UNIT-2

PULSE ANALOG MODULATION

8.1. INTRODUCTION

As we know that broadly, there are two types of signals, continuous time signal and discrete-time signals. Due to some recent advance development in digital technology over the past few decades, the inexpensive, light weight, programmable and easily reproducible discrete-time systems are available. Therefore, the processing of discrete-time signals is more flexible and is also preferable to processing of continuous-time signals.

This means that in practice, although we have a large number of continuous-time signals, but we prefer processing of discrete-time signals. For this purpose we should be able to convert a continuous-time signal into discrete-time signal.

This problem is solved by a fundamental mathematical tool known as sampling theorem. The sampling theorem is extremely important and useful in signal processing. With the help of sampling theorem, a continuous-time signal may be completely represented and recovered from the knowledge of samples taken uniformly. This means that sampling theorem provides a mechanism for representing a continuous-time signal by a discrete-time signal. Therefore, sampling theorem may be viewed as a bridge between continuous-time signals and discrete-time signals.

The concept of sampling provides a widely used method for using discrete-time system technology to implement continuous-time systems and process the continuous-time signals. We utilize sampling to convert a continuous time signal to a discrete-time signal, process the discrete-time signal using a discrete-time system and then convert back to continuous-time signals.

8.2. THE SAMPLING THEOREM

(MD University, Rohtak, 2003)(04 marks)

As discussed earlier, sampling of the signals is the fundamental operation in signal-processing. A continuous time signal is first converted to discrete-time signal by sampling process. The sufficient number of samples of the signal must be taken so that the original signal is represented in its samples completely. Also, it should be possible to recover or reconstruct the original signal completely from its samples. The number of samples to be taken depends on maximum signal frequency present in the signal. Sampling theorem gives the complete idea about the sampling of signals. Different types of samples are also taken like ideal samples, natural samples and flat-top samples.

> **DO YOU KNOW?**
>
> Prior to sampling, a low-pass anti-aliasing filter is used to altenuate those high frequency components of the signal that are not essential to the information being conveyed by the signal.

Let us discuss the sampling theorem first and then we shall discuss different types of sampling processes. The statement of sampling theorem can be given in two parts as:

(*i*) A band-limited signal of finite energy, which has no frequency-component higher than f_m *Hz*, is completely described by its sample values at uniform intervals less than or equal to $\frac{1}{2f_m}$ second apart.

(*ii*) A band-limited signal of finite energy, which has no frequency components higher than f_m *Hz*, may be completely recoverd from the knowledge of its samples taken at the rate of $2\,f_m$ samples per second.

The first part represents the representation of the signal in its samples and minimum sampling rate required to represent a continuous-time signal into its samples.

The second part of the theorem represents reconstruction of the original signal from its samples. It gives sampling rate required for satisfactory reconstruction of signal from its samples.

Combining the two parts, the sampling theorem may be stated as under :

"A continuous-time signal may be completely represented in zits samples and recovered back if the sampling frequency is $f_s \geq 2\,f_m$. Here f_s is the sampling frequency and f_m is the maximum frequency present in the signal".

8.3. PROOF OF SAMPLING THEOREM

(Important)

To prove the sampling theorem, we shall show that a signal whose spectrum is band-limited to f_m *Hz*, can be reconstructed exactly without any error from its samples taken uniformly at a rate $f_s > 2\,f_m Hz$.

Let us consider a continuous time signal $x(t)$ whose spectrum is band-limited to $f_m Hz$. This means that the signal $x(t)$ has no frequency components beyond $f_m Hz$. Therefore, $X(w)$ is zero for $|w| > w_m$, *i.e.*,

$$X(w) = 0 \text{ for } |w| > w_m$$

where $$w_m = 2\pi\,f_m$$

Figure 8.1 (*a*) shows this continuous-time signal $x(t)$. Let $X(w)$ represents its Fourier transform or frequency spectrum as shown in figure 8.1(*b*). Sampling of $x(t)$ at a rate of $f_s Hz$ (f_s samples per second) may be achieved by multiplying $x(t)$ by an impulse train $\delta_{T_S}(t)$. The impulse train $\delta_{T_S}(t)$ consists of unit impulses repeating periodically every T_s seconds, where $T_s = 1/f_s$. Figure 8.1(c) shows this impulse train. This multiplication results in the sampled signal $g(t)$ shown in figure 8.1(e).

This sampled signal consists of impulses spaced every T_s seconds (the sampling interval).

The resulting or sampled signal may be written as

$$g(t) = x(t)\ \delta_{T_S}(t) \quad \text{...(8.1)}$$

Again, since the impulse train $\delta_{T_S}(t)$ is a periodic signal of period T_s, it may be expressed as a Fourier series.

The trigonometric Fourier series expansion of impulse-train $\delta_{T_s}(t)$ is expressed as

$$\delta_{T_S}(t) = \frac{1}{T_s}\,[1 + 2\cos w_s t + 2\cos 2\,w_s t + 2\cos 3\,w_s t + \ldots] \quad \text{...(8.2)}$$

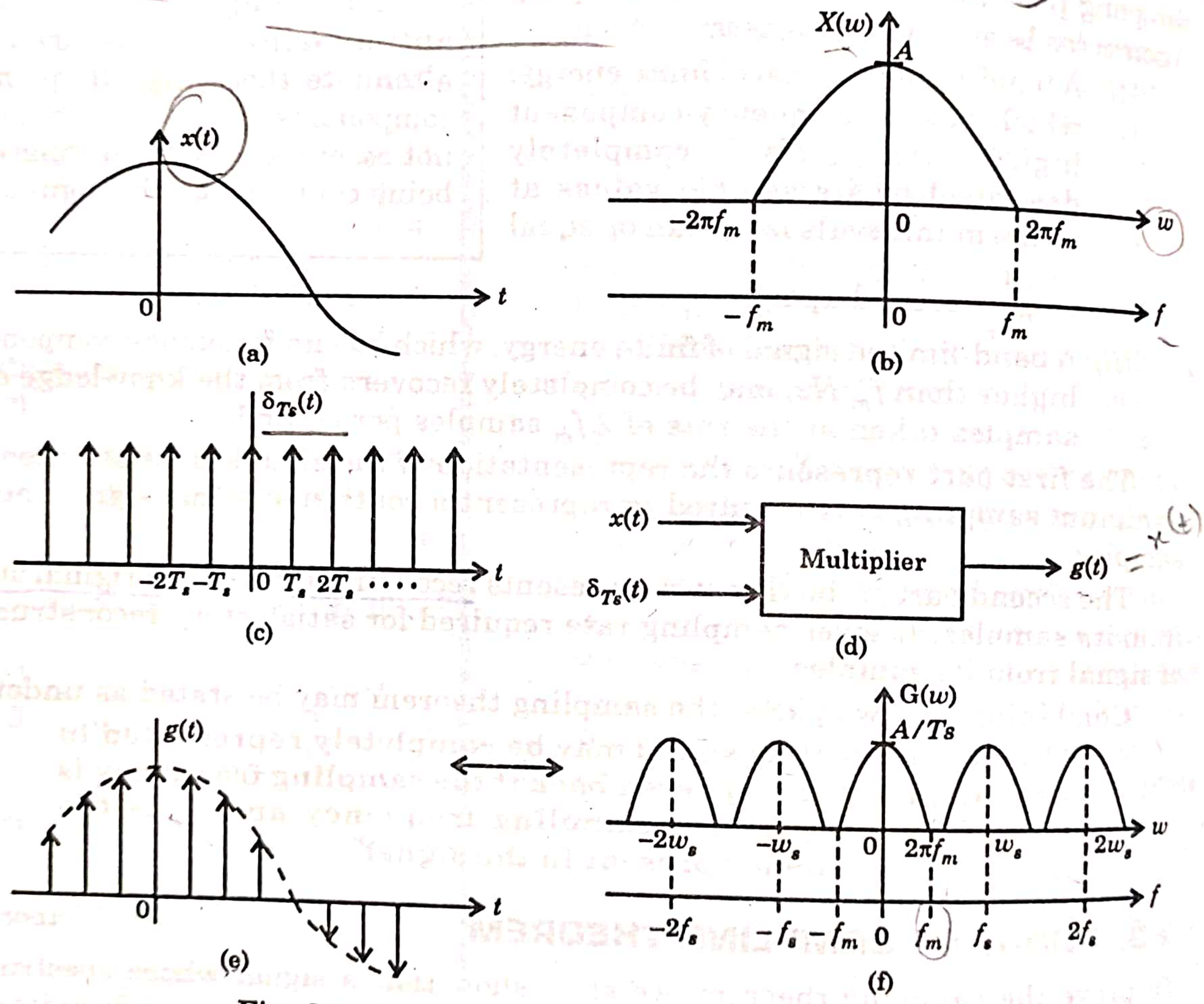


Fig. 8.1. *(a) A continuous-time signal.*
(b) Spectrum of continuous-time signal.
(c) Impulse train as sampling function.
(d) Multiplier.
(e) Sampled signal.
(f) Spectrum of sampled signal.

Here

$$w_s = \frac{2\pi}{T_s} = 2\,\pi f_s$$

Putting the values of $\delta_{T_s}(t)$ from equation (8.2) in equation (8.1), the sampled signal is

$$g(t) = \frac{1}{T_s}\,[x(t) + 2\,x(t)\cos w_s t + 2x(t)\cos 2\,w_s t + 2x(t)\cos 3\,w_s t + \ldots] \qquad \ldots(8.3)$$

Now, to obtain $G(w)$, the Fourier transformation of $g(t)$, we will have to take the Fourier transform of right hand side.

Fourier transform of $x(t)$ is $X(w)$.

Fourier transform of $2x(t)\cos w_s t$ is $[X(w - w_s) + X(w + w_s)]$.

Fourier transform of $2x(t)\cos 2\,w_s t$ is $[X(w - 2w_s) + X(w + 2w_s)]$ and so on.

Therefore, on taking Fourier transformation, the equation (8.3) becomes

$$G(w) = \frac{1}{T_s}\,[X(w) + X(w - w_s) + X(w + w_s) + X(w - 2\,w_s) + X(w + 2\,w_s) + X(w - 3\,w_s) + X(w + 3\,w_s) + \ldots] \qquad \ldots(8.4)$$

or

$$G(w) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(w - n w_s) \qquad \ldots(8.5)$$

From equations (8.4) and (8.5), it is clear that the spectrum $G(w)$ consists of $X(w)$ repeating periodically with period $w_s = \frac{2\pi}{T_s}$ rad/sec. or $f_s = \frac{1}{T_s}$ Hz as shown in figure 8.1 (*f*).

Now if have to reconstruct $x(t)$ from $g(t)$, we must be able to recover $X(w)$ from $G(w)$. This is possible if there is no overlap between successive cycles of $G(w)$. figure 8.1 (*f*) shows that this requires

$$f_s > 2\,f_m \qquad \ldots(8.6)$$

But the sampling interval $T_s = \frac{1}{f_s}$

$$\text{Hence, } T_s < \frac{1}{2\,f_m} \qquad \ldots(8.7)$$

Therefore, as long as the sampling frequency f_s is greater than twice the maximum signal frequency f_m (signal, bandwidth, f_m), G(w) will consist of non-overlapping repetitions of X(w). If this is true, figure 8.1 (f) shows that x(t) can be recovered from its samples g(t) by passing the sampled signal g(t) through an ideal law-pass filter of bandwidth f_m Hz. This proves the sampling theorem.*

> **DO YOU KNOW?**
>
> Samples must be taken fast enough in order for high-frequency components to be recognized and adequately represented.

8.3.1. Few Points About Sampling Theorem

(*i*) Figure 8.1 (*f*) shows the spectrum of sampled signal. According to the figure, as long as, the signal is sampled at rate $f_s > 2\,f_m$, the spectrum $G(w)$ will repeat periodically without overlapping.

(*ii*) The spectrum of sampled signal extends upto infinity and the ideal bandwidth of sampled signal is infinite. But here our purpose is to extract our original spectrum $X(w)$ out of the spectrum $G(w)$.

(*iii*) The original *or* desired spectrum $X(w)$ is centred at $w = 0$ and is having bandwidth *or* maximum frequency equal to w_m. The desired spectrum may be recovered by passing the sampled signal with spectrum $G(w)$ through a low pass filter with cut-off frequency w_m. This means that since a low-pass filter allows to pass only low frequencies up to cut-off frequency (w_m) and rejects all other higher frequencies, the original spectrum $X(w)$ extended upto w_m will be selected and all other successive higher frequency cycles in the sampled-spectrum will be rejected. Therefore, in this way, original spectrum $X(w)$ will be extracted out of spectrum $G(w)$. This original spectrum $X(w)$ can now be converted into time-domain signal $x(t)$.

(iv) It may also be observed from figure that for the case $f_s > 2f_m$, the successive cycles of $G(w)$ are not overlapping each other. Hence in this case, there is no problem in recovering the original spectrum $X(w)$.

(v) For the case $f_s = 2\,f_m$, although the successive cycles of $G(w)$ are not overlapping each other, but they are touching each other. In this case also, the original spectrum $X(w)$ can be recovered from the sampled spectrum $G(w)$ using a low-pass filter with cut-off frequency w_m.

(vi) For the case $f_s < 2\,f_m$, the successive cycles, of the sampled spectrum will overlap each other and hence in this case, the original spectrum $X(w)$ cannot be extracted out of the spectrum $G(w)$.

Hence, for reconstruction without distortion, we must have

$$f_s \geq 2\,f_m$$

8.4. NYQUIST RATE AND NYQUIST INTERVAL

(Important)

When the sampling rate becomes exactly equal to $2\,f_m$ samples per second, then it is called Nyquist rate. Nyquist rate is also called the *minimum* sampling rate. It is given by

$$f_s = 2\,f_m \quad ...(8.8)$$

Similarly, maximum sampling interval is called *Nyquist interval.* It is given by

$$\text{Nyquist Interval } T_s = \frac{1}{2f_m} \text{ seconds} \quad ...(8.9)$$

When the continuous-time band-limited signal is sampled at Nyquist rate ($f_s = 2\,f_m$), the sampled-spectrum $G(w)$ contains non-overlapping $G(w)$ repeating periodically. But the successive cycles of $G(w)$ touch each other as shown in figure 8.2. Therefore, the original spectrum $X(w)$ can be recovered from the sampled spectrum by using a low pass filter with a cut-off frequency w_m.

DO YOU KNOW?

