Engineering Mechanics (3ME3-04)

DEPARTMENT

OF

MECHANICAL ENGINEERING

(Jaipur Engineering College and Research Centre, Jaipur)

UNIT: I

Statics of particles and rigid bodies: Fundamental laws of mechanics, Principle of transmissibility, System of forces, Resultant force, Resolution of force, Moment and Couples, Varignon's theorem, Resolution of a force into a force and a couple, Free body diagram, Equilibrium, Conditions for equilibrium, Lami's theorem.

Plane trusses: Types of structures, Trusses, Support Conditions, Types of Loadings, Classification of trusses, Determinacy of trusses, Basic assumptions of truss analysis, Method of joints, Method of sections.

Virtual work: Principle of Virtual Work, Active forces and active force diagram, Stability of equilibrium.

Faculty: AKHILESH PALIWAL
(Assistant Professor)

SUB - MECHANICS [redacted]

Unit - 1

Engg. Mechanics:- It deals with the study of body at rest or in motion, when subjected to external mechanical disturbance.

Engg. Mechanics
- Statics (rest)
- Dynamics (motion)
  - Kinematics
  - Kinetics

Statics:- Deals with forces in terms of their distribution and effect on a body in equilibrium i.e. at rest.

Dynamics:- Deals with the ~~body~~ study of body in motion.

Kinematics:- is concerned with the description of motion of objects independent of causes of motion. (displacement, time, velocity, Acc.)

Kinetics:- Both the motion and its causes are considered.
Torque, Gravity, friction etc. mass & force M.I

Matter:- Matter is anything that occupies space, possesses mass and offers resistance to any external force. i.e. Iron, stone, wood etc.

Particle:- A particle is an object that has infinitely small volume (occupies negligible space) but has a mass which can be considered to be concentrated at a point.

Body:- A body in which the distance b/w two particles remains constant i.e. the size and shape of the body do not change, is called rigid body.

Space:- is a region which extends in all directions and contains everything in it e.g. sun, moon, stars etc.

Time:- is a measure of succession of events. The unit of time is second.

Motion:- When a body changes its position with respect to other bodies, then body is said to be in motion. It involves both space and time.

Trajectory:- is a path followed by a body during its motion. It may be a straight line or a curve.

Newton's law of motions:-

First law:- Every body continues in it rest or of uniform motion in a straight line if there is no unbalanced force acted upon it.

or

Law of inertia.

Second law:- The rate of change of linear momentum is directly proportional to the force and it takes place in the direction of the impressed force.

Third law:- To every action, there is equal and opposite reaction.

Newton's law of gravitation:- The mutual attraction between two isolated bodies. i.e.

Every body in the universe attracts every other body with a force directly proportional to the product of their masses and inversally proportional to the square of the distance separating them.

$$F \propto \frac{m_1 m}{r^2}$$

$$\Rightarrow \boxed{F = G\frac{m_1 m_2}{r^2}}$$

G = Universal gravitation constant $= 6.67 \times 10^{-11}$ ~~Nm²/kg²~~ $m^3/kg.sec^2$

Scalar:- A quantity is said to be scalar if it is completely defined by its magnitude alone. mass, length, volume etc

magnitude $= |\vec{A}|$

Vector:- Any quantity which possesses magnitude as well as direction is called a vector quantity. A vector quantity needs both magnitude and direction for its complete specification. i.e. displacement, velocity, accⁿ etc.

Mass:- Mass is an indication of the quantity of matter present within a system. (kg) → unit

Force:- force is an external agent which tends to change the speed or direction of a system.

$\boxed{F = ma}$ units → $kg \times \frac{m}{s^2} = kg\,m/s^2 = N$

Weight:- It is a force which the system exerts due to gravitational acceleration. The weight (W) of a system equals the product of mass (m) and local gravitational acceleration (g).

→ mass of a system remains constant.

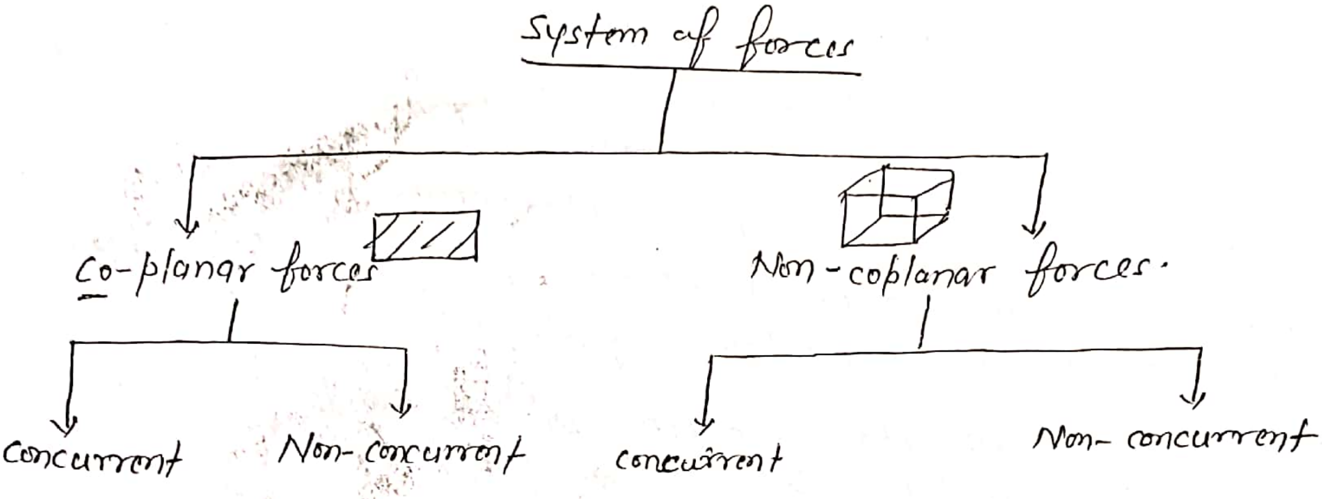
→ weight varies with change in value of gravitational acceleration from one place to another place.

→ generally the value of $\boxed{g = 9.81\ m/s^2}$

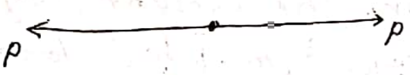
Tensile force:- A member is said to be in tension when it is subjected to two equal and opposite pulls and the members tends to elongate/increase in length.

compressive force:- If a member is subjected to two equal and opposite pushes, and the member tends to shorten/decrease in length, the member is said to be in compression.

System of forces:— when several forces of different magnitude and directions act upon a body, they consitute a force system.

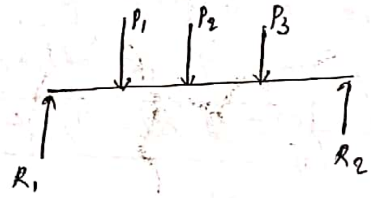


Collinear forces:— The line of action of all forces lie along the same straight line.



Ex:- Forces on a rope in a tug of war.

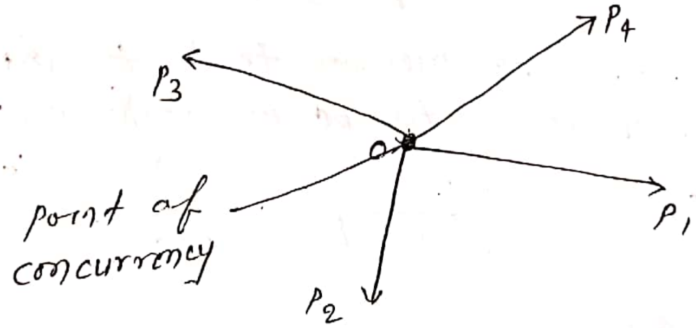
Co-planar parallel forces:— The Lines of action of all forces are parallel to each other and lie in a single plane.



Ex:- System of vertical loads acting on a beam.

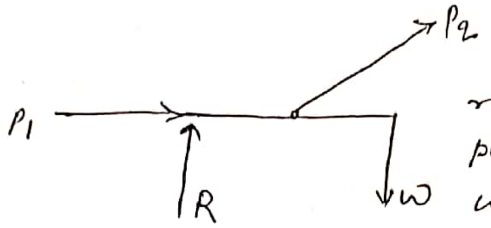
coplanar concurrent forces:— All forces lie in the same plane have different directions but their lines of action act at one point.

→ The point where the lines of action of the forces meet is known as the point of concurrency of the force system.



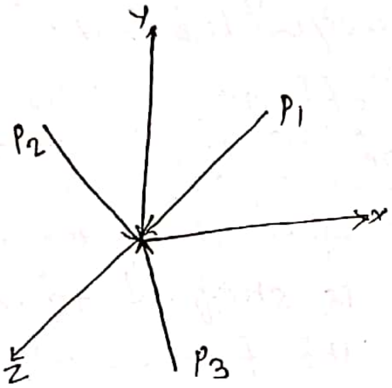
Ex:- Forces on a rod resting against a wall.

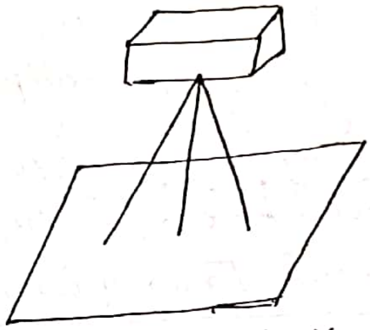
__Coplanar non-concurrent forces:__— All forces lie in the same plane but their lines of action do not pass through a single point.



__Ex:-__ Forces on a ladder resting against wall and a person standing on a ring which is not at its centre of gravity.

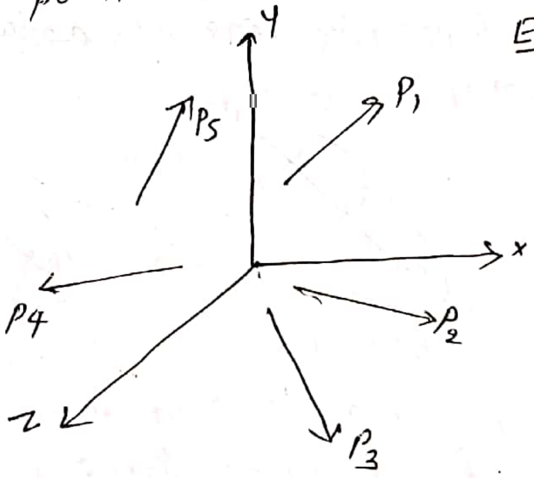
__Non-coplanar concurrent forces:__- All forces do not lie in the same plane but their lines of action pass through a single point.





__Ex:-__ Forces on a tripod carrying a camera.

__Non-coplanar and non-concurrent forces:__— All forces do not lie in the same plane and their lines of action do not meet at a single point.



__Ex:-__ Forces acting on a moving bus.

__Equilibrium:__— When two or more than two forces act on a body in such a way that the body remains in a state of rest or of uniform motion. Then the system of forces is said to be in equilibrium.

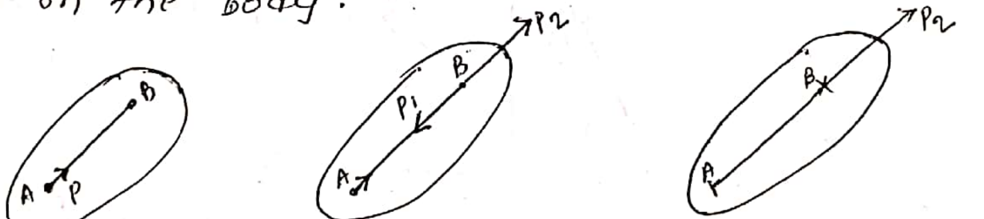
Resultant :- When a body is acted upon by a system of forces, then the vectorial sum of all the forces is known as resultant.

→ Resultant refers to the single force which produces the same effect as is done by the combined effect of several forces.

Equilibrant :- A number of forces may act on a body in such a manner that the body is not in equilibrium. The resultant of several forces may cause a change of state of rest or of uniform motion. A single force may have to be applied to the body to bring it in equilibrium st. That single force is known as equilibrant.

→ Equilibrant is equal and opposite to the resultant of several forces acting on the body.

