Unit 3

"Image Restoration"

Syllabus
- Image degradation and restoration process
- Noise Models, Noise filters, degradation func, Inverse filtering
- Homomorphism filtering.

- The goal of restoration techniques is to improve an image in some predefined sense.

- Restoration (recover an image) attempts to recover an image — that has been degraded by using a prior knowledge — of the degradation phenomenon.

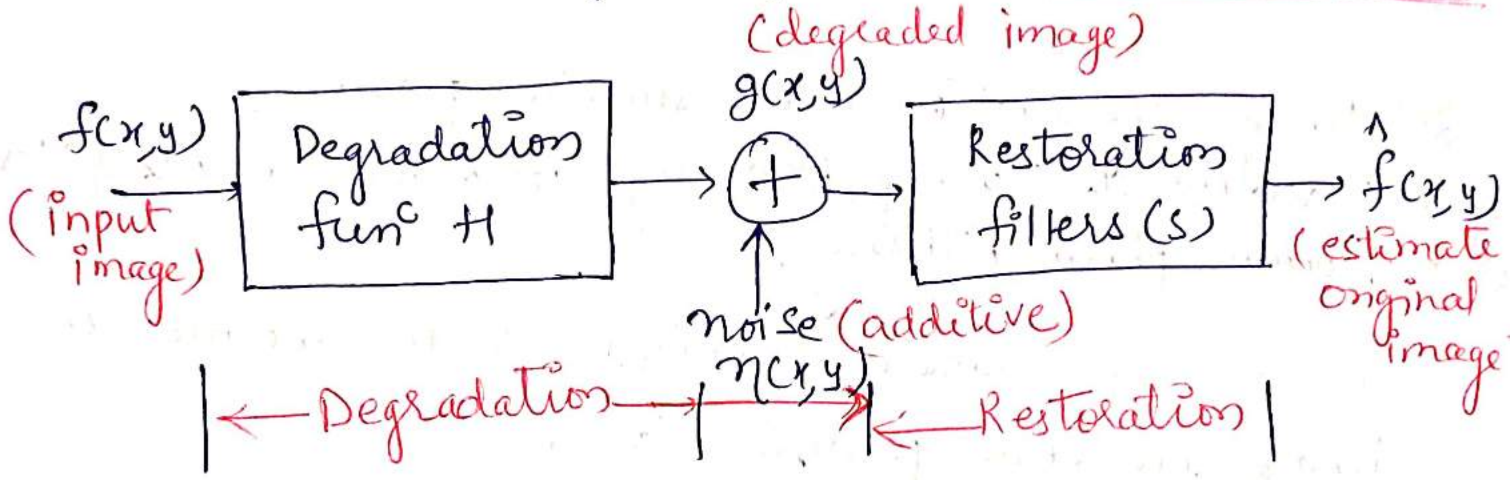
- Restoration techniques are oriented towards modeling the degradation and applying the inverse process in order to recover the original image.

- Restoration techniques are best formulated in

Spatial domain
- when degradation is additive noise
- difficult for blur image using small filter mask.

Frequency domain
- when image blur

A model of Image Degradation/Restoration Process



If H is linear, position-invariant process, degraded image in spatial domain

$$g(x,y) = \underbrace{h(x,y) \ast f(x,y)}_{\text{convolution}} + \eta(x,y) \quad \text{--- (1)}$$

- Convolution in spatial domain is multiplication in freq. domain.
- So in frequency domain eq (1) will be

$$G(u,v) = H(u,v) \cdot F(u,v) + N(u,v) \quad \text{--- (2)}$$

Noise Model :-

- Principal source of noise in digital images arise during → image acquisition / transmission.

eg.- imaging sensor $\xrightarrow[\text{by}]{\text{affected}}$ enviorment condition, senseing element, Light level, sensor temprature

Spatial & Frequency Properties of Noise :→

- Noise is independent of spatial coordinates, no correlation b/w pixel values and values of noise components.
- frequency properties – freq content of noise in fourier sense / fourier transform
  - it is called white noise
  - white light contain all freq in visible spectrum.
- PDF (Probability Distribution func) define the distribution of noise in an image.
- Some common PDFs found in image application

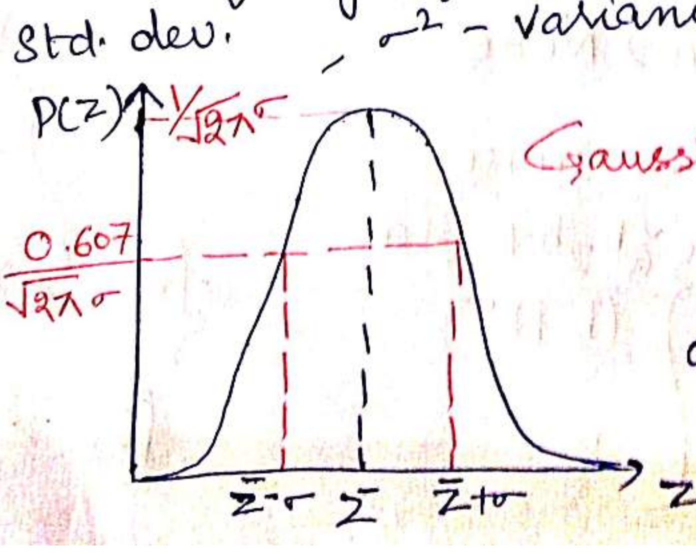
(1) Gaussian Noise [Normal noise model]

PDF of Gaussian random variable, Z / most commonly used due to easy math

$$P(Z) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-(Z-\bar{Z})^2/2\sigma^2}$$

– Guassian noise arise due to electronic circuit noise in camera & poor illumination on high temp.

Z – gray level / intensity,
$\bar{Z}$ – mean of avg of Z values.
σ – std. dev. σ^2 – variance of Z.



70% value in range $[(\bar{Z}-\sigma), (\bar{Z}+\sigma)]$

95% value in range $[(\bar{Z}-2\sigma), (\bar{Z}+2\sigma)]$

2. Rayleigh Noise

PDF of Rayleigh noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

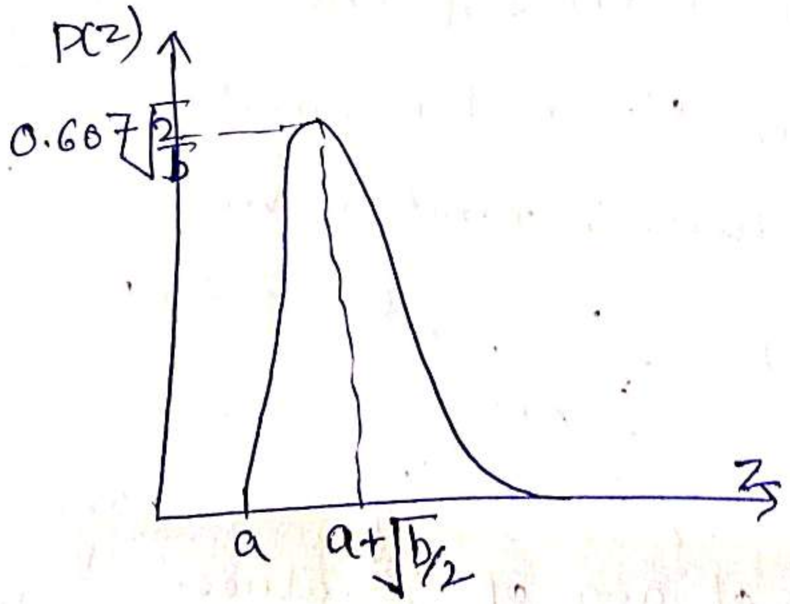
a, b → non negative no.

mean $\bar{z} = a + \sqrt{\pi b / 4}$

variance $\sigma^2 = \frac{b(4-\pi)}{4}$

- Useful in range-imaging
range-imaging is used in digital camera.

→ useful for skewed histogram



3. Erlang (gamma) Noise

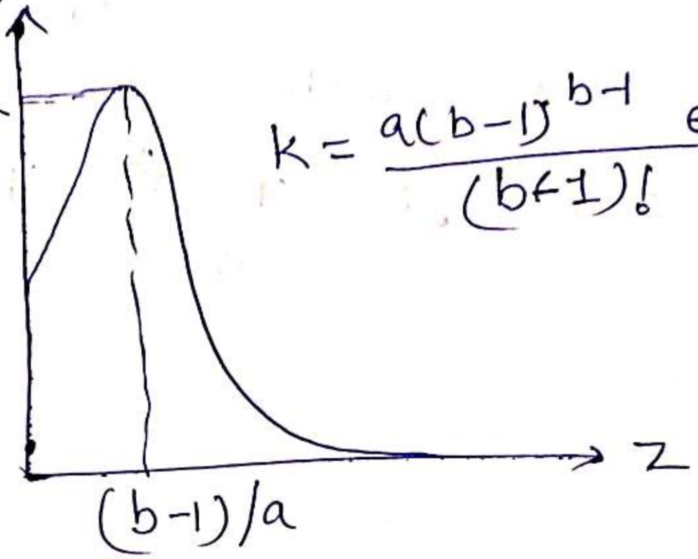
PDF of Erlang noise

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

a, b > 0 positive integer
! - factorial

mean $\bar{z} = b/a$

variance $\sigma^2 = b/a^2$



$$K = \frac{a(b-1)^{b-1}}{(b-1)!} e^{-(b-1)}$$

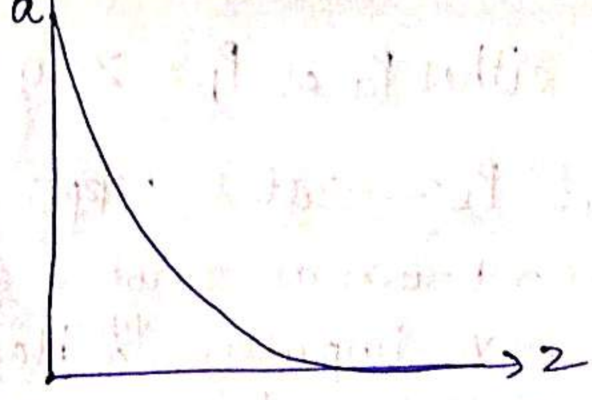
- It is called gamma noise if denominator is $\Gamma(b)$ only.

④ Exponential Noise :-

Erlang & Exponential find the application in lazer imaging

PDF of exponential noise

$$p(z) = \begin{cases} a e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$



a > 0,

Mean $\bar{z} = \frac{1}{a}$

variance $\sigma^2 = \frac{1}{a^2}$

- It is a special case of Erlang PDF (if b = 1)

- Useful in random no. generation, in simulation.

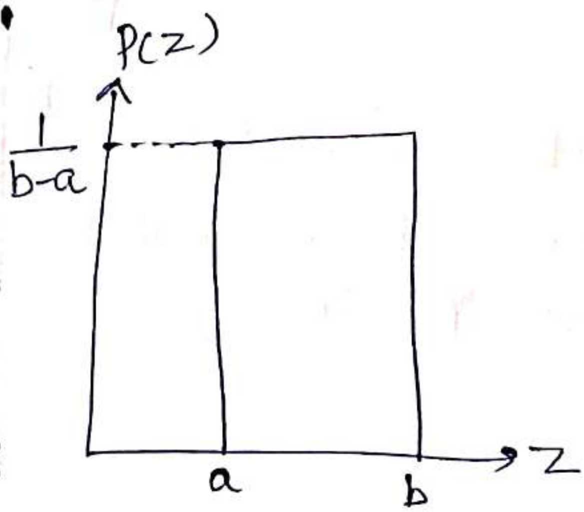
⑤ Uniform Noise :→

The PDF of uniform

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b, \\ 0 & \text{otherwise} \end{cases}$$

mean $\bar{z} = \frac{a+b}{2}$

variance $\sigma^2 = \frac{(b-a)^2}{12}$



⑥ Impulse (Salt & Pepper) Noise :-

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

- If $b > a$, intensity $b \rightarrow$ appears as a light dot in imag. $a \rightarrow$ dark dot.

