int (x^2 + y^2)\, dA$$

$$\Rightarrow I_{zz} = \int x^2 dA + \int y^2 dA$$

$$\Rightarrow \boxed{I_{zz} = I_{yy} + I_{xx}}$$

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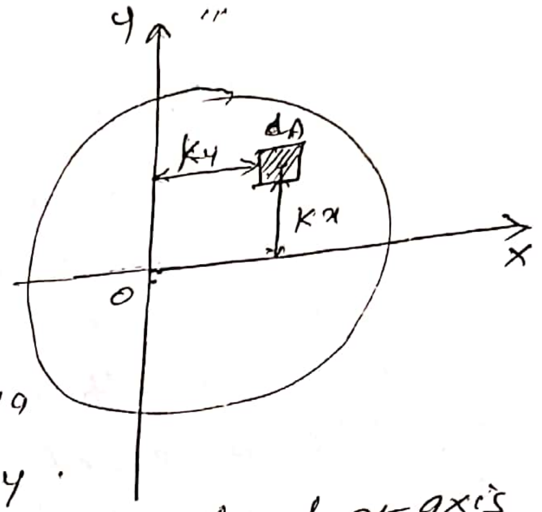
RADIUS OF GYRATION OF AN AREA:-



Consider an area A which has moment of inertia I_{xx} with respect to the x-axis.

Let the distance from x-axis of elementary area is K_x and from y-axis is K_y.

Then, moment of inertia of an area about x-axis

$$I_{xx} = K_x^2 \cdot A$$

$$\boxed{K_x = \sqrt{\frac{I_{xx}}{A}}}$$ ← Radius of gyration

The distance K_x is known as radius of gyration of the area with respect to x-axis.

Similarly with respect to y-axis

$$I_{yy} = K_y^2 \cdot A$$

$$\boxed{K_y = \sqrt{\frac{I_{yy}}{A}}}$$

Also radius of gyration of with respect to the polar axis-

$$\boxed{K_z = \sqrt{\frac{I_{zz}}{A}}}$$

$$I_{zz} = I_{xx} + I_{yy} \Rightarrow A(K_z)^2 = A(K_x)^2 + A(K_y)^2$$

$$\boxed{K_z^2 = K_x^2 + K_y^2}$$

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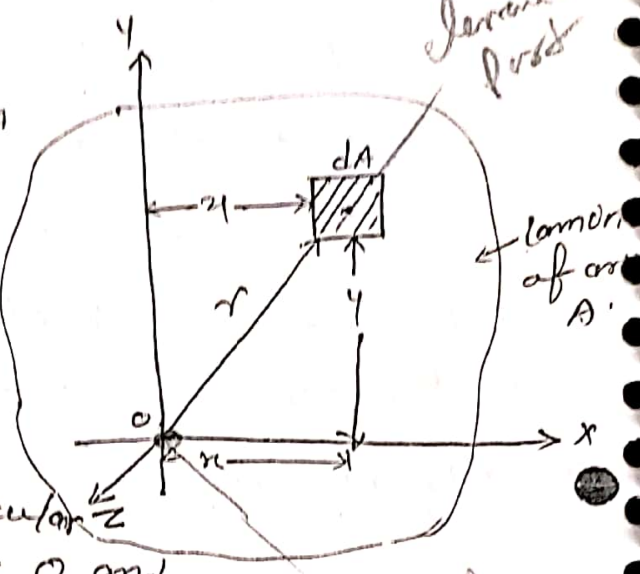
THEOREM OF MOMENT OF INERTIA:-

(A) PERPENDICULAR AXIS THEOREM:-

Only 2D (planer body)

According to perpendicular axis theorem "The moment of inerta of a plane lamina about an axis perpendicular to its plane passing through any point O is equal to the sum of moment of inertia about two mutually perpendicular axes through the same point O and lying in the plane of the lamina.



(all three axis should must lie in same point)

PROOF:-

Consider an area A whose moment of inertia with respect to x-axis and y-axis are the I_{xx} and I_{yy} respectively. If an elemental area dA is located at a distance of r from 'O' and then from fig

moment of inertia of area A about x-axis $(I_{xx}) = \int y^2 dA$

moment of inertia of area A about y-axes $(I_{yy}) = \int x^2 dA$

Also
moment of inertia of area A about the perpendicular z-axis through 'O' $(I_{zz}) = \int r^2 dA$

$$= \int (x^2+y^2)\, dA \qquad \because \boxed{r^2 = x^2+y^2}$$

$$= \int x^2 dA + \int y^2 dA$$

$$\boxed{I_{zz} = I_{yy} + I_{xx}}$$

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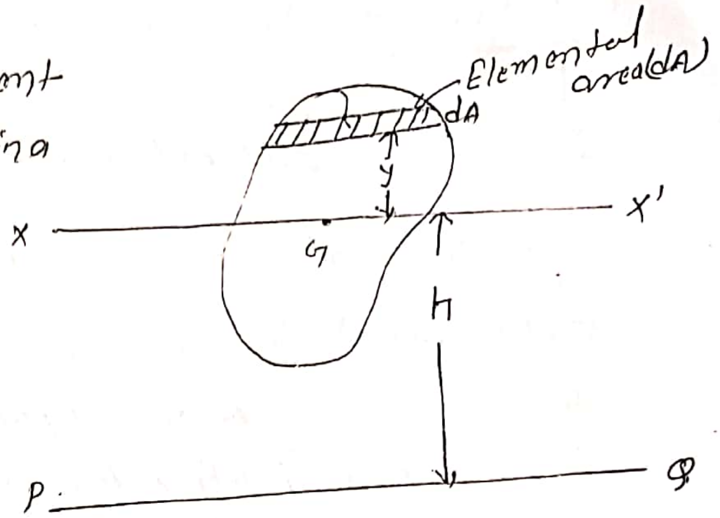
(2D & 3D) apply

(B) PARALLEL AXES THEOREM :-



It states that "the moment of inertia of a plane lamina about any axis lying in the plane of lamina is equals to the sum of moment of inertia about a parallel centroidal axis in the plane of lamina and the product of the area of the lamina and square of the distance b/w the two axis.

