Digital Image Processing (6th Sem)

Aim & Objective of DIP

To study the fundamental concepts in DIP and discuss the operation of DIP and its components.

Objectives:-

1. Learn digital image fundamental
2. Expose simple image processing techniques
3. familiar with image compression and segmentation techniques.
4. Represent image in form of features.

At the end of the course, the student should be able to:

1. Discuss digital image fundamentals.
2. Apply image enhancement and restoration techniques.
3. Use image compression and segmentation techniques.
4. Represent features of images.

Applications:- Area monitoring, Health care monitoring, Enviornmental/Earth sensing, Air pollution monitoring, Forest fire detection, Landslide detection, Water quality monitoring and Natural disaster monitoring and prevention, signal transmission and detection etc.

Industry Connectivity:-
CISSO, IBM, SIEMENS, NORTEL

It is a
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Unit 1:-Introduction to Image Processing

Digital Image processing refers to preprocessing digital images by means of a digital computer. Digital Image is composed of a finite number of elements, each of which:
- particular location
- value.

- Pixel

In image processing input and outputs are images, but in computer vision, input is scence and after preception its output is image.

In image processing three type of process are

(1) Image pre-processing (reduce noise, Contrast enhancement, image sharpening)
I/p - Image
o/p - Image

(2) Segmentation (partitioning an image into regions or objects)
I/p - Image but
o/p → features exraced from images.

(3) Recognized object

→ Widely used application of image processing is Adobe Photoshop.

What is an Image ?

It is a Two dimensional signal. Defined by f(x,y), where x & y are co-ordinate horizontally and vertically.

232	123	321
123	77	89
80	215	255

→ pixel value at co-ordinate (intencity) (0-255)

Relationship b/w Digital Image & Signal.

Signal is a mathematical func, convey some information, like voice.

→ Capturing of an image - using sunlight, object and camera to create digital image.

Computer Vision :- Input is an image and output is information.

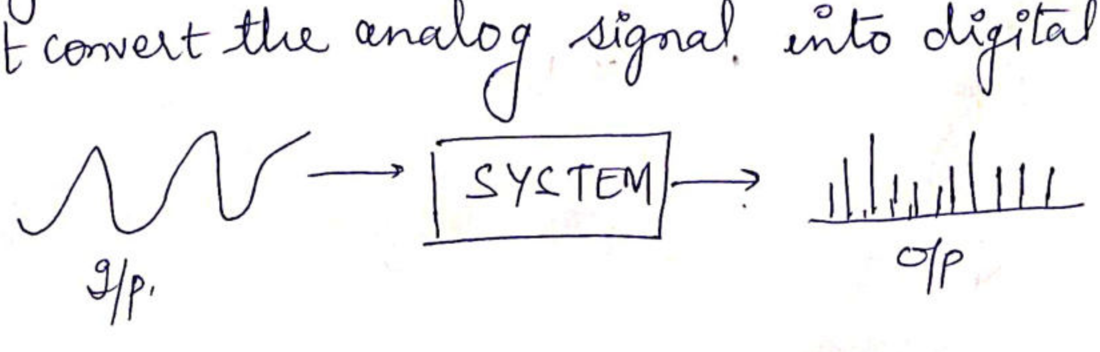
eg:. **Biometric** : input is face/finger print and o/p is attendence or your detail

Computer Graphics :- Input is object model and op is image.

eg:- **Object rendering**:. Generate image using object model.

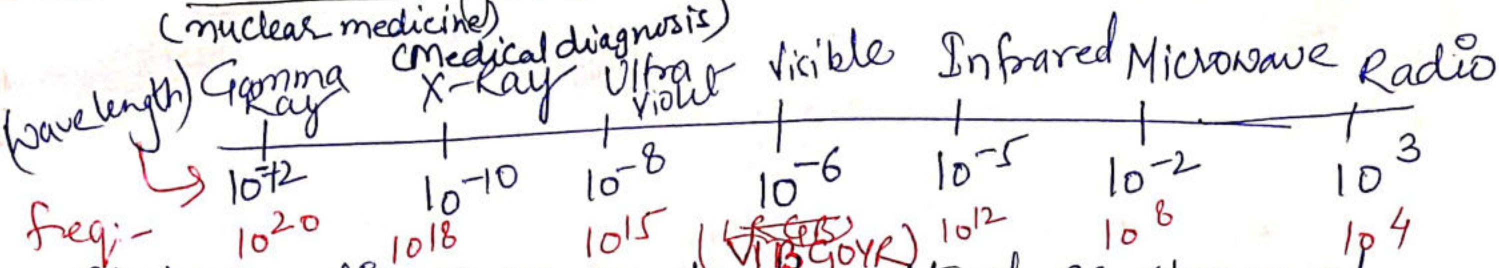
System :→

It convert the analog signal into digital form



Examples of fields that use Digital Image processing:

Electromagnetic Spectrum:-



Electromagnetic waves can be thought of as stream of particles, each particle is moving with the speed of light. Each particle contain a bundle of energy. This bundle of energy is called a photon.

Why do wee need to analyze all that other stuff in EM spectrum too?

Application of DIP:-

- Image Sharpening and restoration
- Medical field
- Remote sensing
- Transmission and encoding
- Machine/Robot Vision
- Color processing
- Pattern recognization
- Video processing
- Microscopic Imaging etc.

— process image : Zooming, blurring, Sharpening, grayscale to color conversion, detecting edges and vice versa, Image retrieval and Image recognition.

Medical field:-

1) Gamma ray imaging
2) PET scan
3) X-ray Imaging
4) Medical CT
5) UV imaging

Fundamental steps in Digital Image Processing

(1) Image acquisition
(2) Image Enhancement
(3) Image restoration
(4) Image Compression
(5) Image Segmentation
(6) Image representation & description

1. Image Acquisition-

It is the first step of image processing. It involve capturing of image using cameras and apply preprocessing and scaling on Image.

2. Image Enhancement:-

It is simple and most appealing area of DIP. Using this step interesting features of an image are ~~highlighted~~. highlight.

e.g:. Improve the contrast of image because image looks better.

3. Image Restoration:-

It improve the appearance of an image. It is used to apply mathematical & probabilistic models of image degradation

Enhancement - is subjective ~~and~~
Restoration - is objective
like color model or ~~wavelets~~ ~~of~~ image

4. Image Compression:-

- Used to reduce the size of image (for storage) or bandwidth (for transmit)

5. Image Segmentation:-

Segmentation method is used to divide the image into subparts to get the Region of Interest (ROI).