The use of sampling rate higher than the Nyquist rate also has the beneficial effect of easing the design of the reconstruction filter used to recover the original signal from its sampled version.

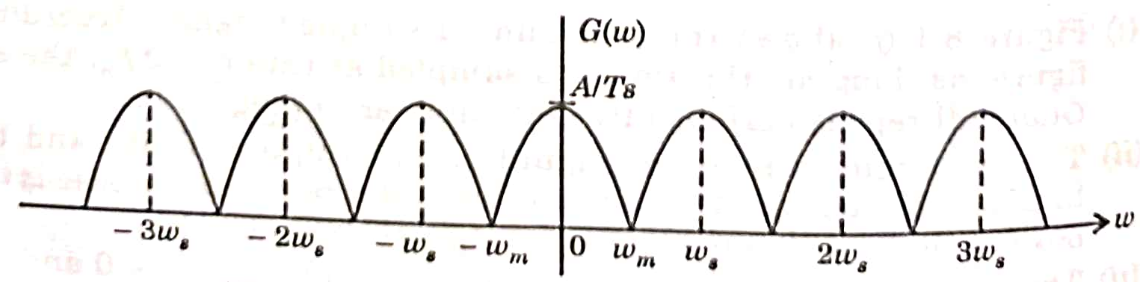


Fig. 8.2. Sampled spectrum at Nyquist rate.

EXAMPLE 8.1. An analog signal is expressed by the equation $x(t) = 3 \cos 50\pi t + 10 \sin 300\pi t - \cos 100\pi t$. Calculate the Nyquist rate for this signal.

Solution : The given signal is expressed as

$$x(t) = 3 \cos 50\pi t + 10 \sin 300\pi t - \cos 100\pi t \quad ...(i)$$

Let three frequencies present be w_1, w_2 and w_3

So that the new equation for signal,

$$x(t) = 3 \cos w_1 t + 10 \sin w_2 t - \cos w_3 t \quad ...(ii)$$

Comparing equations (*i*) and (*ii*) we have

$$w_1 t = 50\,\pi t; \; w_1 = 50\,\pi$$

or $$2\,\pi f_1 = 50\,\pi$$

or $$2\,f_1 = 50$$

$\therefore$ $$f_1 = 25 \text{ Hz}$$

Similarly, for second factor

$$w_2 t = 300\pi t \text{ or } w_2 = 300\pi$$

or $$2\,\pi f_2 = 300\pi \text{ or } 2\,\pi f2 = 300\pi$$

$\therefore$ $$f_2 = 150 \text{ Hz.}$$

Again, for third factor

$$w_3 t = 100\pi t \text{ or } 2\,\pi f_3 t = 100\pi t$$

or $$2\,\pi f_3 = 100\pi$$

$\therefore$ $$f_3 = 50 \text{ Hz}$$

Therefore, the maximum frequency present in $x(t)$ is,

$$f_2 = 150 \text{ Hz}$$

Nyquist rate is given as

$$f_s = 2\,f_m$$

where f_m = Maximum frequency present in the signal.

Here $f_m = f_2 = 150$ Hz

Therefore, Nyquist rate

$$f_s = 2\,f_2 = 2 \times 150 = 300 \text{ Hz} \qquad \textbf{Ans.}$$

EXAMPLE 8.2. Find the Nyquist rate and the Nyquist interval for the signal

$$x(t) = \frac{1}{2\pi} \cos (4000\pi t) \cos (1000\pi t)$$

Solution : Given signal is

$$x(t) = \frac{1}{2\pi} \cos (4000\pi t) \cos (1000\pi t)$$

or $$x(t) = \frac{1}{4\pi} [2 \cos (4000\pi t) \cos 1000\pi t)]$$

or $$x(t) = \frac{1}{4\pi} [\cos (4000\pi t + 1000\pi t) + \cos (4000\pi t - 1000\pi t)]$$

$$[\because 2 \cos A \cos B = \cos (A + B) + \cos (A - B)]$$

or $$x(t) = \frac{1}{4\pi} [\cos 5000\pi t + \cos 3000\pi t] \quad ...(i)$$

Let the two frequencies present in the signal be w_1 and w_2 so that the new equation for the signal will be

$$x(t) = \frac{1}{4\pi} [\cos w_1 t + \cos w_2 t] \quad ...(ii)$$

Comparing equations (*i*) and (*ii*), we have

$$w_1 t = 5000\pi t$$

or $$2\pi f_1 t = 5000\pi t$$

or $$2 f_1 = 5000$$

$\therefore$ $$f_1 = 2500 \text{ Hz}$$

Similarly, for second factor

$$w_2 t = 3000\pi t$$

or $$2\pi f_2 t = 3000\pi t$$

or $$2\pi f_2 = 3000$$

$\therefore$ $$f_2 = 1500 \text{ Hz}$$

Therefore, the maximum frequency present in $x(t)$ is

$$f_1 = 2500 \text{ Hz}$$

Nyquist rate is given as

$$f_s = 2 f_m$$

where f_m = Maximum frequency present in the signal.

Here, $$f_m = f_1 = 2500 \text{ Hz}$$

Therefore Nyquist rate

$$f_s = 2 f_m = 2 \times 2500 = 5000 \text{ Hz} = 5 \text{ KHz} \quad \textbf{Ans.}$$

Nyquist interval is given as

$$T_s = \frac{1}{2f_m} = \frac{1}{2 \times 2500} = \frac{1}{5000}$$

or $$T_s = 0.2 \times 10^{-3} \text{ seconds} = 0.2 \text{ m sec.} \quad \textbf{Ans.}$$

EXAMPLE 8.3. A continuous-time signal is given below :

$$x(t) = 8 \cos 200\, \pi t$$

Determine:

(*i*) Minimum sampling rate *i.e.*, Nyquist rate required to avoid aliasing.

(*ii*) If sampling frequency f_s = 400 Hz. What is the discrete-time sigal $x[n]$ or $x[nT_s]$ obtained after sampling ?

(*iii*) If sampling frequency f_s = 400 Hz. What is the discrete-time signal $x[n]$ or $x[nT_s]$ obtained after sampling ?

(*iv*) What is the frequency $0 < f < f_s/2$ of sinusoidal that yields samples identical to those obtained in part (*iii*) ?

Solution :

(*i*) The highest frequency component of continuous-time signal is f = 100 Hz. Hence minimum sampling rate required to avoid aliasing is called Nyquist rate and is given as

$$\text{Nyquist rate} = 2f = 2 \times 100 = 200 \text{ Hz} \quad \textbf{Ans.}$$

(*ii*) The continuous-time signal $x(t)$ is sampled at f_s = 400 Hz. The frequency of the discrete-time signal will be

$$F = \frac{\text{Frequency of continuous-time signal } f}{\text{Sampling frequency, } f_s} = \frac{100}{400} = \frac{1}{4}$$

Then the discrete-time signal will be given as

$$x[n] = 8 \cos 2\pi Fn = 8 \cos 2\pi \times \frac{1}{4} n$$

or

$$x[n] = 8 \cos \frac{\pi n}{2} \quad \textbf{Ans.}$$

(*iii*) The continuous-time signal $x(t)$ is sampled at $f_s = 150$ Hz. The frequency of discrete-time will be

$$F = \frac{f}{f_s} = \frac{100}{150} = \frac{2}{3}$$

Then, the discrete-time signal will be given as

$$x[n] = 8 \cos 2\pi fn = 5 \cos 2\pi \left(\frac{2}{3}\right) n = 8 \cos \frac{4\pi}{3} n$$

$$= 8 \cos \left(2\pi - \frac{2\pi}{3}\right) n = 8 \cos \frac{2\pi n}{3}$$

or

$$x[n] = 8 \cos \frac{2\pi n}{3} \quad \textbf{Ans.}$$

(*iv*) For sampling rate of $f_s = 150$ Hz

$$F = \frac{f}{f_s} \text{ or } f = f_s \times F = \frac{1}{3} \times 150 = 50 \text{ Hz}$$

Then, the sinusoidal signal will be

$$y(t) = 8 \cos 2\pi ft = 8 \cos 2\pi \times 50 \times t = 8 \cos 100\, \pi t$$

Sampling at $f_s = 150$ Hz, yields indentical samples hence $f = 100$ Hz is an alias of $f = 50$ Hz for sampling rate $f_s = 150$ Hz. **Ans.**

EXAMPLE 8.4. Determine the Nyquist rate for a continuous-time signal

$$x(t) = 6 \cos 50\, \pi t + 20 \sin 300\, \pi t - 10 \cos 100\, \pi t$$

Solution : In a general form, any continuous-time signal may be expressed as

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t + A_3 \cos \omega_3 t \quad (i)$$

And the given signal is

$$x(t) = 6 \cos 50\pi t + 2 \sin 300\pi t - 10 \cos 100\, \pi t \quad (ii)$$

On comparing given signal equation (*ii*) with standard form of a signal equation (*i*), we obtain the frequencies for the given signal as

$$f_1 = \frac{\omega_1}{2\pi} = \frac{50\pi}{2\pi} = 25 \text{ Hz}$$

$$f_2 = \frac{\omega_2}{2\pi} = \frac{300\pi}{2\pi} = 150 \text{ Hz}$$

$$f_3 = \frac{\omega_3}{2\pi} = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

Thus, the highest frequency component of the given meassage signal will be

$$f_{max} = 150 \text{ Hz}$$

Therefore, Nyquist rate $= 2 f_{max} = 2 \times 150 = 300$ Hz **Ans.**

8.5. RECONSTRUCTION FILTER (LOW PASS FILTER)

The low pass filter is used to recover original signal from its samples. This is also known as **interpolation filter.**

A low-pass filter is that type of filter which passes only low-frequencies upto a specified cut-off frequency and rejects all other frequenices above cut-off frequency. Figure 8.3 shows the frequency response of low-pass filter.

From figure 8.3 it may be observed that in case of low-pass filter, there is sharp-change in res-ponse at cut-off frequency, that is amplitude response becomes sud-denly zero at cut-off frequency which is not possible practically. This means that an ideal low-pass filter is not physically realizable. In place of ideal-low pass filter, we use practical filter.

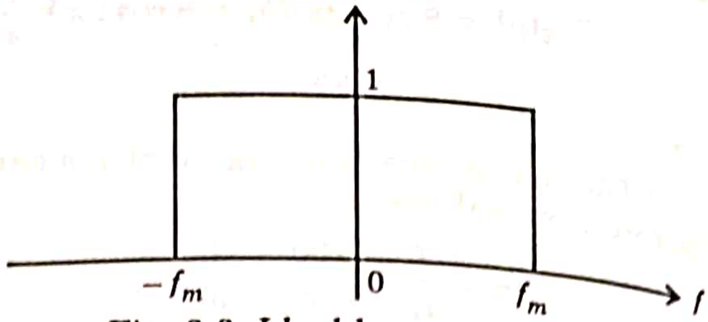


Fig. 8.3. Ideal low-pass filter.

Figure 8.4 shows the frequency response of practical low-pass filter. From figure 8.4, it may be observed that in case of practical filter, the amplitude response decreases slowly to become zero. This means that there is a transition band in case of practical filter. Figure 8.5 shows the use of practical low-pass filter in reconstruction of original signal from its sample.

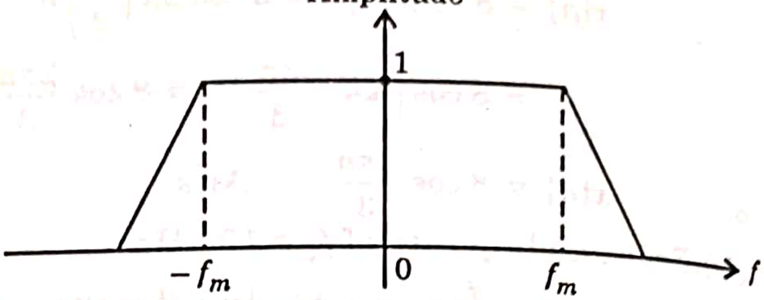


Fig. 8.4. Practical low-pass filter

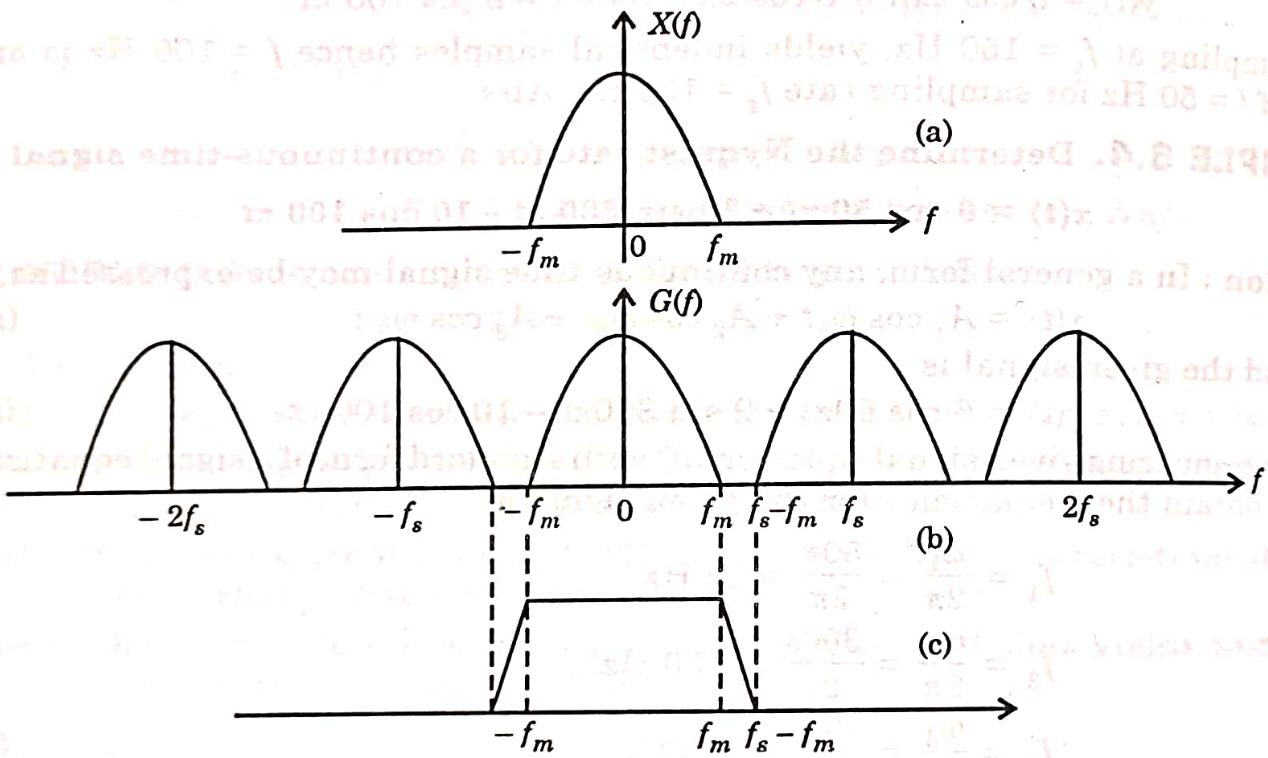


Fig. 8.5 (a) Spectrum of original signal
(b) Spectrum of sampled signal
(c) Amplitude response of practical low-pass filter.