PROOF:- Let a lamina of area(A) has centroid at G positioned on axis xx', another axis (PQ) parallel to (xx') at h distance from xx'. Assumed lamina consists of number of small elemental area (dA). The distance of small area (dA) from xx' is y. Distance of the elemental area from axis (PQ) is $(h \pm y)$. Thus the moment of inertia of elemental area about axis (PQ) is -

$$= dA\,(h+y)^2$$

moment of inertia of whole lamina about PQ can be

$$I_{PQ} = \Sigma\, dA\,(h+y)^2$$

$$\Rightarrow I_{PQ} = \Sigma dA\,(h^2+y^2+2hy)$$

$$\Rightarrow I_{PQ} = h^2\Sigma dA + y^2\Sigma dA + \underbrace{2h\Sigma y dA}_{= 0}$$

(because x-x axis is centroidal axis) ?

$$\Rightarrow \boxed{I_{PQ} = Ah^2 + I_{xx}}$$

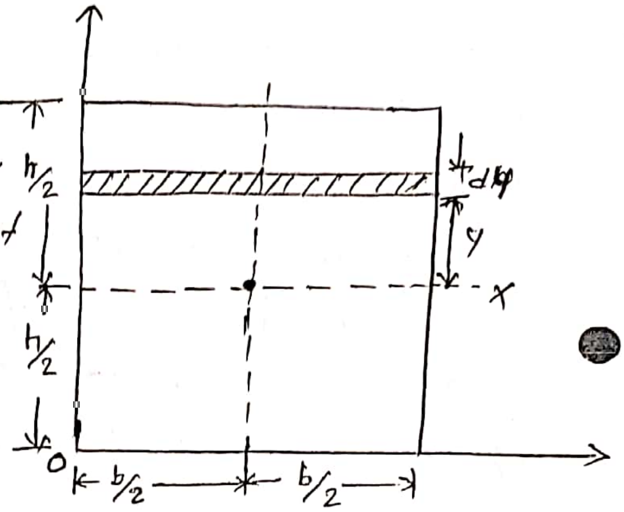
moment of complete body about centroidal axis.

Area ka moment about same axis is zero

MOMENT OF INERTIA OF DIFFERENT SHAPES :-

[1] RECTANGULAR LAMINA :-

moment of inertia about centroidal axis :-



Let the centroid G be the origin with x-axis parallel to base and y-axis perpendicular to it. The differential element is chosen for integration which is parallel to base i.e. x-axis. It is at a distance 'y' from the x-axis and its thickness is dy.

Area of strip = $dA = b\,dy$

As each part of strip is at the same distance y from x-axis, so moment of inertia w.r.to centroidal axis xx

$$dI_{xx} = y^2 dA = y^2(b\,dy)$$

Integrating from $-h/2$ to $h/2$ we get -

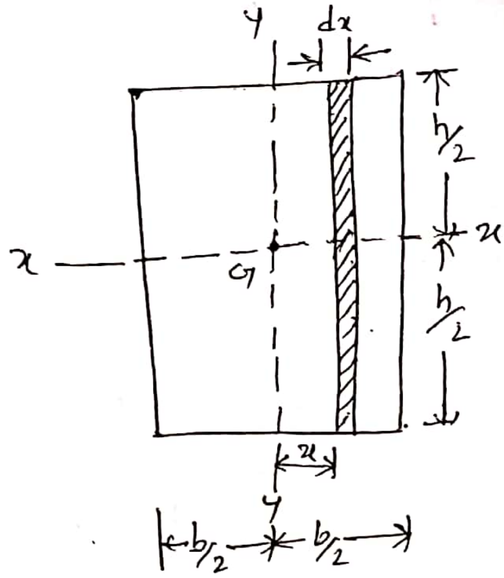
$$\int dI_{xx} = \int_{-h/2}^{h/2} y^2 b\,dy$$

$$\Rightarrow I_{xx} = b\left[\frac{y^3}{3}\right]_{-h/2}^{h/2} = \frac{b}{3}\left[\frac{h^3}{8}+\frac{h^3}{8}\right]$$

$$\Rightarrow I_{xx} = \frac{b}{3} \times \frac{2h^3}{8}$$

$$\Rightarrow \boxed{I_{xx} = \frac{bh^3}{12}}$$

To determine I_{yy}, consider vertical element as shown in figure.