6. Image Representation and description:-

They use segmented image, like edges of the objects differentiating all objects or boundary of the region (separating one image region from other region)

According to the image representation features are extracted from the image. This feature extraction is called description of the image.

Recognization is applied on the extracted features and identify the object based on its description.

—x—

Components of an Image Processing System

① Image Sensors

To capture a image is called image sensors. It require two elements

(1) Physical device - it's sensitive to the energy radiated by the object

(2) Digitizer:- It convert the o/p of physical sensing device into digital form.

(2) Specialized image processing hardware

This hardware consist of digitizer plus ALU that perform arithmetic and logical operations.

(3) Computer

It's a image processing system and it's a general-purpose computer and can range from a PC to supercomputer.

(4) Software

The software for image processing system consist of specialized modules that perform specific task. Software packages are used to write code using specialized modules.

(5) Mass Storage

Mass storage capability is needed if the image is not compressed.

Three principal categories.

- Computer memory, frame buffer are used for short storage during processing. Frame buffer are special board that store more images and accessed rapidly at video rates.
- On-line storage for relatively fast recall e.g. magnetic disk or optical media.
- Archival storage characterized by frequent acess eg: magnetic tapes and optical disks.

6. Image Displays :-

Display are color TV monitors. These monitors are driven by the o/p of image and graphics display cards.

7. Hard copy

Record images and print them. It include Laser printer, film cameras, heat sensitive device, inkjet printers and digital units.

8. Networking

Used for the transmission of images. It include optical fiber and other broad-band technologies

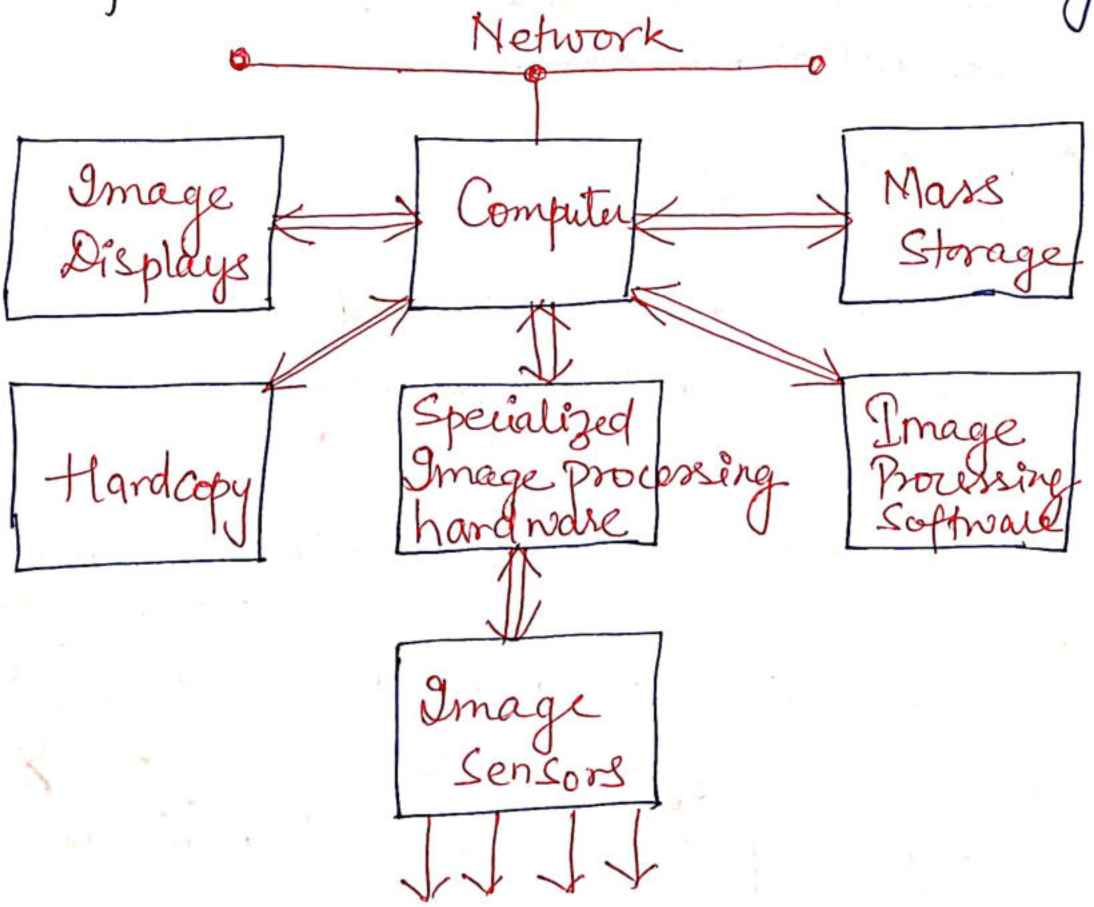
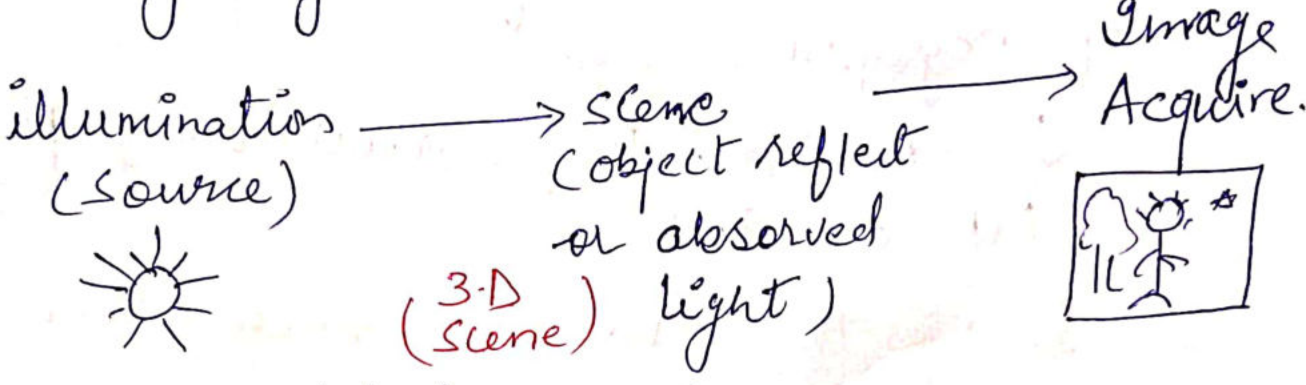