Principle of Transmissibility :- When the point of application of a force acting on a body is shifted to any other point on the line of action of the forces without changing its direction, there occurs no change in the equilibrium state of the body. i.e. This implies that a force acting at any point on a body may also be considered to act at any other point along its line of action without changing its effect on the body.



$P_1 = P_2 = P$

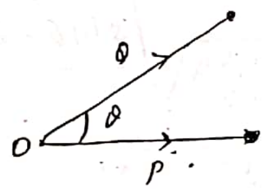
Consider a force P acting at point A on rigid body. B is another point on the line of action of force P. At point B, apply two oppositely directed forces (P_1 and P_2) equal to and collinear with P. Such an application will in no way alter the action of given force P. At point A, forces P and P_1 are equals but opposite and accordingly cancel each other. The leaves a force = P at B. This implies that a force acting at any point on a body may also be considered to act at any other point along its line of action without changing its effect on the body.

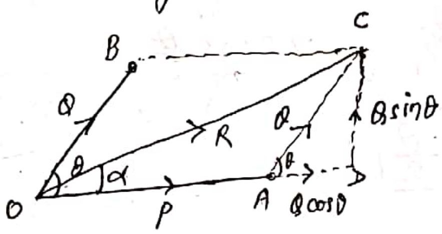
NOTE:- When a number of concurrent forces act on a body to keep the body in equilibrium, then resultant of these forces will be the vector sum of all these forces.

→ The process of determining of resultant is known as composition of forces.

Parallelogram law of forces:- The parallelogram law of forces is used to determine the resultant of two forces acting at a point in a plane and inclined to each other at an angle.

STATEMENT :- If two forces acting at a point be represented in magnitude and direction by the two adjacent sides of parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.





consider two forces 'P' and 'Q' acting on a body at 'O'. The force P is represented in magnitude and direction by $\vec{OA}$ whereas the force Q is represented in magnitude and direction by $\vec{OB}$. Let the angle b/w the two force be θ. The resultant of these two forces is obtained by the diagonal $\vec{OC}$ of the parallelogram OACB.

Drop ⊥ from C and let it meet OA extend at point D. In △CAD, side CA is parallel and equal to OB i.e. it represents force Q. From △OCD.

$OA = P$
$AD = Q\cos\theta$
$CD = Q\sin\theta$

$$R = OC = \sqrt{(OD)^2 + (CD)^2} = \sqrt{(OA + AD)^2 + (CD)^2}$$

$$\Rightarrow R = \sqrt{(P + Q\cos\theta)^2 + (Q\sin\theta)^2} = \sqrt{P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta}$$

$$\Rightarrow R = \sqrt{P^2 + Q^2(\cos^2\theta + \sin^2\theta) + 2PQ\cos\theta}$$

$$\Rightarrow \boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

$$\tan\alpha = \frac{CD}{OD}$$

$$\Rightarrow \tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\Rightarrow \boxed{\alpha = \tan^{-1}\frac{Q\sin\theta}{P+Q\cos\theta}}$$

Special cases:-

① When the two forces are equal and θ is the angle b/w them.

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta} = \sqrt{P^2+P^2+2P^2\cos\theta} = \sqrt{2P^2+2P^2\cos\theta}$$

$$\Rightarrow R = \sqrt{2P^2(1+\cos\theta)} = \sqrt{2P^2\times 2\cos^2\frac{\theta}{2}}$$

$\boxed{1+\cos\theta = 2\cos}$

$\boxed{\sin\theta = 2\sin\frac{\theta}{2}.c}$

$$\Rightarrow R = \sqrt{4P^2\cos^2\theta/2} = 2P\cos\frac{\theta}{2}$$

$$\Rightarrow \boxed{R = 2P\cos\frac{\theta}{2}}$$

$$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta} = \frac{P\sin\theta}{P+P\cos\theta} = \frac{\cancel{P}\sin\theta}{\cancel{P}(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta}$$

$$\Rightarrow \tan\alpha = \frac{\cancel{2}\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{\cancel{2}\cos^2\theta/2} = \frac{\sin\frac{\theta}{2}\cdot\cancel{\cos\theta/2}}{\cancel{\cos\theta/2}\cdot\cos\theta/2}$$

$$\Rightarrow \tan\alpha = \tan\theta/2$$

$$\Rightarrow \alpha = \tan^{-1}(\tan\theta/2)$$

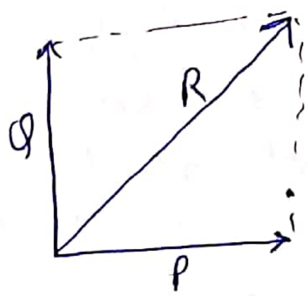
$$\Rightarrow \boxed{\alpha = \theta/2}$$

i.e. the resultant bisects the angle b/w the forces.

Scanned by CamScanner

(II) when the two forces act at right angle i.e. $\theta = 90°$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{P^2 + Q^2 + 2PQ\cos 90}$$



$$\Rightarrow \boxed{R = \sqrt{P^2 + Q^2}}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{Q\sin 90}{P + Q\cos 90} = \frac{Q}{P}$$

$$\Rightarrow \boxed{\alpha = \tan^{-1}\left(\frac{Q}{P}\right)}$$

(III) When the two forces act in the same line and same sense. i.e. $\theta = 0$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{P^2 + Q^2 + 2PQ\cos 0}$$

$$\Rightarrow R = \sqrt{P^2 + Q^2 + 2PQ} = \sqrt{(P+Q)^2}$$

$$\Rightarrow \boxed{R = (P+Q)}$$

$\cancel{\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}}$ The resultant is maximum when the forces are collinear and act in the same direction.

(IV) When the two forces have the same line of action but opposite sense i.e. $\theta = 180$

$\cos 180 = -1$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{P^2 + Q^2 + 2PQ\cos 180}$$

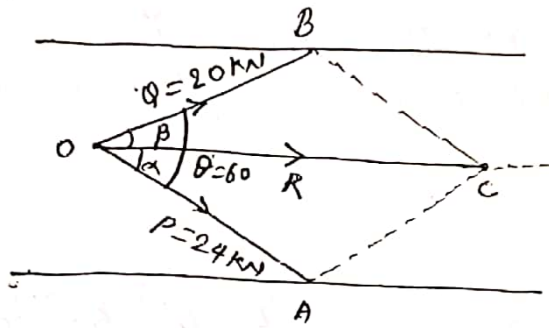
$$\Rightarrow R = \sqrt{P^2 + Q^2 - 2PQ} = \sqrt{(P-Q)^2}$$

$$\Rightarrow \boxed{R = (P - Q)}$$

The resultant is minimum when the two forces are collinear but act in opposite direction.

Q. Two locomotives on opposite banks of a canal pull a vessel moving parallel to the banks by means of two horizontal ropes. The tensions in these ropes have been measured be 20 kN and 24 kN while the angle b/w them is 60°. Find the resultant pull on the vessel and the angle b/w each of the rope and the sides of the canal.

Soln:-



$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{(24)^2 + (20)^2 + 2 \times 24 \times 20 \times \cos 60}$

$= \sqrt{576 + 400 + 480} = \sqrt{1456} = 38.16 \text{ kN}$

$\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta} = \dfrac{20\sin 60}{24 + 20\cos 60} = \dfrac{20 \times 0.866}{24 + 20 \times 0.5} = 0.5094$

$\Rightarrow \alpha = \tan^{-1} 0.509$

$\Rightarrow \alpha = 27°$

$\beta = \theta - \alpha = 60 - 27 = 33°$

Q. The magnitude of two forces is such that when acting at right angles produce a resultant force of $\sqrt{20}$ and when acting at 60° produce a resultant equal to $\sqrt{28}$. Work out the magnitude of the two forces.

Soln:- $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ ——— (I)

case (I) when $\theta = 90$

$R^2 = P^2 + Q^2 + 2PQ\cos 90 \Rightarrow R^2 = P^2 + Q^2$

case (II) when $\theta = 60$

$\Rightarrow (\sqrt{20})^2 = P^2 + Q^2 \Rightarrow 20 = P^2 + Q^2$ ——— (II)

$R^2 = P^2 + Q^2 + 2PQ\cos 60 \Rightarrow (\sqrt{28})^2 = P^2 + Q^2 + \cancel{2}PQ \frac{1}{\cancel{2}}$

$\Rightarrow 28 = P^2 + Q^2 + PQ$ ——— (III)

Scanned by CamScanner

From eqn (II) and (III)

$20 = \cancel{P^2} + \cancel{Q^2}$

$28 = \cancel{P^2} + \cancel{Q^2} + PQ$

~~flat means~~

$+8 = + PQ$

$\Rightarrow PQ = 8$

Squaring both sides we get

$P^2Q^2 = 8^2$

$\Rightarrow Q^2 = \frac{64}{P^2}$

Put the value of Q^2 in eqn (II) we get

$20 = P^2 + Q^2$

$\Rightarrow 20 = P^2 + \frac{64}{P^2}$

$\Rightarrow 20P^2 = P^4 + 64$

$\Rightarrow P^4 - 20P^2 + 64 = 0$

~~$\Rightarrow P^4 - 4P^2 - 16P^2 + 64 = 0$~~

$\Rightarrow (P^2 - 16)(P^2 - 4) = 0$

when $(P^2 - 16) = 0$

$\Rightarrow P^2 = 16 \Rightarrow P = 4$

when $P^2 - 4 = 0$

$\Rightarrow P^2 = 4 \Rightarrow P = 2$

$\therefore P = 2$ or 4.

The corresponding value of force Q will be 4 or 2.

Q. The resultant of two forces P and Q acting at a point is R. The resultant R gets doubled when Q is either doubled or direction is reversed. Show that P, Q and R conform to the ratio.

$$P:Q:R = \sqrt{2}:\sqrt{3}:\sqrt{2}$$

Soln:- $\because$ $R^2 = P^2+Q^2+2PQ\cos\theta$ —— (I)

when Q is double R is also doubled

$$(2R)^2 = P^2+(2Q)^2+2P2Q\cos\theta$$

$$\Rightarrow 4R^2 = P^2+4Q^2+4PQ\cos\theta \text{ —— (II)}$$

when direction of Q is reversed the R is also doubled

$$(2R)^2 = P^2+(-Q)^2+2P(-Q)\cos\theta$$

$$\Rightarrow 4R^2 = P^2+Q^2-2PQ\cos\theta \text{ —— (III)}$$

Adding eqn (I) and (III) we get

$$R^2 = P^2+Q^2+2PQ\cancel{\cos\theta}$$

$$+ \quad 4R^2 = P^2+Q^2-2PQ\cos\theta$$

$$5R^2 = (P^2+Q^2)\times 2$$

$$\Rightarrow 5R^2 = 2P^2+2Q^2 \text{ —— (IV)}$$

Eqn (III)×2 and adding in eqn (II) we get

$$8R^2 = 2P^2+2Q^2-4PQ\cancel{\cos\theta} \text{ —— Eqn (III)×2}$$

$$+ \quad 4R^2 = P^2+4Q^2+4PQ\cos\theta \text{ —— Eqn (II)}$$

$$12R^2 = 3P^2+6Q^2$$

$$\Rightarrow \cancel{3}(4R^2) = \cancel{3}(P^2+2Q^2)$$

$$\Rightarrow 4R^2 = P^2+2Q^2 \text{ —— (V)} \qquad \Rightarrow 2Q^2 = 4R^2-P^2 \text{ —— (VI)}$$

put the value of $2Q^2$ in eqn (IV) we get

$$5R^2 = 2P^2+(4R^2-P^2) \Rightarrow$$

$$\Rightarrow 5R^2 = 2P^2+4R^2-P^2 \Rightarrow 5R^2-4R^2 = 2P^2-P^2 \Rightarrow R^2 = P^2$$

$$\Rightarrow \boxed{R=P}$$

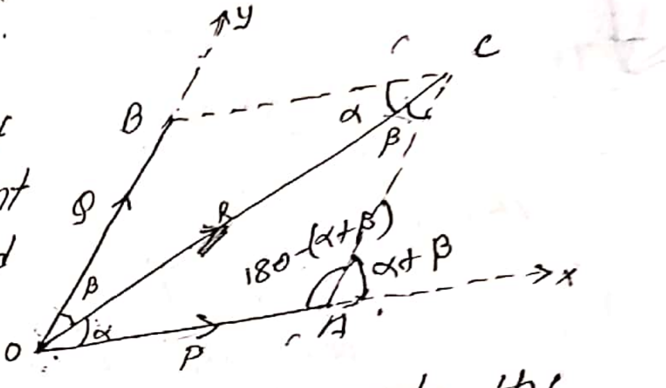
From eqn (V) we get $4P^2 = P^2+2Q^2 \Rightarrow 3P^2 = 2Q^2$

$$\Rightarrow Q = \sqrt{\tfrac{3}{2}P^2} = \sqrt{\tfrac{3}{2}}P \qquad \therefore \boxed{P:Q:R = \sqrt{2}:\sqrt{3}:\sqrt{2}} \text{ Proved.}$$

$$\Rightarrow P:Q:R = P:\sqrt{\tfrac{3}{2}}P:P$$

RESOLUTION OF FORCES:- finding the component of a given force in two directions is called resolution of forces. These component forces will have the same effect on the body as the given single force.