- If either P_a or P_b = zero, called Unipolar

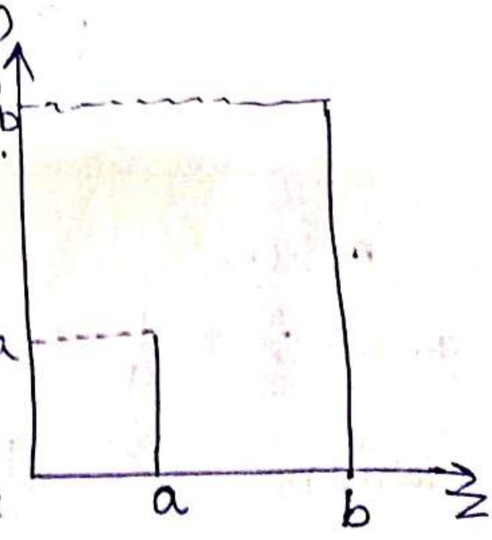
- $P_a \cong P_b \rightarrow$ salt & pepper granules

- also known as shot & spice noise

  -ve impulses appear as black point (pepper)

  +ve impulses ⟶ white point (salt)

- 8-bit image $a = 0$ (black) $b = 255$ (white)



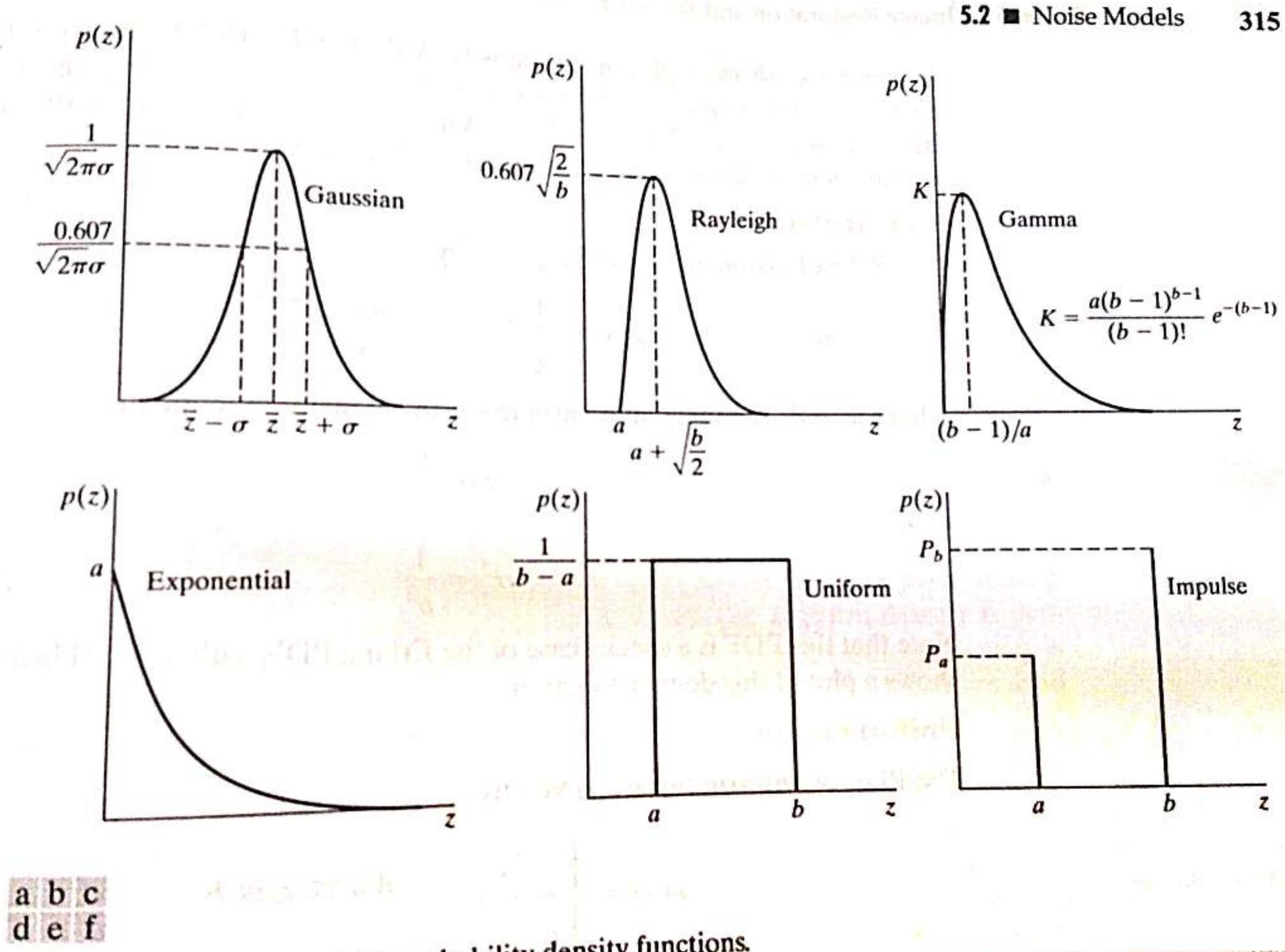


FIGURE 5.2 Some important probability density functions.

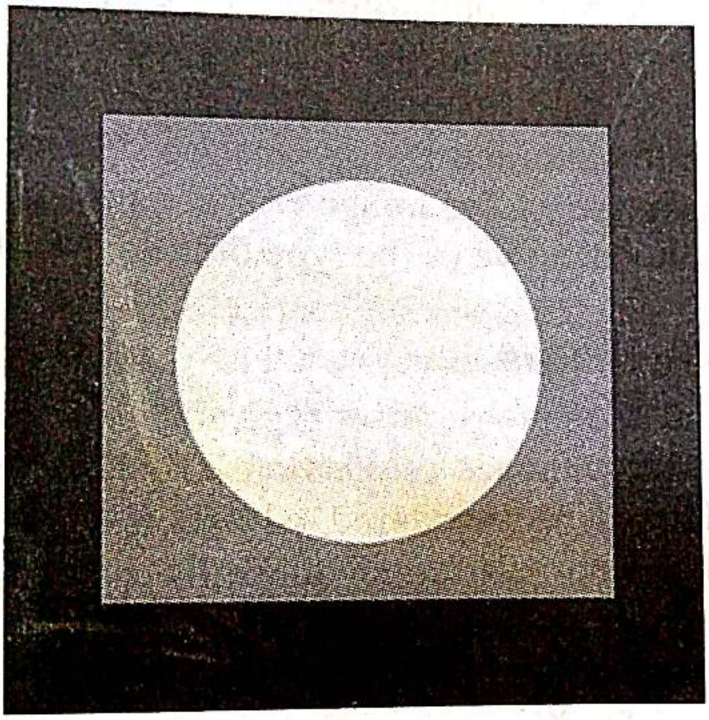
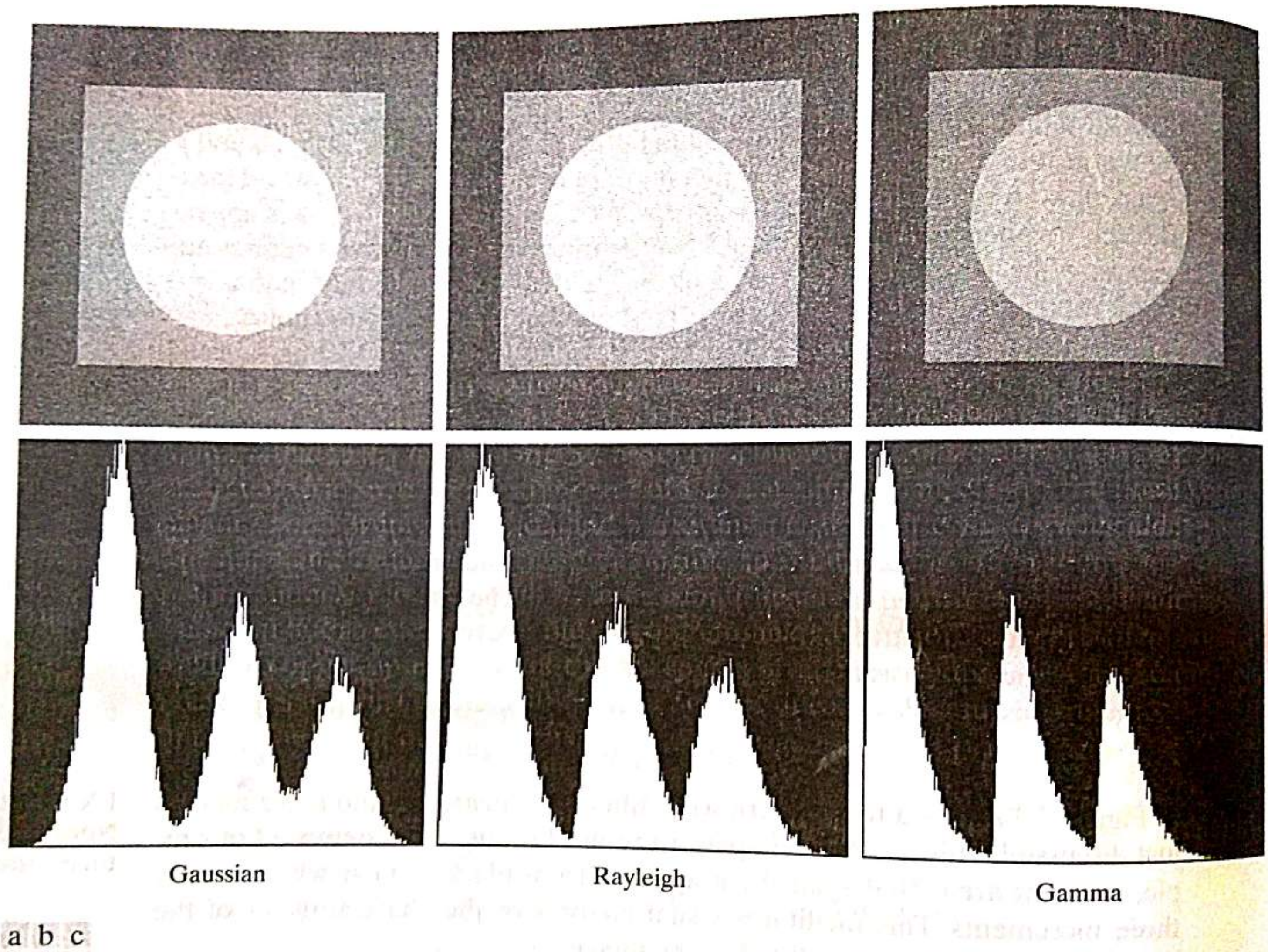
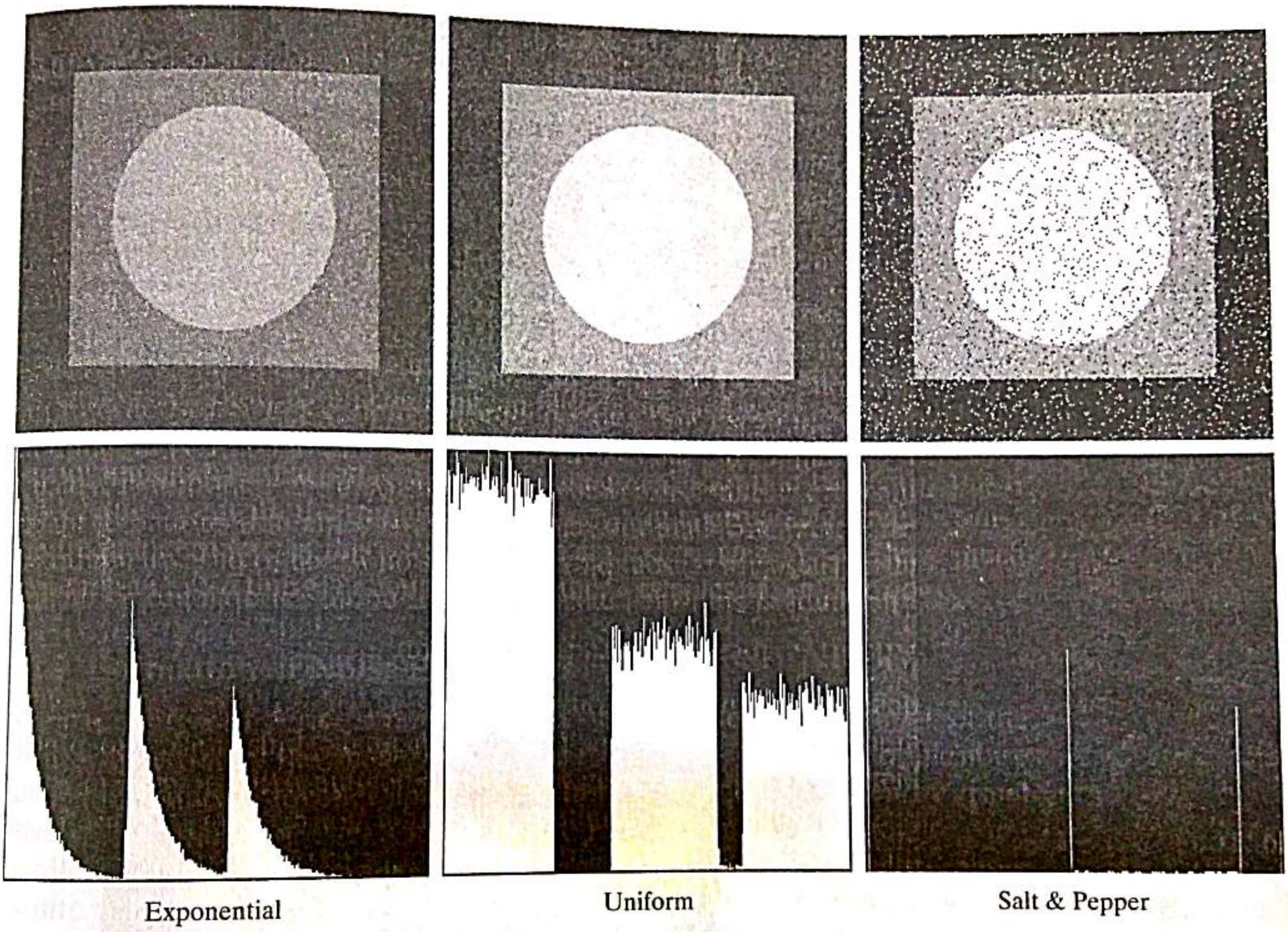


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.



a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.



g h i
j k l

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt-and-pepper noise to the image in Fig. 5.3.