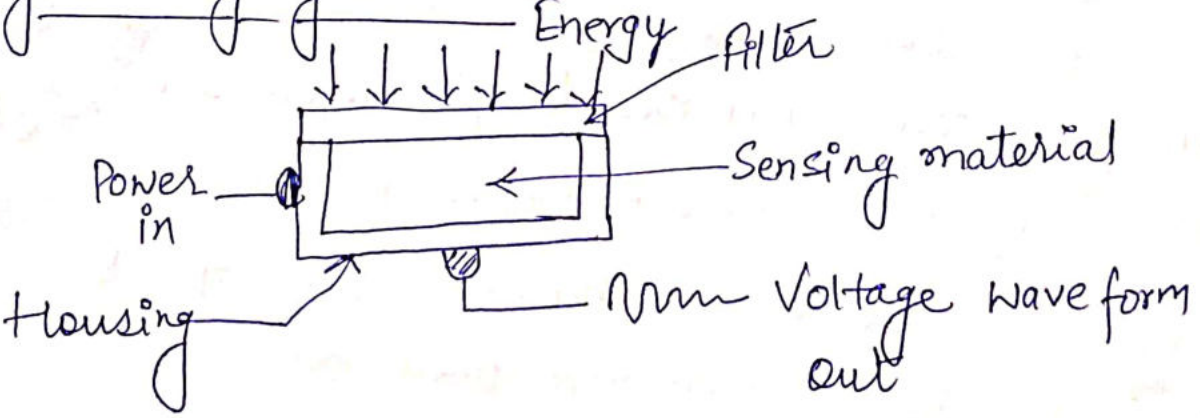
Image Acquisition :-

Image acquisitios and senseing depend on the "illumination" source and reflected or absorvedtion of energy from that source by the elements of the "scene" being imaged



eg:- 1) light reflected from rock
2) X-ray passes through human body

Single imaging Sensor :-

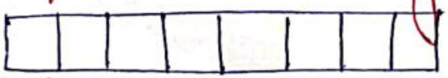


(1) Image Acquisition Using a Single Sensor

Photodiode is the single sensor and constructed by silicon material and its o/p voltage wavelength is proportional to light.

In order to generate 2D image using a single sensor, there has to be relative displacement in both X and Y direction b/w the sensor and the area to be imaged. eg:- In scanner

(2) Image Acquisition using Sensor strips:-



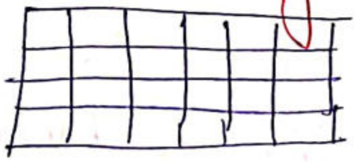
Inline sensors

By using In-line sensor, two dimension images can be captured at a time.

eg:- 1) Sensor strip on aircraft can capture the image of geographical area.

2) In CAT (computerized axial tomography) MRI and PET

(3) Image Acquisition using sensor arrays:-



Used in CCD (chareged-coupled device) array, which ~~or~~ can be manufactured with a broad-range of sensing properties and can be packaged in rugged array of 4000 x 4000 elements or more.

→ Image is a two-dimensional fun^c $f(x,y)$

- It must be nonzero and finite so

$$0 < f(x,y) < \infty$$

- $f(x,y)$ is characterized by two components
  - (1) illumination of source $i(x,y)$
  - (2) amount of illumination reflected $r(x,y)$

so,

$$f(x,y) = i(x,y)\, r(x,y)$$

where $0 < i(x,y) < \infty$ and

$0 < r(x,y) < 1$

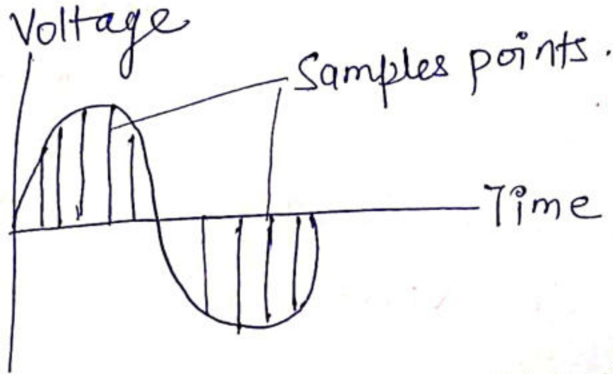
(total absorption) (total reflected)

Conversion of Analog signals to digital Signal ⑤

The two main concept are

(1) Sampling — digitization of co-ordinate value
(2) Quantization - digitization of amplitude value

Sampling :-



Sampling - take sample, over x axis. or conversion of x-axis to digital is done under sampling

-If range of value on x-axis are less then increase the sample of values. → up sampling and its viceversa is ~~ad~~ known as down sampling.