→ Let the given force R and let it be required to find its component in directions making angle α and β with its line of action.



with refrence to parallelogram OACB the sides OA and OB represents the components ~~in directions making angle α~~ of the given force R along ox and oy respectively. i.e.

$OA = P$ and $OB = Q$

$\angle OCA = \angle BOC = \beta$ (By alternate angles)

$\angle OAC = 180 - (\alpha+\beta)$

Applying sine rule to ΔOAC, we get -

$$\frac{OA}{\sin\beta} = \frac{AC}{\sin\alpha} = \frac{OC}{\sin[180-(\alpha+\beta)]}$$

$AC = OB = Q$.

$$\Rightarrow \boxed{\frac{P}{\sin\beta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin(\alpha+\beta)}} \quad \text{(1)}$$

from eq^n (1)

$$\boxed{P = \frac{R\sin\beta}{\sin(\alpha+\beta)}}$$

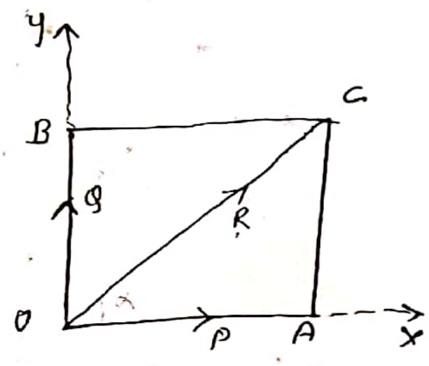
from eq^n (1)

$$\boxed{Q = \frac{R\sin\alpha}{\sin(\alpha+\beta)}}$$

⇒ If ox and oy are at right angles and OACB becomes a rectangle

$\therefore \alpha+\beta = 90°$

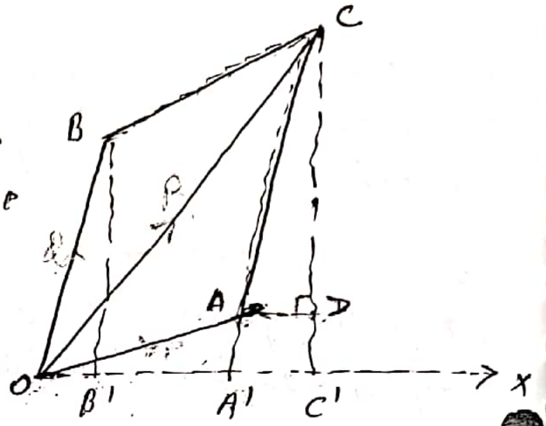
$\Rightarrow \beta = 90 - \alpha$



$$\therefore \; P = \frac{R\sin\beta}{\sin(\alpha+\beta)} = \frac{R\sin(90-\alpha)}{\sin 90} = R\cos\alpha$$

$$Q = \frac{R\sin\alpha}{\sin(\alpha+\beta)} = \frac{R\sin\alpha}{\sin 90} = R\sin\alpha$$

THEOREM OF RESOLVED PARTS:—



STATEMENT:— The algebraic sum of the resolved parts of two forces in a given direction is equal to the resolved part of their resultant in the same direction.

⇒ Let OC represents the resultant of two forces OA and OB acting at point O. Further let OX be the direction along which ~~parts~~ resolved parts of OA, OB and OC are to be worked out.

Drop perpendicular from B, A and C to meet OX at B', A' and C' respectively.

The triangles OBB' and ACD are congruent and that gives $OB' = AD = A'C'$

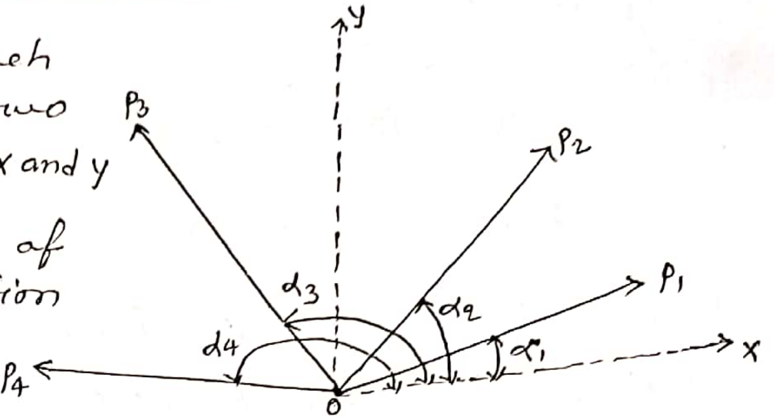
from the geometry, we have—

$$OC' = OA' + A'C' = OA' + AD = OA' + OB'$$

⇒ $\boxed{\text{Resolved parts of force OC on OX = Resolved part of OA on OX + Resolved part of force OB on OX.}}$

RESULTANT OF COPLANAR-CONCURRENT FORCES:-

Analytical method (Principle of resolved parts):-



(i) Find the components of each force in the system in two mutually perpendicular x and y directions.

(ii) Make algebraic addition of components in each direction to get two components ΣF_x and ΣF_y.

(iii) Obtain the resultant both in magnitude and direction by combining the two component forces ΣF_x and ΣF_y which are mutually perpendicular. 1

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

and its inclination θ to x-axis

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

Theorem of resolved parts:- STATEMENT:- The algebraic sum of the resolved parts of two forces in a given direction is equal to the resolved parts of their resultant in the same direction.

From fig. P_1, P_2, P_3 and P_4 are the concurrent forces meeting at point O and making an angles $\alpha_1, \alpha_2, \alpha_3$ and α_4 with OX. 2

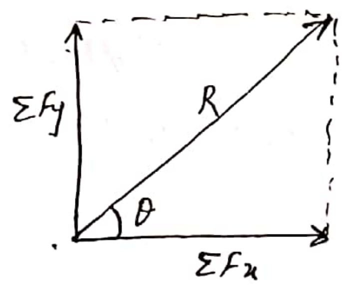
Resolving along x-axis and y-axis, we get-

$$\Sigma F_x = P_1\cos\alpha_1 + P_2\cos\alpha_2 + P_3\cos\alpha_3 + P_4\cos\alpha_4$$

$$\Sigma F_y = P_1\sin\alpha_1 + P_2\sin\alpha_2 + P_3\sin\alpha_3 + P_4\sin\alpha_4$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$ 3



Equilibrium conditions of a particle :-

→ A particle will be in equilibrium when resultant of all the forces acting on it is zero.

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 0$$

$$\Rightarrow (\Sigma F_x)^2 + (\Sigma F_y)^2 = 0$$

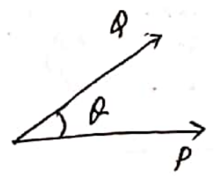
Now $(\Sigma F_x)^2$ and $(\Sigma F_y)^2$ are positive quantities and their sum can not be zero unless each of them is zero.

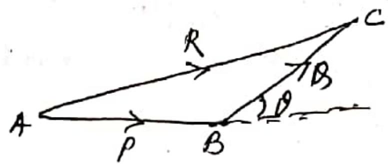
$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

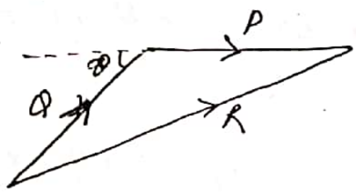
Hence if any number of forces acting at a particle are in equilibrium, then the algebraic sum of their resolved parts in any two perpendicular directions are separately zero.

TRIANGLE LAW OF FORCES :- STATEMENT :- If two forces acting simultaneously on a body are represented by the sides of a triangle taken in order, their resultant is represented by the closing side of the triangle taken in the opposite order.

consider two forces acting on the body as shown in fig.



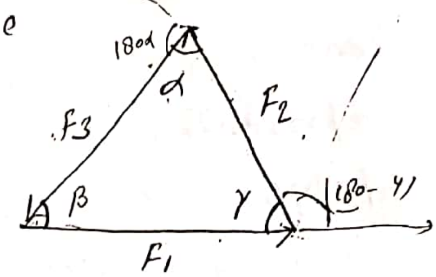




Line AB be drawn to represent force P and BC to represent Q. The triangle ABC is completed by drawin the closing line AC. Line AC represents the resultant in magnitude, line of action and direction.

→ It is to be noted that addition of force P and Q in any order gives the same resultant R.

The following trigonometric relations can be applied while working out solutions by the triangle law of forces.



$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

and

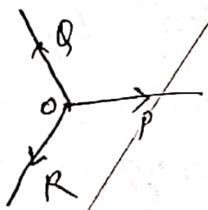
$$F_1^2 = F_2^2 + F_3^2 - 2F_2F_3\cos\alpha$$

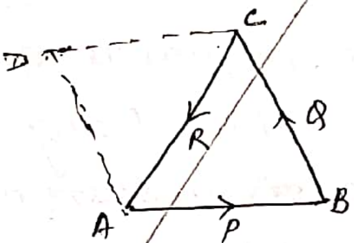
$$F_2^2 = F_1^2 + F_3^2 - 2F_1F_3\cos\beta$$

$$F_3^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\gamma$$

-ve sign because $\cos(180-\gamma) = -(\cos\gamma)$

Triangle law of forces (STATEMENT):- If a system of three forces acting upon a body can be represented in magnitude and direction by the sides of a triangle taken in order, then the system will be in equilibrium.





The three forces P, Q, and R acting at point O have been represented by the sides AB, BC and CA of the triang ABC. construct the parallelogram ABCD. The sides AD, being equal and parallel to BC represents the force Q.

By the parallelogram law of forces

$$\vec{AB} + \vec{AD} = \vec{AC}$$

$$\Rightarrow \vec{P} + \vec{Q} = -\vec{CA} = -\vec{R}$$

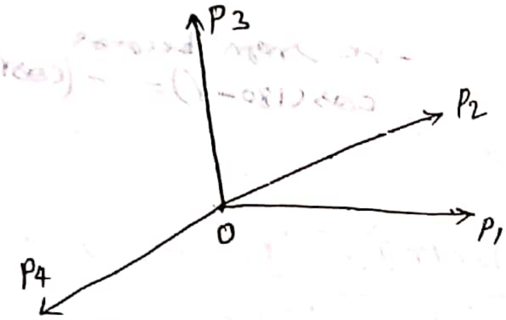
$\therefore$ Resultant of P, Q, and R $= -R + R = 0$

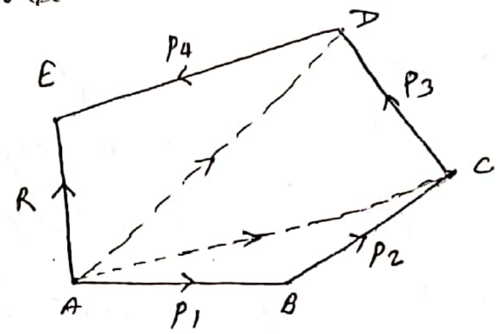
Hence the system is in equilibrium.

Scanned by CamScanner

POLYGON LAW OF FORCES:- (STATEMENT):-

If a number of concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order then the resultant is represented in magnitude and direction by the closing side of the polygon taken in opposite order.





consider force P_1, P_2, P_3 and P_4 acting on the body at a point O. Line AB is drawn to represent force P_1, line BC to represent P_2, line CD to represent P_3 and line DE to represent P_4. The polygon is completed by drawing the closing line AB. This closing line AE represents the resultant of the given system in magnitude, and direction.