Periodic Noise :-

Periodic noise arises in image during image acquisition due to ~~efficit~~ electrical or electromechanical interferences.

- It is spatially dependent noise

→ It can be reduced by frequency domain filters

Estimation of Noise Parameters

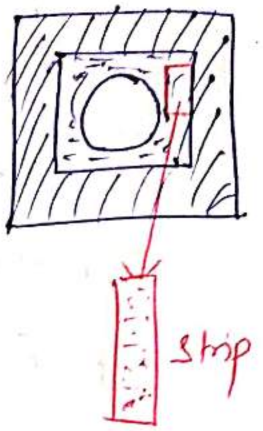
- Periodic noise parameters can be estimated by fourier spectrum of the image. (because periodic noise produce freq spikes)
- Automated analysis possible ~~→~~ When knowledge is available → about general location of frequency components of interference.
- If image systems are avaiable then study the characterstic of system noise to capture a set of images of flat enviorments.

Eg: If we have solid gray board which illuminate uniformaly → then → good indication of system noise in optical sensor

- Use vertical strip from the image and calculate the mean & variance of gray levels.

strip size (150 x 20 pixels.

Strip is a subimage denoted by S.



mean $\bar{z} = \sum_{i=0}^{L-1} z_i \, p_s(z_i)$

variance $\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 \, p_s(z_i)$

$z_i \rightarrow$ gray level values of pixels.

$p(z_i) \rightarrow$ normalized histogram values.

→ Histograph shape $\xrightarrow{\text{identifies}}$ PDF match (closest)

→ If shape is gaussian, then mean & variance are all needed variable to show PDF ~~of~~ Gaussian noise.

Restoration in the Presence of Noise Only

Spatial fitering :-

$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$$

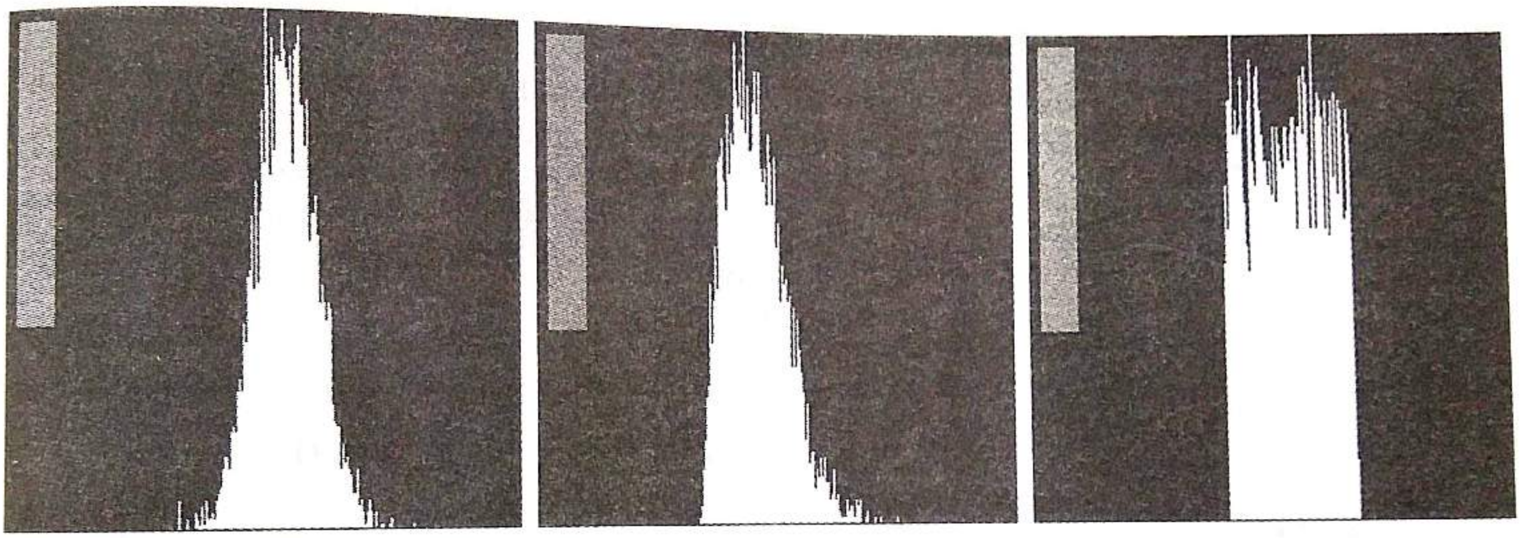
$h(x,y)$ is degradation func.

If degradation is only due to noise

- So, not considering $h(x,y)$ so new equation

$$g(x,y) = f(x,y) + \eta(x,y)$$

& $\quad G(u,v) = F(u,v) + N(u,v)$



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

- If noise is periodic, estimate the noise $N(u,v)$ and subtract it from spectrum $G(u,v)$ to obtain original image $f(u,v)$

$$f(u,v) = G(u,v) - N(u,v)$$

- Spatial filtering is the method when additive noise is present.

I → Median filter

(i) Arithmetic mean filter :→

- Simplest of mean filter

S_{xy} → set of coordinates in a rectangular subimage window of size $m \times n$, centered at point (x,y).

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t) \quad \text{—— (i)}$$

- Size of filter is $m \times n$.
- A mean filter smoothens local variations
- Noise is reduced due to blurring.

ii) Geometric mean filter :-

Π - represents product

$$\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{\frac{1}{mn}} \quad \text{——— (2)}$$

- restored pixel is given by the product of pixels in the subimage window - raised to power 1/mn.

→ It achieves smoothing comparable to the arithmetic mean filter.

→ But lose less image details

(iii) Harmonic mean filter :-

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}} \quad \text{——— (3)}$$

- works well for salt noise. & fails for pepper noise.

- It work well for Gaussian noise.

(iv) Contraharmonic mean filter :-

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

$Q \rightarrow$ order of the filter.

→ Well suited for reducing the effect of salt and pepper noise.

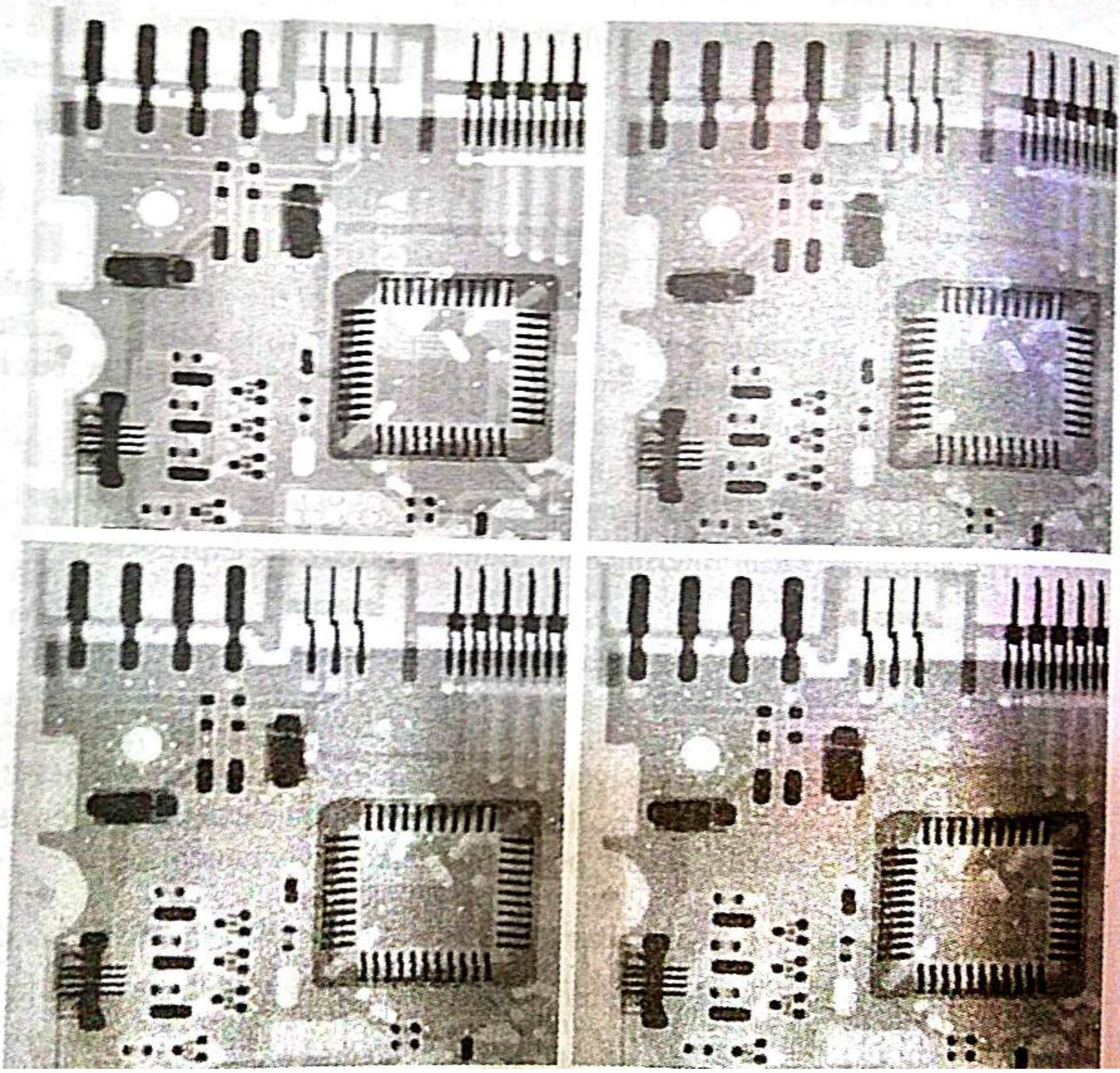
→ $Q \rightarrow$ +ve value – filter eliminate pepper noise.

$Q \rightarrow$ –ve. value – filter eliminate salt noise.

can't do both simultaneously.

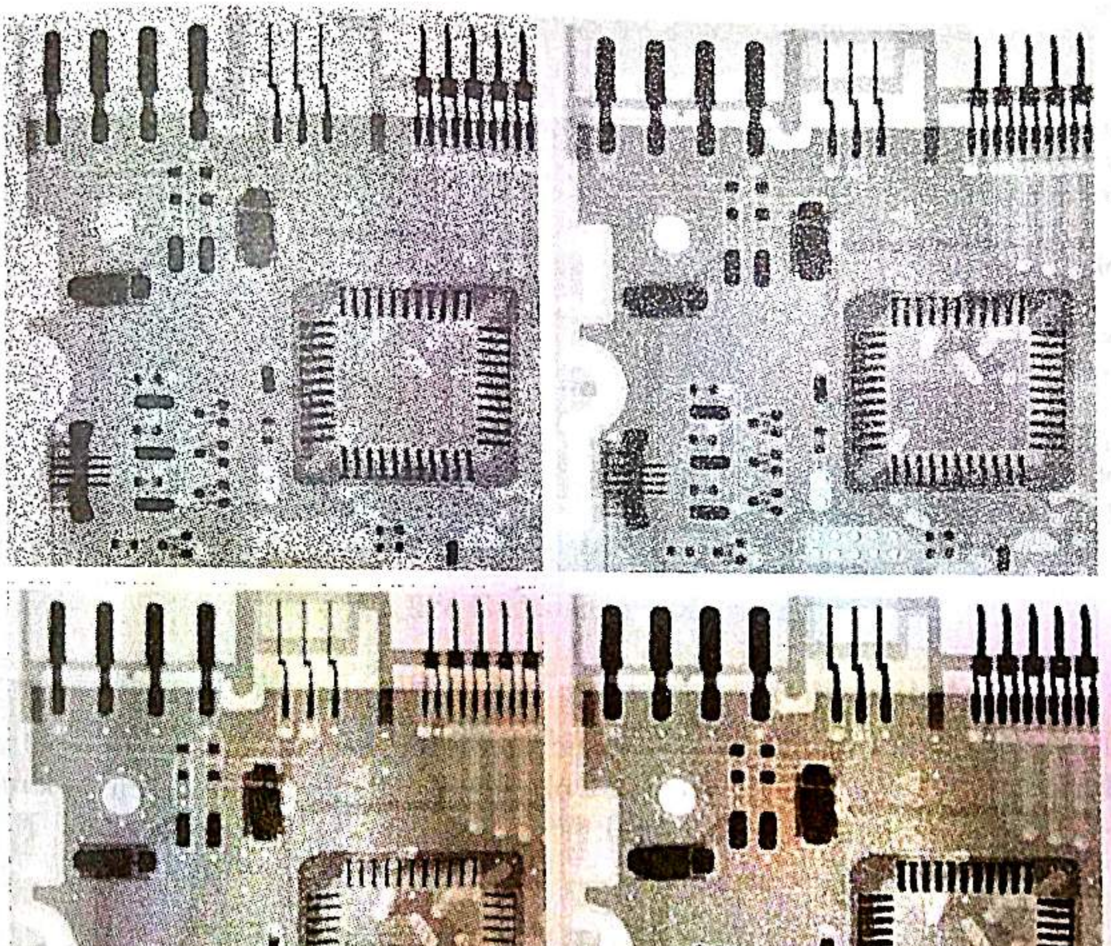
- Contraharmonic filter act as arithmetic filter if $Q = 0$

- harmonic mean filter if $Q = -1$.