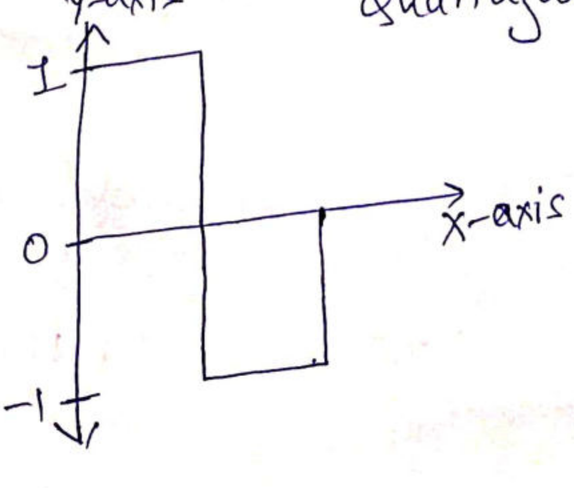
Quantization :-

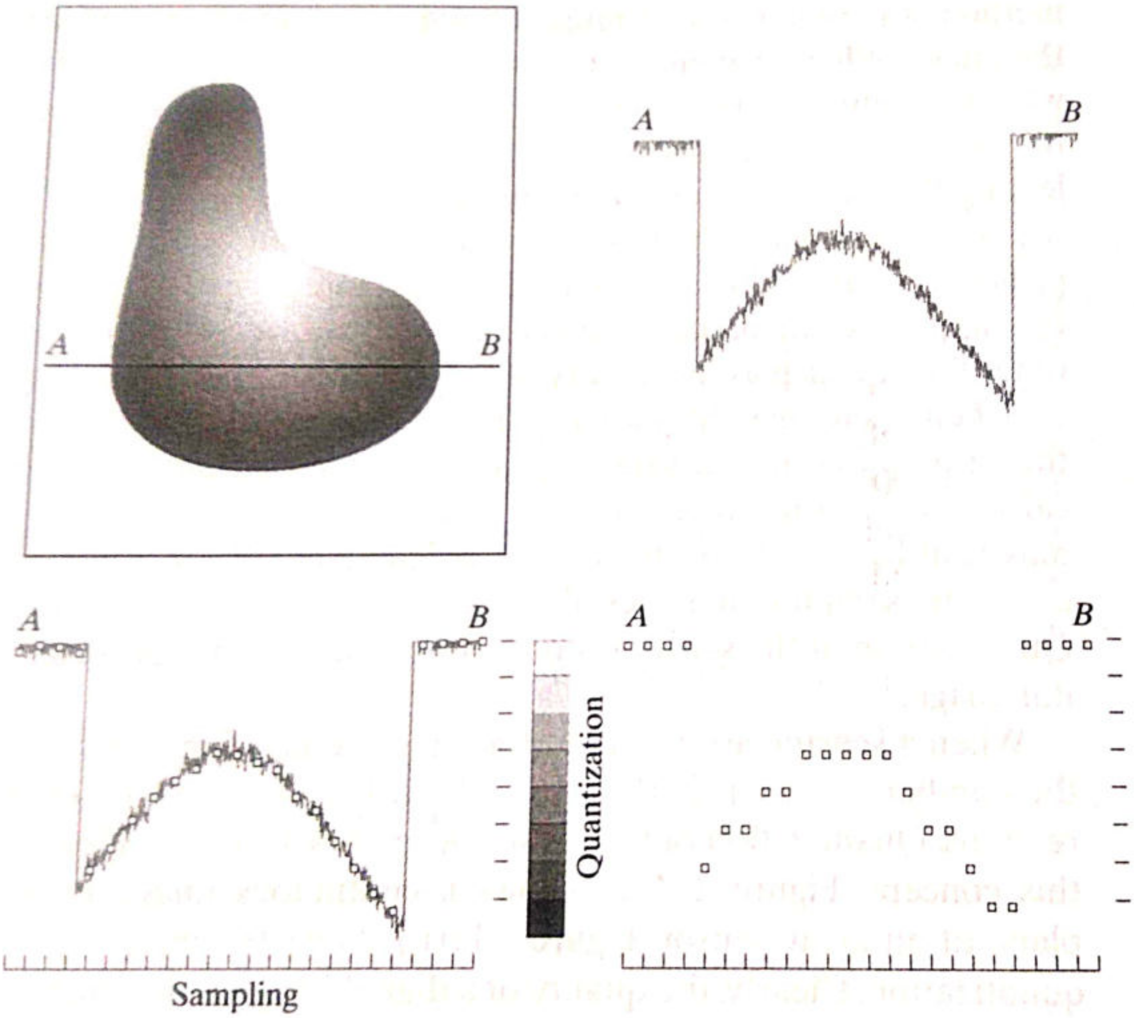
divide into quanta (partitions)

eg:- $y = \sin(x)$

Quantization is done on Y-variable. It is done on y-axis.

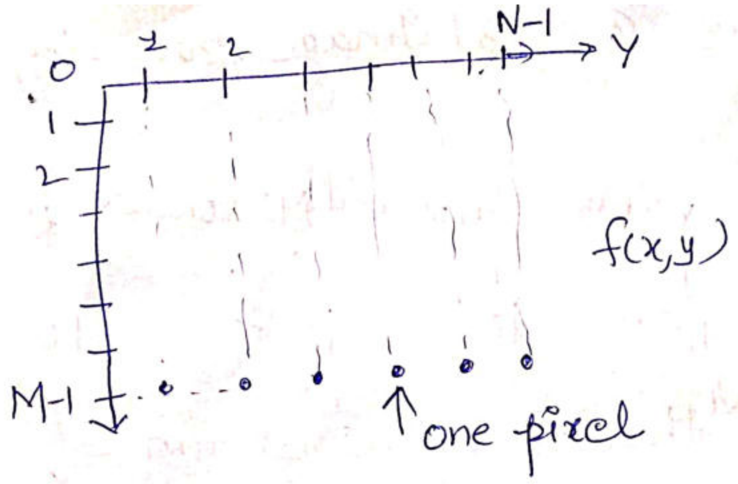
Quantization (1, 0, -1)





a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.



$$f(x,y) = \begin{bmatrix} f(0,0) & \cdots & f(0,N-1) \\ \vdots & & \vdots \\ f(M-1,0) & \cdots & f(M-1,N-1) \end{bmatrix}$$

If sampling freq is increased → o/p image have pleasing effect.

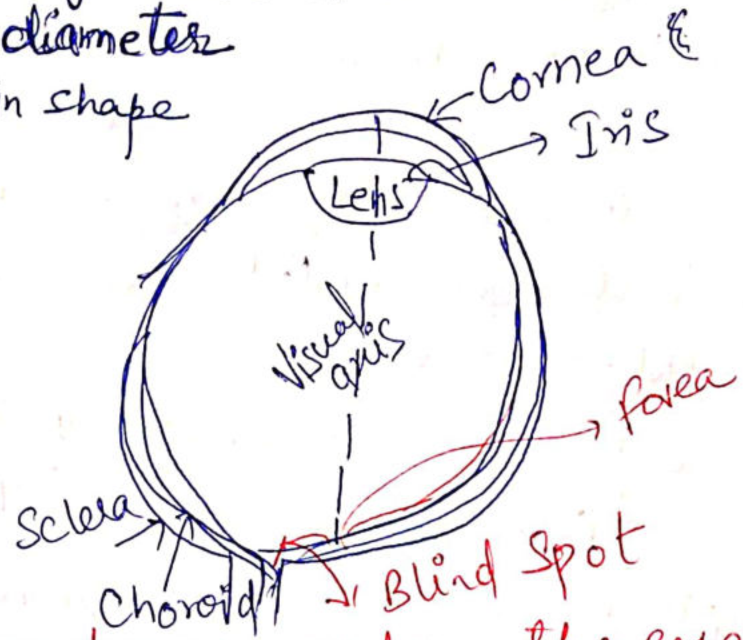
Q1:- What is meant by image sensing? Explain the construction and operation of various image acquisition devices.