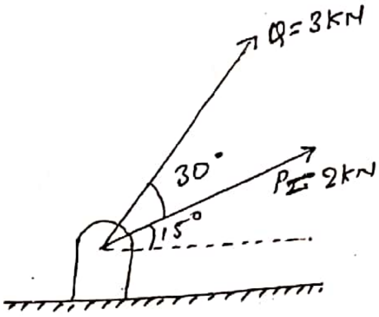
$$\vec{AC} = \vec{AB} + \vec{BC}$$

$$\vec{AD} = \vec{AC} + \vec{CD} = \vec{AB} + \vec{BC} + \vec{CD}$$

$$\vec{AE} = \vec{AD} + \vec{DE} = \vec{AB} + \vec{BC} + \vec{CD} + \vec{DE}$$

$$\Rightarrow \boxed{R = P_1 + P_2 + P_3 + P_4}$$

Q. A stud is acted upon by two forces P = 2 KN and Q = 3 KN as shown in fig. Determine the resultant of these concurrent forces by using:-
(I) Parallelogram law of forces.
(II) Triangle law of forces.



Soln:-

(I) $\because R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$= \sqrt{2^2+3^2+2\cdot 2\cdot 3\cdot \cos 30^\circ}$

$= \sqrt{4+9+10.39}$

$= 4.836$

$\tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta} = \dfrac{3\sin 30}{2+3\cos 30}$

$= \dfrac{3\times 0.5}{2+3\times 0.866} = 0.326$

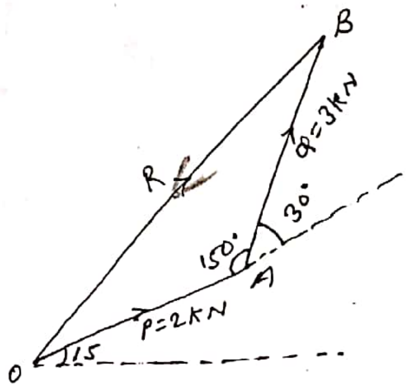
$\Rightarrow \alpha = \tan^{-1} 0.326$

$\Rightarrow \alpha = 18.06^\circ$

$\therefore$ Angle made by resultant with x-axis

$= 18.06+15 = 33.06^\circ$

(II) By co-sine rule



$R^2 = P^2+Q^2-2PQ\cos OAB$

$= 2^2+3^2-2\times 3\times \cos 150$

$R^2 = 4+9+10.39 = 23.39$

$\Rightarrow R = \sqrt{23.39} = 4.836$

Applying the law of sines.

$$\frac{R}{\sin OAB} = \frac{Q}{\sin AOB}$$

$$\Rightarrow \sin AOB = \frac{Q}{R} \sin OAB$$

$$= \frac{3}{4.836} \times \sin 150$$

$$= 0.310$$

$$\Rightarrow \angle AOB = \sin^{-1}(0.310)$$

$$= 18.06^\circ$$

$\therefore$ Angle of inclination of the resultant with the horizontal $= 18.06 + 15 = 33.06$

FREE BODY DIAGRAM (FBD):-

It is essential to identify all external forces acting on a body of interest. "A carefully prepared sketch that shows a body of interest seperated from all interacting bodies is known as free body diagram. It is simple diagram indicating the magnitude and direction of all forces acting upon the Object.

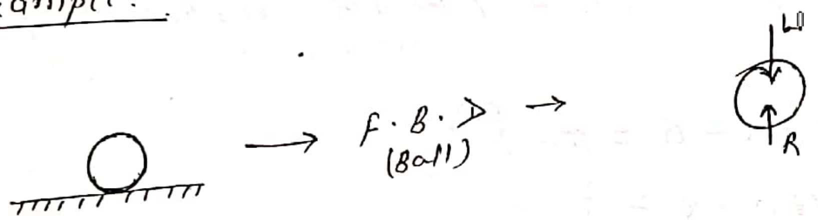
A free body diagram has three essential characteristics:-

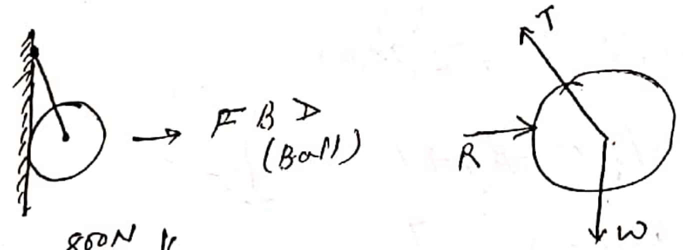
(I) It is a diagram or sketch of the body.

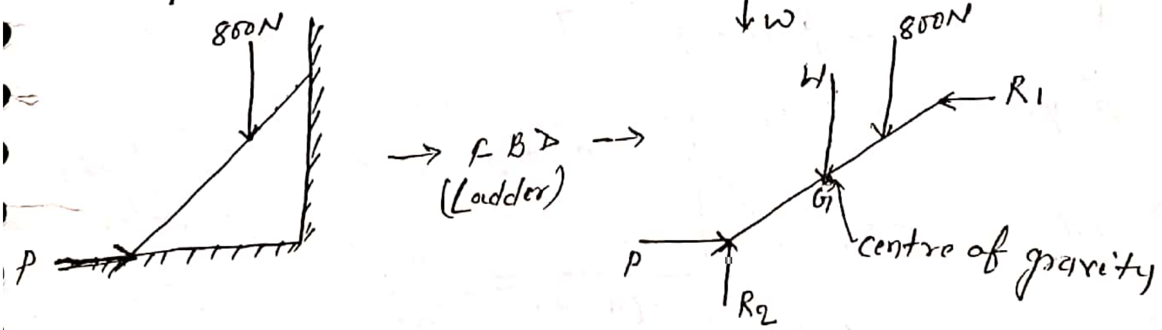
(II) The body is shown completely separated from all other bodies.

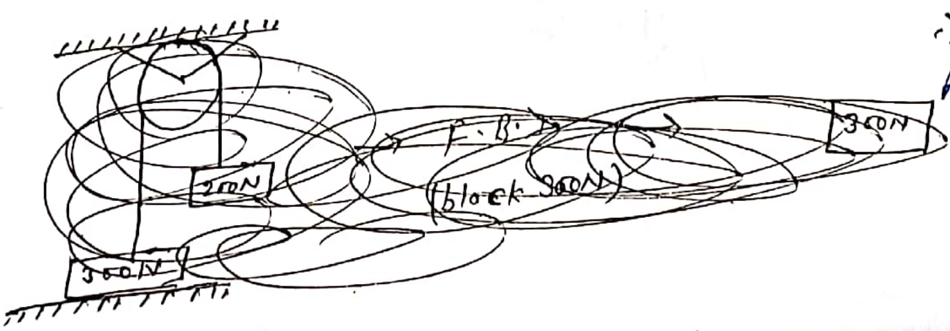
(III) The action on the body due to each body removed in the isolating process is shown as a force or forces in the diagram.

Example:-



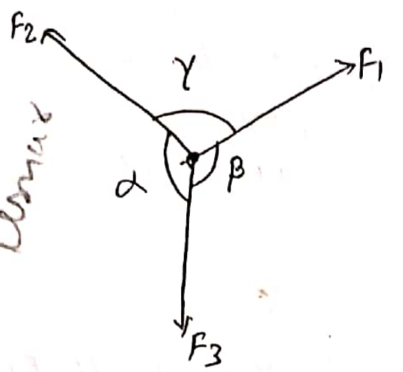


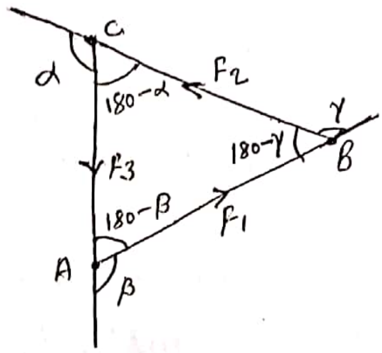




LAMI'S THEOREM :- STATEMENT :- If a body is in equilibrium under the action of three ~~b/o~~ coplanar forces then each force is proportional to the sine of the angle b/w the other two forces.

Diagram for same





Draw the three forces F_1, F_2 and F_3 one after the other in direction and magnitude starting from point A. Since the body is in equilibrium (resultant is zero), the last point must be concide with A. Thus it results in a ~~triang~~ triangle of forces ABC. Now the external angle of A, B and C are equal to β, γ and α respectively.

From ΔABC

$$\angle BAC = 180 - \beta = \pi - \beta$$

$$\angle BCA = 180 - \alpha = \pi - \alpha$$

$$\angle ABC = 180 - (\angle BAC + \angle BCA)$$

$$= \pi - [(\pi - \beta) + (\pi - \alpha)]$$

$$= \pi - [2\pi - \beta - \alpha]$$

$$(\pi - \gamma) = -\pi + \alpha + \beta$$

$$\Rightarrow 2\pi = \alpha + \beta + \gamma$$

$$\Rightarrow \gamma = 2\pi - \alpha - \beta$$

$AB = F_1$

$BC = F_2$

$CA = F_3$

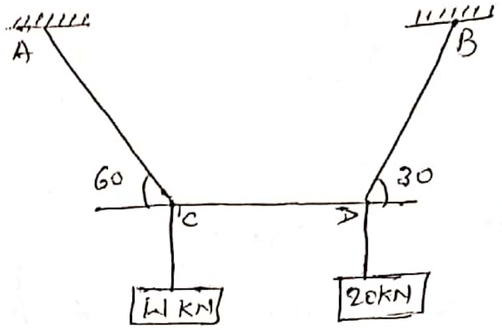
From the sine rule of the triangles

$$\frac{AB}{\sin(180-\alpha)} = \frac{BC}{\sin(180-\beta)} = \frac{CA}{\sin(180-\gamma)}$$

$$\Rightarrow \boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

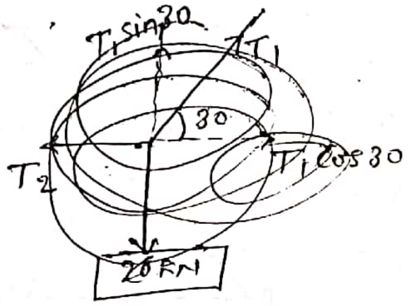
Scanned by CamScanner

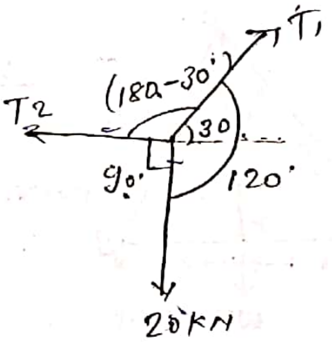
A cord supported at A and B carries a load of 20 kN at D and a load of W at C as shown in fig. Find the value of W so that CD remains horizontal.



Soln.- F.B.D

At point D.





apply lami's theorem

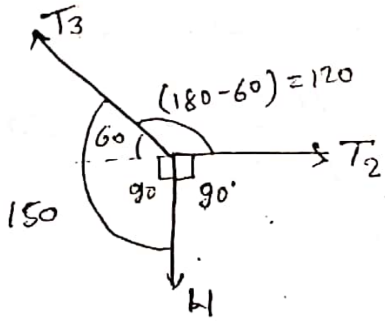
$$\frac{T_1}{\sin 90} = \frac{T_2}{\sin 120} = \frac{20}{\sin(180-30)}$$

$$\Rightarrow \frac{T_1}{\sin 90} = \frac{T_2}{\sin 120} = \frac{20}{\sin 150}$$

$$T_1 = \frac{20 \sin 90}{\sin 150} = \frac{20 \times 1}{0.5} = 40 \text{ kN}$$

$$T_2 = \frac{20 \sin 120}{\sin 150} = \frac{20 \times 0.867}{0.5} = 34.68 \text{ kN}$$

F.B.D at point C.