Here,

$$dA = h \times dx$$

$$dI_{yy} = x^2 dA$$

$$dI_{yy} = x^2 (h\,dx)$$

Integrating with in limits $-b/2$ to $b/2$ we get

$$\int dI_{yy} = \int_{-b/2}^{b/2} x^2 h\,dx.$$

$$\Rightarrow I_{yy} = h\left[\frac{x^3}{3}\right]_{-b/2}^{b/2}$$

$$\Rightarrow I_{yy} = \frac{h}{3}\left[\frac{b^3}{8} + \frac{b^3}{8}\right]$$

$$\Rightarrow I_{yy} = \frac{h}{3} \times \frac{\cancel{2}b^3}{\cancel{8}4}$$

$$\Rightarrow \boxed{I_{yy} = \frac{hb^3}{12}}$$

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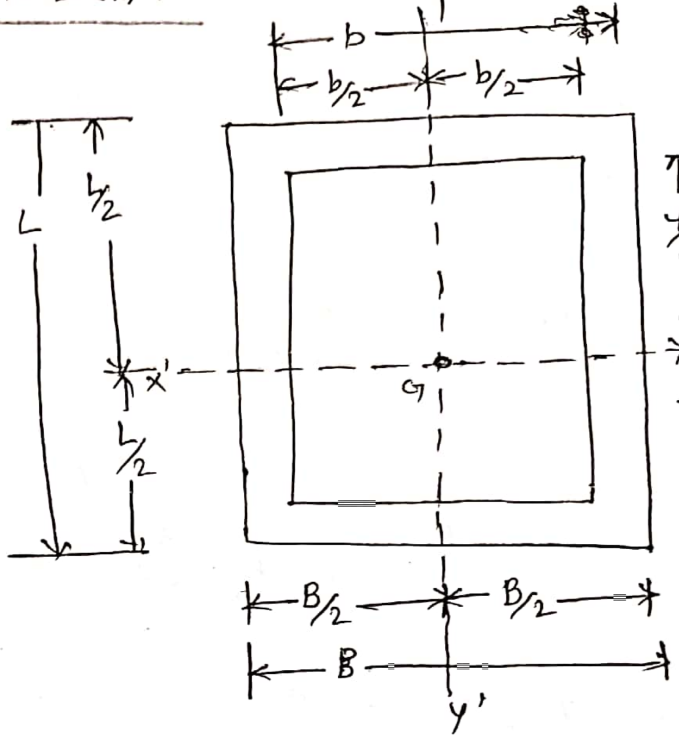
HOLLOW RECTANGULAR LAMINA :-



Rectangular lamina of length (L) and width (B) has rectangular slot of length (l) and width (b). Area moment of inertia about centroidal axis can be given as—

$$\boxed{I_{xx'} = \frac{BL^3}{12} - \frac{bl^3}{12}}$$

$$\boxed{I_{yy'} = \frac{LB^3}{12} - \frac{lb^3}{12}}$$

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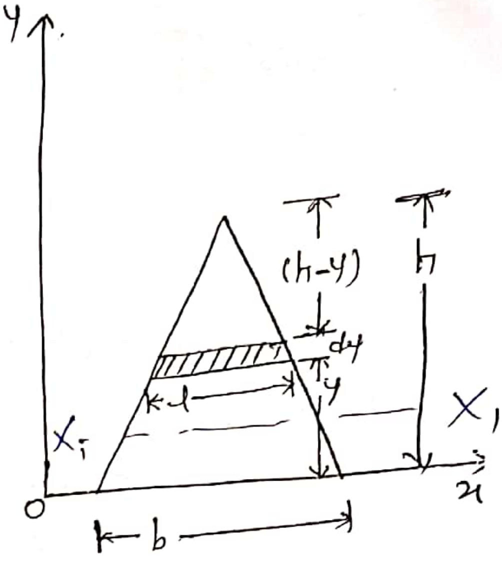
TRIANGULAR LAMINA:-



Consider a triangular lamina of base 'b' and height 'h', choose x-axis to concide with the base. Consider a different strip of thickness 'dy' parallel to x-axis and at a distance of 'y' from it.

Area of strip $= dA = l \times dy$

From the property of similar triangle we get:-

$$\frac{l}{b} = \frac{h-y}{h}$$

$$\Rightarrow l = \frac{h-y}{h} \cdot b$$

Moment of inertia of the strip w.r.to x-axis is -

$$dI_{xx} = y^2 dA = y^2 l\, dy$$

$$dI_{xx} = y^2\left(\frac{h-y}{h}\right) . b\, dy$$

Integrating from $y=0$ to $y=h$ we get

$$\int dI_{xx} = \int_0^h y^2 \frac{b(h-y)}{h} dy$$

$$\Rightarrow I_{xx} = \frac{b}{h}\int_0^h y^2(h-y)\, dy = \frac{b}{h}\int_0^h (y^2 h - y^3)\, dy$$

$$\Rightarrow I_{xx} = \frac{b}{h}\left[h\frac{y^3}{3} - \frac{y^4}{4}\right]_0^h = \frac{b}{h}\left[\frac{h^4}{3} - \frac{h^4}{4}\right]$$

$$\Rightarrow I_{xx} = \frac{b}{\cancel{h}} \times \frac{h^{\cancel{4}3}}{12}$$

$$\Rightarrow \boxed{I_{xx} = \frac{bh^3}{12}}$$

about centroidal axis

$$I_{X_1X_1} = I_{xx(base)} - A y_c^2$$

$$= \frac{bh^3}{12} - \left(\frac{1}{2}b\times h\right)\times\left(\frac{h}{3}\right)^2$$

$$= bh^3/36$$

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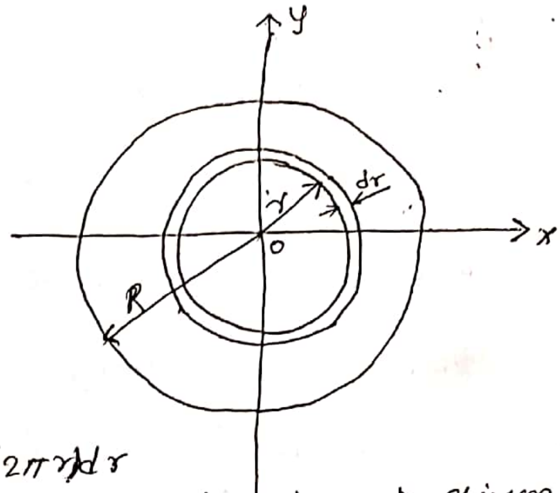
CIRCULAR LAMINA :-