8.6. SIGNAL RECONSTRUCTION : The Interpolation Formula

(WBTU, Kolkata, 2003)(06 marks)

The process of reconstructing a continuous-time signal $x(t)$ from its samples is called as **interpolation**.

As discussed earlier, a signal $x(t)$ band-limited to $f_m Hz$ can be reconstructed (interpolated) completely from its samples. This is achieved by passing the sampled signal through an ideal low-pass filter of cut-off frequency $f_m Hz$.

The expression for sampled signal is written as

$$g(t) = x(t).\delta_{T_s}(t) \quad \text{...(8.10)}$$

$$g(t) = \frac{1}{T_s}[x(t) + 2\,x(t)\cos w_s t + 2\,x(t)\cos 2w_s t + ...] \quad \text{...(8.11)}$$

From above equation, it may be observed that the sampled signal contains a component $\frac{1}{T_s} \times x(t)$.

To recover $x(t)$ or $X(w)$, the sampled signal must be passed through an ideal low-pass filter of bandwidth of $f_m Hz$ and gain T_s.

Therefore, the reconstruction or interpolating filter transfer function may be expressed as

$$H(w) = T_s \times \text{rect}\left(\frac{w}{4\pi f_m}\right) \quad \text{...(8.12)}$$

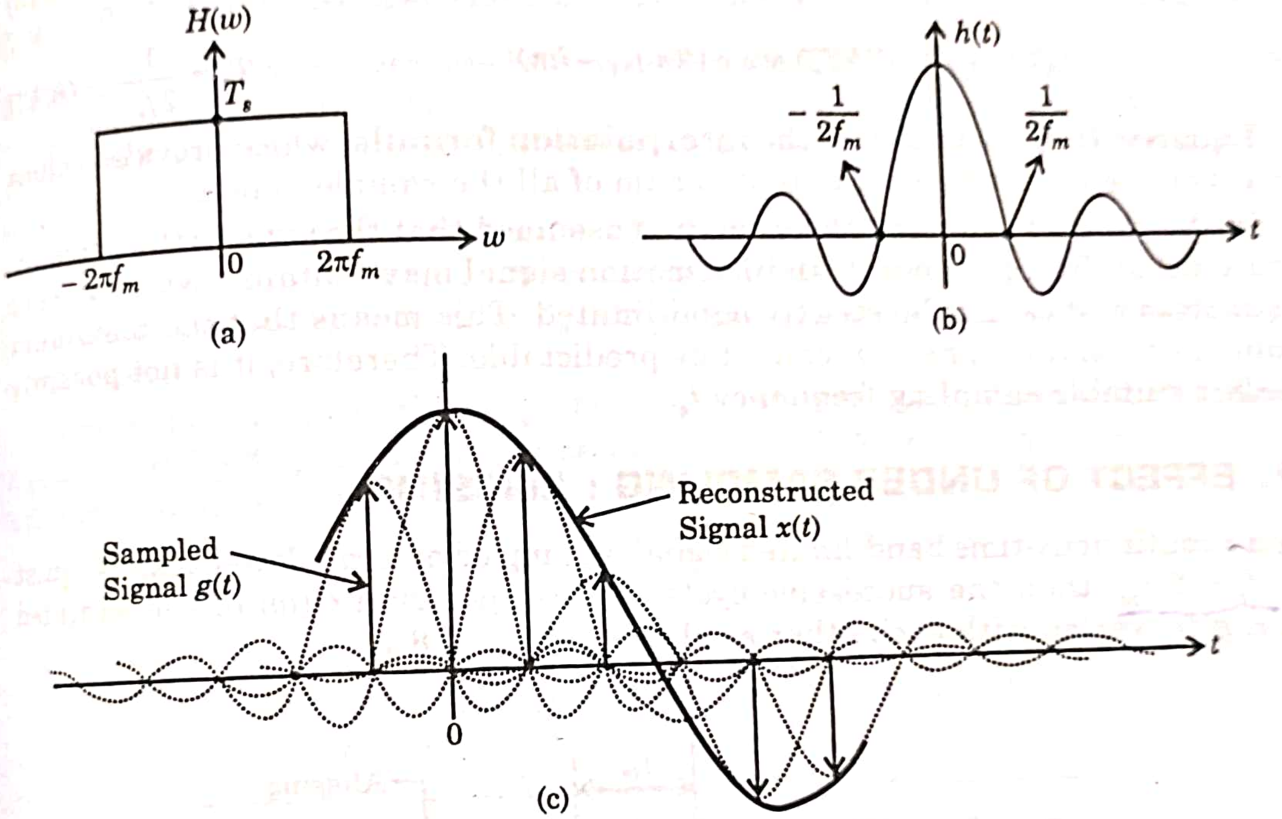


Fig. 8.6.

The impulse response $h(t)$ of this filter is the inverse Fourier transform of $H(w)$, i.e.

$$h(t) = F^{-1}[H(w)] = F^{-1}\left[T_s \text{ rect}\left(\frac{w}{4\pi f_m}\right)\right] \quad \text{...(8.13)}$$

$$h(t) = 2\,f_m\,T_s \sin c(2\pi\,f_m t)$$

Assuming that sampling is done at Nyquist rate, then

$$T_s = \frac{1}{2f_m} \quad \text{...(8.13a)}$$

So that $\quad 2\,f_m\,T_s = 1$

Putting this value of $2\,f_m T_s$ in equation (8.13) we have

$$h(t) = 1 \cdot \sin c\,(2\pi\,f_m t) = \sin c\,(2\pi\,f_m t) \quad \text{...(8.14)}$$

Figure 8.6(*b*) shows the graph of $h(t)$.

From figure, it may be observed that $h(t) = 0$ at all Nyquist sampling instants $t = \pm n/2f_m$ except at $t = 0$.

Now, when the sampled signal $g(t)$ is applied at the input of this filter, the output will be $x(t)$.

Each sample in $g(t)$, being an impulse, produces a sin c pulse of height equal to the strength of the sample.

Addition of the sin c pulses produced by all the samples results in $x(t)$.

For instant, the kth sample of the input $g(t)$ is the impulse $x(kT_s)\,\delta(t - kT_s)$.

The filter output of this impulse will be $x(kT_s)\, h(t - kT_s)$.

Therefore, the filter output to $g(t)$, which is $x(t)$, may be expressed as a sum

$$x(t) = \sum_k x(kT_s)\, h(t - kT_s) \quad \text{...(8.15)}$$

$$= \sum_k x(kT_s) \operatorname{sin c}[2\pi f_m (t - kT_s)] \quad \text{...(8.16)}$$

$$x(t) = \sum_k x(kT_s) \operatorname{sin c}(2\pi f_m - k\pi) \qquad \because T_s = \frac{1}{2f_m} \quad \text{...(8.17)}$$

Equation (8.17) is known as the **interpolation formula,** which provides values of $x(t)$ between samples as a weighted sum of all the sample values.

In the proof of sampling theorem, it is assumed that the signal $x(t)$ is strictly band-limited. But, in general, an information signal may contain a wide range of frequencies and cannot be strictly band-limited. This means that the maximum frequency f_m in the signal $x(t)$ cannot be predictable. Therefore, it is not possible to select suitable sampling frequency f_s.

8.7. EFFECT OF UNDER SAMPLING : ALIASING

When a continuous-time band-limited signal is sampled at a rate lower than Nyquist rate $f_s < 2f_m$, then the successive cycles of the spectrum $G(w)$ of the sampled signal $g(t)$ overlap with each other as shown in figure 8.7.

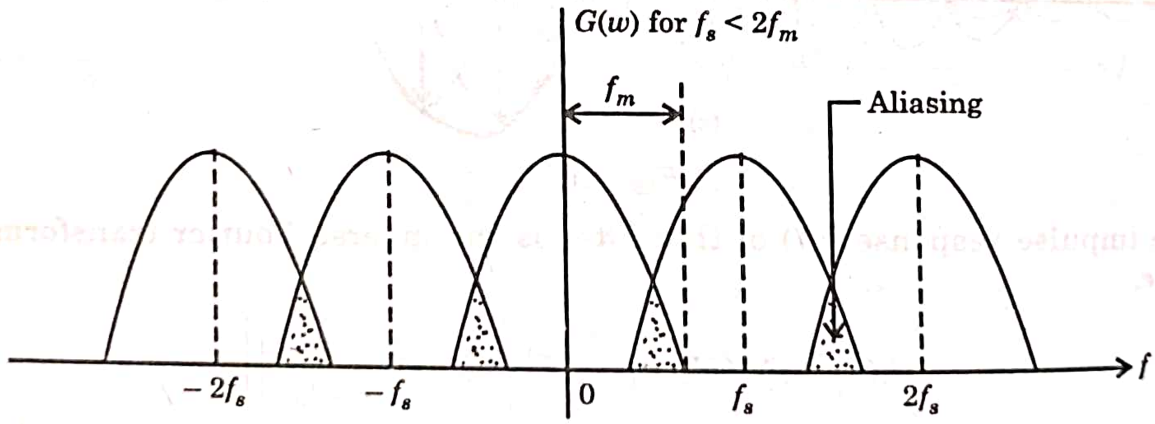


Fig. 8.7. Spectrum of the sampled signal for the case $f_s < 2f_m$

Hence, the signal is under-sampled in this case ($f_s < 2f_m$) and some amount of aliasing is produced in this under-sampling process. In fact, aliasing is the phenomenon in which a high frequency component in the frequency-spectrum of the signal takes identity of a lower-frequency component in the spectrum of the sampled signal.

From figure 8.7, it is clear that because of the overlap due to aliasing phenomenon, it is not possible to recover original signal $x(t)$ from sampled signal $g(t)$ by low-pass filtering since the spectral components in the overlap regions add and hence the signal is distorted.

Since any information signal contains a large number of frequencies, so, to decide a sampling frequency is always a problem. Therefore, a signal is first passed through a low-pass filter. This low-pass filter blocks all the frequencies which are above f_m Hz. This process is known as band limiting of the original signal $x(t)$. This low-pass filter is called **prealias filter** because it is used to prevent aliasing effect. After band-limiting, it becomes easy to decide sampling frequency since the maximum frequency is fixed at f_mHz.

In short, to avoid aliasing :

(i) Prealias filter must be used to limit band of frequencies of the signal to f_mHz.

(ii) Sampling frequency 'f_s' must be selected such that

$$f_s > 2 f_m$$

8.8. SAMPLING OF BANDPASS SIGNALS *(Important)*

In previous sections, we discussed sampling theorem for low-pass signals. However, when the given signal is a bandpass signal, then a different criteria must be used to sample the signal. Therefore, the sampling theorem for bandpass signals may be expressed as under :

The bandpass signal $x(t)$ whose maximum bandwidth is $2f_m$ can be completely represented into and recovered from its samples if it is sampled at the minimum rate of twice the bandwidth. Here, f_m is the maximum frequency component present in the signal.

Hence if the bandwidth is $2f_m$, then the minimum sampling rate for bandpass signal must be $4 f_m$ samples per second. Figure 8.8 shows the spectrum of an arbitary bandpass signal.

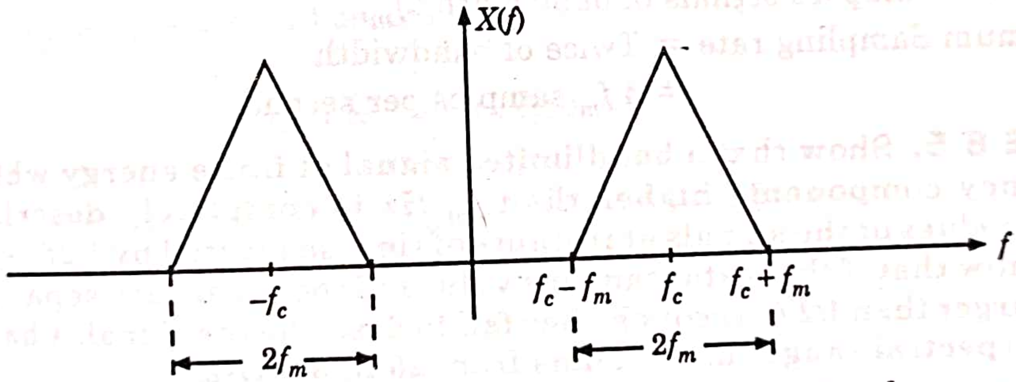


Fig. 8.8. Spectrum of an orbitary bandpass signal.

The spectrum in figure 8.8 is centred around frequency f_c. The bandwidth is $2f_m$. Thus, the frequencies present in the bandpass signal are from $f_c - f_m$ to $f_c + f_m$. This means that the highest frequency present in the bandpass signal is $f_c + f_m$. Generally the centre frequency $f_c > f_m$.

This bandpass signal is first represented in terms of its inphase and quadrature components

Let $\qquad x_I(t)$ = Inphase component of $x(t)$

and $x_Q(t)$ = Quadrature component of $x(t)$

Thus, the signal $x(t)$ in terms of inphase and quadrature components will be expressed as

$$x(t) = x_I(t) \cos(2\pi f_c t) - x_Q(t) \sin(2\pi f_c t) \qquad ...(8.18)$$

The inphase and quadrature components are obtained by multiplying $x(t)$ by $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ and then suppressing the sum frequencies by means of low-pass filters. Thus, inphase $x_I(t)$ and quadrature $x_Q(t)$ components contain only low frequency components. The spectrum of these components is limited between $-f_m$ to $+f_m$. This is shown in figure 8.9.