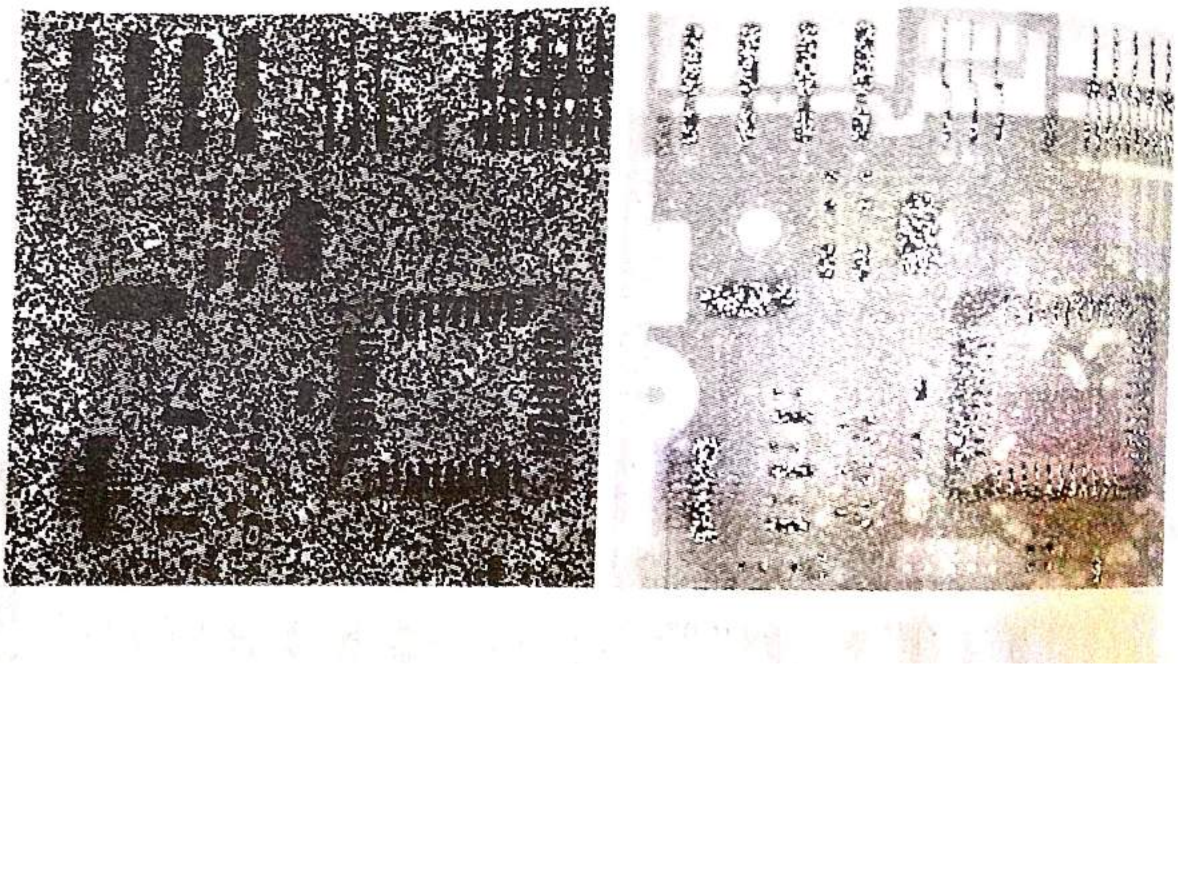
a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Spatial filter
(1) Mean filter
(2) Order-statistic filters

Order-statistic filters

- These types of filters response is based on ordering (ranking) the values of the pixels contained in the image area encompassed by the filter.
- Response is determined by the ranking results.

(i) Median filter:-

- best known order statistic filter.

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}} \{g(s,t)\} \quad \text{—— (i)}$$

- Replaces the value of pixel by the median of gray levels
- Excellent noise reduction with less blurring
- Particular effective in the presence of both bipolar, unipolar and impulse noise.

(ii) Min & Max filters:-

Min filter

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\} \quad \text{—— (ii)}$$

Max filter

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\} \quad \text{—— (iii)}$$

Spatial fiter
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- Excellent noise reduction with less blurring
- Particular effective in the presence of both bipolar, unipolar and impulse noise.

(ii) Min & Max filters :-

Min filter

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{ g(s,t) \} \quad \text{——— (ii)}$$

Max filter

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{ g(s,t) \} \quad \text{——— (iii)}$$

(iii) Midpoint filter :-

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

- Compute the mid point b/w max & min values
- filter combines order statistics & averaging
- Work best for randomly distributed noise like gaussian or uniform noise.

(iv) Alpha-trimmed mean filter :-

- Remove $\frac{d}{2}$ lowest intensity values &

  $d/2$ highest intensity values of $g(s,t)$

Now $g_r(s,t)$ have $(mn-d)$ pixels.

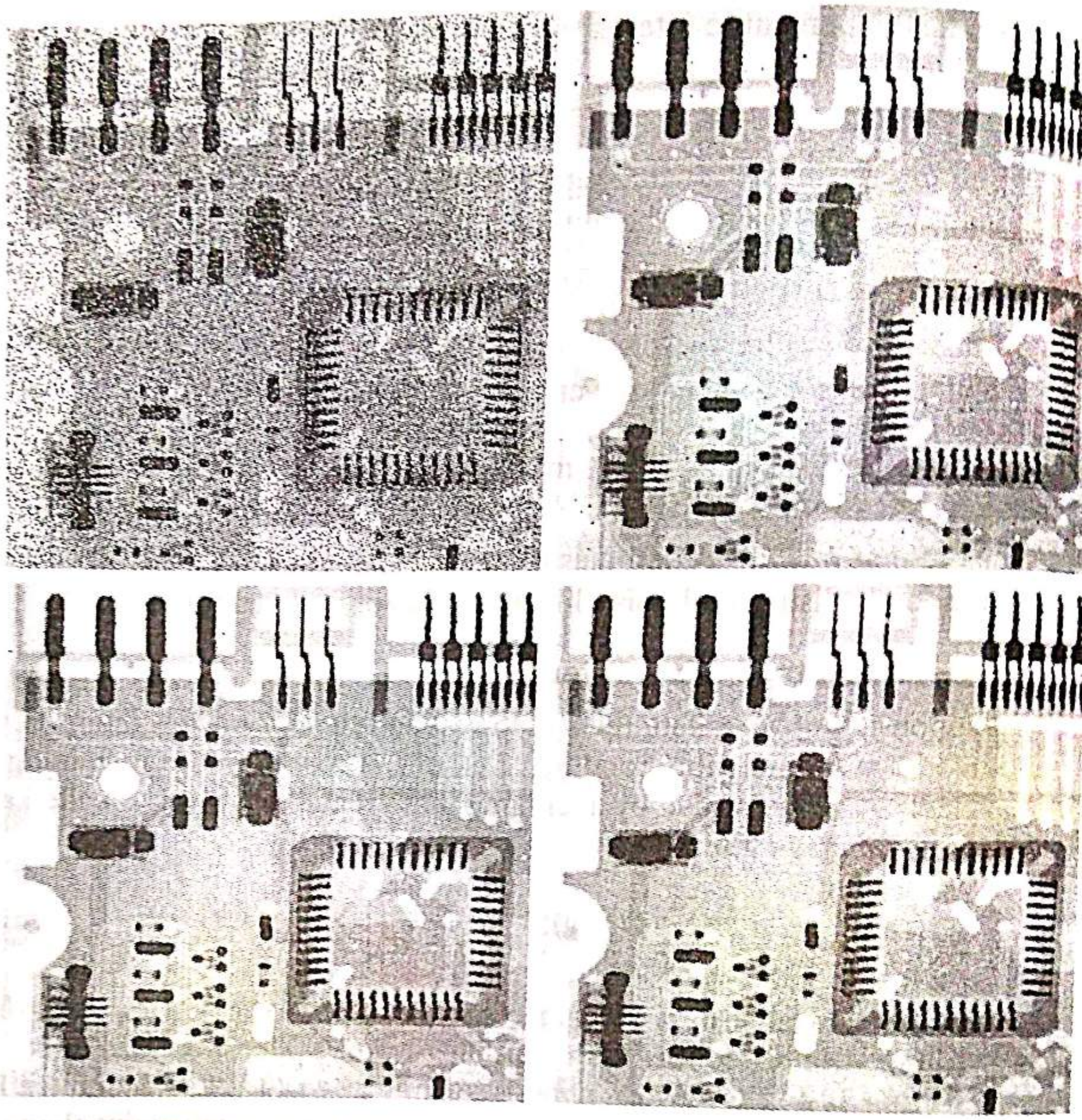
$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

d - range from 0 to $mn-1$

- When $d=0$ - It's work as arithmetic mean filter
- If $d = mn-1$, it's become a median filter.
- It's useful in different type of noise, such as combination of salt & pepper and Gaussian noise.