Scanned by CamScanner

applying Lami's theorem:-

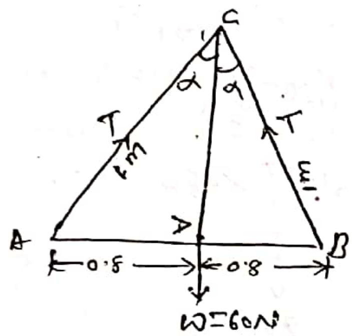
$$\frac{T_2}{\sin 150} = \frac{T_3}{\sin 90} = \frac{W}{\sin 120}$$

$$\Rightarrow \frac{34.68}{\sin 150} = \frac{T_3}{\sin 90} = \frac{W}{\sin 120}$$

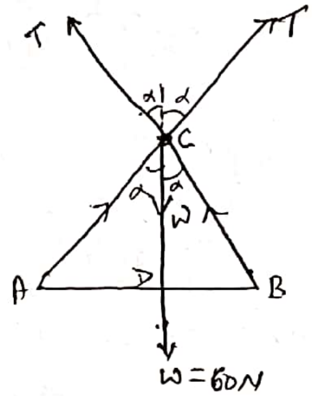
$$\Rightarrow W = \frac{34.64 \sin 120}{\sin 150} = \frac{34.68 \times 0.867}{0.5}$$

$$= 60.13 \text{ KN} \simeq 60 \text{ KN}$$

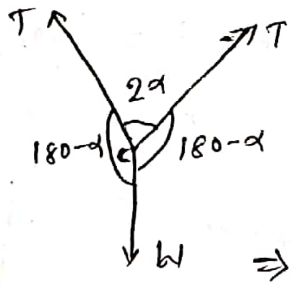
Q A string 2m long is tied to the end of a uniform rod that weighs 60N and is 1.6m long. The string passes over a nail, so that the rod hangs horizontally. find (T)?



Soln:-



applying Lami's theorem at point C.



$$\frac{W}{\sin 2\alpha} = \frac{T}{\sin[180-\alpha]} = \frac{T}{\sin(180-\alpha)}$$

$$\Rightarrow \frac{60}{\sin 2\alpha} = \frac{T}{\sin\alpha} = \frac{T}{\sin\alpha}$$

from Δ ACD

$\sin\alpha = \frac{0.8}{1} = 0.8$, ~~cos α~~ $CD = \sqrt{1^2 - (0.8)^2} = 0.6$ m

$(\alpha = 53.13)$ $\cos\alpha = \frac{0.6}{1} = 0.6$

$$\therefore T = \frac{60 \times \sin\alpha}{\sin 2\alpha}$$

$$\Rightarrow T = \frac{60 \times \sin\alpha}{2\sin\alpha \cdot \cos\alpha} = \frac{60 \times \cancel{0.8}}{2 \times \cancel{0.8} \times 0.6} = 50 \text{ N}$$

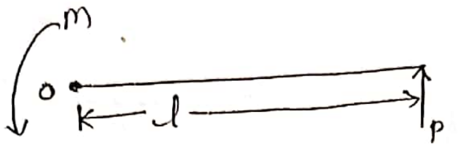
$$\Rightarrow \boxed{T = 50 N}$$

Scanned by CamScanner

MOMENT:- The moment of a force about a point is defined as the tendency of the force to rotate a body about that point.

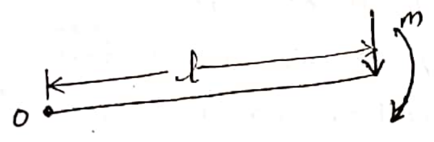
→ A moment has both magnitude and a direction.

→ Mathematically moment is defined as the product of the force and perpendicular distance.



anticlockwise moment = -ve

$m = -P.l$

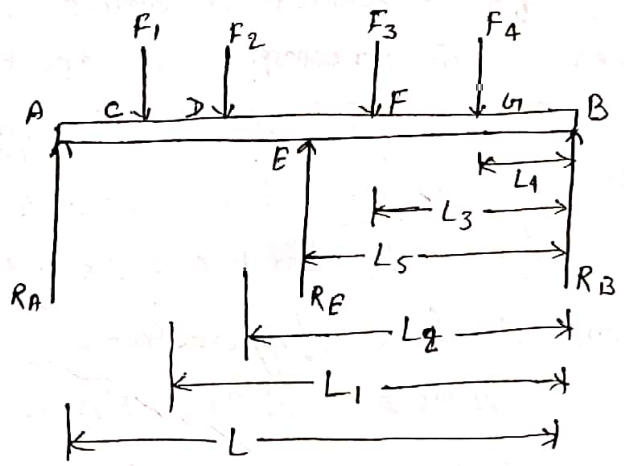


clockwise moment +ve

$m = P l$

PRINCIPLE OF MOMENTS:- The algebraic sum of all the clockwise moments about a point must be equal to sum of all anticlockwise moments. i.e.

clock wise moments = anticlockwise moments.



If we take moments about point B. then

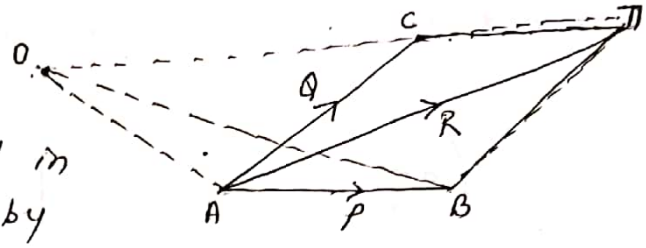
$\Sigma m = 0$

$R_A \times L - F_1 \times L_1 - F_2 \times L_2 - F_3 \times L_3 - F_4 \times L_4 + R_E \times L_5 = 0$

$\Rightarrow \underbrace{R_A \times L + R_E \times L_5}_{\text{clockwise moments}} = \underbrace{F_1 \times L_1 + F_2 \times L_2 + F_3 \times L_3 + F_4 \times L_4}_{\text{anticlockwise moments.}}$

add One Topic from D.S. kumar Page No (68)

VARIGNON'S THEOREM :– STATEMENT:- Moment of a resultant of two forces about a point lying in the plane of the forces is equal to the algebraic sum of moments of the two forces about the same point.



consider two concurrent forces P and Q represented in magnitude and direction by AB and AC respectively. Let O be the point about which moment is to be taken.

Through O, draw a line parallel to the direction of force P and let this line meet ~~the~~ line of action of force Q at point C. ~~that~~ with AB and AC as adjacent sides, comp[lete] the parallelogram ABCD. The diagram ~~AB~~ of this parallelogram represents in magnitude and direction th[e] resultant of force P and Q.

Join O with point A and B.

Moments of force about a point is equal to twice of the area of the triangle. so that -

moment of force P about O = 2 × Area of Δ AOB = 2 × Δ A[OB] — (I)

moment of force Q about O = 2 × Δ AOC — (2)

moment of force R about O = 2 × Δ AOD — ~~(I)~~ (II)

from geometrical configuration:-

Δ AOD = Δ AOC + Δ ACD

The Δ AOB and Δ ABD are on the same base AB and b/w the same lines and as such are equal in area, then

Δ AOD = Δ AOC + Δ AOB

→ Δ ADB = Δ ACD = Δ ABD
AB = CD
OB = AD

The moment of force R about 'O' may be written as
= 2 × (Δ AOC + Δ AOB) — (III)

From eqn (I), (II) and (III) we get:-

$\boxed{\text{Moment of force P and Q about O = moment of resultant R about O}}$

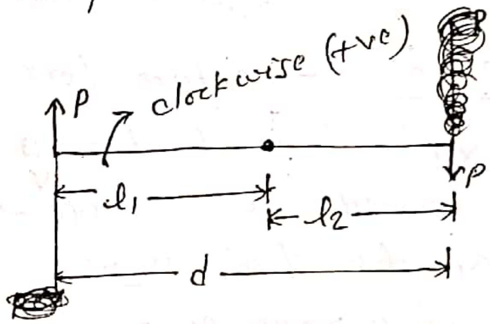
PARALLEL FORCES:- Forces which do not meet at a point but lie in the same plane are called coplanar parallel forces.

→ Like parallel forces (Same direction)
→ Unlike parallel forces (All forces do not act in the same direction)

COUPLE:- Two parallel forces equal in magnitude but opposite in direction and separated by a finite distance are said to form a couple.

→ Since sum of the forces of a couple in any direction is zero.
→ A couple has no tendency to translate a body in any direction but tends only to rotate the body on which it acts.
→ The perpendicular distance separating the two forces is called arm of the couple.

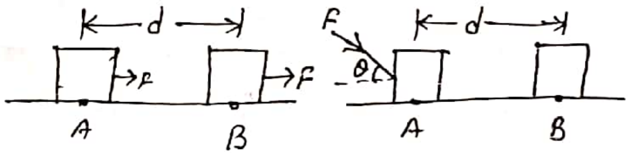
→ moment of a couple
$= (P \times d)$ N-m



$M = +P \times l_1 + P \times l_2$
$= +P(l_1 + l_2)$

$$\boxed{M = +(P \times d)\ N\text{-}m}$$

WORK:- When a force acts on a body and body under goes some displacement, then some work is done by force and it is equal to the force multiplyed by distance travelled in the direction of force.



→ if force is acting horizontally then work done

$$\boxed{W = F \times d\ N\text{-}m}$$

→ if force F acting at a point at an angle θ then workdone by the body

$$\boxed{W = F\cos\theta \times d \ \ Nm}$$

CONCEPT OF VIRTUAL WORK:- If a body is in equilibrium under the applied forces, the workdone is zero. If we assume that the body in equilibrium, undergoes an infinitely small imaginary displacement (virtual displacement) then some work will be imagined to be done. Such an imaginary work is called virtual work.

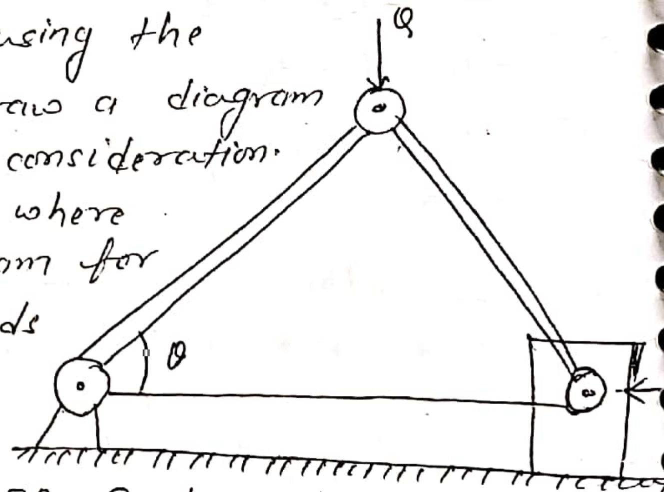
PRINCIPLE OF VIRTUAL WORK:- STATEMENT:- If the virtual work done of particals or rigid bodies on a frictionless system by all external forces and couples is zero all virtual displacements then the system is in equilibrium.

Check

ACTIVE FORCE:- Active forces are external forces capable of doing virtual work during possible virtual displacement

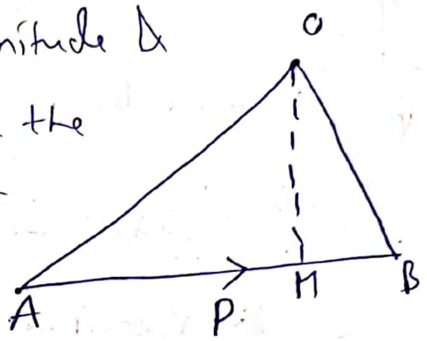
→ force P and Q are active forces because they would do work as the link move.

ACTIVE FORCE DIAGRAM:- when using the method of virtual work, we draw a diagram that isolates the system under consideration. Unlike the free body diagram, where all forces are shown, the diagram for the method of virtual work needs to show only the active forces since the reactive forces do not enter into the application of $\delta U = 0$. Such a drawing will be termed as active force diagram. above figure is an active force diagram for the system shown.



Graphical Representation of Moment:-

Let force 'P' be represented in magnitude & direction by vector AB. Let 'O' be the point about which moment is to be determined.