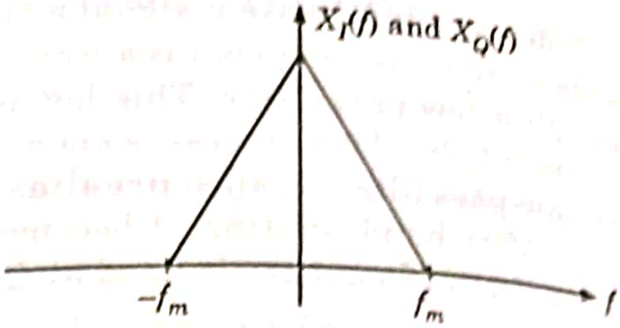


Fig. 8.9. Spectrum of Inphase and Quadrature components of bandpass signal x(t).

After few mathematical manipulation in equation (8.18), we obtain the reconstruction formula as

$$x(t) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{4f_m}\right) \operatorname{sin c}\left(2f_m t - \frac{n}{2}\right) \cos\left[2\pi f_c\left(t - \frac{n}{4f_n}\right)\right] \quad ...(8.19)$$

Comparing this reconstruction formula with that of low-pass signals given in equation (8.17), we observe that $x(t)$ is replaced by $x\left(\frac{n}{4f_m}\right)$.

Here, $x\left(\frac{n}{4f_m}\right) = x(nT_s)$ = Sampled version of bandpass signal

and
$$T_s = \frac{1}{4f_m}$$

Thus, if $4f_m$ samples per second are taken, then the bandpass signal of bandwidth $2f_m$ can be completely recovered from its samples.

Hence, for bandpass signals of bandwidth $2f_m$,

Minimum Sampling rate = Twice of bandwidth

= $4\,f_m$ samples per second.

EXAMPLE 8.5. Show that a bandlimited signal of finite energy which has no frequency components higher than f_m Hz is completely described by specifying values of the signals at instants of time separated by $1/2f_m$ seconds and also show that if the instantaneous values of the signal are separated by intervals larger than $1/2\,f_m$ seconds, they fail to describe the signal. A bandpass signal has spectral range that extends from 20 to 82 KHz.

Find the acceptable range of sampling frequency f_s.

Solution : Let $x(t)$ be the bandlimited signal which has no frequency components higher than f_m Hz. Let this signal be sampled by a sampling function given as

$$\delta_{Ts}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

The sampling function is the train of impulses with T_s as distance between successive impulses. Let $x(nT_s)$ be the instantaneous amplitude of signal $x(t)$ at instant $t = T_s$. The sampled version of $x(t)$ may be given as multiplication of $x(nT_s)$ and $\delta_{Ts}(t)$ *i.e.*,

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s) \quad ...(i)$$

Now, Fourier transform of this sampled signal may be obtained as

$$G(f) = FT\{g(t)\} = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

Here, f_s is the sampling rate which is given as

$$f_s = \frac{1}{T_s}$$

and $$X(f - nf_s) = X(f) \text{ at } nf_s = 0, \pm f_s, \pm 2f_s \pm 3f_s + ...$$

Hence, the same spectrum $X(f)$ appears at $f = 0$,

$$f = \pm f_s, f = \pm 2f_s \text{ etc.}$$

This means that a periodic spectrum with period equal to f_s is generated in frequency domain because of sampling $x(t)$ in time-domain.

Thus, equation (*i*) may be written as

$$G(f) = f_s \times (f) + f_s \times (f \pm f_s) + f_s \times (f \pm 2f_s) + f_s \times (f \pm 3f_s) + f_x \times (f \pm 4f_s) + ... \quad ...(ii)$$

or $$G(f) = f_s \times (f) + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} f_s . \times (f - nf_s) \quad ...(iii)$$

By definition of Fourier transform, we have

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

For sampled version of $x(t)$, we have $t = nT_s$.

Then the equation (*iii*) becomes

$$G(f) = \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s} \quad ...(iv)$$

Now, given that the signal is band limited to f_m Hz and

$$T_s = \frac{1}{2f_m} \text{ seconds}$$

$$...(v)$$

Therefore, $$f_s = \frac{1}{T_s} = 2f_m$$

From equation (*ii*), it may be observed that $G(f)$ is periodic with a period f_s.

Thus the spectrum $X(f)$ and $G(f)$ are shown in figure 8.10.

Now, since $f_s = 2f_m$

therefore,

$$f_s - f_m = f_m$$

and $$f_s + f_m = 3f_m$$

Hence, the periodic spectrums $X(f)$ just touch $\pm f_m, \pm 3f_m, \pm 5f_m$ ... etc.

Thus, there is no aliasing.

Using equation (*iii*), we write

$$X(f) = \frac{1}{f_s} G(f) - \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} X(f - nf_s) \quad ...(vi)$$

Substituting $f_s = 2f_m$ in equation (*vi*), we get

$$X(f) = \frac{1}{2f_m} G(f) - \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} X(f - nf_s) = \frac{1}{2f_m} G(f) \text{ for } -f_m \leq f \leq f_m \quad ...(vii)$$

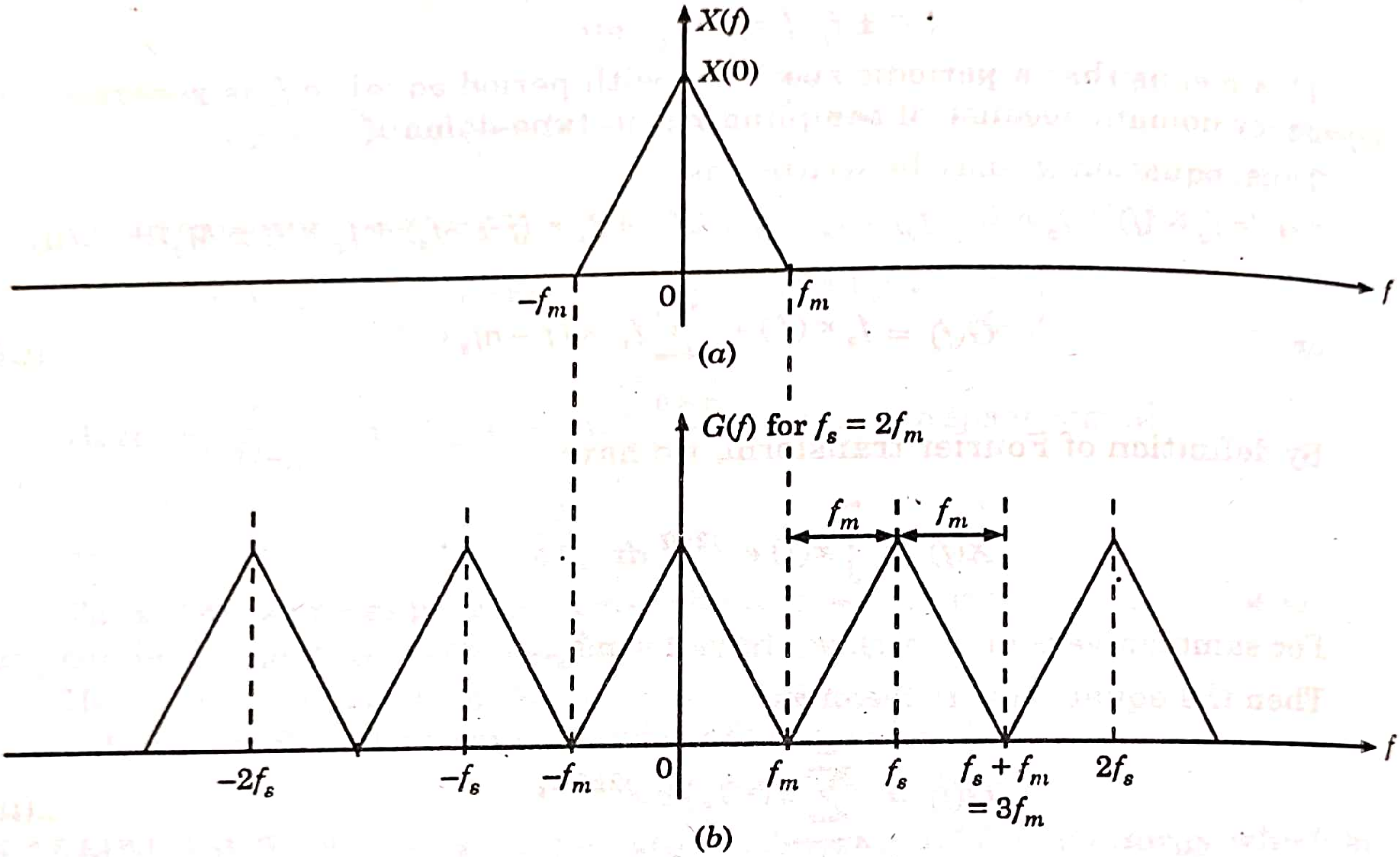


Fig. 8.10. (a) Spectrum of x(t) (b) Spectrum of g(t) with $f_s = 2f_m$.

Now putting the value of $G(f)$ from equation (*iv*) to equation (*vii*), we get

$$X(f) = \frac{1}{2f_m} \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi fn.T_s}$$

Since $\quad T_s = \frac{1}{2f_m}$

i.e.,

$$X(f) = \frac{1}{2f_m} \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2f_m}\right) e^{-j\pi fn/f_m} \quad ...(viii)$$

$x(t)$ may, be recovered from $X(f)$ by taking inverse Fourier transform of last equation, *i.e.,*

$$x(t) = IFT\left\{\frac{1}{2f_m} \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2f_m}\right) e^{-j\pi f_n/f_m}\right\} \quad ...(ix)$$

This equation indicates that $x(t)$ is represented completely by its samples $x\left(\frac{n}{2f_m}\right)$ for $-\infty < n < \infty$. This means that the sequence $x\left(\frac{n}{2f_m}\right)$ has all the information contained in $x(t)$.

Reconstruction of signal form samples :

Let us consider equation (*ix*) as

$$x(t) = IFT\left\{\frac{1}{2f_m}\sum_{n=-\infty}^{\infty} x\left(\frac{n}{2f_m}\right)e^{-j\pi fn/f_m}\right\}$$

or
$$x(t) = \int_{-f_m}^{f_m} \frac{1}{2f_m}\sum_{n=-\infty}^{\infty} x\left(\frac{n}{2f_m}\right)e^{-j\pi fn/f_m}e^{j2\pi ft}\,dt$$

Interchanging the order of summation and integration, we get

$$x(t) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2f_m}\right)\frac{1}{2f_m}\int_{-f_m}^{f_m} e^{j2\pi f\left(t-\frac{n}{2f_m}\right)}\,dt$$

or
$$x(t) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2f_m}\right)\frac{\sin(2\pi f_m t - n\pi)}{(2\pi f_m t - n\pi)} = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2f_m}\right)\frac{\sin\pi(2f_m t - n\pi)}{\pi(2f_m t - n)}$$

Since, $\text{sinc}\,\theta = \frac{\sin \pi\theta}{\pi\theta}$

Therefore, $x(t) = \sum_{n=-\infty}^{\infty}{}' x\left(\frac{n}{2f_m}\right)\text{sinc}\,(2f_m t - n)\quad -\infty < n < \infty$

Hence, this is the interpolation formula to reconstruct $x(t)$ from its samles $x(nT_s)$. Therefore, from all above, it is clear that the signal may be completely represented into and recovered from its samples if the spacing between the successive samples is $\frac{1}{2f_m}$ seconds i.e., $f_s = 2f_m$ samples per second.

Sampling Frequency for Bandpass Signal :

Since the spectral range of the bandpass signal is 20 kHz to 82 kHz

Therefore

Bandwidth $= 2f_m = 82\text{ kHz} - 20\text{ kHz} = 62\text{ kHz}$

Hence, Minimum Sampling rate $= 2 \times$ bandwidth

$= 2 \times 62 = 124$ kHz

Generally, the range of minimum sampling frequencies is specified for bandpass signals.

It lies between $4f_m$ to $8\,f_m$ samples per second.

Therefore,

Range of minimum sampling frequencies

$= (2 \times \text{bandwidth})$ to $(4 \times \text{bandwidth})$

$= 2 \times 62$ kHz to 4×62 kHz

$= 124$ kHz to 248 KHz Ans.

8.9. SAMPLING TECHNIQUES
(JNTU, Hyderabad, 2004)(06 marks)

In the last article, we discussed how sampling of a continuous-time signal is done. This sampling of a signal is done in several ways. Therefore in this section we shall discuss different types of sampling *i.e.*, sampling techniques.

Basically, there are three types of sampling techniques as under :

(*i*) Instantaneous sampling or ideal sampling

(*ii*) Natural sampling

(*iii*) Flat top sampling.

Out of these three, instantaneous sampling is called ideal sampling whereas natural sampling and flat-top sampling are called practical sampling methods. Now, let us discuss three different types of sampling techniques in detail.

8.9.1. Ideal Sampling or Instantaneous Sampling or Impulse Sampling

In the proof of sampling theorem, we used ideal or impulse sampling. In this type of sampling, the sampling function is a train of impulses. Figure 8.11(*b*) shows this sampling function.

$x(t)$ is the input signal (*i.e.*, signal to be sampled) as shown in figure 8.11(*a*).

Figure 8.11(*c*) shows a circuit to produce instantaneous or ideal sampling. This circuit is known as the **switching sampler.**

The working principle of this circuit is quite easy. The circuit simply consists of a switch. Now if we assume that the closing time 't' of the switch approaches zero, then the output $g(t)$ of this circuit will contain only instantaneous value of the input signal $x(t)$. Since the width of the pulse approaches zero, the instantaneous sampling gives a train of impulses of height equal to the instantaneous value of the input signal $x(t)$ at the sampling instant.

We know that the train of impulses may be represented as

$$\delta T_s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad ...(8.20)$$

This is known as sampling function and its waveform is shown in figure 8.11(*b*). The sampled signal $g(t)$ is expressed as the multiplication of $x(t)$ ans $\delta_{T_s}(t)$. Thus

$$g(t) = x(t) \cdot \delta_{T_s}(t) \quad ...(8.21)$$

$$= x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad ...(8.22)$$

or

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s) \quad ...(8.23)$$

The Fourier transform of the ideally sampled signal given by above equation may be expressed as

$$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \quad ...(8.24)$$

Note :
This equation gives the spectrum of ideally sampled signal. It shows that the spectrum $X(f)$ is periodic in f_s and weighted by f_s. However, it may be noted that ideal or instantaneous sampling is possible only in theory since it is impossible to have a pulse whose width approaches zero. Ideal sampling was used in last article to prove sampling theorem. Practically flat-top sampling and nature sampling are used.