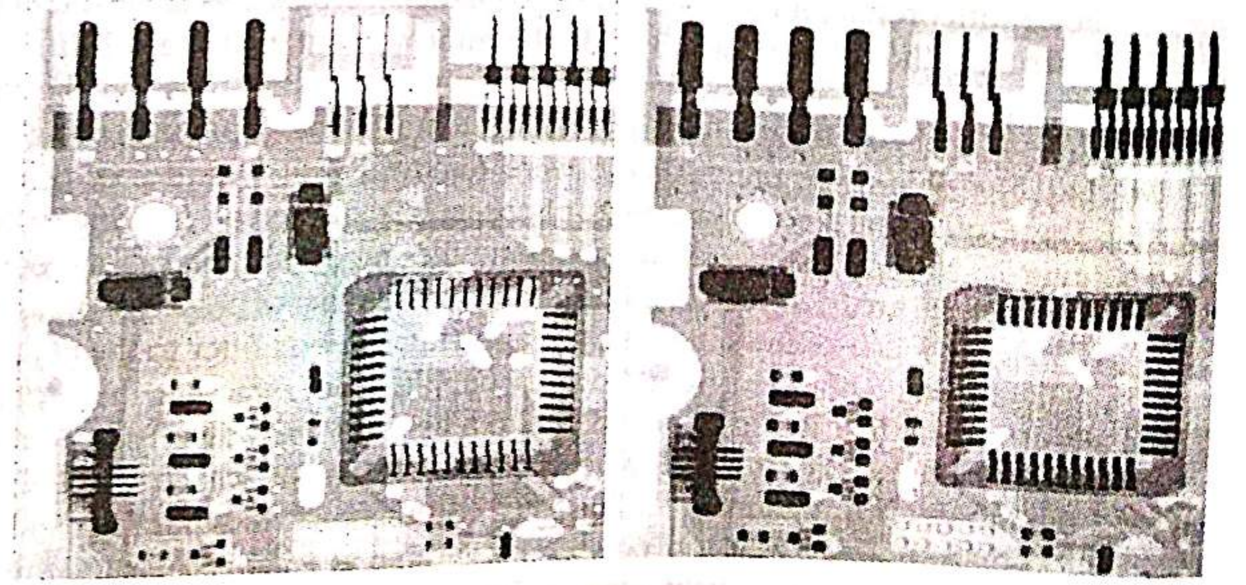
a b
c d

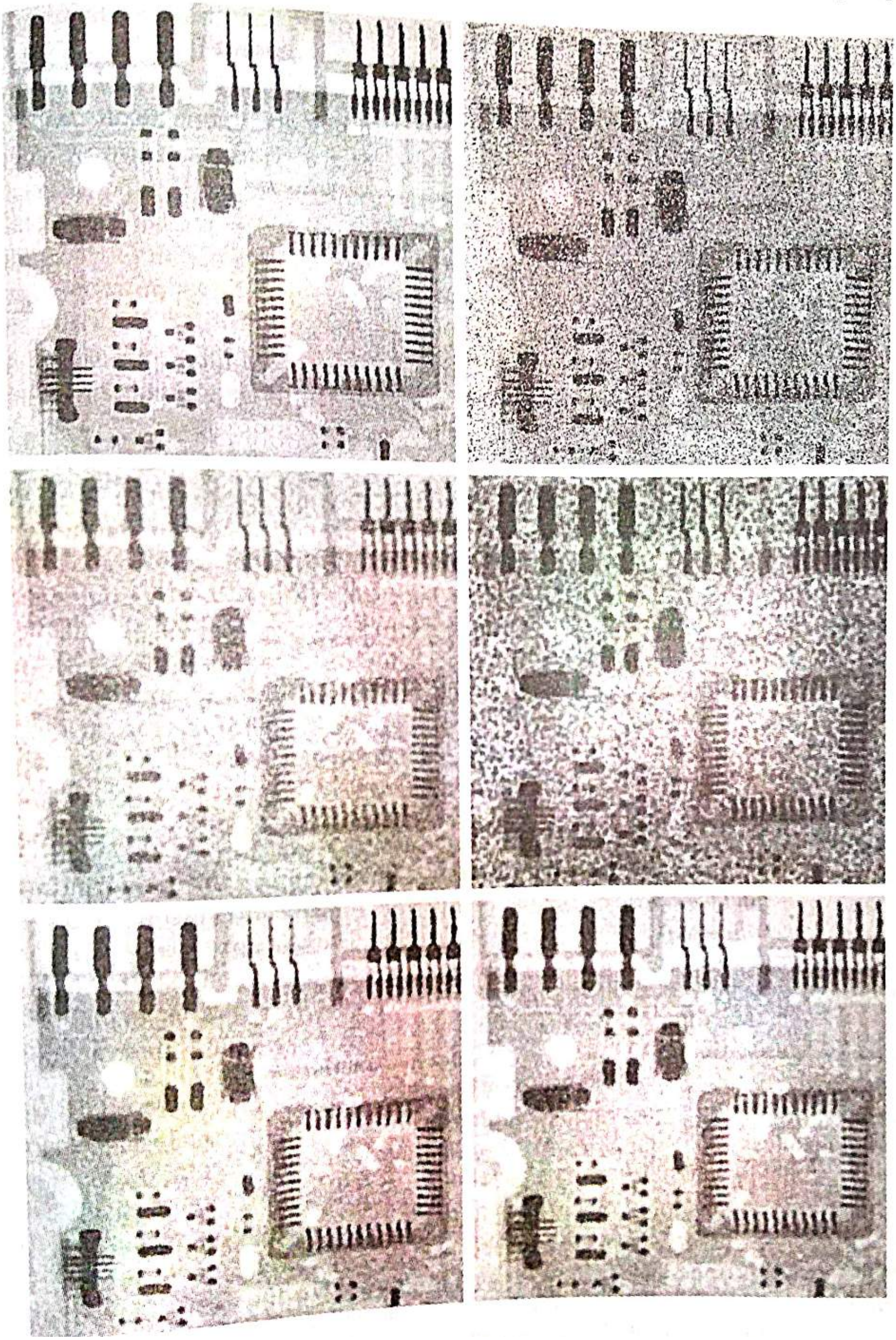
FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.



a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.





a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5: (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

(iii) Adaptive filter (spatial domain)

(i) Adaptive, local noise reduction filter

Statistical measures of a random variable are

(i) Mean
- measure of avg intensity in image

(ii) Variance
- it measures the contrast in that region.

- Region of image is S_{xy} and (x,y) is center value of filter.
- Respone of filter based on four quantities
  1. $g(x,y)$ value of noise at image (x,y) point
  2. σ^2_η - variance of noise corrupted $f(x,y)$ to form $g(x,y)$
  3. m_L - local mean of pixel in S_{xy}.
  4. σ^2_L, local variance of pixel in S_{xy}.

So the equation of adaptive filter can be written as follow by using above variables.

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma^2_\eta}{\sigma^2_L}\left[g(x,y) - m_L\right]$$

- The behaviour of adaptive filter change depending on the characteristics of the image inside the filter region.

- It's purpose is to preserve the edges.
- Behaviours of filter or concept

1. If $\sigma_\eta^2 = 0$, filter will return the value of $g(x,y)$, means $\boxed{\text{zero noise}}$ → zero

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

2. $\sigma_L^2 > \sigma_\eta^2$ → preserve edges

- filter will return the value close to $g(x,y)$

3. If $\sigma_L^2 = \sigma_\eta^2$ → Area inside objects.

- filter should return the arithmetic mean value m_L

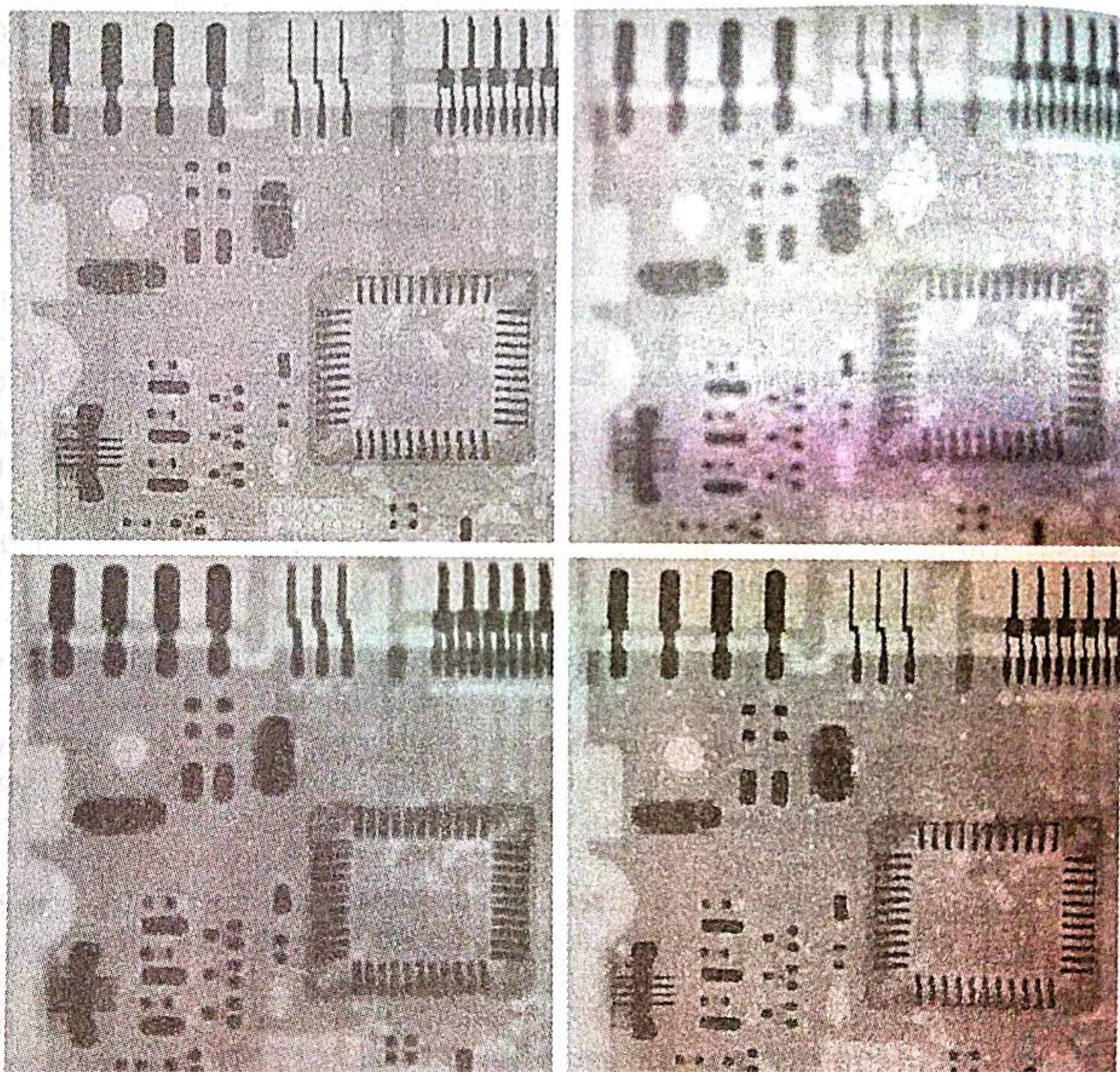
$$\hat{f}(x,y) = g(x,y) - 1 \cdot [g(x,y) - m_L]$$
$$= g(x,y) - g(x,y) + m_L$$
$$= m_L$$

Local mean $(m_L) = \frac{1}{m \cdot n} \sum_{(s,t) \in S_{xy}} g(s,t)$

Local variance $\sigma_L^2 = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} (g(s,t) - m_L)^2$

a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.



Ex:-

$$B = \begin{bmatrix} 0.9361 & 1.000 & 1.000 & 0.8871 \\ 1.000 & 1.000 & 0.9184 & 1.000 \\ 0.9868 & 1.000 & 1.000 & 0.9591 \\ 0.000 & 0.8987 & 0.9400 & 1.000 \end{bmatrix}$$

a. Define 3x3 - window (mxn)

b. Pad matrix with 0

c. Compute local mean & local variance

$\boxed{\text{mean}} = 0+0+0+0+0.9361+1+0+1+1/9$

$= 3.9361/9 \rightarrow \boxed{0.4373}$

Calculate for all by shifting the window.

$$m_L = \begin{bmatrix} 0.4373 & 0.6505 & 0.6451 & 0.4228 \\ 0.6581 & 0.9824 & 0.9738 & 0.6405 \\ 0.5428 & 0.8604 & 0.9685 & 0.6464 \\ 0.3206 & 0.5362 & 0.6442 & 0.4332 \end{bmatrix}$$

Mean(B)

$\text{mean} = 0+0+0+.9361+1+1+1+1+.9184/9$

$= 5.8545/9 \rightarrow \boxed{0.6505}$

d. Calculate local variance

$\sigma^2_L = \text{Mean}(B^2) - [\text{Mean}(B)]^2$

$=$

$$[\text{Mean}(B)]^2 = \begin{bmatrix} 0.1913 & 0.4232 & 0.4161 & 0.1788 \\ 0.4331 & 0.9650 & 0.9484 & 0.4103 \\ 0.2947 & 0.7403 & 0.9379 & 0.4178 \\ 0.1028 & 0.2875 & 0.4150 & 0.1877 \end{bmatrix}$$

Square of m_L

Calculate Mean (B^2)

(9)

$$B^2 = \begin{bmatrix} 0.8763 & 1.000 & 1.000 & 0.7869 \\ 1.000 & 1.000 & 0.8435 & 1.000 \\ 0.9738 & 1.000 & 1.000 & 0.9199 \\ 0 & 0.8077 & 0.8836 & 1.000 \end{bmatrix}$$

Calculate mean(B^2) using last step formula.

$$\text{mean}(B^2) = \begin{bmatrix} 0.4307 & 0.6355 & 0.6256 & 0.4034 \\ 0.6500 & 0.9659 & 0.9500 & 0.6167 \\ 0.5313 & 0.8343 & 0.9394 & 0.6274 \\ 0.3090 & 0.5183 & 0.6285 & 0.4226 \end{bmatrix}$$

$$\sigma_L^2 = \begin{bmatrix} 0.2394 & 0.2124 & 0.2095 & 0.2246 \\ 0.2169 & \underline{0.0009} & \underline{0.0017} & 0.2064 \\ 0.2366 & \underline{0.0939} & \underline{0.0015} & 0.2096 \\ 0.2063 & 0.2309 & 0.2085 & 0.2349 \end{bmatrix}$$

d. σ_n^2 = avg. of all σ_L^2 = 0.1709

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_n^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

$\{$ if ($\sigma_n^2 \geq \sigma_L^2$) then
do $\sigma_n^2 = \sigma_L^2$ ~~then~~
else copy σ_L^2 value as it is $\}$

$$\sigma_L^2 = \begin{bmatrix} - & - & - & - \\ - & 0.1709 & 0.1709 & - \\ - & 0.1709 & 0.1709 & - \\ - & - & - & - \end{bmatrix}$$

same as in above table but these value changed

$$\text{Restored image} = \begin{bmatrix} 0.5802 & 0.7188 & 0.7105 & 0.5339 \\ 0.7307 & 0.9824 & 0.9738 & 0.7025 \\ 0.6662 & 0.8604 & 0.9685 & 0.7042 \\ 0.2656 & 0.6304 & 0.6975 & 0.5878 \end{bmatrix}$$

Adaptive Median Filter →

- Median filter performs relatively well on impulse noise.
- The adaptive median filter can handle much more spatially dense impulse noise, and also performs some smoothing for non-impulse noise.
- In median filter if P_a & P_b less than 0.2 → work well but for above value → not → AMP can handle
- Adaptive median filter change the size of window (S_{xy}) depending on the characteristics of the image.
- Adaptive median filter has three purposes
  1. Remove impulse noise (salt & pepper)
  2. Provide smoothing of other noise
  3. Reduce distortion

Notation in AMF

1. Z_{min} - min intensity value in S_{xy}
2. Z_{max} - max intensity value in S_{xy}
3. Z_{med} - median of intensity value in S_{xy}
4. Z_{xy} - intensity at (x, y)
5. S_{max} - max size of S_{xy}

Algorithm of AMF :→

Stage A: $A1 = Z_{med} - Z_{min}$

$A2 = Z_{med} - Z_{max}$

If $A1 > 0$ & $A2 < 0$, go to stage B.

else

increase window size

If window size $\leq S_{max}$ repeat stage A

else o/p Z_{med}.

