Spline Representations:

→ In drafting terminology a spline is flexible strip used to produce a smooth curve through a designated set of points.

⇒ Spline curve:- We can mathematically describe such a curve with a piecewise cubic polynomial function whose first and second derivatives are continuous across the various curve sections.

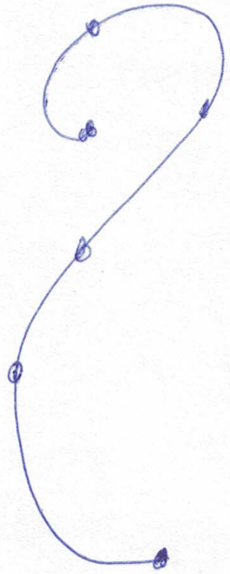
⇒ In computer graphics, the term spline curve now refers to any composite curved formed with polynomial sections satisfying specified continuity conditions at the boundary of pieces.

⇒ A spline surface can be described with two sets of orthogonal spline curves.

⇒ Spline are used in

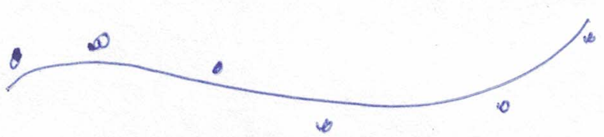
1) design curves & surface shapes to digitize drawings for computer graphics storage.
2) To specify animation path to the objects
3). design of automobile bodies (CAD), aircraft & spacecraft surfaces

⇒ Interpolation & Approximation splines.



1) We specify a spline curve by giving a set of coordinate positions called control points, which indicate the general shape of the curve.

2) These control points are then fitted with piecewise continuous parametric polynomial functions in one of two ways.
- Interpolate
- Approximate



⇒ a) Interpolate: when polynomial sections are fitted, so that the curve passes through each control point.

b) Approximate: – without necessarily passing through each control point.

⇒ Uses: –

Interpolation curves	Approximation
a) used to digitize drawings or to specify animation path	used to structure surfaces

⇒ spline curves can be modified with option on control points

⇒ CAD packages can also insert extra control points to aid a designer in adjusting

⇒ Spline Specifications:

⇒ There are three equivalent methods for specifying a particular spline representation.

1) We can state the set of boundary conditions that are imposed on the spline.
2) We can state the matrix that characterize the spline.
3) We can state the set of blending functions (or basis functions)

$$x(u) = a_x u^3 + b_x u^2 + c_x u + d_x \quad 0 \le u \le 1$$

⇒ Boundary Conditions: $x(0)$, $x(1)$, $x'(0)$ & $x'(1)$

⇒ are sufficient to calculate a_x, b_x, c_x & d_x

⇒ Cubic spline: → ~~are often~~ →

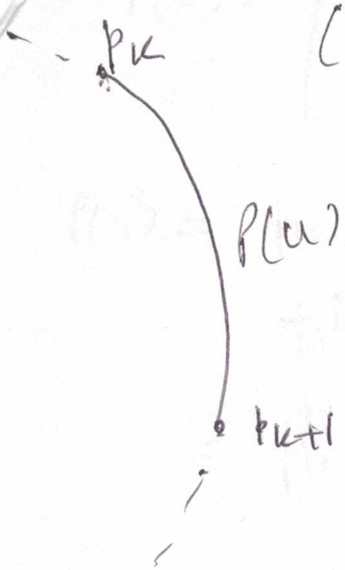
↓

⇒ advantages → 1) Compared to higher order polynomials, cubic splines require less calculations & memory & they are more stable.

⇒ Compared to lower order polynomials, cubic splines are more flexible for modelling arbitrary curve shapes.

Cubic Spline Interpolation method ①

(Hermite Spline) → Interpolation piecewise cubic spline polynomial & each curve section is only dependent on its endpoint constraints.



$P(0) = P_k$

$P(1) = P_{k+1}$

$P'(0) = DP_k$

$P'(1) = DP_{k+1}$

$$P(u) = (x(u), y(u), z(u))$$

$$P(u) = au^3 + bu^2 + cu + d \quad 0 \le u \le 1$$

$$P(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$P(u) = U \cdot C$

$$P'(u) = \begin{bmatrix} 3u^2 & 2u & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$C = M_{spline} \cdot M_{geom}$

M_{spline} → 4x4

M_{geom} → 4-element col. matrix

$$\begin{bmatrix} P_k \\ P_{k+1} \\ DP_k \\ DP_{k+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} P_k \\ P_{k+1} \\ DP_k \\ DP_{k+1} \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_k \\ P_{k+1} \\ DP_k \\ DP_{k+1} \end{bmatrix}$$

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$$P(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \cdot M_{spline} \cdot M_{geom}$$

$$P(u) = p_k(2u^3 - 3u^2 + 1) + p_{k+1}(-2u^3 + 3u^2) + Dp_k(u^3 - 2u^2 + u) + Dp_{k+1}(u^3 - u^2)$$

$$= p_k H_0(u) + p_{k+1} H_1(u) + Dp_k H_2(u) + Dp_{k+1} H_3(u)$$

$$P(u) = \sum_{k=0}^{3} g_k \cdot BF_k(u)$$

Constant Parameters.

Polynomial Blending function.