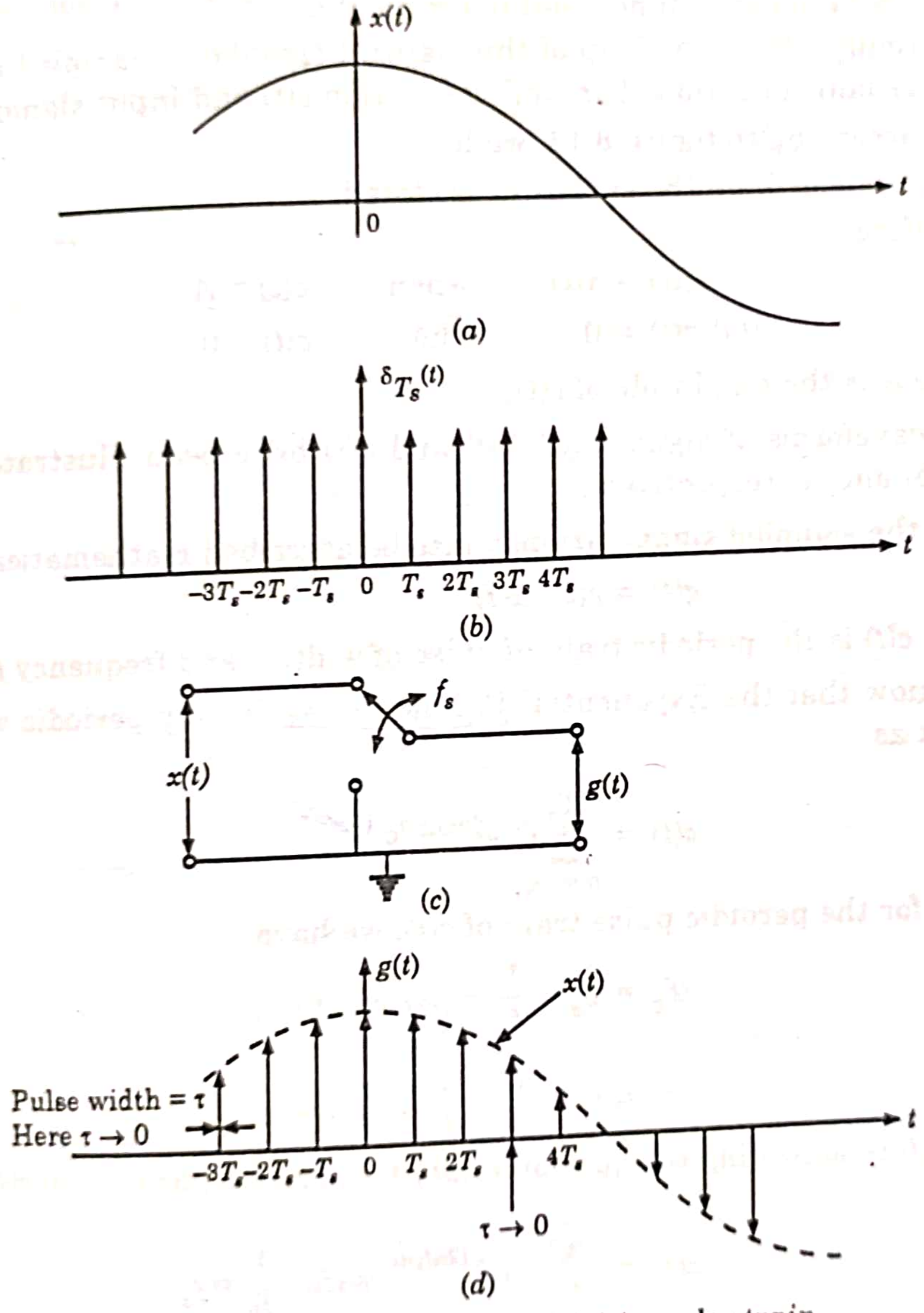


Fig. 8.11. (a) Baseband signal (b) impulse train (c) functional diagram of a switching sampler (d) sampled signal.

8.9.2. Natural Sampling

As discussed in last article, the instantaneous sampling results in the samples whose width τ approaches zero. Due to this, the power content in the instantaneously sampled pulse is negligible. Thus, this method is not suitable for transmission purpose. Natural sampling is a practical method and will be discussed in this section.

In natural sampling the pules has a finite width equal to τ.

Let us consider an analog continuous-time signal $x(t)$ to be sampled at the rate of f_s Hertz. Here it is assumed that f_s is higher than Nyquist rate such that simpling theorem is satisfied.

Again, let us consider a sampling function $c(t)$ which is a train of periodic pulses of width τ and frequency equal to f_s Hz.

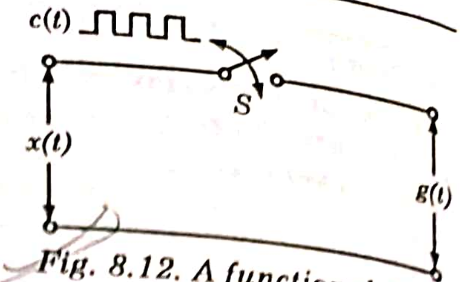


Fig. 8.12. A functional diagram of a Natural Sampler.

Figure 8.12 shows a functional diagram of a natural sampler. With the help of this natural sampler, a sampled signal $g(t)$ is obtained by multiplication of sampling function $c(t)$ and input signal $x(t)$.

Now, according to figure 8.12, we have

when $c(t)$ goes high the switch 'S' is closed.

Therefore

$$g(t) = x(t) \quad \text{when} \quad c(t) = A \qquad ...(8.25)$$

$$\text{and } g(t) = 0 \quad \text{when} \quad c(t) = 0 \qquad ...(8.26)$$

where A is the amplitude of $c(t)$.

The waveforms of signals $x(t)$, $c(t)$ and $g(t)$ have been illustrated in figure 8.13(a), (b) and (c) respectively.

Now, the sampled signal $g(t)$ may also be described mathematically as

$$g(t) = c(t) \cdot x(t) \qquad ...(8.27)$$

Here, $c(t)$ is the periodic train of pulse of width τ and frequency f_s.

We know that the Exponential Fourier series for any periodic waveform is expressed as

$$s(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi nt/T_0} \qquad ...(8.28)$$

Also, for the peroidic pulse train of $c(t)$, we have

$$T_0 = T_s = \frac{1}{f_s} = \text{period of } c(t)$$

or

$$f_0 = f_s = \frac{1}{T_0} = \frac{1}{T_s} = \text{frequency of } c(t)$$

Therefore according to equation (8.28) for periodic pusle train $c(t)$, we have

$$c(t) = \sum_{n=-\infty}^{\infty} C_n . e^{j2\pi f_s nt} \text{ with } \frac{1}{T_0} = f_s \qquad ...(8.29)$$

Now, it may be noted that since $c(t)$ is a rectangular pulse train, therefore C_n for this waveform will be expressed as

$$C_n = \frac{TA}{T_0} \sin c\,(f_n . T) \qquad ...(8.30)$$

here $\quad T$ = pulse width = τ

and $\quad f_n$ = harmonic frequency

But here, $\quad f_n = nf_s$

or $\quad f_s = \dfrac{n}{T_0} = nf_0$

Hence,
$$C_n = \frac{\tau . A}{T_s} \cdot \sin c(f_n . \tau) \quad ...(8.31)$$

Therefore, using equations (8.29) and (8.30) the Fourier series representation for $c(t)$ will be given as

$$c(t) = \sum_{n=-\infty}^{\infty} \frac{\tau . A}{T_s} \cdot \sin c(f_n . \tau)\, e^{j2\pi f_s \cdot nt} \quad ...(8.32)$$

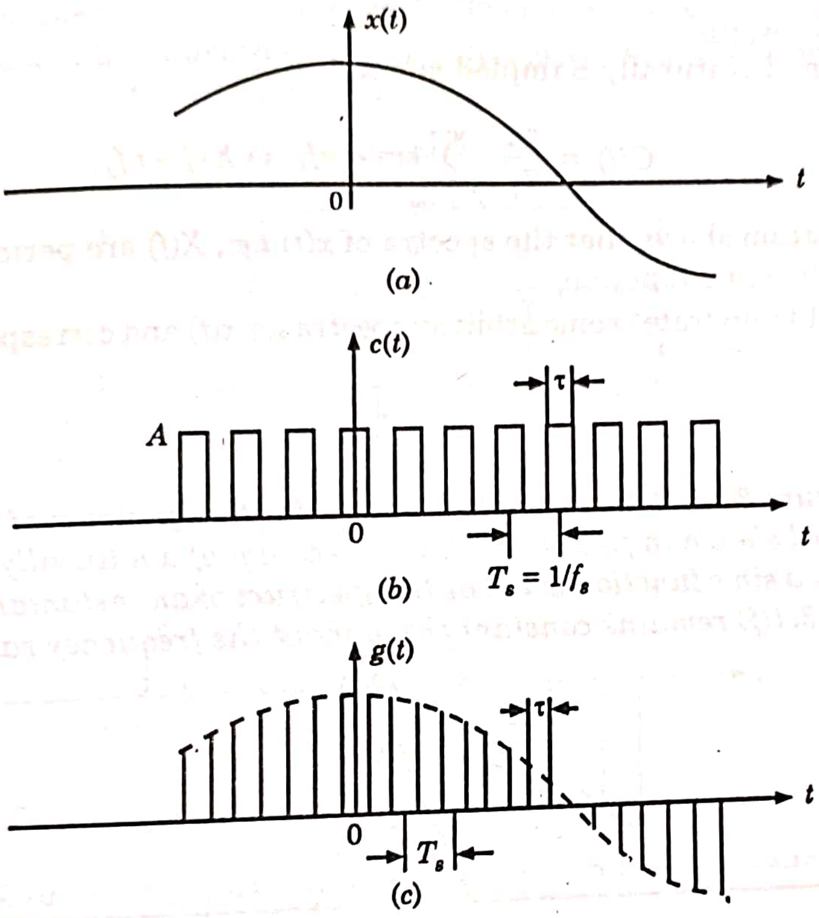


Fig. 8.13. (a) Continuous time signal x(t).
(b) Sampling function wavefrom i.e. periodic pulse train
(c) Naturally sampled signal waveform s(t).

Now, substituting the value of $c(t)$ from equation (8.32) to equation (8.27) we get

$$g(t) = \frac{\tau A}{T_s} \cdot \sum_{n=-\infty}^{\infty} \sin c(f_n . \tau) . e^{j2\pi f_s nt} . x(t) \quad ...(8.33)$$

This is required time-domain representation for naturally sampled signal $g(t)$.

Now, to get the frequency-domain representation of the naturally sampled signal $g(t)$, let us take its Fourier transform as

$$G(f, = FT[g(t)]$$

or
$$G(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \sin c(f_n . \tau)\, FT[e^{j2\pi f_s nt} . x(t)] \quad ...(8.34)$$

Recall the frequency-shifting property of Fourier transform which states that

$$e^{j2\pi f_s n(t)} . x(t) \longleftrightarrow X(f - f_s . n) \quad ...(8.35)$$

Therefore,

$$G(f) = \frac{\tau A}{T_s} \cdot \sum_{n=-\infty}^{\infty} \sin c(f_n.\tau)\, X(f - f_s n) \quad ...(8.36)$$

Now, since $f_n = f_s n$ = harmonic frequency

Therefore, equation (8.36) becomes

$$G(f) = \frac{\tau A}{T_s} \cdot \sum_{n=-\infty}^{\infty} \sin c(nf_s.\tau)\, X(f - nf_s) \quad ...(8.37)$$

Hence, we write

Spectrum of Naturally Sampled signal:

$$G(f) = \frac{\tau A}{T_s} \cdot \sum_{n=-\infty}^{\infty} \sin c(nf_s.\, \tau)\, X(f - nf_s) \quad ...(2.38)$$

This equation shows that the spectra of $x(t)$ *i.e.*, $X(f)$ are periodic in f_s and are weighed by the sin *c* function.

Figure 8.14 illustrates some arbitray spectra for $x(t)$ and corresponding spectrum $G(f)$.

Note :

Thus form figure 8.14, it may be noted that unlike the spectrum of instantaneously sampled signal shown in figure 8.1(f), the spectrum of a naturally sampled signal is weighted by a sin c function whereas the spectrum of an instantaneously sampled signal figure 8.1(f) remains constant throughout the frequency range.

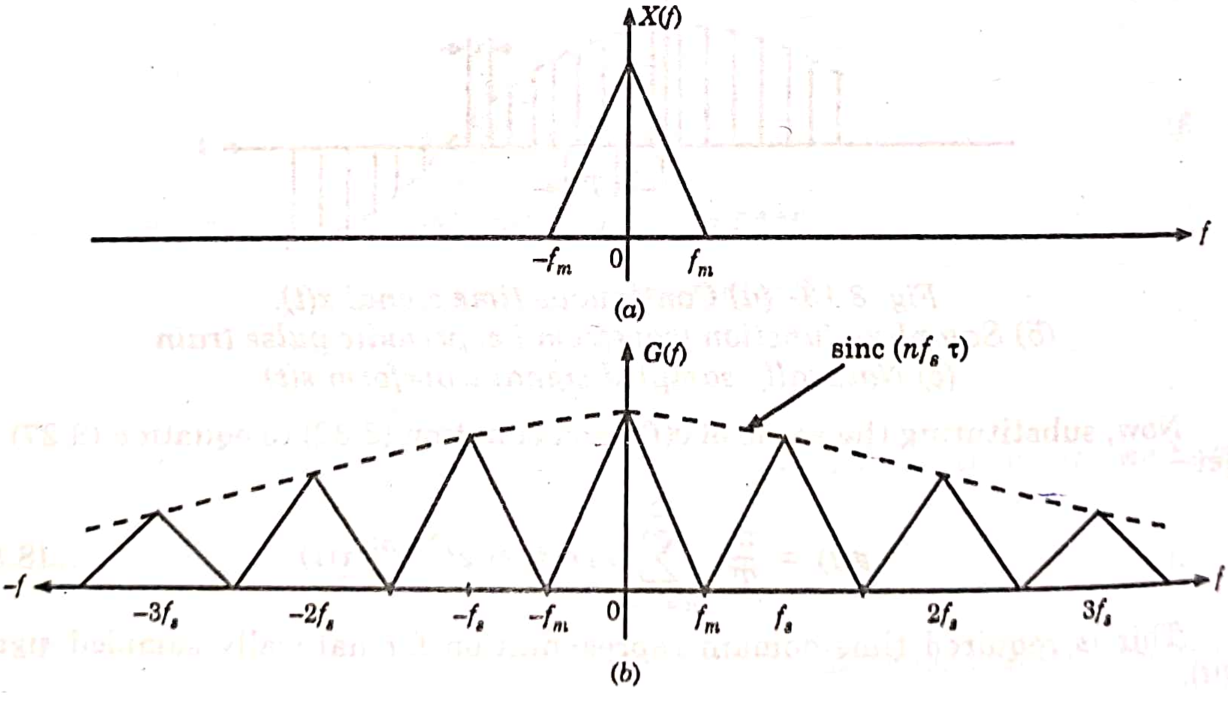


Fig. 8.14. (a) Spectrum of continuous-time signal x(t), (b) Spectrum of naturally sampled signal

8.9.3. Flot Top Sampling or Rectangular Pulse Sampling

(Rajasthan University, 2003)(05 marks)

Flot top sampling like natural sampling is also a practically possible sampling method. But natural sampling is little complex whereas it is quite easy to get flat top samples.

