JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE	**JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE** Shri Ram ki Nangal, via Sitapura RIICO Jaipur- 302 022.	**Academic year 2020-21**

Department of Mathematics

Question Bank

Academic Year – 2020-21

Subject: Engineering Mathematics-1

Course Outcomes	
CO1	Understand fundamental concepts of improper integrals, beta and gamma functions and their properties. Evaluation of Multiple Integrals in finding the areas, volume enclosed byseveral curves after its tracing and its application in proving certain theorems.
CO2	Interpret the concept of a series as the sum of a sequence, and use the sequence of partialsums to determine convergence of a series. Understand derivatives of power, trigonometric, exponential, hyperbolic, logarithmic series.
CO3	Recognize odd, even and periodic function and express them in Fourier series using Euler's formulae.
CO4	Understand the concept of limits, continuity and differentiability of functions of several variables. Analytical definition of partial derivative. Maxima and minima of functions of several variables Define gradient, divergence and curl of scalar and vector functions.

BETA –GAMMA FUNCTION

Q.1 Find the value of $\int_0^{\frac{\pi}{2}} Sin^6\theta Cos^7\theta \, d\theta$.

Q.2Prove that $\Gamma n\Gamma(1-n) = \frac{\pi}{\text{Sin}n\pi}$ $0 < n < 1$

Q.3 Prove that $B(m,n) = \frac{\Gamma m\Gamma n}{\Gamma(m+n)}$ $, m > 0, n > 0$

OR

Show that $B(m,n) = a^m b^n \int_0^\infty \frac{x^{m-1}}{(ax+b)^{m+n}} = \Gamma m \Gamma n / \Gamma(m+n)$

OR

Show that $B(m,n) = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\, dy$

Q.4 Show that $\int_0^2 (8-x^3)^{-\frac{1}{3}} dx = \frac{2\pi}{3\sqrt{3}}$.

Q.5 Evaluate $\int_0^\infty \frac{\mathbf{1}}{\mathbf{1+x^4}} dx$

Q.6 Show that $\int_0^1 \sqrt{1-x^4}\, dx = \frac{\left(\sqrt{\frac{1}{4}}\right)^2}{6\sqrt{2\pi}}$

Q.7 Prove that $\int_0^\infty \frac{x^2}{(1+x^4)^3} dx = \frac{5\pi}{128}$

Partial Differentiation

Q1. If e^{xyz}, than show that

$$\frac{\delta^3 u}{\delta x \delta y \delta z} = (1+3xyz+x^2y^2z^2),$$

Q2. $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} + \frac{\delta^2 u}{\delta z^2} = -\frac{1}{3(x+y+z)^2}$,

Q3. If u=x $f(\frac{y}{x}) + g\left(\frac{y}{x}\right), prove\ that$

$X^2 \frac{\delta^2 u}{\delta x^2} + 2xy \frac{\delta^2 u}{\delta x \delta y} + y^2 \frac{\delta^2 u}{\delta y^2} = 0,$

Q4. If $f(x,y) = \begin{cases} \frac{x^3-y^3}{x^3+y^3}, & when\ x \neq 0, y \neq 0 \\ 0 & when\ x = 0, y = 0, \end{cases}$ then discuss the continuity of f(x, y)at the origin.

Q5. Find the equation of tangent plane and the normal line to the surface $x^2+2y^2+3z^2=12$ at P(1,2,-1).

Q6. $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = \frac{\delta^2 u}{\delta \eta^2} + \frac{\delta^2 u}{\delta \xi^2}$, Where x=ξ cosα- ηsinα, y=ξsinα + ηcosα.

Q7.Find the values of a, b and c such that

A= (x+2y+az) $\hat{\imath}$ +(bx-3y-z) $\hat{\jmath}$ +(4x+cy+2z) k is irrotational vector field. Also, find its scalar potential.