(a) Polar moment of inertia :-



Consider an annular differential element of thickness (dr) situated at a distance of (r) from the centre "O" as shown in fig :-

Area of the elemental ring $(dA) = (2\pi r)dr$

Polar moment of inertia of this element about o is given by

$$dJ_o = r^2 dA$$
$$= r^2 (2\pi r) dr$$

Integrating from $r=0$ to $r=R$ we get, :-

$$\int dJ_o = \int_0^R 2\pi r^3 dr = 2\pi \left[\frac{r^4}{4}\right]_0^R$$

$$J_o = 2\pi \times \frac{1}{4} \times R^4$$

$$\Rightarrow \boxed{J_o = \frac{\pi}{2} R^4 \text{ or } \frac{\pi}{32} D^4}$$

(b) moment of inertia about centroidal axis :-

The centroidal axis in the plane of lamina coincide with diameters. Because of symmetry of the circular area, we have :

$$\bar{I}_{xx} = \bar{I}_{yy}$$

Using perpendicular axis theorem at O we get

$$J_o = I_{xx} + I_{yy}$$
$$J_o = 2 I_{xx}$$
$$\Rightarrow I_{xx} = I_{yy} = \frac{J_o}{2}$$
$$\Rightarrow I_{xx} = I_{yy} = \frac{\frac{\pi}{2} R^4}{2} \text{ or } \frac{\frac{\pi}{32} D^4}{2}$$

$$\Rightarrow \boxed{I_{xx} = I_{yy} = \frac{\pi}{4} R^4 \text{ or } \frac{\pi}{64} D^4}$$

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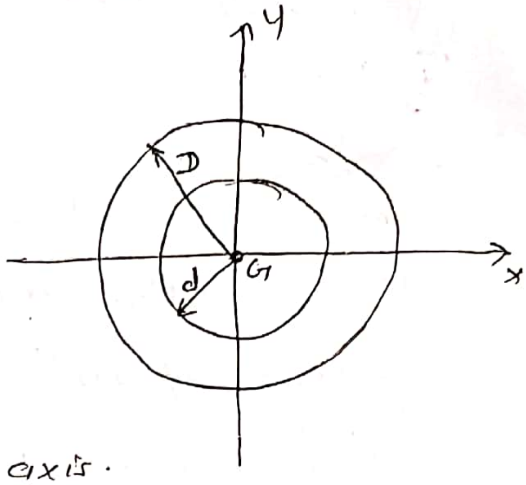
HOLLOW CIRCULAR LAMINA :-

moment of inertia of circular lamina with a circular hole at the centre can be obtained as:-



$$I_G = \frac{\pi}{34} D^4 - \frac{\pi}{32} d^4$$

$$\Rightarrow \boxed{I_G = \frac{\pi}{32}(D^4 - d^4)}$$ ← about polar axis.

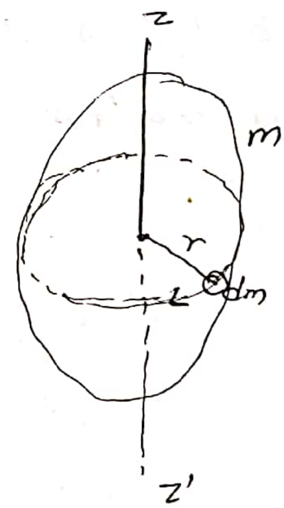
$$I_{xx} = I_{yy} = \frac{I_G}{2}$$

$$\Rightarrow \boxed{I_{xx} = I_{yy} = \frac{\pi}{64}(D^4 - d^4)}$$

MASS MOMENT OF INERTIA :-

The mass moment of inertia of a body is property that measure the resistance of the body to angular acceleration. i.e. It is a measure of inertia for rotational motion.

For a particle of mass (m) situated at a distance (r) from axis of rotation moment of inertia is defined as:-



$$I = mr^2$$

For a system of particles:-

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2$$

$$\Rightarrow I = \sum m_i r_i^2$$

Consider a rigid body of mass M. Take an element of a mass (dm) at a distance (r) from the axis as shown in fig. Here (r) is the moment of arm.

moment of inertia of elemental mass about axis zz' is

$$dI_{zz'} = r^2 dm \quad \text{——— (1)}$$

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For whole body

$$\int dI_{zz'} = \int r^2 dm \quad \text{——} (11)$$

If the axis passes through centre of gravity of the body then moment of inertia of the body is denoted by I_G.

Mass-moment of inertia of a body is always positive and has a unit of $kg\text{-}m^2$.

RADIUS OF GYRATION:—

Radius of gyration is the distance from a axis at which entire mass is assumed to be concentrated. Such that moment of inertia of the actual body and concentrated mass is same.