Q2: Write the difference b/w analog and digital signal (in term of analysis, representation, accuracy, storage, subject to noise, eg.)

Elements of Visual Perception

① Human Eye & It's structure

- 20mm diameter
- Sphere in shape



Three membrances enclose the eye

① Cornea & Sclera (outer cover)
② Choroid
③ retina

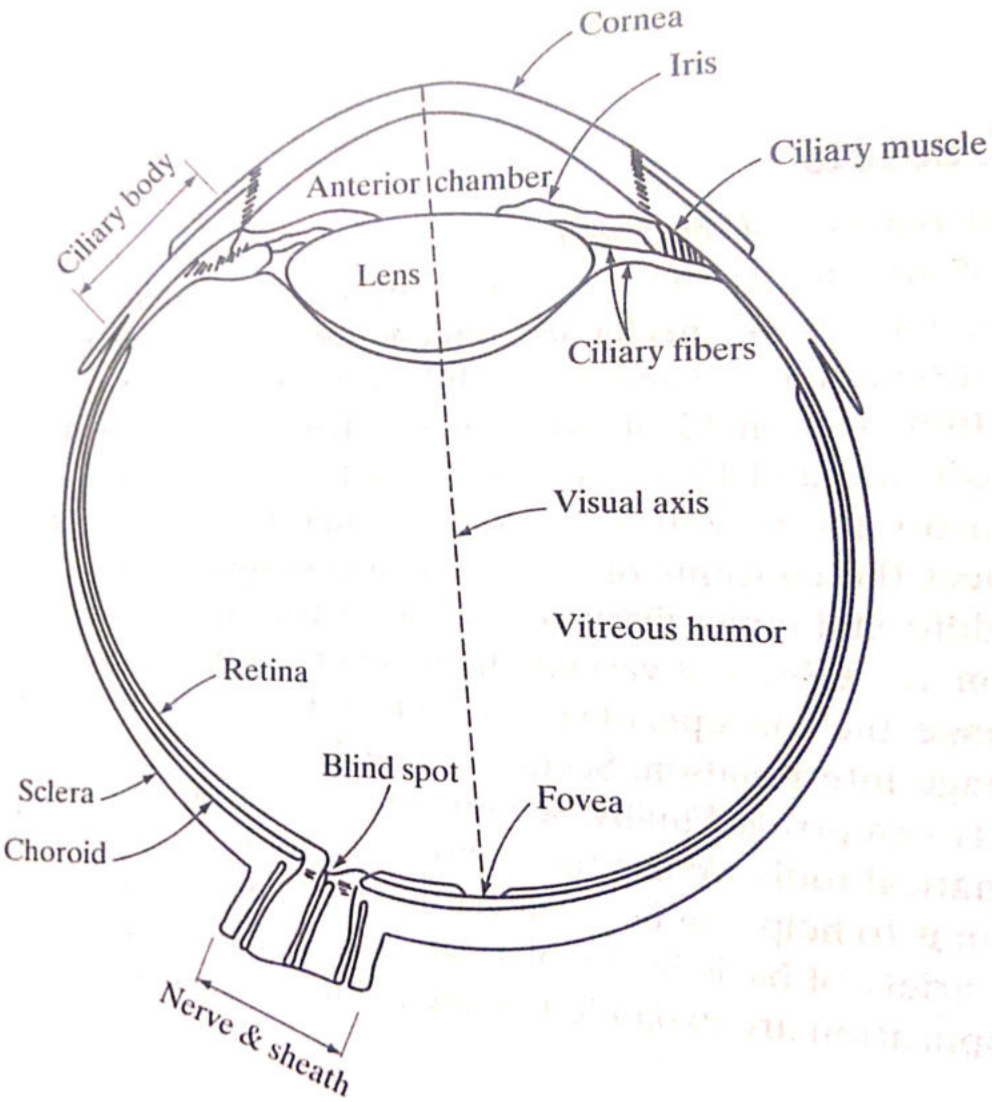
→ tough, transparent tissue that cover anterior surface of the eye

Sclera:- The sclera is continuation of cornea and also covers remain portion of the optic globe.

→ Choroid (below sclera): contains n/w of blood vessels. The major source of nutrition to the eye is divided into ciliary body and iris diaphragm. The Iris diaphragm is used to control the amount of light enter into the eye.

Lens :- The lens is made up of fibrous cell. It contains 60 to 70% water about 7% fat and more protein. It is colored with a slightly yellow pigmentation.

FIGURE 2.1 Simplified diagram of a cross section of the human eye.



Inner most membrance of the eye is the retina.

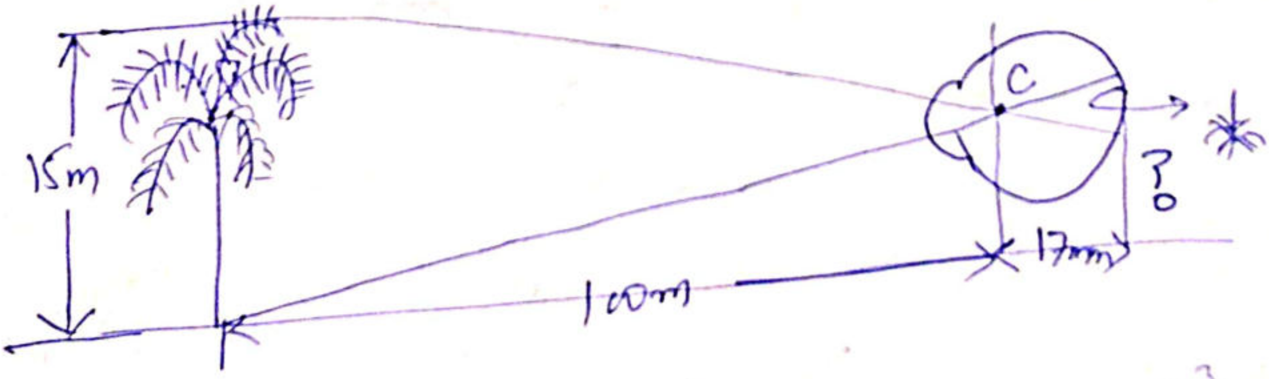
Retina:– The retina consist of two type of light receptors – cones and rods

Cones:– There are 6 to 7 milion no. of cones in each eye. The cones are located at the central portion of the eye (retina) called fovea and it highly sensitive to color. Each Cone connected to its nerve end. The cone vision is called photopic or bright-light vision.