Moment of force 'P' about point 'O'.

$$= P \times OM$$

OM is $\perp$ to AB

Then $= \quad AB \times OM = 2 \times \left[\frac{1}{2} AB \times OM\right]$

$$= 2(\text{Area of } \triangle AOB).$$

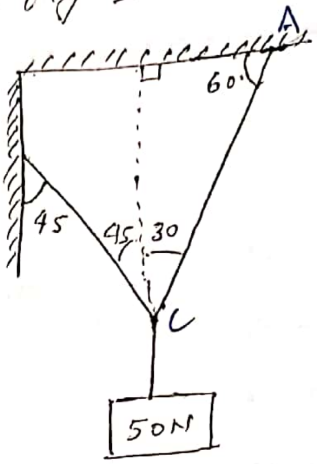
Statement :- The work done on a rigid body or a system of rigid bodies in equilibrium, when acted upon a set of forces, is zero for any virtual displacement compatible with constraints on system.

Sign Conventions

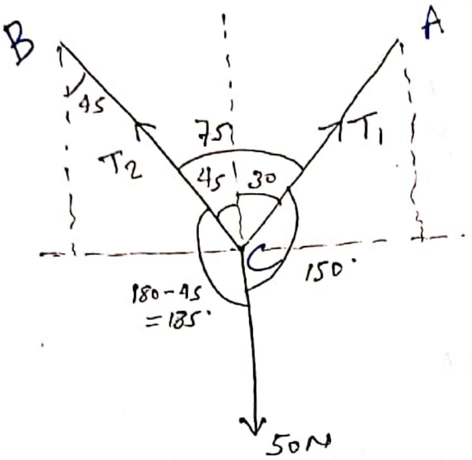
1) Upward force, forces acting towards right and forces in C c.w direction are considered (+ve)

2) D.F, L., C A.c.w direction are (-ve)

Q. An electric light ~~fixture weighing~~ 50N hangs from point C by two strings AC and BC as shown in fig. Determine the forces in the strings AC and BC.



Soln:-

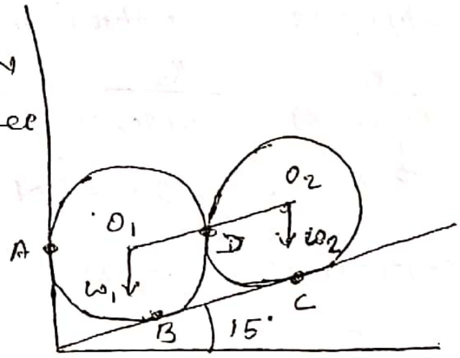


$$\frac{T_1}{\sin 135} = \frac{T_2}{\sin 150} = \frac{50}{\sin 75}$$

$$T_1 = \frac{50 \sin 135}{\sin 75} = 36.594 \text{ N}$$

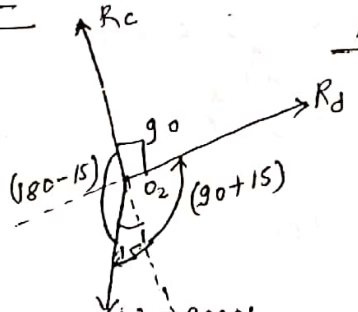
$$T_2 = \frac{50 \sin 150}{\sin 75} = 25.88 \text{ N}$$

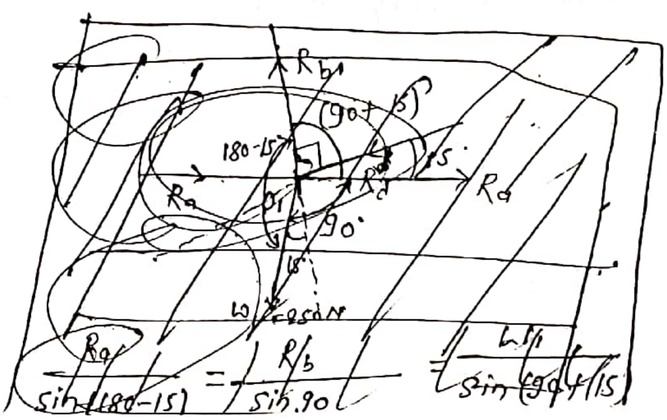
Q. Two rollers of the same diameter are supported by an inclined plane and a vertical wall as shown in fig. The upper and the lower rollers are respectively 200 N and 250N in weights. Assuming smooth surface Find the reactions induced at the points of support A, B, C and D.



Soln:-

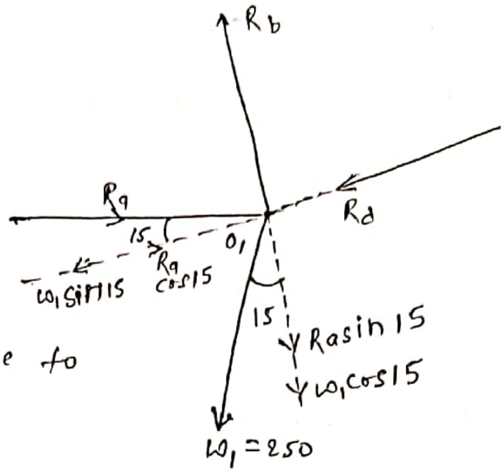
F.B.D at O_2:





$$\frac{R_d}{\sin(180-15)} = \frac{R_c}{\sin(90+15)} = \frac{W_2}{\sin 90}$$

$R_c = 193.18\,N$
$R_d = 51.76\,N$



Later

Resolving the forces paralle to the O_1O_2.

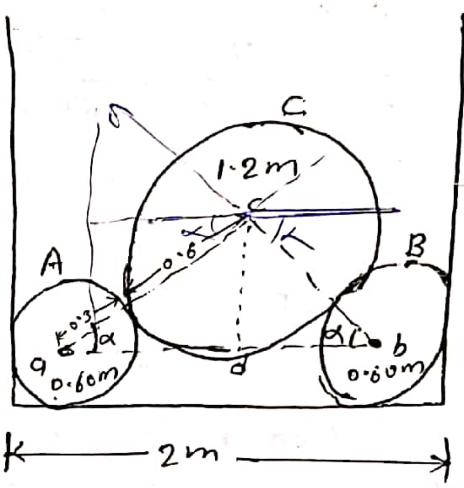
$R_a \cos 15 - w_1 \sin 15 - R_d = 0$

$R_a = 120.61\,N$

Resolving the forces ⊥ to the O_1O_2.

$R_b = R_a \sin 15 + w_1 \cos 15 = 272.68\,N$.

Q. Refer to the system of cylinders arranged as depicted in fig. The cylinders A and B weigh 1000N each and the weight of the cylinder C is 2000N. Determine the forces exerted at the contact points.



Soln:-

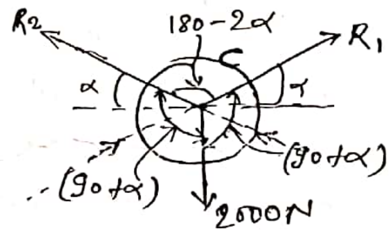
$ab = 2 - \frac{0.6}{2} - \frac{0.6}{2} = 1.4\,m$

$ac = 0.3 + 0.6 = 0.9\,m$

$\cos\alpha = \frac{1.4/2}{0.9} = \frac{ad}{ac} = 0.7777$, $\alpha = 38.94°$

applying Lami's theorem to the forces acting on sphere C.

$\frac{R_1}{\sin(90+\alpha)} = \frac{R_2}{\sin(90+\alpha)} = \frac{2000}{\sin(180-2\alpha)}$

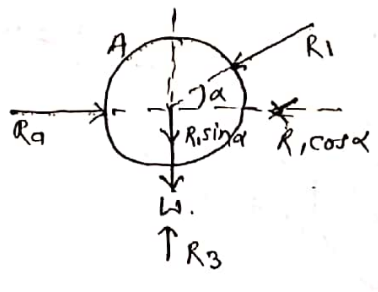


$\Rightarrow R_1 = R_2 = 1590.87\,N$

consider the free body diagram of cylinder A.

$\Sigma F_x = 0$

$\Rightarrow R_A = R_1 \cos\alpha = 1237.38\,N$



$\Sigma F_y = 0$

$R_3 = w + R_1 \sin\alpha$

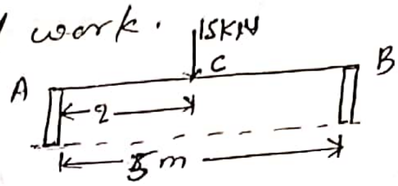
$= 1000 + 1590.87 \sin 38.94$

$= 1999.87\,N$

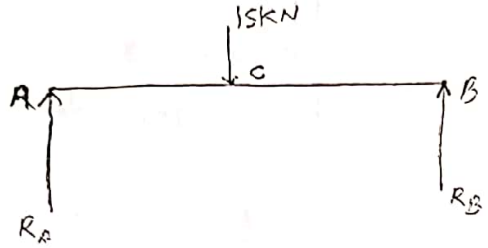
8 /

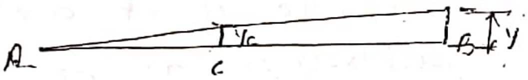
VIRTUAL WORK

Q. A simple supported beam at the ends, 5m span carries a load of 15 kN at a distance of 2m from one end. Determine the reaction using the principle of virtual work.



Soln:-





Assume the virtual displacement given in vertical upper direction at point B is y and point (c) is y_c.

Then from the geometry

$$\frac{y_c}{y} = \frac{AC}{AB}$$

$$\Rightarrow y_c = \frac{AC}{AB} y \Rightarrow \frac{2}{5} y.$$

Here beam in equilibrium,

Total workdone by these forces due to virtual work must be zero.

$$0 \times R_A - y_c \times 15 + y \times R_B = 0$$

$$\Rightarrow -\frac{2}{\cancel{5}} \times y \times \cancel{15}^{3} + y \times R_B = 0$$

$$\Rightarrow \cancel{y} \times R_B = 6 \cancel{y}$$

$$\Rightarrow \boxed{R_B = 6 \text{ kN}}$$

The workdone by R_A is zero because no displacement.

Now resolving forces vertically we get

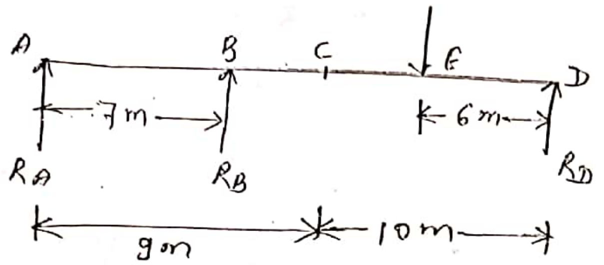
$$R_A + R_B = 15 \Rightarrow R_A = 15 - R_B$$

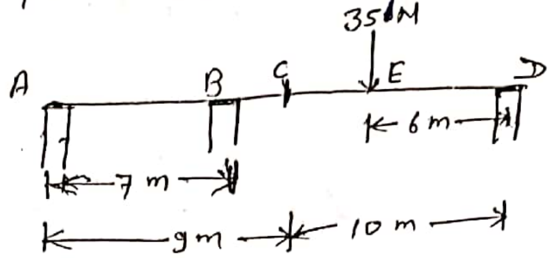
$$\Rightarrow \boxed{R_A = 15 - 6 = 9 \text{ kN}}$$

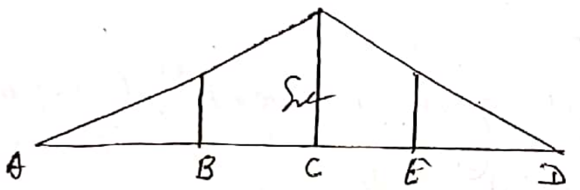
Scanned by CamScanner

Q. Two beam AC and CD of length 9 m and 10 m respectively are hinged at C. These are simply supported at A and B. Another supported is given at point B. The loads on the beam are shown in fig. Using the principle of virtual work find the reactions at the point B.

Solⁿ:-







Assume that a vertically displacement at the hinged point C in upward direction is y.

From geomentry the displacement at point B and E may be calculated as-

$$\frac{y_B}{y} = \frac{AB}{AC}$$

$$\Rightarrow y_B = \frac{AB}{AC} \times y \Rightarrow y_B = \frac{7}{9} y$$

and

$$\frac{y_E}{y} = \frac{ED}{CD}$$

$$\Rightarrow y_E = \frac{ED}{CD} \times y \Rightarrow y_E = \frac{6}{10} y$$

As per principle of virtual work the algebraic sum of the total workdone is zero.