So if I is moment of inertia of a body of mass M about a given axis and K is radius of gyration then

$$I = mk^2$$

$$\Rightarrow \boxed{k = \sqrt{\frac{I}{m}}}$$

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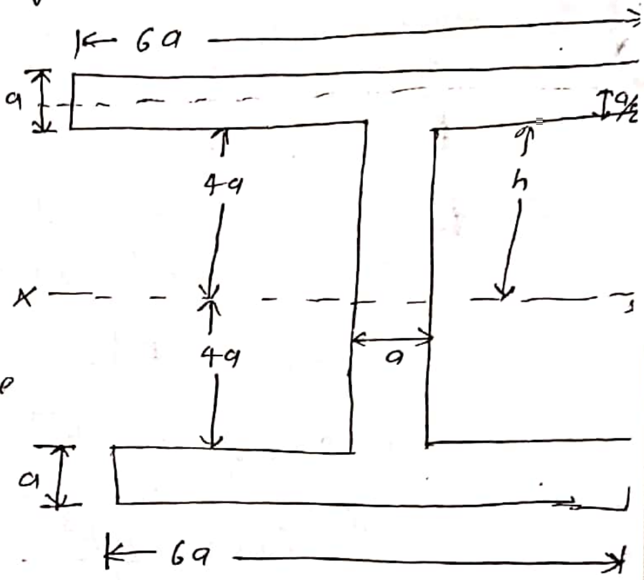
Q. Find the moment of inertia of a rolled steel joist girder of symmetrical I-section shown in fig.

Sol^n:- Three rectangle



Upper flange $A_1 = 6a \times a = 6a^2$

web $A_2 = 8a \times a = 8a^2$

lower flange $A_3 = 6a \times a = 6a^2$

MOI about (x-x) axis of upper flange (Using parallel axis theorem)

$= I_{xx} + Ah^2$

$= \frac{bh^3}{12} + Ah^2$

$= \frac{6a \times a^3}{12} + 6a^2(4a + a/2)^2$

$= \frac{6a^4}{12} + 6a^2\left(\frac{9a}{2}\right)^2 =$ ~~$\frac{6a^4}{12} + \frac{6a^2 \times 81a^2}{4} = 512$~~

~~$= \frac{a^4}{2} + 27a^4$~~

$= \frac{6a^4}{12} + \frac{3 \times 6a^2 \times 81a^2}{4\;2} = \frac{a^4}{2} + \frac{243a^4}{2}$

$= \frac{244a^2}{2} = 122a^4$ —— (I)

MOI about (x-x) axis of web

$= \frac{bh^3}{12} = \frac{a \times (8a)^3}{12}$

$= a \times \frac{512a^3}{12} = a \times \frac{128a^3}{3}$

$= \frac{128a^4}{3}$ —— (II)

MOI about (x-x) axis of lower flange (By using parallel axis theorem)

$= I_{xx} + \frac{bh^3}{12} + Ah^2$

$= \frac{bh^3}{12} + Ah^2$

$= \frac{6a \times a^3}{12} + 6a^2\left(4a + \frac{a}{2}\right)^2$

$= \frac{a^4}{2} + \frac{243a^4}{2}$

$= 122a^4$ —— (III)

∴ MOI ~~about~~ of I section about (x-x) axis

$= 122a^4 + \frac{128a^4}{3} + 122a^4$

$= \frac{860}{3}a^4$

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:- MOMENT OF INERTIA :-

Q: The moment of inertia of a rectangular section beam about x-x and y-y axis passing through the centroid are 250×10^6 mm^4 and 40×10^6 mm^4 respectively. Calculate the size of the section.

Sol^n:- Let, 'b' and 'h' be the breadth and depth respectively of the rectangular section beam

$$\because \; I_{xx} = \frac{bh^3}{12}$$

$$\Rightarrow \; 250 \times 10^6 = \frac{bh^3}{12} \quad \text{——— (i)}$$

and $$I_{yy} = \frac{hb^3}{12}$$

$$\Rightarrow \; 40 \times 10^6 = \frac{hb^3}{12} \quad \text{——— (ii)}$$

Eq^n (i) ÷ Eq^n (ii) we get

$$\frac{250 \times 10^6}{40 \times 10^6} = \frac{\cancel{b} h^{\cancel{3}\,2} / \cancel{12}}{\cancel{h} b^{\cancel{3}\,2} / \cancel{12}}$$

$$\Rightarrow \; 6.25 = \left(\frac{h}{b}\right)^2$$

$$\Rightarrow \; \frac{h}{b} = 2.5$$

$$\Rightarrow \; h = 2.5\,b \quad \text{——— (iii)}$$

Put the value of h in eq^n (i) we get

$$\frac{b(2.5b)^3}{12} = 250 \times 10^6$$

$$\Rightarrow \; b^4 = \frac{250 \times 10^6 \times 12}{(2.5)^3} = 1.92 \times 10^8$$

$$\Rightarrow \; b = 117.7 \text{ mm and}$$

$$h = 2.5 \times 117.7 = 294.25 \text{ mm}$$

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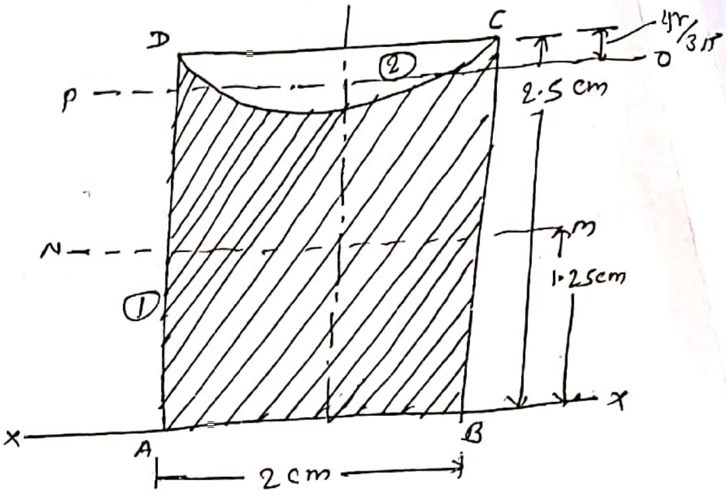
Q. Determine the moment of inertia of the area shown in the shaded in figure. about axis (x-x) which coincides with the base edge AB.