Q8.If $\overrightarrow{F}$ is a solenoid vector, show that

Curl curl curl curl $\overrightarrow{F}$ = $\nabla^2(\nabla^2\overrightarrow{F})$= $\nabla^4\overrightarrow{F}$

Q9. If u= f(x,y), where x=rcosΘ and y =rsinΘ. Prove that the equation $\frac{\delta^2 u}{\delta x^2}$ + $\frac{\delta^2 u}{\delta y^2}$ =0, transformed into $\frac{\delta^2 u}{\delta r^2}$ + $\frac{1}{r}\frac{\delta u}{\delta r}$ $+\frac{1}{r^2}$ + $\frac{\delta^2 u}{\delta \Theta^2}$=0.

Q10. If u=Sin-1 $\left(\frac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}\right)$prove that x $\frac{\delta u}{\delta x}$ + y $\frac{\delta u}{\delta y}$=$\frac{1}{20}\tan u$

SURFACES AND VOLUMES OF REVOLUTION

Q.1 The part of the parabola $y^2 = 4ax$ cut off by the latus rectum revolves about the tangent at the vertex. Find the surface area and volume of the reel thus generated.

Q.2 Find the volume of the solid generated by revolution of the curve $(a-x)y^2 = a^2x$ about Its asymptote.

Q.3 Find the volume of the solid generated by revolution of the arc of the Cycloid $x = a(\theta - sin\theta), y = a(1 - cos\theta)$ about Its base.

Q.4 Find the surface area and the volume of the spindle shaped solid generated by revolution Of curve astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about x axis.

Q.5 Prove that the surface area and volume of the solid generated by revolution of the loop Of the curve $x = t^2\,, y = \left(t - \frac{t^3}{3}\right)$ about the X axes are 3π and $\frac{3\pi}{4}$.

Q.6 Find the volume of the solid generated by revolution of the Cissoid $y^2(2a - x) = x^3$ About its asymptote.

Q.7 Find the volume and surface area of the solid generated by revolution of the curve cardioid $r = a(1 + cos\theta)$ about the initial line and about the line $\theta = \frac{\pi}{2}$.

Double and Tripple Integral

Q.1 The Cardioid $r = a(1 + cos\theta)$ revolves about the initial line. Find the volume of solid generated.

Q.2 Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 3$ and z=0.

Q.3 Evaluate $\int_0^1 \int_0^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dx\, dy$ by changing the order of Integration.

Q.4 Find the surface area of the solid generated by the revolution of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis.

Q.5 Find by Double Integration the area of the Region enclosed by $x^2 + y^2 = a^2$ and $x + y = a$ (In the First Quadrant)

Q.6 Find the centre of Gravity of the arc of the curve $x = asin^3\theta, y = acos^3\theta$ lying in the first Quadrant.

Q.7 $\int_0^{log2} \int_0^{x} \int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz$

Q.8 Evaluate $\iiint \frac{dx\, dy\, dz}{x^2+y^2+z^2}$ throughout the volume of the sphere $x^2+y^2 + z^2 = a^2$.

Q.9 Use Green's Theorem in a Plane to evaluate $\oint_C (2xy - y)dx + (x + y)dy$Where C is the boundary of the circle $x^2 + y^2 = a^2$ in the XY-Plane.

Q.10 Evaluate $\oint_C \vec{F}.d\vec{r}$ by Stoke's Theorem , Where $\vec{F} = y^2\hat{\imath} + x^2\hat{\jmath} - (x + z)\hat{k}$ and C is the boundary of the triangle with vertices at (0,0,0),(1,0,0) and (1,1,0).

Q.11 Verify Divergence Theorem for the function $\vec{F} = y\hat{\imath} + x\hat{\jmath} + z^2\hat{k}$ over the cylindrical region bounded by $x^2 + y^2 = a^2, z = 0\ and\ z = h.$

Q.12 Using triple integration find the volume bounded by the coordinates planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Q.13 Use Stoke's theorem to evaluate $\int_C \vec{F}.d\vec{r}\ where\ \vec{F} = (\sin x - y)\hat{\imath} - \cos x\hat{\jmath}$ and C is the boundary of the triangle whose vertices are (0,0) , (π/2, 0) and (π/2 , 1).