Rods:– There are 75 to 150 milion distributed over retinal surface. The rods are connected to a single nerve, hence not involved in color vision. These are sensitive to low levels of illumination. Rod vision is called scotopic or dim-light vision.

Image formation in the Eye :–

17 mm = center of lens and retina



$$\frac{100\,\cancel{m}}{15\,\cancel{m}} = \frac{17\ mm}{x}$$

$$\Rightarrow x = \frac{17\times15}{100}$$

$$\Rightarrow x = 2.55mm$$

3
17×15
85
17×
255

Color image Representation

Types of Images:→

① Binary Image:→ 0 & 1 — Monochrome
↓ 0 = Black, ↓ 1 = White

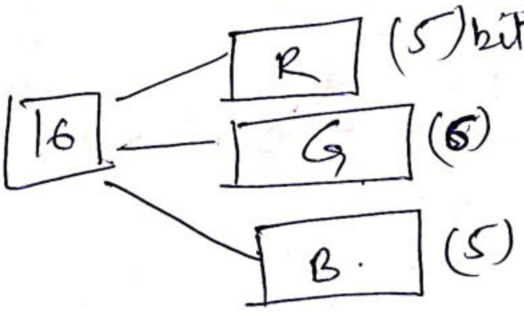
② 8 bit color format:-

$2^8 = 256$ Colors

0 --- 127 --- 255
↓ 0 = Black Color, ↓ 127 = gray color, ↓ 255 = White color

③ 16-bit color

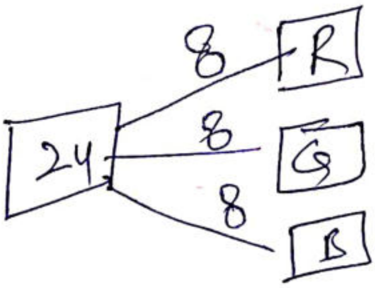
$2^{16} = 65,536$ — High color format



— 1 bit assign ~~to~~ green color because it is most soothing to eyes.

(4) 24-bit Color

True color



Black (0,0,0)
White (255, 255, 255)
Red (255, 0, 0)
Green (0, 255, 0)
Blue (0, 0, 255)
Grey (128, 128, 128)

(5) CMYK Color model –

C - cyan (0,255,255)
(255,0,255) M - magenta.
Y - yellow (255,255,0)
K - black.

in printers
two carters
CMY
Black.

Conversion

RGB → to Hex

(1) Take color (255, 255, 255) white
(2) Take first portion 255
(3) Divide by 16

16 | 255 | 15 → F
16
95
80
15 → F

FF

(4) apply same process for remaining two parts

$\boxed{\text{FF FF FF}}$ Ans

Hex to RGB

(1) Take Hex no FFFFFF
(2) Break it into three part

F F
(1111) (1111) = 256

Answer (255, 255, 255)

RGB to Gray

Two methods
- (1) Average method
- (2) Weighted method or luminosity method.

Average Method :-

$$\text{Gray image} = (R+G+B)/3$$

Outcome is not as expected, because three different colors have different wavelength, so we have to take average according to their contribution.

Weighted method or luminosity method . -

- Red color has more wavelength of all three
- Green less wavelength but also soothing effect to eyes.

Decrease contribution of Red color and increase contribution of green color.

$$\text{New gray image} = (\cdot 3 * R) + (0.59\, G) + (\cdot 11$$

mean 30% of Red
59% of Green
11% of Blue

Sampling & Quantization relationship with Pixels:-

- If we are doing sampling only, the signal is not converted into digital format.
- Quantization is also needed.

if 5 row and 5 column

Pixel (total no) = total no. of row × total no. of column

25 = 5×5

Total pixel = 25

In CCD

- no of pixels equal to the no. of pixels.

sampling
- up sampling (over sampling) or Zooming
- down sampling

Q: What is the difference b/w sampling & zooming?

Sampling on signals

Zooming on the digital images.

Resolution:- n×m

if resolution high ↑ Quality of image ↑ high

Mega pixel

m×n / 1 milion

Size of image = no. of pixel × $\boxed{bpp}$ bits/pixel.

eg:. Image dimension = 2500×3192

$$\text{Resolution} = \frac{\cancel{2500}\times\overset{\overset{798}{\cancel{1596}}}{\cancel{3192}}}{\underset{\underset{\underset{100}{\cancel{200}}}{\cancel{400}}}{\cancel{1000000}}} = 7.98 \cong 8 \text{ mega Pixel}$$

Aspect Ratio:-

Another important concept with the pixel resolution is aspect ratio.

$$\text{aspect ratio} = \frac{\text{width of image}}{\text{length of image}} = \frac{c}{r}$$

like 1.33:1, 1.66:1, 2.00:1 etc.

Eg:- Aspect ratio = 6:2

Pixel resolution = 480000

Bits per pixel grayscale image = 8 bpp

find: Number of rows? Size of

Number of column?

Ans:: $A.R = \frac{6}{2} = \frac{C}{R} \Rightarrow C = \frac{6R}{2}$ —①

Pixel resolution = 480000 = R×C

$\Rightarrow C = \frac{480000}{R}$ —②

Using ① & ②

$$\frac{6R}{2} = \frac{48000}{R}$$

$$R^2 = \frac{\overset{8}{\cancel{48}}0000\times 2}{\cancel{6}}$$

$$R = \sqrt{2\times2\times2\times2\times100\times100} = 2\times2\times100 = 400$$

$$C = \frac{6}{2} \times R$$

$$= \frac{6 \times \cancel{400}^{200}}{\cancel{2}} = 1200$$

Size of image = r×c×bpp

$$= 400 \times 1200 \times 8 \text{ bits}$$

$$= \frac{480000 \times \cancel{8}}{\cancel{8\text{-}00000}} \text{ bytes}$$

$$= \frac{480000}{1024} \text{ KB}$$

$$\cong 48 \text{ (Kb)}$$

* * *

Zooming — (Two type of zooming)

(1) Optical Zoom	(2) Digital Zoom
(Before taking an particular image)	(After an image hes been captured.
- known as pre-processing Zoom	- known as processing of image
- True zoom	- pixels get expand
- Camera lens is physically. extended to zoom or magnify an object	- Quality of image compromised.
	- Different algorithms as applied to make zoom of image.