Stage B: $B1 = Z_{xy} - Z_{min}$

$B2 = Z_{xy} - Z_{max}$

if $B1 > 0$ & $B2 < 0$ → o/p - Z_{xy}

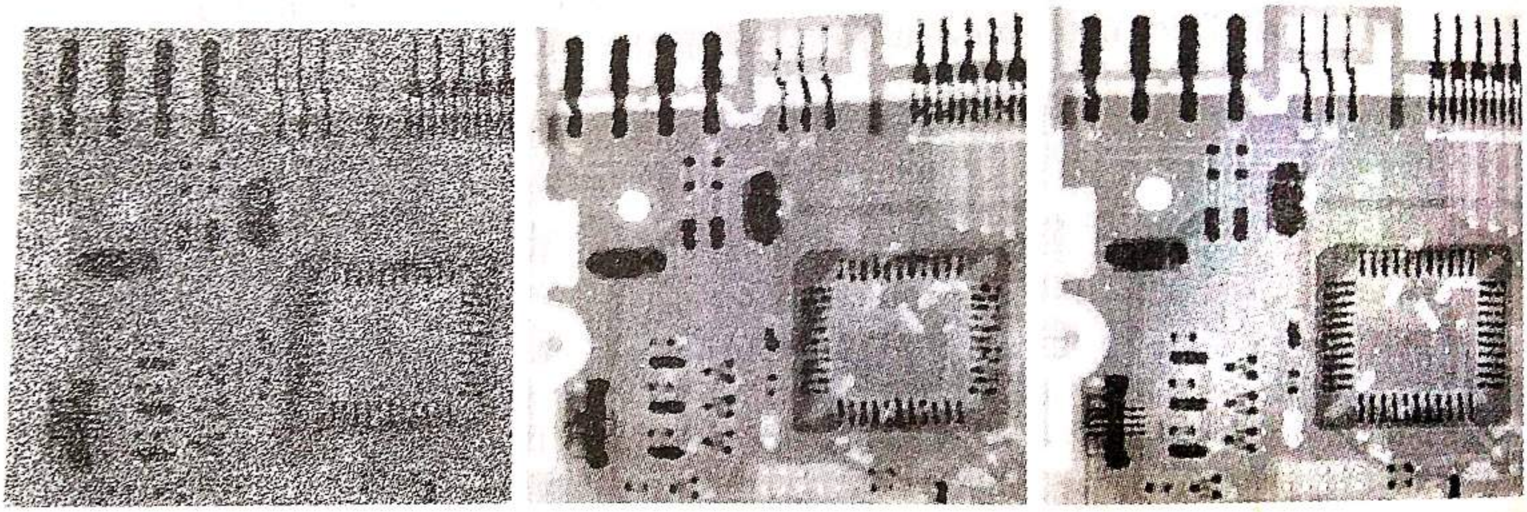
else o/p Z_{med}.

- Purpose of stage A determine if median filter o/p. Z_{med} → impulse (black or white) or not.

if $Z_{min} < Z_{med} < Z_{max}$ → Z_{med} is not a impulse.

→ In this case → go to stage B → check Z_{xy} is impulse.

→ If $B1 > 0$ & $B2 < 0$ → true then $Z_{min} < Z_{xy} < Z_{max}$ and Z_{xy} is not an impulse.



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.

Periodic Noise Reduction by Frequency Domain filtering:-

- Periodic noise can analyzed and filtered effectively using freq. domain technique.
- The approach is to use three types of filters
  1. Band reject
  2. Band pass
  3. Notch

1. Bandreject filters :-

Band reject filters remove or attenuate a band of frequencies about the origin of the fourier transform.

Ideal Bandreject filters :-

$D(u,v) < D_0 - \frac{W}{2}$

$$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$$

or $D(u,v) > D_0 + W/2$ or

D_0 - radial center of band (cutoff freq), $D(u,v)$ - distance from the center (origin)

W - width of band , n - order of Butterworth filter.

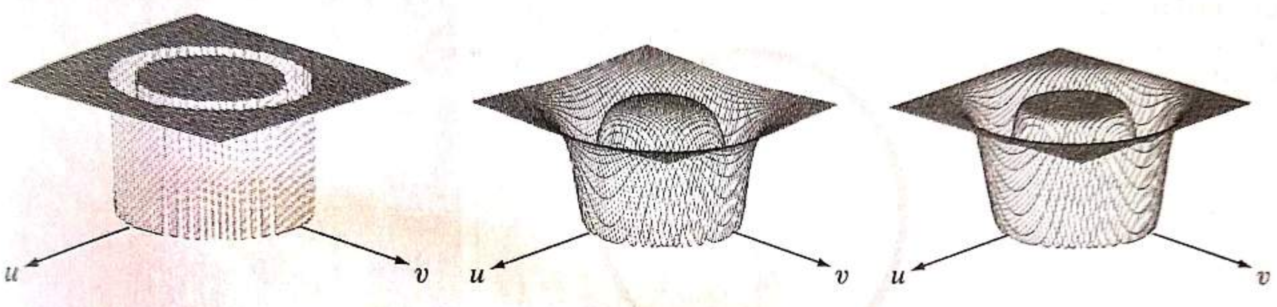
Butterworth Bandreject filters :-

D is $D(u,v)$

$$H(u,v) = \frac{1}{1 + \left[\frac{DW}{D^2 - D_0^2}\right]^{2n}}$$

Gaussian Bandreject filters :-

$$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$$

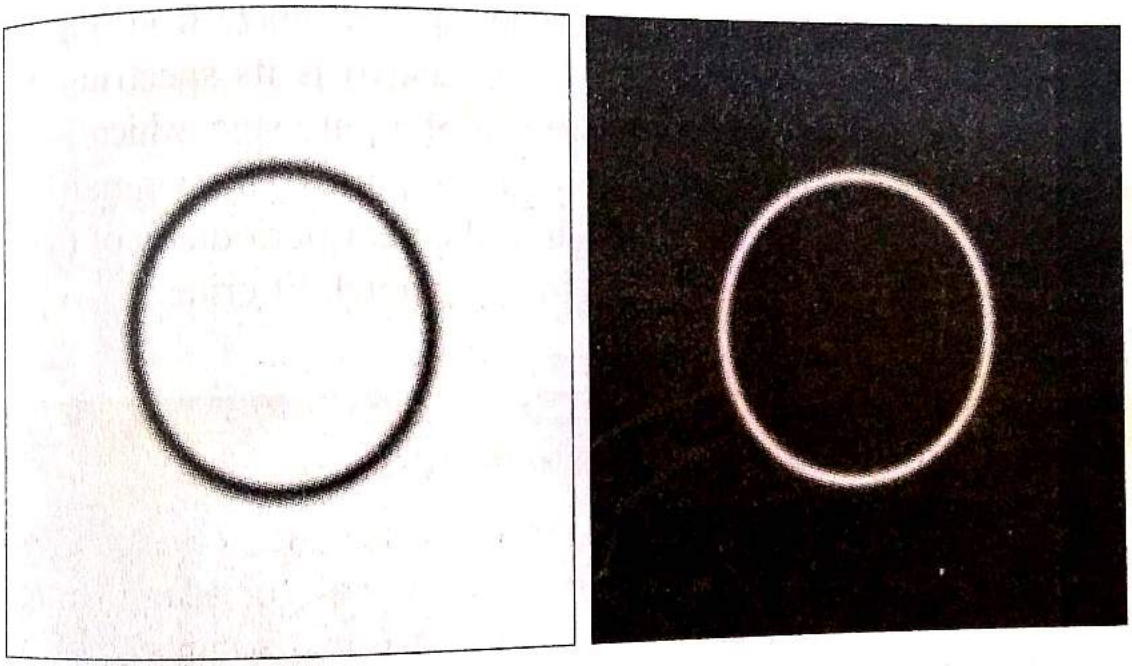


a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

TABLE 4.6
Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$



a b

FIGURE 4.63
(a) Bandreject Gaussian filter. (b) Correspondin bandpass filter. The thin black border in (a) was added for clarity; is not part of the data.

2. Bandpass filters :-

Bandpass filter performs the opposite operation of a bandreject filter.

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

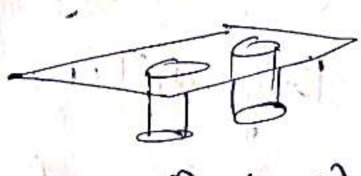
3. Notch filters :-

- A notch filters rejects or passes frequencies in predefined neighborhoods about a center frequency.
- Due to the symmetry of the fourier transform, notch filters must appear in symmetric pairs about the origin (except the one at the origin)

eg: if notch with center (u_0, v_0) — corresponding notch at location $(-u_0, -v_0)$

Notch Reject filters

Ideal Notch filter



$$H(u,v) = \begin{cases} 0 & \text{if } D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0 \\ 1 & \text{otherwise.} \end{cases}$$

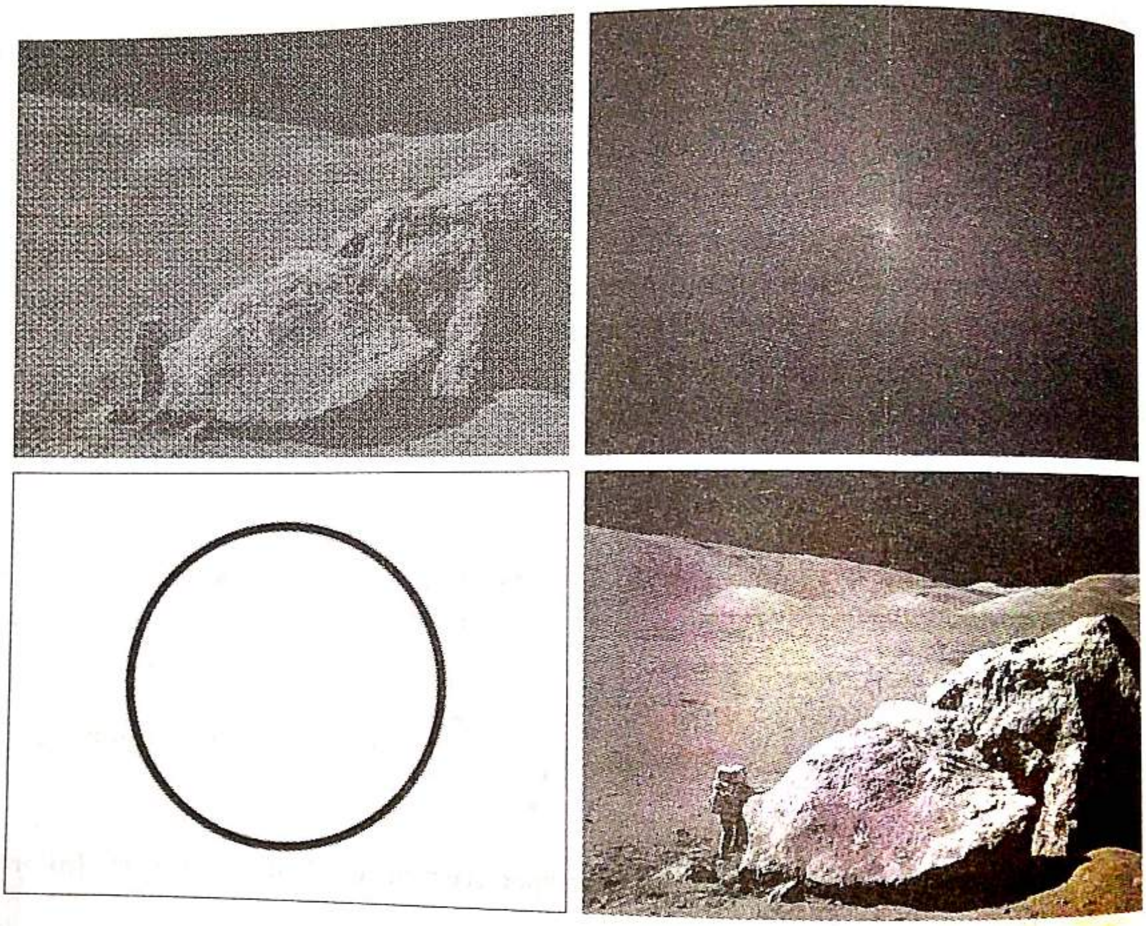
Butterworth Notch filter

Smooth transformati

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v)\, D_2(u,v)}\right]^n}$$

Gaussian Notch filter

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D_1(u,v)\, D_2(u,v)}{D_0^2}\right]}$$