Sol^n:-

MOI of rectangle about its centroidal axis MN.

$$I_{MN} = \frac{bh^3}{12} = \frac{2\times(2.5)^3}{12} = 2.60 \text{ cm}^4$$

MOI of rectangle about (x-x) axis

$$I_{xx_1} = I_{MN} + Ah^2$$

$$= (2.60) + (2\times2.5)(1.25)^2$$

$$= 10.41 \text{ cm}^4$$

MOI of semi circle about CD axis

$$I_{CD} = \frac{\pi}{8}R^4 = \frac{\pi}{8}\times1^4 = 0.3927$$

MOI of semicircle about its centroidal axis OP.

$$I_{CD} = I_{OP} + Ah^2$$

$$A = \frac{\pi}{2}\times r^2 = \frac{\pi}{2}\times1^2 = 1.57 \text{ cm}^2$$

$$I_{OP} = I_{CD} - Ah^2$$

$$h = \frac{4r}{3\pi} = \frac{4\times1}{3\pi} = 0.4244 \text{ cm}$$

$$= 0.3927 - (1.57)(0.4244)^2$$

$$= 0.3927 - 0.2827$$

$$= 0.11$$

MOI of semi circle about (x-x) axis.

$$I_{xx_2} = I_{OP} + Ah^2$$

$$h = 2.5 - \frac{4r}{3\pi} = 2.5 - \frac{4\times1}{3\pi}$$

$$= 0.11 + (1.57)(2.0756)^2$$

$$= 2.5 - 0.4244 = 2.0756$$

$$= 0.11 + 6.76 = 6.87 \text{ cm}^4$$

Now MOI of shadded area about (x-x) axis are.

$$I_{xx} = I_{xx_1} - I_{xx_2}$$

$$= 10.41 - 6.87$$

$$= 3.54 \text{ cm}^4$$

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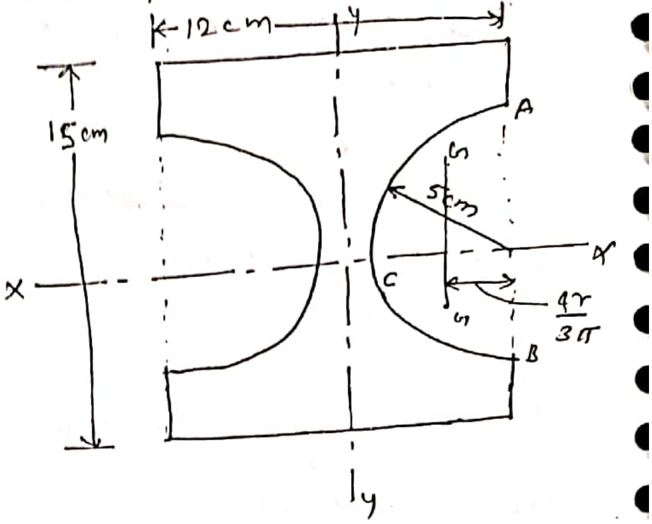
Q. Determine I_{xx} and I_{yy} of the cross-section of a cast iron beam shown in figure.



Soln:- MOI of the beam about (x-x) axis.

$I_{xx} = I_{xx}$ of rectangle $- 2 \times I_{xx}$ of semicircle

$$= \frac{bh^3}{12} - \cancel{2} \times \frac{\pi}{\cancel{8}\,4} R^4$$

$$\Rightarrow I_{xx} = \frac{\cancel{12} \times 15^3}{\cancel{12}} - \frac{\pi}{4} \times 5^4 = 3375 - 490.873 = 2884.127 \text{ cm}^4$$

MOI of rectangle about (YY) axis

$$I_{yy_1} = \frac{hb^3}{12} = \frac{15 \times 12^3}{12} = 2160 \text{ cm}^4$$

MOI of semi circle about its base AB

$$I_{AB} = \frac{\pi}{8} R^4 = \frac{\pi}{8} \times 5^4 = 245.437$$

MOI of semi circle about its centroidal axis (GG)