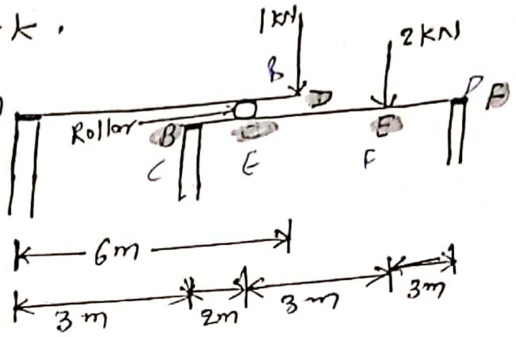
$$\therefore R_A \times 0 + y_B \times R_B - y_E \times 35\cancel{5} + R_D \times 0 = 0$$

$$\Rightarrow y_B \times R_B = y_E \times 350$$

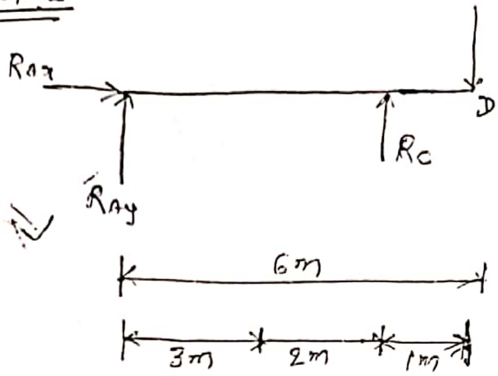
$$\Rightarrow \frac{7}{9} \cancel{y} \times R_B = \frac{6}{1\cancel{0}} \cancel{y} \times 35\cancel{0}$$

$$\Rightarrow \boxed{R_B = \frac{6 \times \cancel{35}^{5} \times 9}{7} = 270\ N}$$

Two beam AB and BF are supported on rollar at shown in fig. Determine the reaction at the rollar B and C using the method of virtual work.



Soln.



First of all consider the beam AB with rollar support at C. Let y be virtual displacement in upward direction at point B then from geometry we get

$$\frac{AD}{AC} = \frac{y}{y_c} \Rightarrow y_c = \frac{AC}{AD} \times y$$

$$\Rightarrow y_c = \frac{5}{6} y$$

The algebraic sum of the total virtual work must be zero

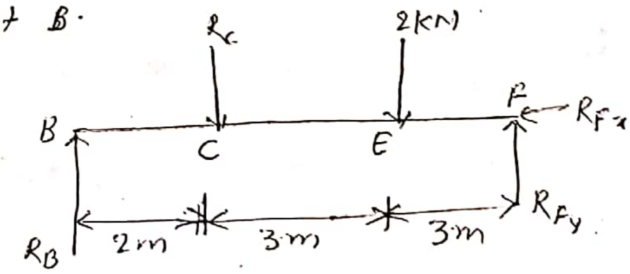
$$\therefore R_A \times 0 + R_c \times y_c - 1 \times y_B = 0$$

$$\Rightarrow R_c \times y_c = 1 \times y_B \qquad \Rightarrow R_c \times \frac{5}{6} \cancel{y} = \cancel{y}$$

$$\Rightarrow R_c = \frac{6}{5} = 1.2 \text{ kN} \qquad \Rightarrow R_c = \frac{6}{5} \text{ kN}$$

Now consider the beam BF, Let y be the virtual upward displacement at point B.

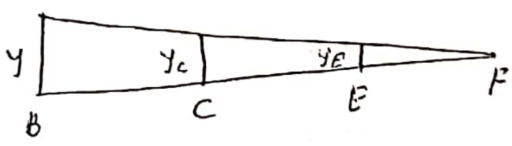
$\therefore$ from geometry



$$\frac{y_c}{y} = \frac{CF}{BF} \Rightarrow y_c = \frac{CF}{BF} \times y$$

$$\Rightarrow y_c = \frac{\cancel{6}^{3}}{\cancel{8}_{4}} y$$

$$\Rightarrow y_c = \frac{3}{4} y$$



Scanned by CamScanner

and $\frac{y_E}{y} = \frac{EF}{BF}$

$\Rightarrow y_E = \frac{EF}{BF} \times y$

$\Rightarrow y_E = \frac{3}{8} y$

The algebraic sum of the total virtual work is zero.

$R_B \times y - R_C \times y_C - 2 \times y_E + R_F \times 0 = 0$

$\Rightarrow R_B \times y = R_C \times \frac{\cancel{6}}{8} y + \cancel{2} \times \frac{3}{\cancel{8}_4} y$

$\Rightarrow R_B \times \cancel{y} = \left(R_C \times \frac{\cancel{6}3}{\cancel{8}4} + \frac{3}{4}\right) \cancel{y}$

$\Rightarrow R_B = \frac{3}{4}(R_C + 1)$

$\Rightarrow R_B = \frac{3}{4} \times (1.2 + 1)$

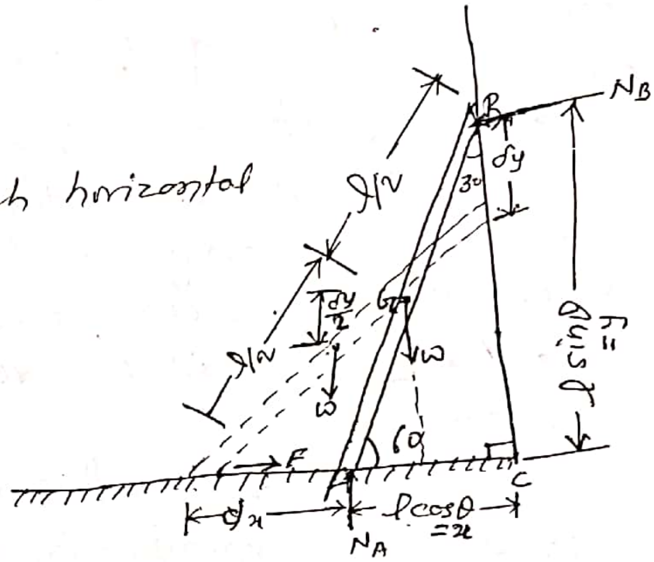
$\Rightarrow R_B = \frac{3 \times 2.2}{4} = 1.65$

Q. A uniform ladder of wt. of 600N rest against a smooth vertical wall and a rough horizontal floor making an angle of 60° with the horizontal. Determine the force of friction at the floor using method of virtual work.

Soln:—

Let, L → Length of the ladder.

f_a → Friction force at A.



Let, virtual displacement on rough horizontal floor is dx

$$x = l\cos\theta$$

virtual displacement

$$dx = -l\sin\theta \cdot d\theta$$

Height of 'G' above base line AC

$\Rightarrow y = AG\sin\theta \Rightarrow y = \frac{l}{2}\sin\theta$

Virtual displacement at wall

$$dy = \frac{l}{2}\cos\theta \cdot d\theta$$

There is also some virtual displacement at centre of gravity of uniform ladder.

$$dy = \frac{l}{2}\cos\theta \, d\theta$$

According to the principle of virtual work

$\underbrace{F_a \times dx + W \times dy}_{\text{Total workdone}} = 0$

$F_a dx$ → workdone by friction force

$W dy$ → workdone by the wt. of ladder.

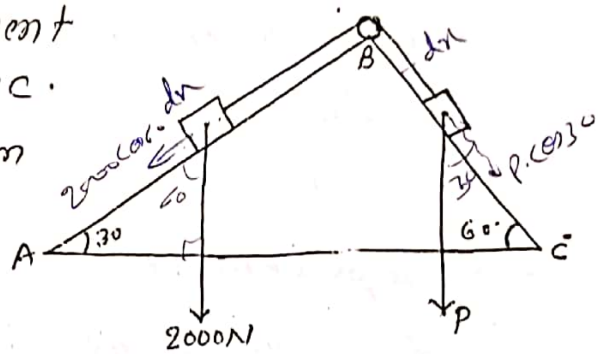
$$\Rightarrow F_a(-l\sin\theta \cdot d\theta) + W\left(\frac{l}{2}\cos\theta \cdot d\theta\right) = 0$$

$$\Rightarrow F_a = \frac{\frac{W}{2}\cancel{l}\cos\theta \cdot \cancel{d\theta}}{\cancel{l}\sin\theta \cdot \cancel{d\theta}} \Rightarrow F_a = \frac{W}{2}\cot\theta$$

$$\Rightarrow F_a = \frac{600}{2} \times \cot 60° = \boxed{173.2\ N = F_a}$$

Q. A block of wt. 2000N rests on a smooth inclined plane that makes an angle of 30° with the horizontal. The block is supported by load P lying on another smooth plane of inclination 60° as shown in fig. The block and the load have been connected by an inelastic string. Determine the value of load P by using the method of virtual work.

Soln:- → small displacement "δx" of 2000N parallel to plane AB equals the displacement δy of load P along the plane BC.



→ When the load 2000N moves down the plane, the load P moves up the plane.

→ The resolved parts of 2000N parallel and perpendicular to plane BC are "P sin 60°" and "P cos 60°"

∴ Total virtual work $= (2000 \times \cancel{\sin 30°}\ \cos 60) \times dx - (P\ \cancel{\sin 60}\ \cos 30 \times dy)$

−ve sign because of movement of the two loads are in opposite directions.

∴ From the principle of virtual work

$$2000 \sin 30 \times dx - P \sin 60 \times dy = 0$$

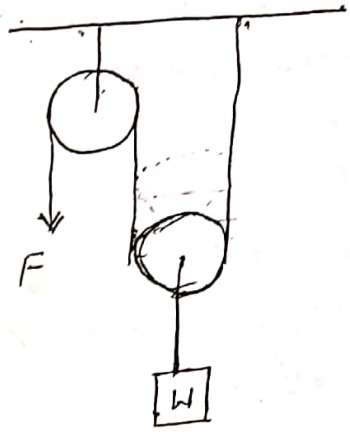
$$\Rightarrow 2000 \sin 30 \times \cancel{dx} = P \sin 60 \times \cancel{dy} \qquad \boxed{dx = dy}$$

$$\Rightarrow P = 2000 \times \frac{\sin 30}{\sin 60} = 2000 \times \frac{0.5}{0.866}$$

$$\Rightarrow \boxed{P = 1154.73\ N}$$

3. A wt. of 10 KN is raised by two pulley system as shown in fig. Determine the force F required to hold the weight in equilibrium.

Soln:-



Assume that F goes down through a distance y. From the geometry of fig. it may be easily seen that weight moves upward by $\frac{1}{2}y$ distance.

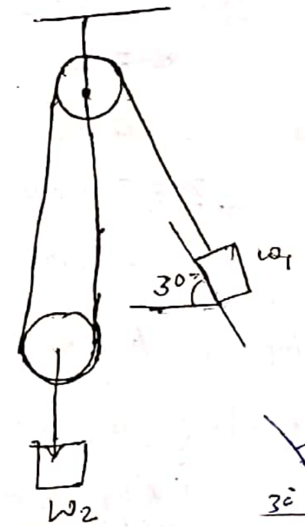
Using the principal of virtual work

$$F \times y - W \times \frac{y}{2} = 0$$

$$\Rightarrow F \times \cancel{y} = W \times \frac{\cancel{y}}{2}$$

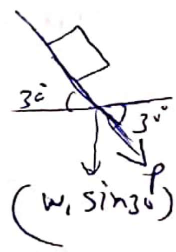
$$\Rightarrow F = \frac{W}{2} = \frac{10}{2} = 5\,KN$$

4. A block of $W_1 = 6$ KN rests on the smooth surface inclined at $\theta = 30°$ with the horizontal. The block is supported by an weight W_2 hung from a pulley as shown in fig. Using the principal of virtual work determine the required W_2 for equilibrium condition.



Soln:- Assume that a virtual displacement y is given to block W_1 in the direction of inclined plane. From fig. it may be easily seen that displacement of wt. W_2 is $y/2$ in upwar direction.

The displacement of block in vertical directions is $y\sin\theta$ in downward direction.