a b
c d

FIGURE 5.16
(a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

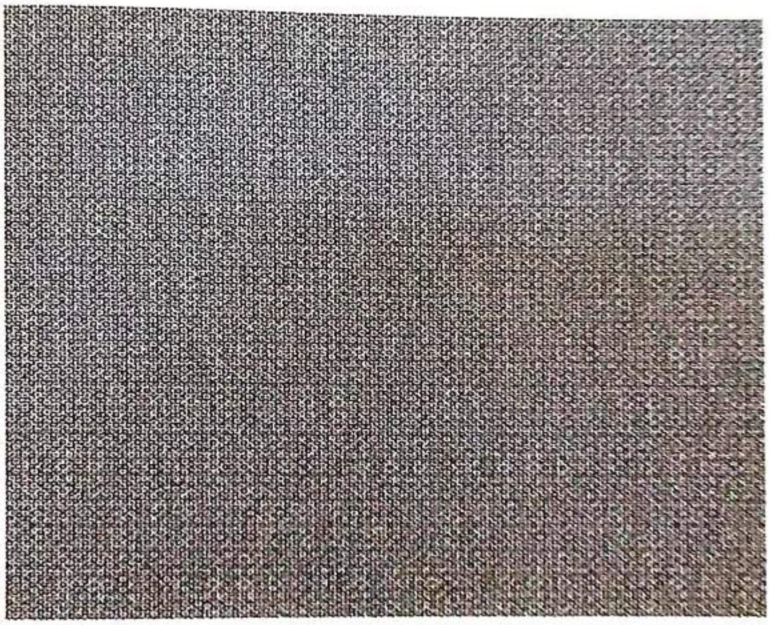
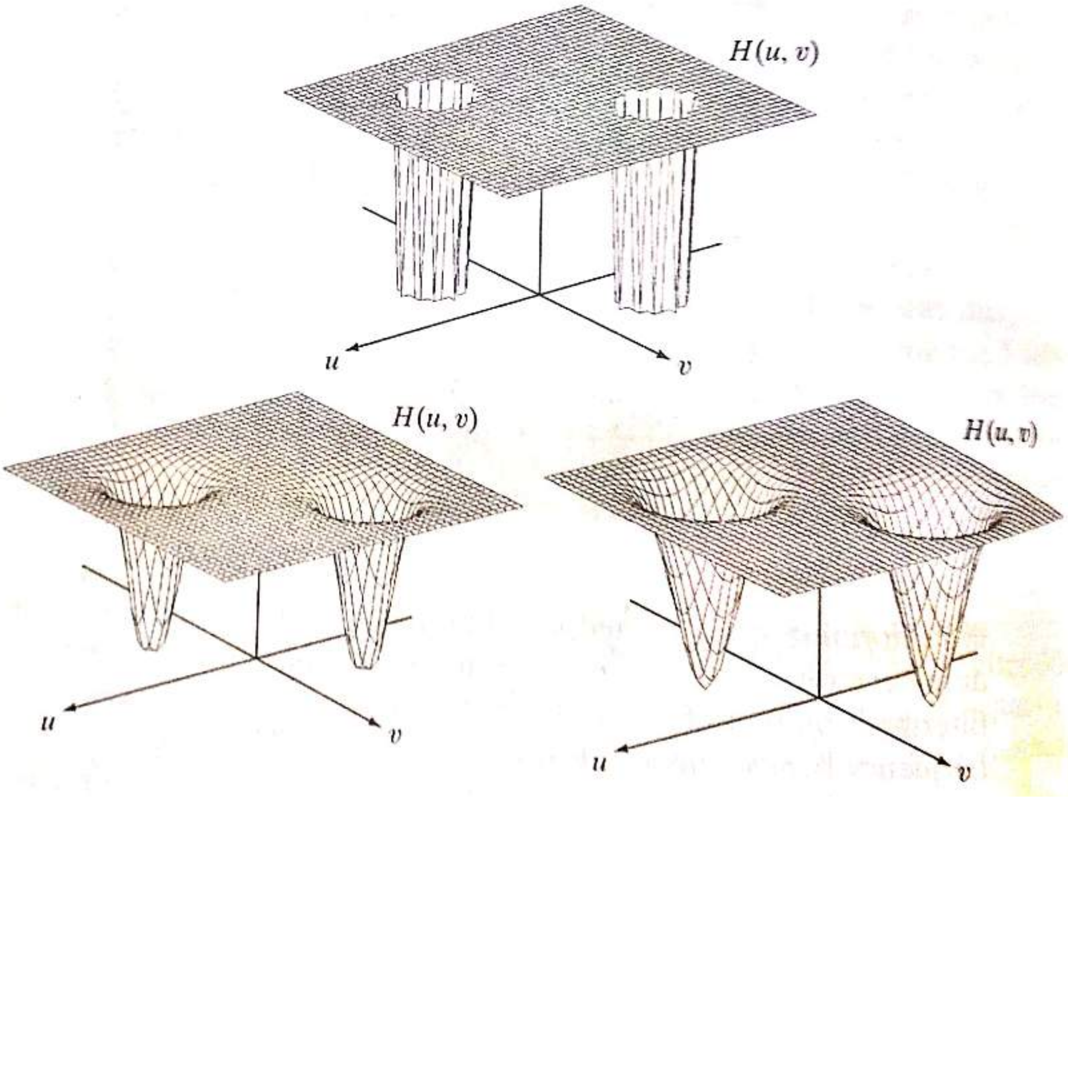
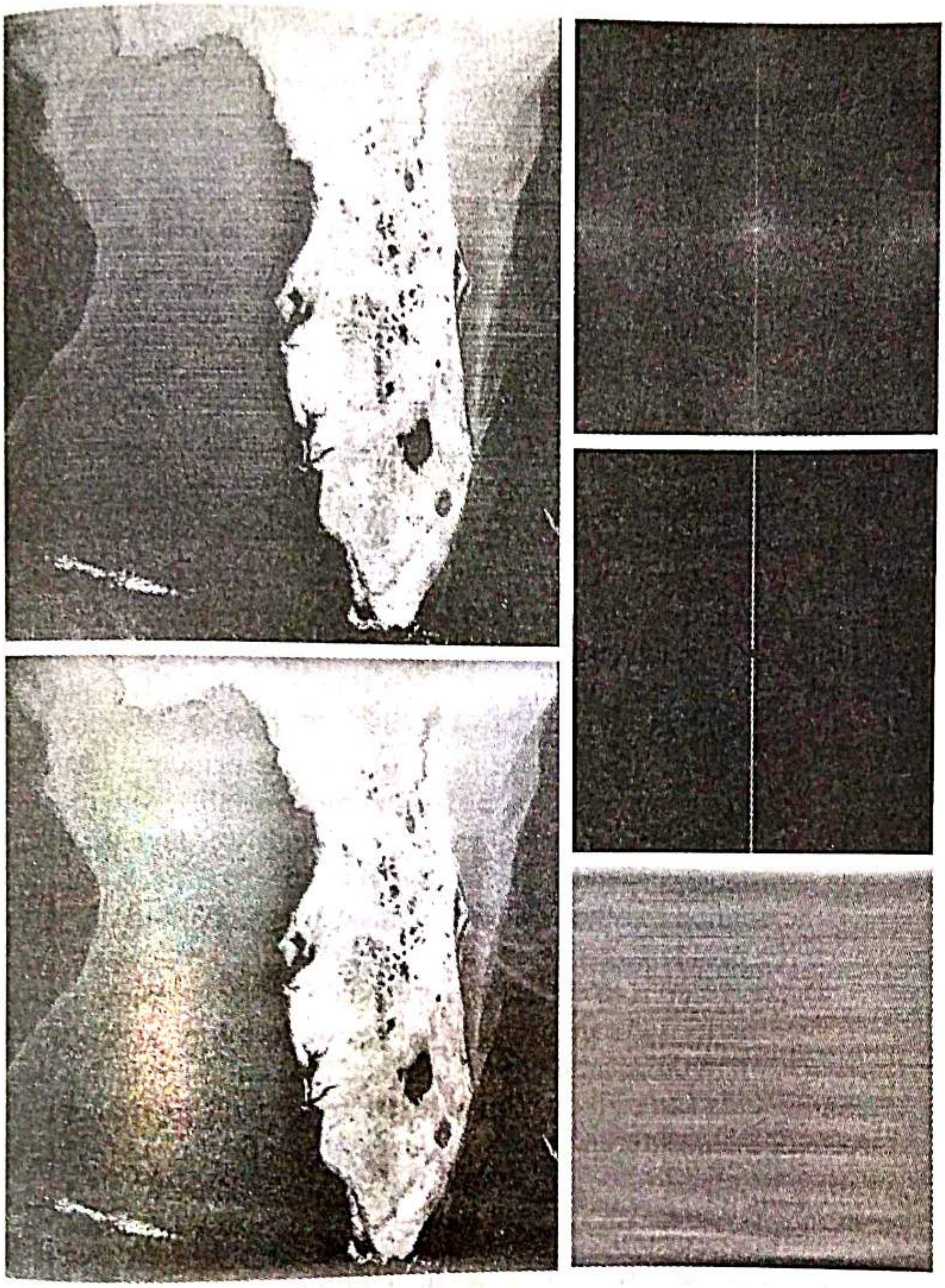


FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.

a
b c

FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.





a b
c
e d

FIGURE 5.19
(a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

Similar to the bandpass / bandreject filters, the Notch reject filters can be turned into Notch pass filters with relation

$$H_{NP}(u,v) = 1 - H_{NR}(u,v)$$

$H_{NP} \rightarrow$ transfer func of notch pass filter
$H_{NR} \rightarrow$ transfer func of notch reject filter

→ if $u_0 = 0, v_0 = 0$
- the Notch reject filter becomes a highpass filter &
- Notch pass filter becomes a lowpass filter

Estimating the Degradation funcn

Degradation $\xrightarrow{\text{due to}}$ noise, blurring

- Degradation function can be estimated by three principal methods.

(1) Estimation by observation

(2) Estimation by experimentation

(3) Estimation by mathematical modeling

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Estimating the Degradation func^n

Degradation $\xrightarrow{\text{due to}}$ noise, blurring

- Degradation function can be estimated by three principal methods.
  (1) Estimation by observation
  (2) Estimation by experimentation
  (3) Estimation by mathematical modeling
- The process of restoring an image by using a degradation func that has been estimated in some way called blind deconvolution.

(1) Estimation by Image Observation :-

- We have a degraded image, but no knowledge of degradation fun^c H.

- H can be estimated by gathering information from image itself.
- eg: Image is blurred, we don't know degradation func, The degradation fun^c (H) can be estimated by visually looking into a small section of the image. (like part of object & background).
- Subimage $g_s(x,y) \rightarrow$ processed subimage $\hat{f}_s(x,y)$
- Here additive noise is negligible in such an area with a strong signal content.

$$\boxed{H(u,v) = \frac{G(u,v)}{\hat{F}(u,v)}}$$

$\hat{g}(x,y) = f(x,y) * h(x,y) + \underbrace{n(x,y)}_{\text{negligible}}$

$G(x,y) = F(x,y) \cdot H(u,v)$

- $H_s(u,v)$ estimated for such a small sub image, the shape of this degradation fun^c can be used to get an estimated of $H(u,v)$ for the entire image.