Fourier Series

Q 1 (1.1) Define Fourier series f(x) in the interval$[-\pi, \pi]$. State Dirichlet's conditions for convergence of Fourier series f(x).

(1.2) Write Dirichlet's conditions for Fourier expansion of a function.

(1.3) Define (Write about)even and odd function with examples.

(1.4) The value of integral $\int_{\alpha}^{\alpha+2\pi} cos^2 nx\, dx = \cdots$

(1.5) Explain Dirichlet's condition for any function f(x) developed as a Fourier series.

(1.6) Define a Fourier series.

(1.7) State Euler's formulae.

(1.8) Fourier expansion of an odd function has only.............. terms.

(1.9) The function $f(x) = \begin{cases} 1 - x \; in - \pi < x < 0 \\ 1 + x \; in \quad 0 < x < \pi \end{cases}$ is an odd function. Is the above function is true or false?

(1.10) Using sine series for $f(x) = 1 \; in \; 0 < x < \pi$, find the value of $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots \infty = \cdots$

(1.11) Determine the Fourier coefficient of a_0 in the Fourier series expansion.

Q 2

(2.1) Find a series of sines and cosines of multiples of x which will represent the function $f(x) = x + x^2$ in the interval $-\pi < x < \pi$. Hence show that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$

(2.2) Find the Fourier series of the function $f(x)$, where $f(x) = \begin{cases} -1, -1 < x < 0 \\ 2x, \; 0 < x < 1 \end{cases}$ and $f(x + 2) = f(x)$

(2.3) An alternating current after passing through a rectifier has the force $I = \begin{cases} I_0 sin\, x, 0 < x < \pi \\ 0, \quad \pi < x < 2\pi \end{cases}$ where I_0 is the maximum current and the period is 2π. Express I as a Fourier series.

(2.4) Expand the function $f(x) = x \sin x$ as a Fourier series in $-\pi \le x \le \pi$. Deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots \infty = \frac{\pi - 2}{4}$

(2.5) Obtain a Fourier series for $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ \pi(2 - x), & 1 < x < 2 \end{cases}$. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty = \frac{\pi^2}{8}$

(2.6) Obtain Fourier's series in the interval $(-\pi, \pi)$ for the function $f(x) = x \cos x$

(2.7) A sinusoidal voltage $E \sin \omega t$ is passed through a half-wave rectifier which clips the negative portion of the wave. Develop the resulting periodic function $V(t) = \begin{cases} 0, \; \frac{-T}{2} < t < 0 \\ E \sin wt\,, \; 0 < t < \frac{T}{2} \end{cases}$ and $T = \frac{2\pi}{w}$, in a Fourier series.

(2.8) If $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$ show that $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left\{\frac{1}{1^2} \cos 2x + \frac{1}{3^2} \cos 6x + \frac{1}{5^2} \cos 10x + \cdots\right\}$

(2.9) If $f(x) = \begin{cases} 0, \; -\pi \le x \le 0 \\ sin\, x, \; 0 \le x \le \pi \end{cases}$ then prove that

$f(x)=\frac{1}{\pi}+\frac{\sin x}{2}-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2nx}{4n^2-1}$.Hence show that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\cdots-\infty=\frac{\pi-2}{4}$

(2.10) Obtain the Fourier series for $f(x)=e^x$ in the interval $0< x<2\pi$

(2.11) Find a Fourier series to represent $x-x^2$ from $x=-\pi$ to $x=\pi$

(2.12) Find the Fourier series expansion for $f(x)$if $(x)=\begin{cases}-\pi, -\pi<x<0\\ x,\ 0<x<\pi\end{cases}$. Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\infty=\frac{\pi^2}{8}$

(2.13) Expand $f(x)=x\sin x$ in the range $0< x<2\pi$ as a Fourier series

(2.14) Expand $f(x)=\sqrt{1-\cos x}, 0<x<2\pi$ in a Fourier series. Hence evaluate $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\cdots$