$$I_{AB} = I_{GG} + Ah^2$$

$$\Rightarrow I_{GG} = I_{AB} - Ah^2$$

$$A = \frac{\pi}{2} \times r^2 = \frac{\pi}{2} \times 5^2 = 39.27$$

$$h = \frac{4r}{3\pi} = \frac{4 \times 5}{3\pi} = 2.122$$

$$= 245.437 - (39.27)(2.122)^2$$

$$= 245.437 - 176.828$$

$$= 68.609 \text{ cm}^4$$

MOI of semi circle about (Y-Y) axis

$$I_{yy_2} = I_{GG} + Ah^2$$

$$= 68.609 + (39.27)(6 - 2.122)^2$$

$$= 68.609 + \cancel{3831.765}\; 590.576$$

$$= 659.186 \text{ cm}^4$$

Now MOI of the beam about (Y-Y) axis

$I_{yy} = I_{yy}$ of rectangle $- 2\, I_{yy}$ of semicircle

$$= 2160 - (2 \times 659.186)$$

$$= 841.628 \text{ cm}^4$$

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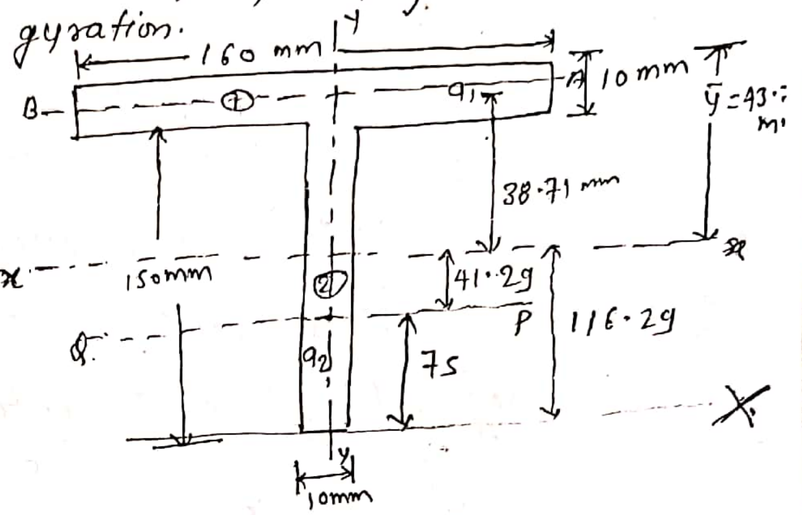
Q: Determine the moment of inertia of the T-section shown in fig. About an axis passing through the centroid and parallel to topmost fibre of the section. Proceed to determine the moment of inertia about axis of symmetry and hence find Out the radius of gyration.



Soln:-

~~Centroid of~~ Rectangle ①

$A_1 = 160 \times 10 = 1600\ mm^2$

$\bar{y}_1 = 150 + \frac{10}{2} = 155\ mm$

Rectangle ②

$A_2 = 150 \times 10 = 1500\ mm^2$

$y_2 = 150/2 = 75\ mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(1600 \times 155) + (1500 \times 75)}{(1600 + 1500)} = \frac{248000 + 112500}{3100}$$

$\Rightarrow \bar{y} = 116.29$ mm From bottom

$\bar{y} = 160 - 116.29 = 43.71$ mm from top.

Now we draw a (x-x) axis on this centroid point.

Now

moI about AB axis of rectangle ①

$$I_{AB} = \frac{bh^3}{12} = \frac{160 \times 10^3}{12} = \frac{16000}{12} = 1333.33$$

Now moI about (x-x) axis of rectangle ①

$$I_{xx_1} = I_{AB} + Ah^2 = 1333.33 + 1600 \times (38.71)^2$$

$= 1333.33 + 2397542.56$

$= 2398875.9$

Now MOI of rectangle (2) about PQ axis.

$$I_{PQ} = \frac{bh^3}{12} = \frac{10\times(150)^3}{12} = 2812500$$

Now MOI about rectangle (II) about (x-x) axis

$$I_{xx_2} = I_{PQ} + Ah^2$$

$$= 2812500 + 1500\times(41.29)^2$$

$$= 2812500 + 2557296.15$$

$$= 5369796.15$$

Now MOI of whole T section about (x-x) axis.

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= 2398875.9 + 5369796.15$$

$$= 7768672.05 \text{ mm}^4$$

$$I_{yy} = \frac{hb^3}{12} + \frac{hb^3}{12}$$

$$= \frac{10\times(160)^3}{12} + \frac{150\times10^3}{12}$$

$$= 3425833 \text{ mm}^4$$

Radius of gyration is given by $k = \sqrt{\frac{I}{A}}$

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} = 50.1 \text{ mm}$$

$$k_{yy} = \sqrt{\frac{I_{yy}}{A}} = 34.24 \text{ mm}$$

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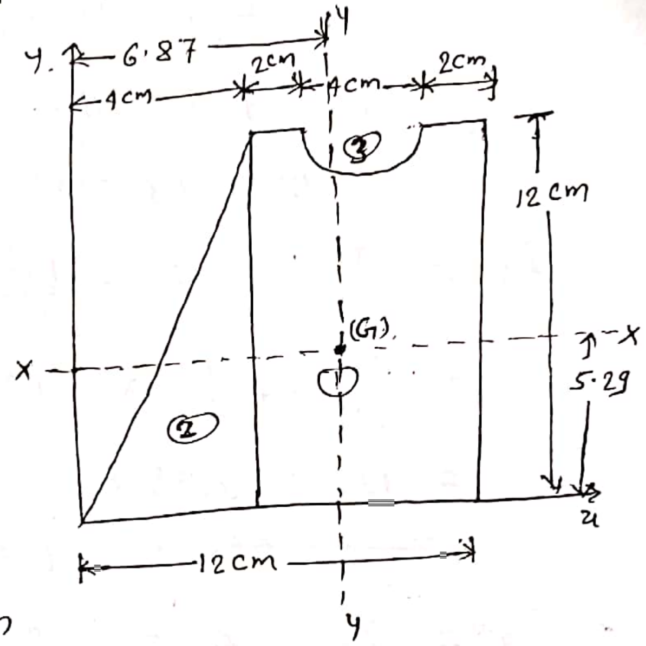
Q. Determine the moment of inertia of the plane area shown in figure about its centroidal axis.