In flat-top sampling or rectangular pulse sampling, the top of the samples remains constant and is equal to the instantaneous value of the baseband signal $x(t)$ at the start of sampling. The duration or width of each sample is τ and sampling rate is equal to $f_s = \frac{1}{T_s}$. Figure 8.15(*a*) shows the functional diagram of a sample and hold circuit which is used to generate the flot top samples. Figure 8.15(*b*) shows the general waveform of flat top samples.

From figure 8.15(*b*), it may be noted that only starting edge of the pulse represents instantaneous value of the baseband signal $x(t)$. Also the flat top pulse of $g(t)$ is mathematically equivalent to the convolution of instantaneous sample and a pulse $h(t)$ as depicted in figure 8.16.

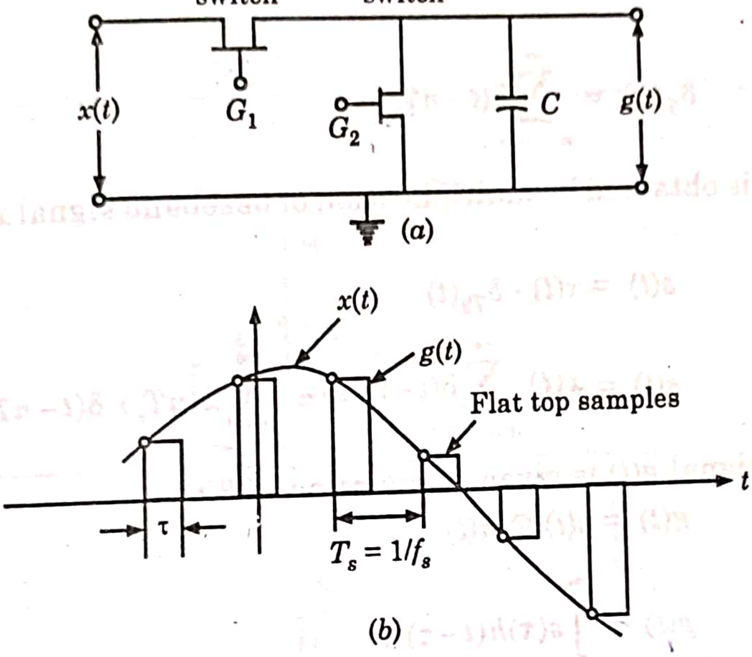


Fig. 8.15. (a) A sample and hold circuit to generate flat top samples.
(b) A general waveform of flat top sampling.

This means that the width of the pulse in $g(t)$ is determined by the width of $h(t)$ and the sampling instant is determined by delta function.

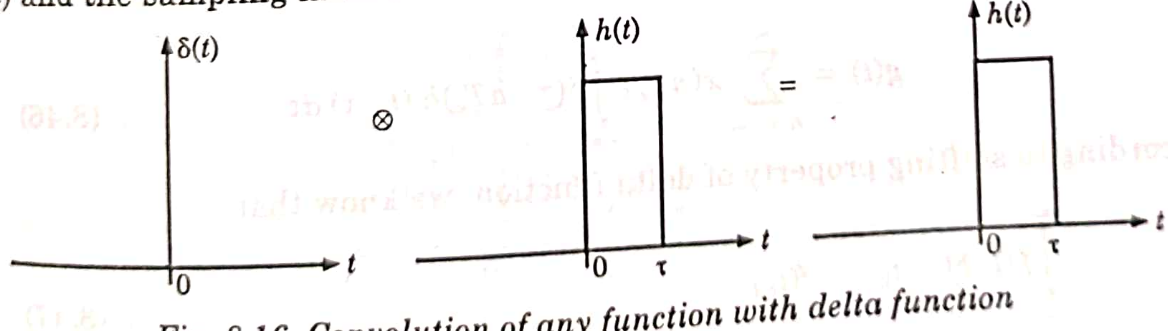


Fig. 8.16. Convolution of any function with delta function is equal to that function.

In figure 8.15(*b*), the starting edge of the pulse represents the point where baseband signal is sampled and width is determined by function $h(t)$. Therefore $g(t)$ will be expressed as

$$g(t) = s(t) \otimes h(t) \quad \text{...(8.39)}$$

This equation has been explained in figure 8.16.

Now, from the property of delta function, we know that for any function $f(t)$

$$f(t) \otimes \delta(t) = f(t) \quad \text{...(8.40)}$$

This property is used to obtain flat top samples. It may be noted that to obtain flat top sampling, we are not applying the equation (8.40) directly here *i.e.* we are applying a modified form of equation (8.40). This modified equation is equation (8.39).

Thus, in this modified equation, we are taking $s(t)$ in place of delta function $\delta(t)$. Observe that $\delta(t)$ is a constant amplitude delta function whereas $s(t)$ is a varying amplitude train of impulses. This means that we are taking $s(t)$ which is an istantaneously sampled signal and this is convolved with function $h(t)$ as in equation (8.39).

Therefore, on convolution of $s(t)$ and $h(t)$, we get a pulse whose duration is equal to $h(t)$ only but amplitude is defined by $s(t)$.

Now, we know that the train of impulses may be represented mathematically as

$$\delta_{Ts}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad \text{...(8.41)}$$

The signal $s(t)$ is obtained by multiplication of baseband signal $x(t)$ and $\delta_{T_s}(t)$. Thus,

$$s(t) = x(t) \cdot \delta_{Ts}(t) \quad \text{...(8.42)}$$

$$s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nTs) = \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s) \quad \text{...(8.43)}$$

Now, sampled signal $g(t)$ is given as equation (8.39)

$$g(t) = s(t) \otimes h(t) \quad \text{...(8.44)}$$

or

$$g(t) = \int_{-\infty}^{\infty} s(\tau) h(t - \tau)\, d\tau \quad \text{...(8.45)}$$

or

$$g(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(\tau - nT_s) h\,(t - \tau)\, d\tau$$

or

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s) h\,(t - \tau)\, d\tau \quad \text{...(8.46)}$$

According to shifting property of delta function, we know that

$$\int_{-\infty}^{\infty} f(t)\, \delta(t - t_0) = f(t_0) \quad \text{...(8.47)}$$

using equations (8.46) and (8.47), we get

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t - nT_s)$$

This equation represents value of $g(t)$ in terms of sampled value $x(nT_s)$ and function $h(t - nT_s)$ for flat top sampled signal.

Now, again from equation (8.39), we have

$$g(t) = s(t) \otimes h(t)$$

Taking Fourier transform of both sides of above equation, we get

$$G(f) = S(f)\, H(f) \quad \text{...(8.48)}$$

We know that $S(f)$ is given as

$$S(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \quad \text{...(8.49)}$$

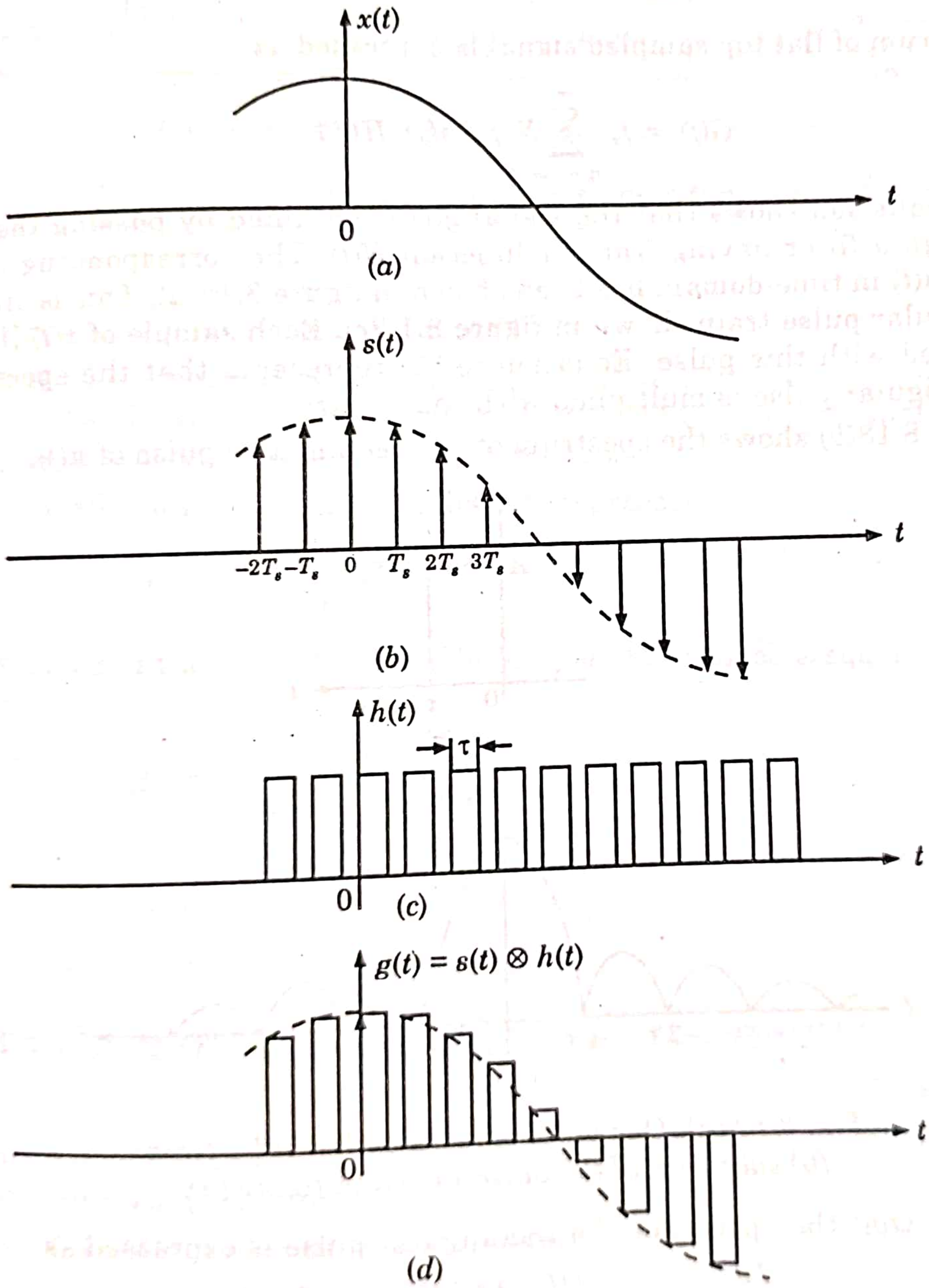


Fig. 8.17. (a) Baseband signal $x(t)$
(b) Instantaneously sample single $s(t)$
(c) Constant pulse width function $h(t)$
(d) Flat top sampled signal $g(t)$ obtained through convolution of $h(t)$ and $s(t)$

Therefore, equation (8.48) becomes

$$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \cdot H(f) \qquad \text{...(8.50)}$$

Thus, spectrum of flat top sampled signal :

$$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)\, H(f) \qquad \text{...(8.51)}$$

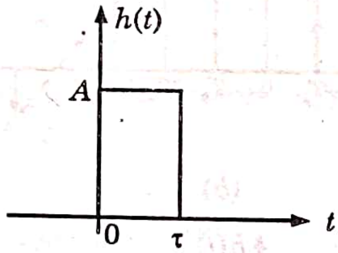
8.10. APERTURE EFFECT

The spectrum of flat top sampled signal is expressed as

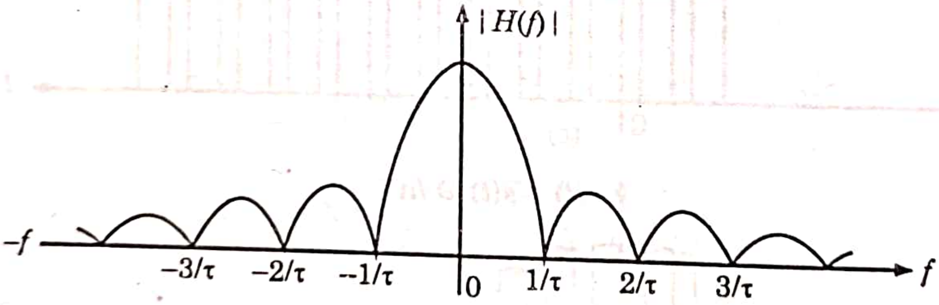
$$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \cdot H(f) \qquad \text{...(8.52)}$$

This equation shows that the signal $g(t)$ is obtained by passing the signal $s(t)$ through a filter having transfer function $H(f)$. The corresponding impulse response $h(t)$ in time-domain has been shown in figure 8.18(*a*). This is one pulse of rectangular pulse train shown in figure 8.17(*c*). Each sample of $x(t)$ [*i.e.*, $s(t)$] is convolved with this pulse. Equation (8.52) represents that the spectrum of this rectangular pulse is multiplied with that of $s(t)$.

Figure 8.18(*b*) shows the spectrum of one rectangular pulse of $h(t)$.



(*a*)



(*b*)

Fig. 8.18.(a). One pulse of rectangular pulse train (b) spectrum of the pulse shown in figure (a).