Zooming methods:-

(1) Pixel replication or (~~neighbo~~ nearest neighbor interpolation)
(2) Zero order hold method
(3) Zooming k times

(i) Pixel replication:-

- As name suggest, just replicate the neighboring pixels.
- eg: Suppose we have image 2x2

1	2
3	4

(2x2)

Zoom in →
row wise
(2 times

1	1	2	2
3	3	4	4

↓

1	1	2	2
1	1	2	2
3	3	4	4
3	3	4	4

(4

x 2 times Zoom → (4x4)

Advantage:- Very simple

Disadvantage:- output image get blur and if zooming factor increased, the image got more and more blurred.

(ii) Zero Order hold

It's known as zoom twice (i) Pick two adjacent elements from the rows respectively, add them and divide the result by two (ii) Place the result in b/w those two elements.

eg:- Image (2×2)

1	2
3	4

Row wise zooming

1	1	2
3	3	4

$(2+1) = 3/2 = 1.5 \simeq 1$

$(4+3) = 7/2 = 3.5 \simeq 3$

Column wise zooming

1	1	2
2	2	3
3	3	4

$(1+3) = 4/2 = 2$

$(4+2) = 6/2 = 3$

Size of image:- $(2(\text{no. of row}) - 1) \times (2(\text{no. of colu}) - 1)$

$= (2\times2-1) \times (2\times2-1) = 3\times3$

Advantage:- Does not create blurry image as compare to nearest neighbor interpolation method

Disadvantage:- It only run on the power of 2. Like if we have image of 2×2 and want to zoom it 6 times then

New image size = $(6\times2-1)\times(6\times2-1)$
$= 11\times11$

Now after applying this method

$2\times2 \rightarrow 3\times3 \rightarrow 5\times5 \rightarrow 9\times9$
$\downarrow$
17×17

So, you are not getting the dimensions accordingly.

(iii) k-Times Zooming: —

- One of the perfect zooming algorithm

eg:- Steps: (i) Take two adjacent pixel and then subtract smaller from the greater one (OP)

(ii) Divide the output (OP) with zooming factor (k). Now, add result in the smaller value in between the adjacent pixel.

(iii) Repeat step (ii) for place (k-1) values. Add OP in the current calculated value.

(iv) Repeat all steps for all rows and colums

(v) get zoomed image.

Row wise Zooming

eg:-

15	30	15
30	15	30

Zoom three times
K=3

(i) 15 30 = (30-15) = 15/3 = 5 (OP) same for ~~col~~ 2 and3

30 15 = (30-15) = =15/3 = 5 (OP)

Adjacent pixel

(iii) now add 5 into 15, because 15 is the smaller value.

	k=1	k=2		k=1	k=2	
15	20	25	30	20	25	15
30	20	25	15	20	25	30

add OP to last value

Add OP to smaller value = 15 + 5 = 20

(iV) Repeat same procedure for last two pixel values.

So, Row wise zooming is present

15	20	25	30	20	25	15
30	20	25	15	20	25	30

After inserting the values sort the inserted values in ascending order according to smaller value. Start sorting from smaller value

So, New table

15	20	25	30	25	20	15
30	25	20	15	20	25	30

Column wise Zooming :-

(i) Calculate the different b/w adjacent pixel value

	(30-15)	(25-20)	(25-20)	(30-15)	(25-20)	(25-20)	(30-15)
	15/3	5/3	5/3	15/3	5/3	5/3	15/3
OP	5	1	1	5	1	1	5

New table

	15	20	25	30	25	20	15
k=1	20	21	21	25	21	21	20
k=2	25	22	22	20	22	22	25
	30	25	20	15	20	25	30

New image size:-

Old image size = 2×3

New image size = 4×7

So

formula = $(K(\text{no. of row} - 1) + 1) \times (K(\text{no. of colum} - 1) + 1)$

$= (3 \times (2-1)+1) \times (3 \times (3-1)+1)$

$= (3 \times 1 + 1) \times (3 \times 2 + 1)$

$= \boxed{4 \times 7}$

Advantage:- It gives improved result and comprises the power of the two algorithms.

Disadvantage:- In the end, sort the pixel value, which increase the cost of computation.

Basic Relationship b/w Pixels:-

An image is denoted by f(x,y). To represent a pixel, we are using term p and q.

① Neighbors of a Pixel:-

A pixel p at co-ordinates (x,y) has fous horizontol and vertical neighbors.

	(x,y-1)	
(x-1, y)	P(x,y)	(x+1,y)
	(x, y+1)	

- This set of pixels, called the 4-neighbors of p, is denoted by $N_4(p)$.
- Each pixel is a unit distance from (x,y)

• If four diagonal neighbors of p have coordinate

(x-1, y-1)		(x+1, y-1)
	p (x,y)	
(x+1, y-1)		(x+1, y+1)

• So, denoted by $N_D(P)$

• These fous and four points together are called 8-neighb
of p, denoted by $N_8(P)$.

② Adjacency, Connectivity, Regions and Boundaries

Let V be the set of intensity values used to define adjacency. In a binary image V = {1}, if adjancy of pixels with value 1. for gray image V could be {0, 255}

<u>Three type of adjacency</u> :-

(1) **4-adjacency** :- Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

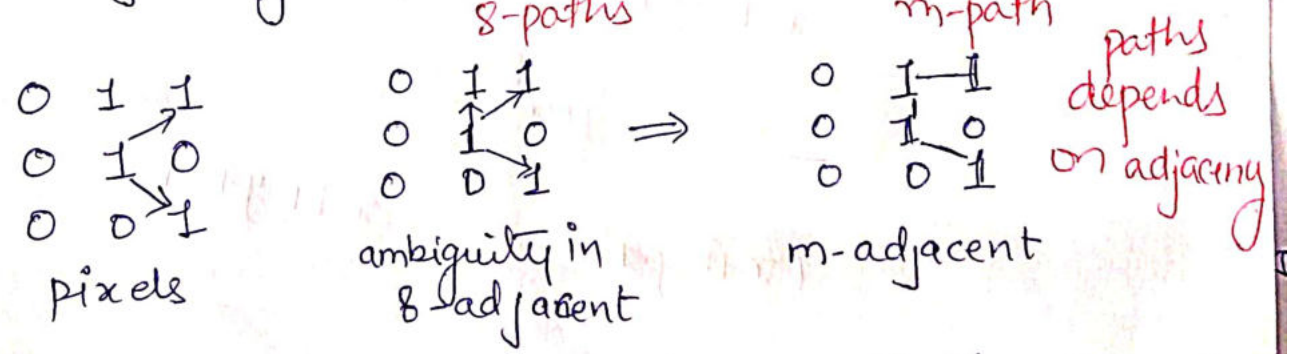
(2) **8-adjacency** :- Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

(3) **(mixed) m-adjacency** :- Two pixels p and q with values from V are m-adjacent if

(i) q is in $N_4(p)$, or

(ii) q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

- Mixed adjacency is modified of 8-adjancency



pixels   ambiguity in 8 adjacent   m-adjacent

- A (digital) Path from pixel p with coordinates (x, y) to pixel q with coordinate (s, t) is a sequence of distinct pixel with coordinates where $(x_0, y_0) = (x, y)$ & $(x_n, y_n) = (s, t)$

$(x_0, y_0), (x_1, y_1) \text{ ---- } (x_n, y_n)$ { n is the length of path.