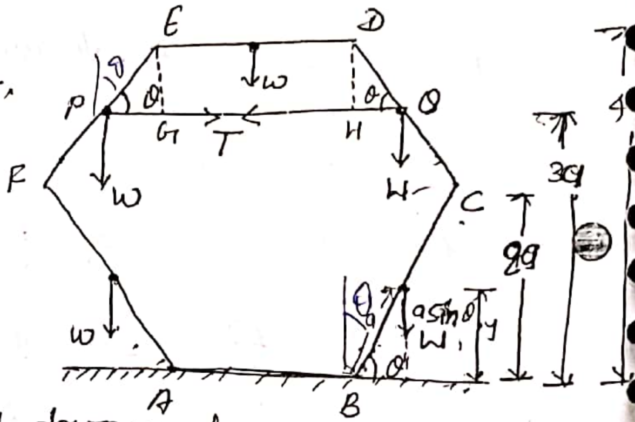
By using the principal of virtual work

$$W_2 \times \frac{y}{2} - W_1 \times y\sin\theta = 0$$

$$\Rightarrow W_2 \times \frac{\cancel{y}}{2} = W_1 \times \cancel{y}\sin\theta$$

$$\Rightarrow W_2 = 2W_1\sin\theta = 2 \times 6 \times \sin 30 = \cancel{2} \times 6 \times \frac{1}{\cancel{2}} = 6\,KN$$

Q: A Hexagonal frame ABCDE is made up of six bars of equal length and weight. One of the bar is fixed on a horizontal plane and the system lies in the vertical plane as shown in fig. The mid point of the two upper non-horizontal bars are connected by a string PQ. Using the principal of virtual work show the tension in the string is "$6W\cot\theta$" where 'w' is the wt. of each bar and θ is the inclination of the inclined bars with the horizontal.



Soln:- Let "2a" be the length of each bar.

Length of string (PQ) = PG + GH + HQ

$= a\cos\theta + 2a + a\cos\theta$

$= 2a + 2a\cos\theta$

differentiating

$d_{PQ} = -2a\sin\theta \cdot d\theta$

The wt. of each bar acts vertically downwards at its mid point. Let x, y, z and h be the heights of the mid points of AB, BC and AF, CD and FE, and ED as measured from line AB.

Then

$x = 0 \qquad dx = 0$

$y = a\sin\theta \qquad dy = a\cos\theta\, d\theta$

$z = 3a\sin\theta \qquad dz = 3a\cos\theta \cdot d\theta$

$h = 4a\sin\theta \qquad dh = 4a\cos\theta \cdot d\theta$

By the principal of virtual work

$$(W\times dx) + (\underbrace{w\times dy}_{\text{w.d. by bar BC}} + \underbrace{w\times dy}_{\text{w.d. by bar AF}}) + (\underbrace{w\times dz}_{\text{w.d. by bar CD}} + \underbrace{w\times dz}_{\text{w.d. by bar FE}}) + (w\times dh) + (T\times d_{PQ}) = 0$$

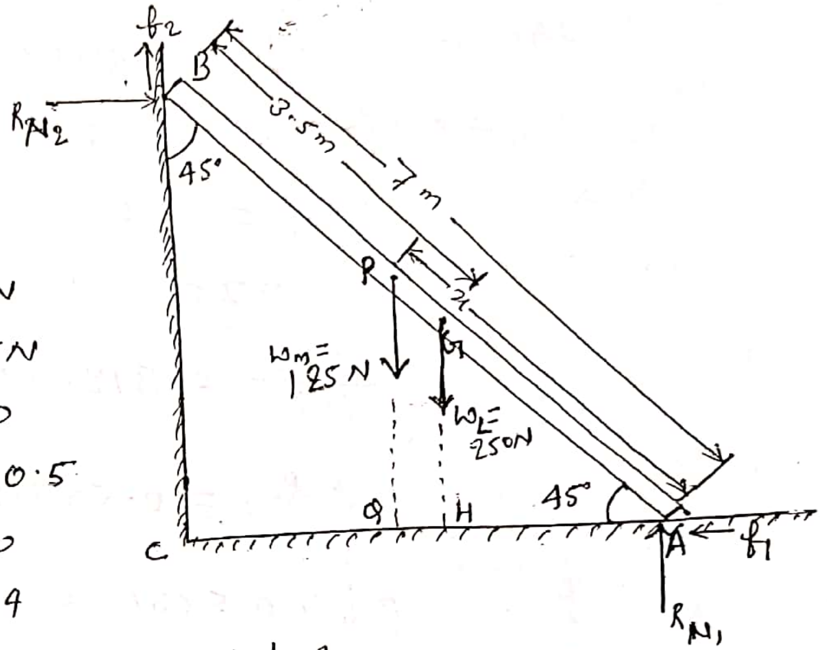
$\Rightarrow w\times 0 + (2w\times a\cos\theta \cdot d\theta) + (2w\times 3a\cos\theta \cdot d\theta) + (w\times 4a\cos\theta\, d\theta) + T\times(-2a\sin\theta\, d\theta) = 0$

$\Rightarrow 2wa\cos\theta d\theta + 6wa\cos\theta\, d\theta + 4wa\cos\theta\, d\theta = T\times 2a\sin\theta d\theta$

$\Rightarrow \overset{6}{\cancel{12}} w \cancel{a}\cos\theta \cdot \cancel{d\theta} = T\times \cancel{2a}\sin\theta\, \cancel{d\theta}$

$\Rightarrow T = \dfrac{6w\cos\theta}{\sin\theta} \Rightarrow \boxed{T = 6w\cot\theta}$

Q. A Ladder of 7m long 250N weight rests against a vertical wall with which it makes an angle of 45°, the co-efficient of friction b/w the ladder and the wall is 0.4 and that b/w ladder and the floor 0.5. If a man whose weight is one half of that of the ladder ascends it, Determine at what position will be induce sliping.



Soln:-

Given:-

AB = 7 m

$\angle ABC = 45°$, $\angle BAC = 45°$

weight of ladder (W_L) = 250N

weight of man (W_m) = 125N

Co-efficient of friction b/w ladder and floor (μ_1) = 0.5

coefficient of friction b/w ladder and wall (μ_2) = 0.4

Let, x = Distance b/w a man and point A.

G is the mid point of the ladder at which the wt. 250N is acting.

The man of wt 125N is standing at point P which is at a distance x from A.

$F_1 = \mu_1 R_{N_1}$ where, $R_{N_1} \rightarrow$ Normal reaction at A.

$F_2 = \mu_2 R_{N_2}$ where $R_{N_2} \rightarrow$ Normal reaction at B.

Resolving forces horizontally:-

$R_{N_2} - f_1 = 0$

$\Rightarrow R_{N_2} = f_1$

$\Rightarrow R_{N_2} = \mu_1 R_{N_1}$

$\Rightarrow R_{N_2} = 0.5 R_{N_1}$ —— (1)

Resolving forces vertically

$$R_{N_1} + f_2 - 250 - 125 = 0$$

$$\Rightarrow R_{N_1} + \mu_2 R_{N_2} = 375$$

$$\Rightarrow R_{N_1} + 0.4 R_{N_2} = 375 \quad \text{——— (ii)}$$

Put the value of R_{N_2} from eqn (I) in eqn (II) we get -

$$R_{N_1} + 0.4(0.5 R_{N_1}) = 375$$

$$\Rightarrow R_{N_1} + 0.2 R_{N_1} = 375$$

$$\Rightarrow 1.2 R_{N_1} = 375$$

$$\Rightarrow R_{N_1} = \frac{375}{1.2} = 312.5\,N$$

$$\therefore R_{N_2} = 0.5 R_{N_1} = 0.5 \times 312.5 = 156.25\,N$$

$$\therefore f_1 = \mu_1 R_{N_1} = 0.5 \times 312.5 = 156.25\,N$$

$$f_2 = \mu_2 R_{N_2} = 0.4 \times 156.25 = 62.5\,N$$

In ΔAGH

$$\cos 45^\circ = \frac{AH}{AG} \Rightarrow AH = AG \cos 45^\circ = 3.5 \times \frac{1}{\sqrt{2}} = 2.47\,m$$

In ΔAPQ

$$\cos 45^\circ = \frac{AQ}{AP} \Rightarrow AQ = AP \cos 45^\circ = x \times \frac{1}{\sqrt{2}}$$

Now taking moments about point A.

$$\Sigma M_A = 0$$

$$(125 \times AQ) + (250 \times AH) - (R_2 \times BC) - (F_2 \times AC) = 0$$

$$\Rightarrow \left(125 \times x \times \frac{1}{\sqrt{2}}\right) + (250 \times 2.47) - (156.25 \times 7 \sin 45^\circ) - (62.5 \times 7 \cos 45) = 0$$

$$\Rightarrow 88.38x + 617.5 - 773.39 - 309.35 = 0$$

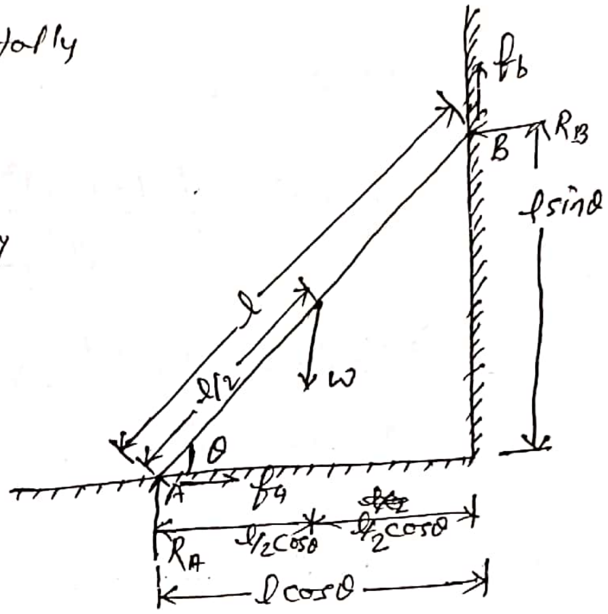
$$\Rightarrow 88.38x = 465.24$$

$$\Rightarrow x = 5.26\,m$$

Q: Determine the minimum angle θ at which a uniform ladder can be placed against a wall without sliping under its own weight. The co-efficient of friction for all surfaces is 0.2.

Soln:-

Resolving forces along horizontally



$R_b = f_a$

$\Rightarrow R_b = \mu R_a$

Resolving forces along vertically

$R_a + f_b = W$

$\Rightarrow R_a + \mu R_b = W$

$\Rightarrow R_a + \mu(\mu R_a) = W$

$\Rightarrow R_a + \mu^2 R_a = W$

$\Rightarrow R_a(1+\mu^2) = W$

$\Rightarrow R_a = \frac{W}{(1+\mu^2)}$

Taking moments about end B.

$R_a \times l\cos\theta = W \times \frac{l}{2}\cos\theta + f_a \times l\sin\theta$

$\Rightarrow \frac{W}{(1+\mu^2)} l\cos\theta = \frac{Wl}{2}\cos\theta + \mu R_a\, l\sin\theta$

$\Rightarrow \frac{Wl}{(1+\mu^2)}\cos\theta = \frac{Wl}{2}\cos\theta + \frac{\mu W l}{(1+\mu^2)}\sin\theta$

$\Rightarrow Wl\left[\frac{1}{1+\mu^2} - \frac{1}{2}\right]\cos\theta = \frac{\mu Wl}{1+\mu^2}\sin\theta$

$\Rightarrow \frac{2-(1+\mu^2)}{2(1+\mu^2)}\cos\theta = \frac{\mu}{(1+\mu^2)}\sin\theta$

$\Rightarrow \frac{(1-\mu^2)}{2(1+\mu^2)}\cos\theta = \frac{\mu}{1+\mu^2}\sin\theta$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\frac{1-\mu^2}{2(1+\mu^2)}}{\frac{\mu}{(1+\mu^2)}}$$

$$\Rightarrow \tan\theta = \frac{1-\mu^2}{2\cancel{(1+\mu^2)}} \times \frac{\cancel{(1+\mu^2)}}{\mu}$$

$$\Rightarrow \tan\theta = \frac{1-\mu^2}{2\mu}$$

$$\Rightarrow \tan\theta = \left[\frac{1-(0.2)^2}{2\times 0.2}\right]$$

$$\Rightarrow \theta = \tan^{-1} \qquad = 67.38^\circ$$