We know that the spectrum of a rectangular pulse is expressed as

$$H(f) = \tau.\sin c(f \cdot t)\, e^{-j\pi f\tau} \qquad [\because A = 1] \qquad \text{...(8.53)}$$

Hence, from figure 8.18(*b*), it may be observed that by using flat top samples an amplitude distortion is introduced in the reconstructed signal $x(t)$ from $g(t)$. In fact, the high frequency roll-off of $H(f)$ acts like a low-pass filter and thus attenuates the upper portion of message signal spectrum. These

DO YOU KNOW?

The distortion caused by the use of pulse-amplitude modulation to transmit an analog information bearing signal is referred to as the aperture effect.

high frequencies of $x(t)$ are affected. This type of effect is known as **aperture effect**.

Now, as the duration 'τ' of the pulse increases, the aperture effect is more prominent. Hence, during reconstruction an equalizer is needed to compensate for this effect. As depicted in figure 8.19, the receiver contains a low-pass reconstruction filter with cutoff frequency slightly higher than the maximum frequency present in the message signal. The equalizer compensates for the aperture effect. It also compensates for the attenuation by a low-pass reconstruction filter.

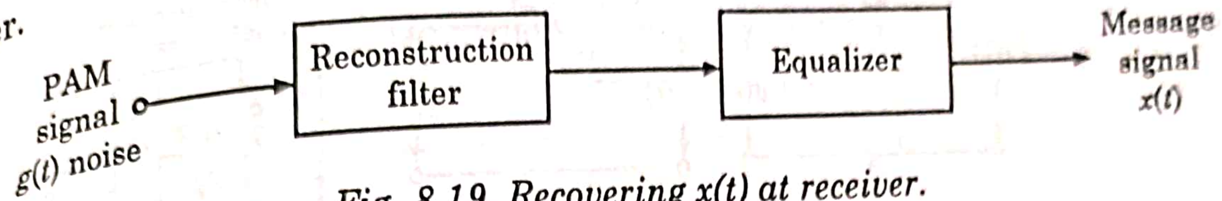


Fig. 8.19. Recovering x(t) at receiver.

From equation (8.53), it may be noted that the sample function $h(t)$ acts like a low-pass filter where fourier transform as expressed as

$$H(f) = \tau.\ \text{sinc}\ (f.\tau)\ e^{-j\pi f\tau} \quad ...(8.54)$$

This spectrum has been plotted in figure (8.18).

Equalizer used in casecade with the reconstruction filter has the effect of decreasing the inband loss of the reconstruction filter as the frequency increases in such a way as to compensate for the aperture effect.

Also, the transfer function of the equalizer is expressed as

$$H_{eq}(f) = \frac{K \cdot e^{-j2\pi f t_d}}{H(f)} \quad ...(8.55)$$

Here 't_d' is known as the delay introduced by low-pass filter which is equal to $\tau/5$.

Therefore,

$$H_{eq}(f) = \frac{K \cdot e^{-j\pi f\tau}}{\tau \sin c(f\tau)\, e^{-j\pi f\tau}} \quad ...(8.56)$$

or

$$H_{eq}(f) = \frac{K}{\tau \sin c(f\tau)} \quad ...(8.57)$$

which is the transfer function of an equalizer.

8.11. COMPARISON OF VARIOUS SAMPLING TECHNIQUES

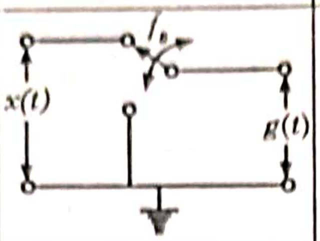
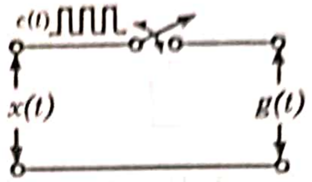
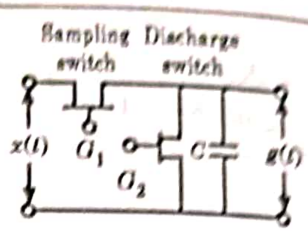
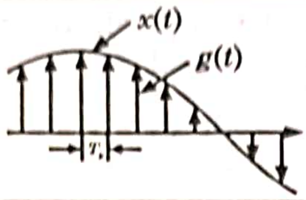
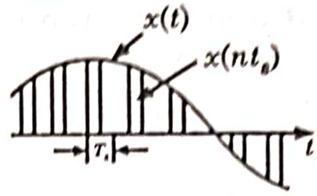
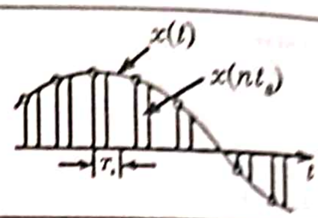
(Important)

We can compare various sampling techniques on the basis of their method, noise interference and spectral properties etc.

The Table 8.1 lists some of the important points of comparison of generation.

Table 8.1. Comparison of three sampling techniques

(PTU Examination, 2003-04)

Sr.	Parameter of comparison	Ideal or instantaneous sampling	Natural sampling	Flat top sampling
1.	Sampling principle	It uses multiplication	It uses chopping principle	It uses sample and hold circuit
5.	Generation circuit			
3.	Waveforms involved			
4.	Feasibility	This is not a practically possible method	This method is used practically	This method is also used practically
5.	Sampling rate	Sampling rate tends to infinity	Sampling rate satisfies Nyquist criteria	Sampling rate satisfies Nyquist criteria
6.	Noise interference	Noise interference is maximum	Noise interference is minimum	Noise interference is maximum
7.	Time domain representation	$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$	$g(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t) \text{sin}\,c(nf_s\tau) e^{j2\pi n f_s t}$	$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)h(t - nT_s)$
8.	Frequency domain representation	$G(f) = f_x \sum_{n=-\infty}^{\infty} X(f - nf_s)$	$G(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sin}\,c(nf_s\tau) X(f - nf_s)$	$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) H(f)$

EXAMPLE 8.6. Figure 8.20 shows the spectrum of an arbitary signal $x(t)$. This signal is sampled at the Nyquist rate with a periodic train of rectangular pulses of duratin $\frac{50}{3}$ milliseconds.

Determine the spectrum of the sampled signal for frequencies upto 50 Hz giving relevant expression.

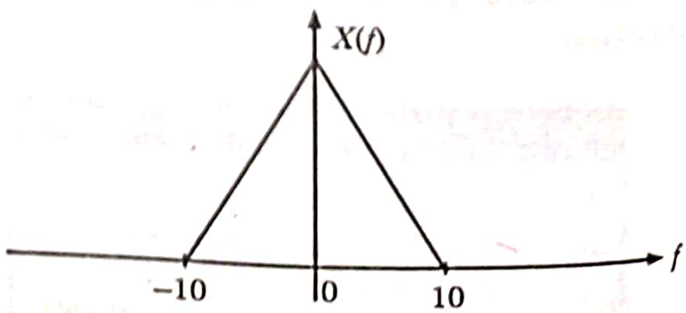


Fig. 8.20.

Solution : From figure 8.20, it may be observed that the signal is band limited to 10 Hz.

$$f_m = 10 \text{ Hz}$$

thus

So the Nyquist rate is $= 2 f_m = 2 \times 10 = 20$ Hz

Since the signal is sampled at the Nyquist rate, the sampling frequency would be

$$f_s = 20\text{Hz}$$

Given that the rectangular pulses are used for sampling *i.e.*, flat top sampling is used.

The spectrum of the flat top sampled signal is given as

$$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)H(f) \qquad (i)$$

value of $H(f)$ is expressed as

$$H(f) = \tau \sin c\,(f\,\tau)\, e^{-j\pi f \tau} \qquad (ii)$$

Here τ is the width of the rectangular pulse used for sampling.

The given value of rectangular sampling pulse duration is $\frac{50}{3}$ milliseconds *i.e.*,

$$\tau = \frac{50}{3} \times 10^{-3} = \frac{0.05}{3} \text{ sec}$$

Substituting the value of τ in equation (*ii*), we get

$$H(f) = \frac{0.05}{3} \sin c\left(\frac{0.05f}{3}\right) e^{-j0.05\pi f/3}$$

Again, putting this value of $H(f)$ and f_s in equation (*i*), we get

$$G(f) = 20 \sum_{n=-\infty}^{\infty} X(f - 20n) \frac{0.05}{3} \sin c\left(\frac{0.05f}{3}\right) e^{-j0.05\pi f/3}$$

$$(\because f_s = 20)$$

$$G(f) = \frac{1}{3} \sum_{n=-3}^{3} X(f - 20n) \sin c\left(\frac{0.05f}{3}\right) e^{-j0.05\pi f/s}$$

This expression gives the spectrum up to 60 Hz (since $n = \pm 3$) for the sampled signal. **Ans.**

EXAMPLE 8.7. **A flot top sampling system samples a signal of maximum frequency 1Hz with 9.5 Hz sampling frequency. The duration of the pulse is 0.2 seconds. Compute the amplitude distortion due to aperture effect at the highest signal frequency. Also determine the equalization characteristic.**

Solution: Given that sampling frequency

$$f_s = 5.5 \text{ Hz}$$

Maximum signal frequency,

$$f_{max} = 1\text{Hz}$$

and pulse width $\tau = 0.2$ seconds.

We know that the aperture effect is expressed by a transfer function $H(f)$ as

$$H(f) = \tau \sin c\,(f\,\tau)\, e^{-j\pi f \tau}$$

The magnitude of this equation will be

$$|H(f)| = \tau \sin c(f\,\tau) \qquad ...(i)$$

$$|H(f)| = 0.2 \sin c(f \times 0.2)$$

Now, aperture effect at the highest frequency will be obtained by putting $f = f_{max} = 1$ Hz in equation (i) i.e.,

$$|H(1)| = 0.2 \sin c\,(0.2) = 0.18709$$

or

$$|H(1)| = 18.70\% \quad \textbf{Ans.}$$

Also, the equalizer characteristic is expressed as

$$H_{eq}(f) = \frac{K}{\tau \sin c(f\tau)}$$

Substituting, $\tau = 0.2$ second and assuming $K = 1$, the last equation becomes

$$H_{eq}(f) = \frac{1}{0.2 \sin c(0.2f)}$$

This equation is the plot of $H_{eq}(f)$ versus f and it represents the equalization characteristic to overcome aperture effect.

8.12. ANALOG PULSE MODULATION METHODS

We know that in analog modulation systems, some parameter of a sinusoidal carrier is varied according to the instantaneous value of the modulating signal. In pulse modulation methods, the carrier is no longer a continuous signal but consists of a pulse train. Some parameter of which is varied according to the instantaneous value of the modulating signal. There are two types of pulse modulation systems as under :

(i) Pulse Amplitude Modulation (PAM)

(ii) Pulse Time Modulation (PTM)

In pulse amplitude modulation (PAM), the amplitude of the pulses of the carrier pulse train is varied in accordance with the modulating signal whereas in Pulse time modulation (PTM), the timing of the pulses of the carrier pulse train is varied.

They are two types of PTM as under :

(i) Pulse width modulation (PWM)*

(ii) Pulse position modulation (PPM)

In pulse width modulation, the width of the pulses of the carrier pulse train is varied in accordance with the modulating signal whereas in pulse position modulation (PPM), the position of pulses of the carrier pulse train is varied. Figure 8.21 shows three types of pulse analog modulation methods.

According to the sampling theorem, if a modulating signal is band limited to f_m Hz*, the sampling frequency must be at least $2f_m$ Hz and, hence the frequency of the carrier pulse train must also be least $2f_m$ Hz.

At this point, it may be noted that all the above pulse modulation methods (i.e., PAM, PWM and PPM) are called analog Pulse modulation methods because the modulating signal is analog in nature in PAM, PWM and PPM.

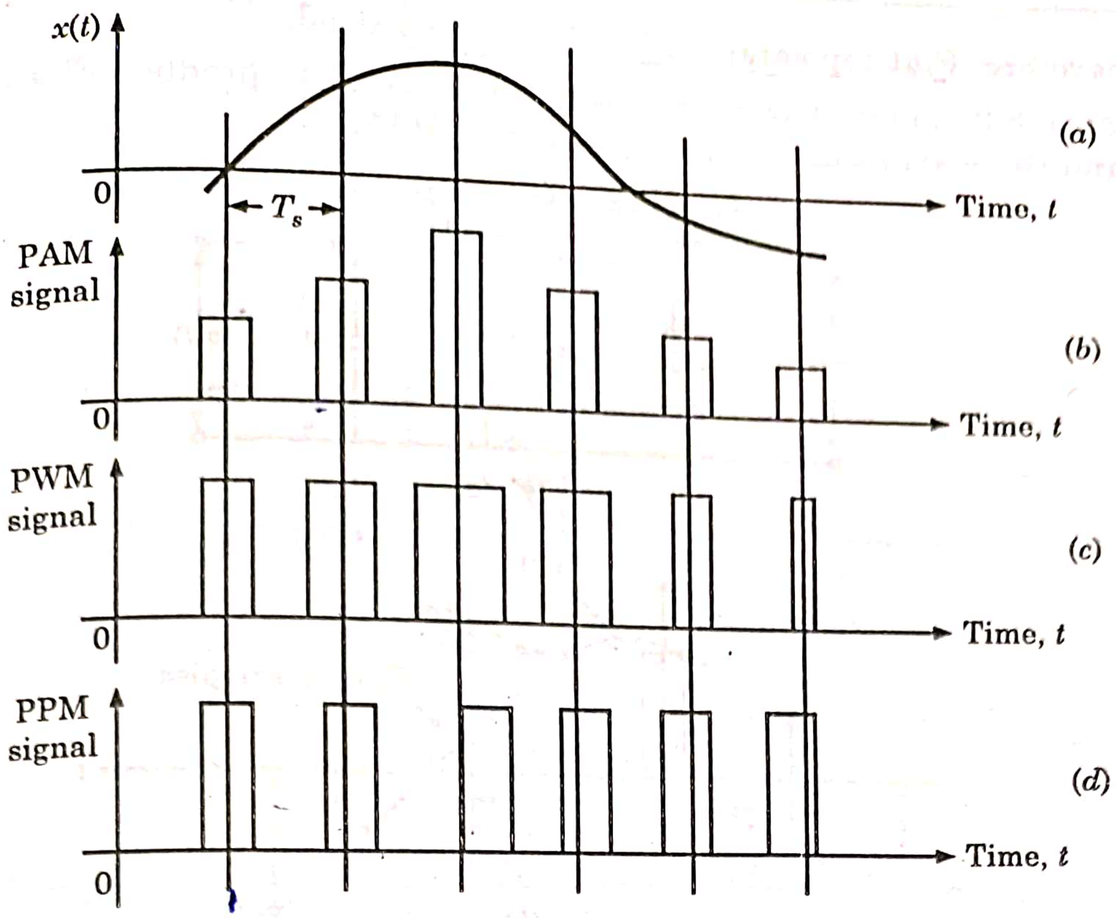


Fig. 8.21. Different types of pulse analog modulation methods.