Sol^n:-

(I) Rectangle

$a_1 = 8 \times 12 = 96\ cm^2$

$x_1 = 4 + 8/2 = 8\ cm$

$y_1 = \frac{12}{2} = 6\ cm$

(II) Triangle

$a_2 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 4 \times 12 = 24\ cm^2$

$x_2 = b - \frac{b}{3} = \frac{2b}{3} = \frac{2 \times 4}{3} = 2.67\ cm$

$y_2 = \frac{h}{3} = \frac{12}{3} = 4\ cm$

(III) Semicircle

$a_3 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2} \times 2^2 = 2\pi = 6.28\ cm^2$

$x_3 = 4 + 2 + 4/2 = 8\ cm$

$y_3 = 12 - \frac{4r}{3\pi} = 12 - \frac{4 \times 2}{3 \times \pi} = 11.15\ cm$

$\therefore \bar{x} = \frac{a_1 x_1 + a_2 x_2 - a_3 x_3}{a_1 + a_2 - a_3} = \frac{(96 \times 8) + (24 \times 2.67) - (6.28 \times 8)}{96 + 24 - 6.28}$

$= \frac{768 + 64.08 - 54.24}{113.72} = 6.87\ cm$

$\bar{y} = \frac{A_1 y_1 + A_2 y_2 - a_3 y_3}{A_1 + A_2 - A_3} = \frac{(96 \times 6) + (24 \times 4) - (6.28 \times 11.15)}{96 + 24 - 6.28}$

$= \frac{576 + 96 - 70.02}{113.72} = 5.29\ cm$

MOI about horizontal centroidal axis

$I_{xx} = I_1 + I_2 - I_3$

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$I_1 = I_{G_1} + A_1 h_1^2 = \frac{bh^3}{12} + Ah^2$

$= \frac{8 \times 12^3}{12} + (12 \times 8)(6 - 5.29)^2$

$= 1152 + 48.393$

$= 1200.393 \text{ cm}^4$

$I_2 = I_{G_2} + A_2 h_2^2 = \frac{bh^3}{36} - \frac{1}{2} \times b \times h\,(5.29 - h/3)^2$

$= \frac{4 \times 12^3}{36} + \frac{1}{2} \times 4 \times 12 (1.29)^2$

$= 192 + 39.938$

$= \cancel{152.062} = 231.938 \text{ cm}^4$

For triangle

$I_{base} = \frac{bh^3}{12}$

$I_{centroidal} = I_{base} - Ah^2$

$= \frac{bh^3}{12} - \frac{1}{2} \times b \times h \times \left(\frac{h}{3}\right)^2$

$= \frac{bh^3}{12} - \frac{bh^3}{18}$

$I_{centroid} = \frac{bh^3}{36}$

$I_3 = I_{G_3} + A_3 h_3^2$

$= 0.11 R^4 + \frac{\pi}{2} R^2 \left(6.71 - \frac{4 \times 2}{3\pi}\right)^2$

$= 0.11 R^4 + \frac{\pi}{2} \times 2^2 \left(6.71 - \frac{8}{3 \times 3.141}\right)^2$

$= 0.11 \times 2^4 + \frac{3.141 \times 4}{2} \left(6.71 - \frac{8}{9.423}\right)^2$

$= 1.76 + (6.282)(34.352)$

$= 217.562 \text{ cm}^4$

For semi circle

$I_{base} = \frac{\pi}{8} R^4$

$I_{centroidal} = I_{base} - Ah^2$

$= \frac{\pi}{8} R^4 - \frac{\pi}{2} R^2 \left(\frac{4R}{3\pi}\right)^2$

$= \frac{\pi}{8} R^4 - \frac{\pi}{2} R^2 \times \frac{16 R^2}{9\pi^2}$

$= \frac{\pi}{8} R^4 - \frac{8}{9\pi} R^4$

$\boxed{I_{xx} = 0.11 R^4}$

$\therefore I_{xx} = I_1 + I_2 - I_3$

$= 1200.393 + 231.938 - 217.562$

$= 1214.769 \text{ cm}^4$

MOI about vertical centroidal axis

$I_{yy} = I_1 + I_2 - I_3$

$I_1 = I_{G_1} + A_1 h_1^2$

$= \frac{hb^3}{12} + (12 \times 8)(8 - 6.87)^2$

$= \frac{\cancel{12} \times 8^3}{\cancel{12}} + 96(1.2769)$

$= 634.582 \text{ cm}^4$

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$$I_2 = I_{G_2} + A_2 h_2^2$$

$$= \frac{bh^3}{36} + \left(\frac{1}{2}\times b\times h\right)\left(6.87 - \frac{8}{3}\right)^2$$

$$= \frac{12\times 4^3}{36} + \left(\frac{1}{\cancel{2}}\times \cancel{4}^{2}\times 12\right)\left(6.87 - 2.66\right)^2$$

$$= \frac{12\times 64}{36} + 24(17.72)$$

$$= 21.333 + 425.378$$

$$= 446.711 \text{ cm}^4.$$

$$I_3 = I_{G_3} + A_3 h_3^2$$

$$= \frac{\pi}{8} R^4 + \frac{1}{\cancel{2}}\pi\times 2^{\cancel{2}}(8 - 6.87)^2$$

$$= \frac{3.141}{8}\times 2^4 + 2\pi(1.13)^2$$

$$= 6.282 + 8.021$$

$$= 14.3 \text{ cm}^4$$

$$\therefore I_{yy} = I_1 + I_2 - I_3$$

$$= 634.582 + 446.711 - 14.3$$

$$= 1066.993 \text{ cm}^4$$

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