a b

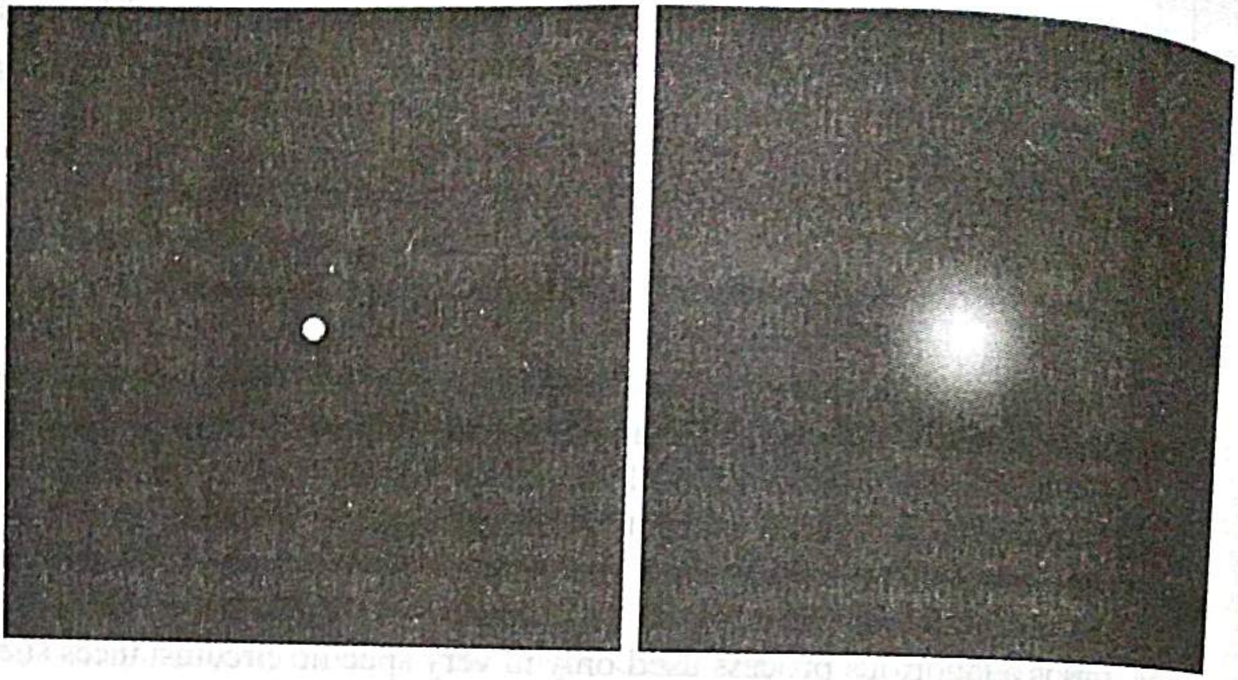


FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

a b
c d

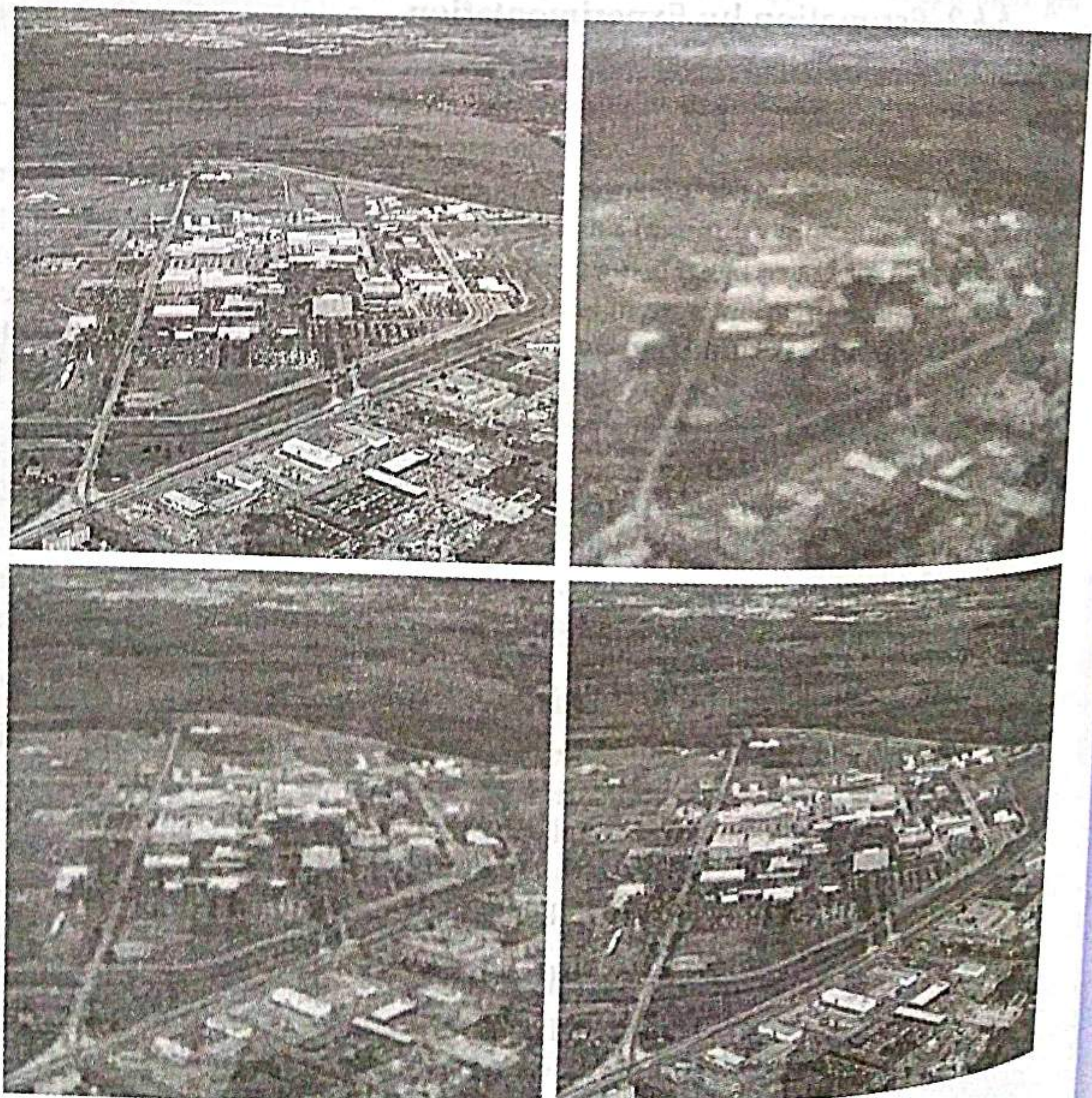


FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

(ii) Estimation by Experimentation :-

- It is possible to obtain accurate estimation of degradation if equipment similar to equipment used for the acquisition of image is available.
- This can be achieved by applying an impulse (bright dot) as an input image.
- The fourier transform of an impulse is constant therefore

$$\boxed{H(u,v) = \frac{G(u,v)}{A}}$$

$G(u,v)$ — fourier transform of observed image

A — constant describ the strength of the impulse.

(iii) Estimation by Modeling :-

- A model is a set of equations that approximate a real system.
- Many types of models have been developed for modelling degradation like motion blurs and atmospheric turbulence
- Two types of techniqus are available based on estimation of blurring funᶜ

(1) Direct Estimation Techniques

(2) Indirect Estimation Technique.

Direct Estimation Technique –

- Blur fun^c is directly measured by isolating the image of an object pixel.
- By observing the point source object, we may get the impulse response of blur because it is the image of point source object.
- If we take many such samples and by observation and estimation, a covariance fun^c can be estimated for a relatively uniform region of the image.

Indirect Estimation Techniques:-

- Many techniques can be formulated.
- Degradation model that has been constructed is the convolution of the image $f(x,y)$ blurring fun^c $h(x,y)$ and additive noise

$$g(x,y) = f(x,y) * h(x,y) + \underbrace{\eta(x,y)}_{0}$$

- Assume no noise

$$G(u,v) = F(u,v) \cdot H(u,v)$$

$$H(u,v) = \int_0^T e^{-2\pi j u x_0(t)}\, dt$$

or estimation of H (m)-dimension

$$\log H = \frac{1}{M}\left[\sum_{k=1}^{M} |\log V_k) - \log(U_k|\right]$$

degraded image

good part of image

Inverse filtering :-

- The process of removing blurs and noise is known as deconvolution or inverse filtering.
- Simple deconvolution starts with an assumption that a blur is characterized by the impulse response of the system.
- Most blurs are linear and o/p of imaging System is the convolution of impulse response and input image.

degraded model:

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$$

fourier Transform

$$G(u,v) = F(u,v) \cdot H(u,v) + \eta(u,v) \quad \text{--- (1)}$$

if noise is zero $\eta(u,v) = 0$

$$G(u,v) = F(u,v) \cdot H(u,v)$$

$$F(u,v) = \frac{G(u,v)}{H(u,v)} \quad \text{or} = \frac{1}{H(u,v)} \cdot G(u,v) \quad \text{--- (2)}$$

- Inverse filter acts as HPF causing blurring & increasing noise

eq. (1) & (2)

$$F(u,v) = \frac{F(u,v) \cdot H(u,v) + \eta(u,v)}{H(u,v)}$$

$$= F(u,v) + \frac{\eta(u,v)}{H(u,v)} \quad \text{--- (3)}$$

Homomorphic filtering :-

"similar nature / similarity"

- Homomorphic filtering is a generalized technique for signal and image processing.

- It involve a nonlinear mapping to a different domain in which linear filter techniques are applied and then mapping back to the original domain.

- Homomorphic filtering simultaneously normalize the brightness across an image and increase contrast.

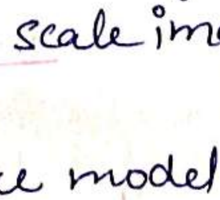
Homomorphic filtering used at (application)

(1) Removing multiplicative noise that has certain characteristics (identify easily)

(2) Correcting non uniform illumination in images.

(3) Improve the appearance of a grey scale image.

- With the help of illumination - reflectance model we will calculate Homomorphic filter.



$$\boxed{f(x,y) = i(x,y) \cdot r(x,y)} \quad (1)$$

- The illumination - reflectance model can be used to address the problem of improving the quality of an image that has been acquired under poor illumination conditions.

The idea of homomorphic filter is to separate i and r components and apply different transfer function

- This equation (1) can be written in Fourier Transform directly

$$F[f(x,y)] \neq F[i(x,y) \cdot r(x,y)]$$

- logarithmic fun is applied. (product → sum)

(1) $$Z(x,y) = \ln f(x,y)$$
$$= \ln i(x,y) + \ln r(x,y)$$

In fourier Transform

(2) $$F[Z(x,y)] = F[\ln i(x,y)] + F[\ln r(x,y)]$$

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

(3) We have filter $H(u,v)$

$$\underline{S(u,v)} = Z(u,v) \cdot H(u,v)$$

↓ in Fourier Domain

$$= \left(F_i(u,v) + F_r(u,v)\right) H(u,v)$$
$$= H(u,v) \cdot F_i(u,v) + H(u,v) \cdot F_r(u,v)$$

In Spatial Domain

$$s(x,y) = F^{-1}\{S(u,v)\}$$
$$= F^{-1}\{H(u,v) \cdot F_i(u,v)\} + F^{-1}\{H(u,v) \cdot F_r(u,v)\}$$

$$i'(x,y) = \quad\quad r'(x,y)$$

$$s(x,y) = i'(x,y) + r'(x,y)$$

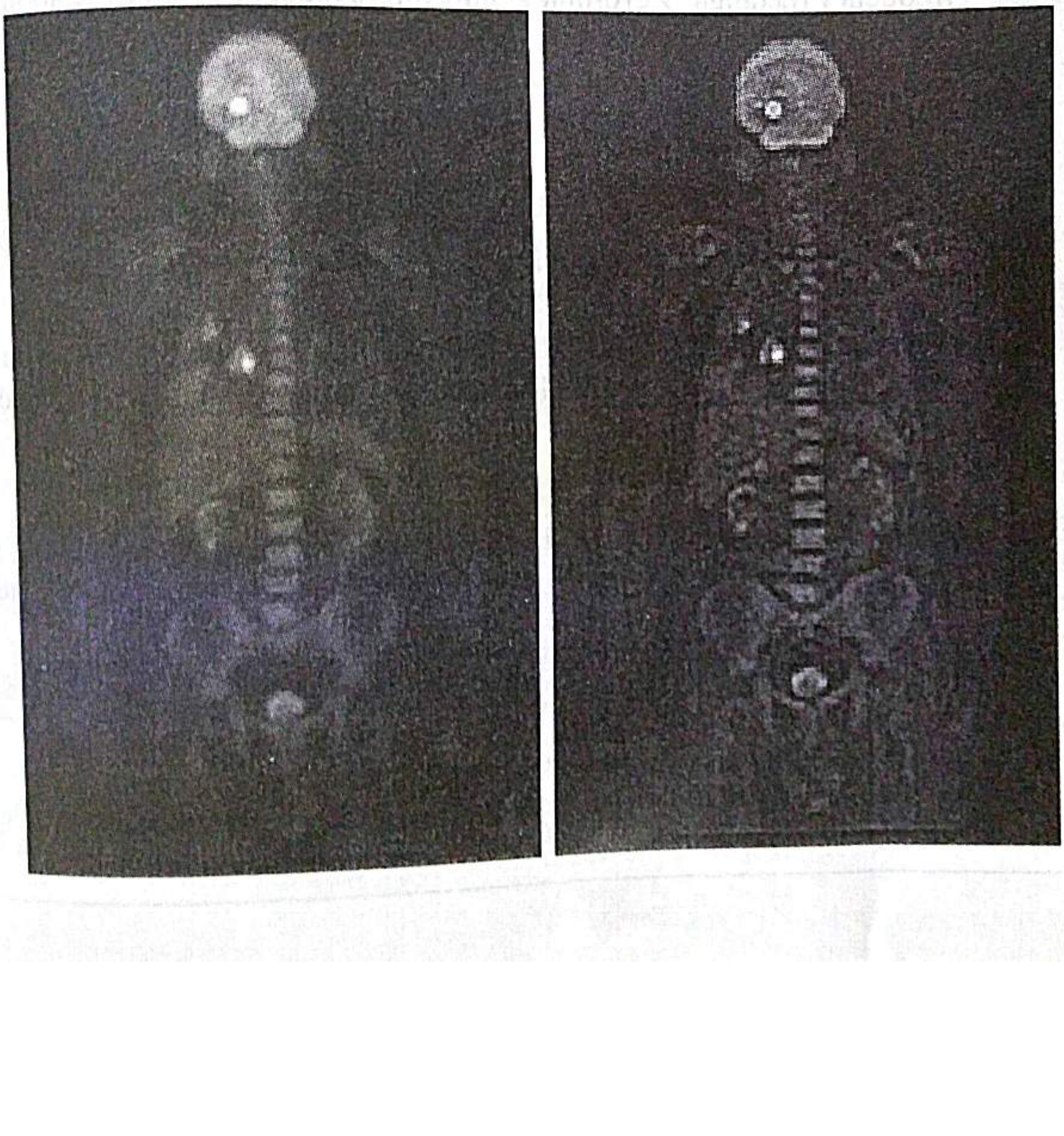
(5) $Z(x,y) \rightarrow$ log fun so apply anti log (e) on image

$$g(x,y) = e^{s(x,y)}$$
$$= e^{i'(x,y)} \cdot e^{r'(x,y)}$$
$$= i_0(x,y)\, r_0(x,y)$$

$i_0(x,y)$ - illumination component
$r_0(x,y)$ - reflection component.

- The enhancement approach using homomorphic filtering is summarized in given figure.

$$f(x,y) \rightarrow \boxed{\ln} \rightarrow \boxed{DFT} \xrightarrow{Z(u,v)} \boxed{H(u,v)} \xrightarrow{S(u,v)} \boxed{DFT^{-1}} \xrightarrow{s(x,y)} \boxed{exp} \rightarrow g(x,y)$$



a b

FIGURE 4.62 (a) Full body PET scan. (b) Image enhanced using homomorphic filtering. (Original image courtesy of Dr. Michael E. Casey, CTI PET Systems.)