Closed Path if $(x_0, y_0) = (x_n, y_n)$

②

Let S represents a subset of pixels in an image. Two pixels p and q are said to be connected in S if there exist a path b/w them consisting entirely of pixels in S. (connected set)

for 4-adjacency these regions are not connected set

$$\left.\begin{array}{ccc}1&1&1\\1&0&1\\0&1&0\end{array}\right\}R_i$$

$$\left.\begin{array}{ccc}0&0&1\\1&1&1\\1&1&1\end{array}\right\}R_j$$

✓ 8-adjacency {connected set}

- Two regions R_i and R_j are said to be adjacent if their union forms a connected set.
- Regions that are not adjacent are said to be disjoint.

Boundaries:-

eg:-

$$\begin{array}{ccccc}0&0&0&0&0\\0&1&1&0&0\\0&1&1&0&0\\0&1&\textcircled{1}&1&0\\0&1&1&1&0\\0&0&0&0&0\end{array}$$

Suppose image have K regions R_k.

$k = 1, 2, 3, \ldots K$

Ru = union of all K regions — foreground

$(Ru)^c$ = ~~of~~ Complement of Ru → Background.

Complement

- The boundary (contour) of a region R is the set of points that are adjacent to points in the complement of R.
- see in above example - for $\boxed{\text{4-adjacent}}$

Distance Measures:-

for pixels p, q and z with co-ordinates
(x,y) (s,t) (v,w) respectively, D is a
distancefun^c or metric if

(i) $D(p,q) \geq 0$ $(D(p,q)=0$ iff $p=q)$,
(ii) $D(p,q) = D(q,p)$ and
(iii) $D(p,z) \leq D(p,q) + D(q,z)$

The Euclidean distance b/w p and q is defined as

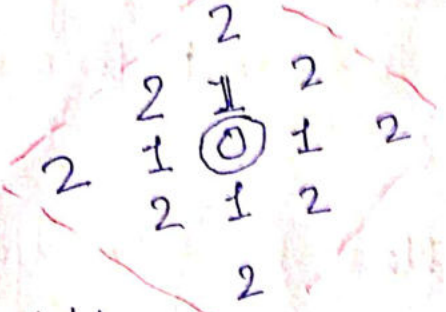
$$D_e(p,q) = \left[(x-s)^2 + (y-t)^2\right]^{1/2}$$

D_4 distance (or city-block distance) b/w p and q

$$D_4(p,q) = |x-s| + |y-t|$$

So, the pixels having a D_4 distance from (x,y) less than or equal to some value r & form a diamond centered at (x,y)

eg:- D_4 distance ≤ 2, (x,y) – centerpoint



		2		
	2	1	2	
2	1	⓪	1	2
	2	1	2	
		2		

if $D_4=1 \rightarrow$ 4 neighbours of (x,y)

- D_8 distance (chessboard distance) b/w p and q ①

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

So, the pixels with D_8 distance form (x,y) less than or equal to some value r form a Square centered at (x,y)

eg. D_8 distance ≤ 2, (x,y) – center point

$$\begin{matrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & 2 & 2 \end{matrix}$$

The pixel $D_8 = 1$ are 8 neighbours of (x,y)

<u>Example</u> –

$p(2,2)$
$q = (1,1)$

	1	2	3
1	q		
2		p	
3			

Euclidian distance = ~~$((2-1)^2 + (2-1)^2)^{1/2}$~~

$$((1-2)^2 + (1-2)^2)^{1/2} = (1+1)^{1/2} = (2)^{1/2}$$

$$D_4 = |1-2| + |1-2| = 2$$

$$D_8 = \max(|1-2|, |1-2|) = 1$$

Example:-

Use D_4 distance

Pixel A: $|2-2| + |1-2| = 1$

Pixel B: $|3-2| + |2-2| = 1$

Pixel C: $|2-2| + |3-2| = 1$

Pixel D: $|1-2| + |2-2| = 1$

	1	2	3
1		d	
2	a	p	c
3		b	

a (2,1)
b = (3,2)
c = (2,3)
d = (1,2)
p = (2,2)

Do:- D_8 distance

Distance Measures:-

- D_m distance: is defined as the shortest m-path b/w the points
- In this case, the distance b/w two pixels will depends on the values of the pixels along the path, as well as the values of their neighbors.

eg:-

Consider the following arrangement of pixels

$$\begin{array}{ccc} & P_3 & P_4 \\ P_1 & P_2 & \\ P & & \end{array}$$

Assume P, P_2 and P_4 have value 1 and P_1 and P_3 can have value 0 or 1

$$\begin{array}{ccc} & 0 & 1 \\ 0 & 1 & \\ 1 & & \end{array}$$

Compute D_m b/w p and P_4?

$V = \{1\}$

Case 1:-
if $P_1 = 0$ & $P_3 = 0$
The length of shortest path m-path is 2 (P, P_2, P_4)

Case 2:
if $P_1 = 1$ and $P_3 = 0$

$$\begin{array}{ccc} & 0 & 1 \\ 1 & 1 & \\ 1 & & \end{array}$$

Then length of the Shortest path 3 (P, P_1, P_2, P_4)

Case3:
if $P_1 = 0$ & $P_3 = 1$

$$\begin{array}{ccc} & 1 & 1 \\ 0 & 1 & \\ 1 & & \end{array}$$

Same 3 (P, P_2, P_3, P_4)

Case4:
if $P_1 = 1$, $P_3 = 1$

$$\begin{array}{ccc} & 1 & 1 \\ 1 & 1 & \end{array}$$

length 4 $(P, P_1, P_2, P_3, P_4$