Q 3

(3.1) Obtain a half range cosine series for $f(x)=\begin{cases}kx,\ 0\le x\le\frac{l}{2}\\ k(l-x),\ \frac{l}{2}\le x\le l\end{cases}$ and deduce that $\frac{\pi^2}{8}=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$

(3.2) Obtain a half-range sine series for $f(x)=\begin{cases}\frac{1}{4}-x,\ when 0<x<\frac{1}{2}\\ x-\frac{3}{4},\ when\ \frac{1}{2}<x<1\end{cases}$

(3.3) Obtain the Fourier sine series for $f(x)=e^x$ for $0<x<1$

(3.4) Express $f(x)=x$ as a half range cosine series in $0<x<2$

(3.5) Given $f(x)=\begin{cases}1-x, -\pi\le x\le 0\\ 1+x,\ \ 0\le x\le\pi\end{cases}$

Is the function even or odd? Find the Fourier series for $f(x)$ and deduce the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\infty$

(3.6) If the function $f(x)$ is defined by $f(x)=|x|, -\pi<x<\pi$. Obtain a Fourier series of $f(x)$.Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots=\frac{\pi^2}{8}$

(3.7) Obtain a Fourier series for the function f(x) given by $f(x)=\begin{cases}1+\frac{2x}{\pi}, -\pi\le x\le 0\\ 1-\frac{2x}{\pi},\ \ 0\le x\le\pi\end{cases}$

Deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{8}$

(3.8) If $f(x)=|\cos x|$,expand $f(x)$ as a Fourier series in the interval $(-\pi,\pi)$

(3.9) If $f(x)=|\sin x|$,expand $f(x)$ as a Fourier series in the interval $(-\pi,\pi)$

(3.10) Find a Fourier series to represent x^2 in the interval $(-l,l)$

Sequence and Series

Q.1 Test the Series for Convergence : $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \cdots \infty$

Q.2 Test the Series for Convergence : $\sum_{n=0}^{\infty} \frac{2n^3+5}{4n^5+1}$

Q.3 Test the Series for Convergence : $\sum_{n=0}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$

Q.4 Test the Series for Convergence : $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \cdots$

Q.5 Prove that the series $\frac{sinx}{1^3} - \frac{sin2x}{2^3} + \frac{sin3x}{3^3} - \cdots$ converges absolutely.

Q.6 Discuss the absolute convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

Q.7 For what values of x are the series convergent: $x - \frac{x^2}{2^2} + \frac{x^3}{3^2} - \frac{x^4}{4^2} + \cdots \infty$

Q.8 Test the convergence of $\sum_{n=1}^{\infty} \frac{n}{n+1}$.

Q.9 Test the convergence of the series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ ....

Q.10 Test the convergence of Geometric series $1 + r + r^2 \ldots\ldots + r^n + \cdots$ for |r| <1

Q.11 Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5}$.+......

Q.12 Find the curl of the vector $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$

Q13. Mention the condition for the vector field A to be solenoidal. [EC 2020 gate]

Q14. Find the partial derivative of the function(x,y,z) = $e^{1-x\cos z} + xze^{-1/1+y}$ m
[EC 2020 gate]

Q15. Find the value of $\lim_{x\to\infty} \frac{x^2-5x+4}{4x^2+2x}$. [CE 2020 gate]

Q16. If $f = 2x^3 + 3y^2 + 4z, find \int gradf.dr$ along the path (-3, -3,2) to (2,-3,2) to (2,6,2) to (2,6,-1). [EE 2019 gate]

Q17. Find the value of line integral $\int_1^2 2xy^2dx + 2yx^2dy + dz$along a path joining the origin (0,0,0) and the point (1,1,1). **[GATE EE 2016 Set 2]**

Q18. Find the line integral of function F=yz iin the counterclockwise direction, along the circle$x^2 + y^2 = 1$ at z=1, **[GATE EE 2014 Set 1]**

Q19. Compute $\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$, [**GATE CSE 2019]**