8.13. PULSE AMPLITUDE MODULATION (PAM)

(PTU, 2004)(03 marks)

Pulse amplitude modulation may be defined as that type of modulation in which the amplitudes of regularly spaced rectangular pulses vary according to instantaneous value of the modulating or message signal. In fact, the pulses in a PAM signal may be of flat top type or natural type or ideal type. Actually all the sampling methods which have been discussed in last sections are basically pulse amplitude modulation methods.

Out of these three pulse amplitude modulation methods, the flat top PAM is most popular and is widely used. The reason for using flat top PAM is that during the transmission, the noise interferes with the top of the transmitted pulses and this noise can be easily removed if the PAM pulse has flat top.

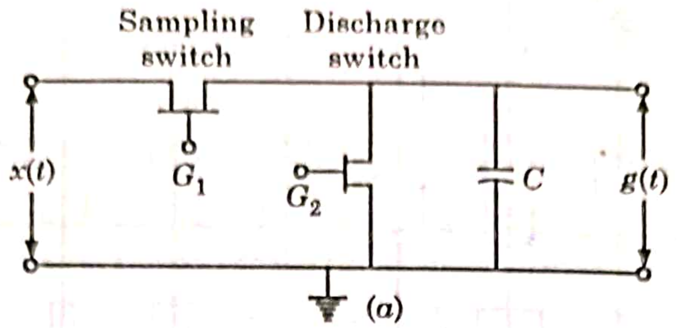
However, in case of natural smaples PAM signal, the pulse has varying top in accordance with the signal variation. Now, when such type of pulse is received at the receiver, it is always contaminated by noise. Then it becomes quite difficult to determine the shape of the top of the pulse and thus amplitude detection of the pulse is not exact. Due to this, errors are introduced in the received signal.

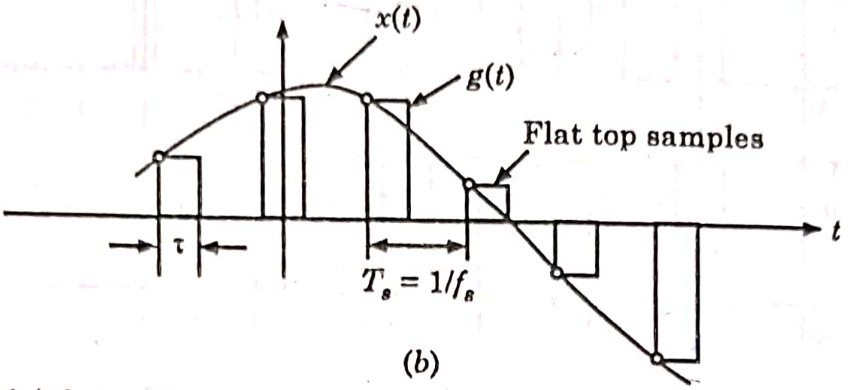
DO YOU KNOW?

Pulse-amplitude modulation is produced by sampling the analog signal. This is done by periodically opening a gate for a brief period, allowing a narrow portion of the analog signal to pass through.

Therefore, Flat top sampled PAM is widely used.

Figure 8.22 shows the sample and hold circuit to produce Flat top sampled PAM and the waveform for flat top sampled PAM.





(b)

Fig. 8.22. (a) Sample and hold circuit generating flat top sampled PAM (b). Waveforms of flat top sampled PAM

Working Principle

A sample and hold circuit shown in figure 8.22 is used to produce Flat top sampled PAM. The working principle of this circuit is quite easy. The sample and Hold (S/H) circuit consists of two field effect transistors (FET) switches and a capacitor. The sampling switch is closed for a short duration by a short pulse applied to the gate G_1 of the transistor. During this period, the capacitor 'C' is quickly charged upto a voltage equal to the instantaneous sample value of the incoming signal $x(t)$. Now, the sampling switch is opened and the capacitor 'C' holds the charge. The discharge switch is then closed by a pulse applied to gate G_2 of the other transistor. Due to this, the capacitor 'C' is discharged to zero volts. The discharges switch is then opened and thus capacitor has no voltage.

Hence, the output of the sample and hold circuit consists of a sequence of flat top samples as shown in figure 8.23.(b).

Mathematical Analysis

In a flat top PAM, the top of the samples remains constant and is equal to the instantaneous value of the baseband signal $x(t)$ at the start of sampling. The duration or width of each sample is τ and sampling rate is equal to $f_s = \frac{1}{T_s}$. From figure 8.22 (b), it may be noted that only starting edge of the pulse represents instantaneous value of the baseband signal $x(t)$. Also, the flat top pulse of $g(t)$ is mathematically equivalent to the convolution of instantaneous sample and a pulse $h(t)$ as depicted in figure 8.24.

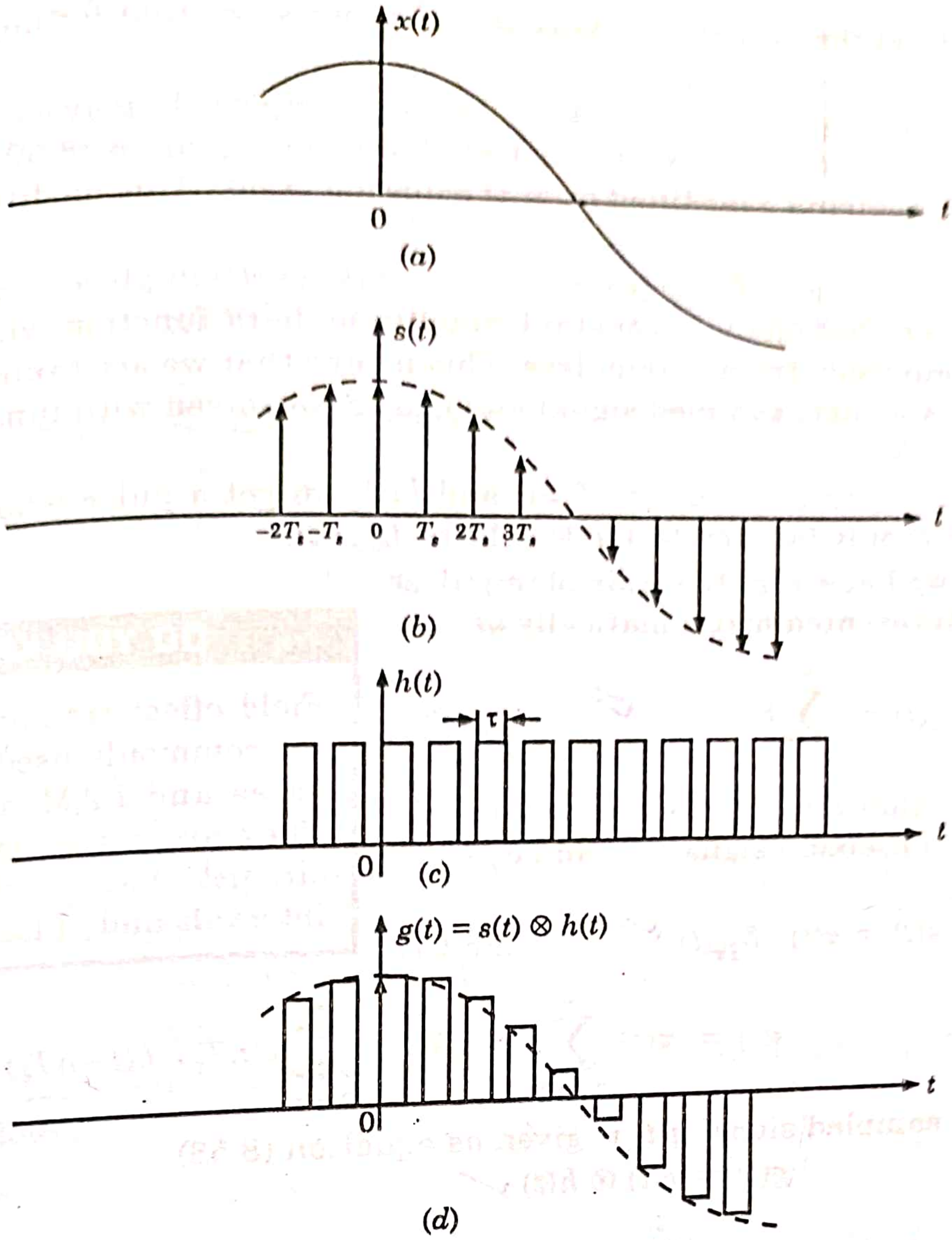


Fig. 8.23. (a) Baseband signal $x(t)$ (b) Instantaneously sample single $s(t)$ (c) Constant pulse width function $h(t)$ (d) Flat top sampled PAM signal $g(t)$ obtained through convolution of $h(t)$ and $s(t)$.

This means that the width of the pulse in $g(t)$ is determined by the width of $h(t)$ and the sampling instant is determined by the delta function.

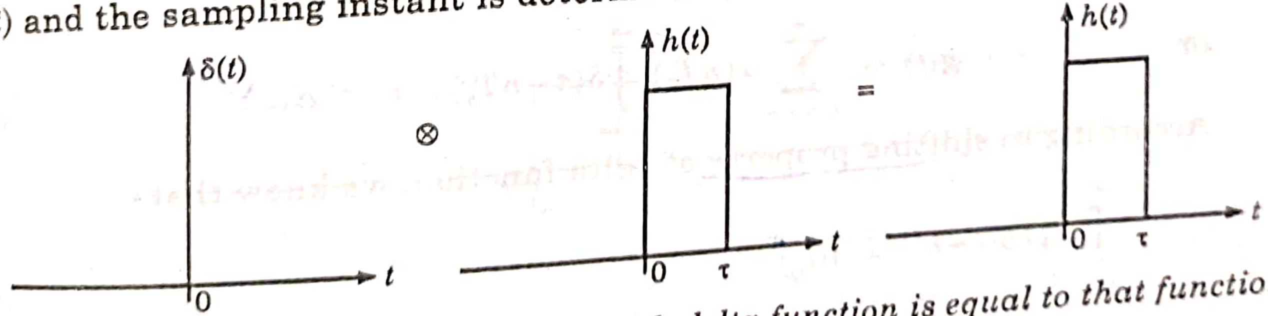


Fig. 8.24. Convolution of any function with delta function is equal to that function.

In figure 8.22 (b), the starting edge of the pulse represents the point where baseband signal is sampled and width is determined by function $h(t)$.

Therefore, $g(t)$ will be expressed as

$$g(t) = s(t) \otimes h(t) \quad ...(8.58)$$

This equation has been explained in figure 8.23.

Now, from the property of delta function, we know that for any function $f(t)$

$$f(t) \otimes s(t) = f(t) \quad ...(8.59)$$

This property is used to obtain flat top samples. It may be noted that to obtain flot top sampling, we are not applying the equation (8.59) directly here i.e., we are applying a modified from of equation (8.59). This modified equation is equation (8.58).

Thus, in this modified equation, we are taking $s(t)$ in place of delta functions $\delta(t)$. Observe that $\delta(t)$ is a constant amplitude delta function whereas $s(t)$ is a varying amplitude train of impulses. This means that we are taking $s(t)$ which is an instantaneously sampled signal and this is convolved with function $h(t)$ as in equation (8.58).

Therefore, on convolution of $s(t)$ and $h(t)$, we get a pulse whose duration is equal to $h(t)$ only but amplitude is defined by $s(t)$.

Now, we know that the train of impulses may be represented mathematically as

$$\delta_{Ts}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad ...(8.60)$$

The signal $s(t)$ is obtained by multiplication of baseband signal $x(t)$ and $\delta_{Ts}(t)$.

Thus,

$$s(t) = x(t) \cdot \delta_{Ts}(t) \quad ...(8.61)$$

> **DO YOU KNOW?**
>
> Field effect transistor switches are commonly used in sampling gates and PAM multiplexers. They are controlled by digital circuits that set the sampling intervals and pulse rates.

or

$$s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s) \quad ...(8.62)$$

Now, sampled signal $g(t)$ is given as equation (8.58)

$$g(t) = s(t) \otimes h(t) \quad ...(8.63)$$

or

$$g(t) = \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau \quad ...(8.64)$$

or

$$g(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(\tau - nT_s)\, h(t-\tau)\, d\tau$$

or

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s)\, h(t-\tau)\, d\tau \quad ...(8.65)$$

According to shifting property of delta function, we know that

$$\int_{-\infty}^{\infty} f(t)\, \delta(t - t_0) = f(t_0) \quad ...(8.66)$$

Using equations (8.65) and (8.66), we get

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t - nT_s)$$

This equation represents value of $g(t)$ in terms of sampled value $x(nT_s)$ and function $h(t - nT_s)$ for flat top sampled signal.

Now, again from equation (8.58), we have

$$g(t) = s(t) \otimes h(t)$$

Taking Fourier transform of both sides of above equation, we get

$$G(f) = S(f)\, H(f) \quad ...(8.67)$$

We know that $S(f)$ is given as

$$S(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \quad ...(8.68)$$

Therefore, equation (8.67) becams

$$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)\, H(f) \quad ...(8.69)$$

Thus, spectrum of flat top PAM signal :

$$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)\, H(f) \quad ...(8.70)$$

Here, $H(f)$ is the Fourier transform of the rectangular pulse. The spectrum of this rectangular pulse is shown in figure 8.18(*b*). Let the spectrum of $s(t)$ be the rectangular pulse train as shown in figure 8.25(*a*) and the spectrum of $h(t)$ *i.e.*, $H(f)$ is shown in figure 8.25(*b*).

By equation (8.67), we know that

$$G(f) = S(f) \cdot H(f)$$

Thus, according to above equation, we can plot the spectrum $G(f)$ as shown in figure 8.25(*b*).

Note :

It may be observed in figure 8.25(b) that higher frequencies in S(f) are attenuated due to roll-off characteristics of the 'sinc' pulse. This effect is popularly known as aperture effect.

An equalizer is needed to overcame this effect.

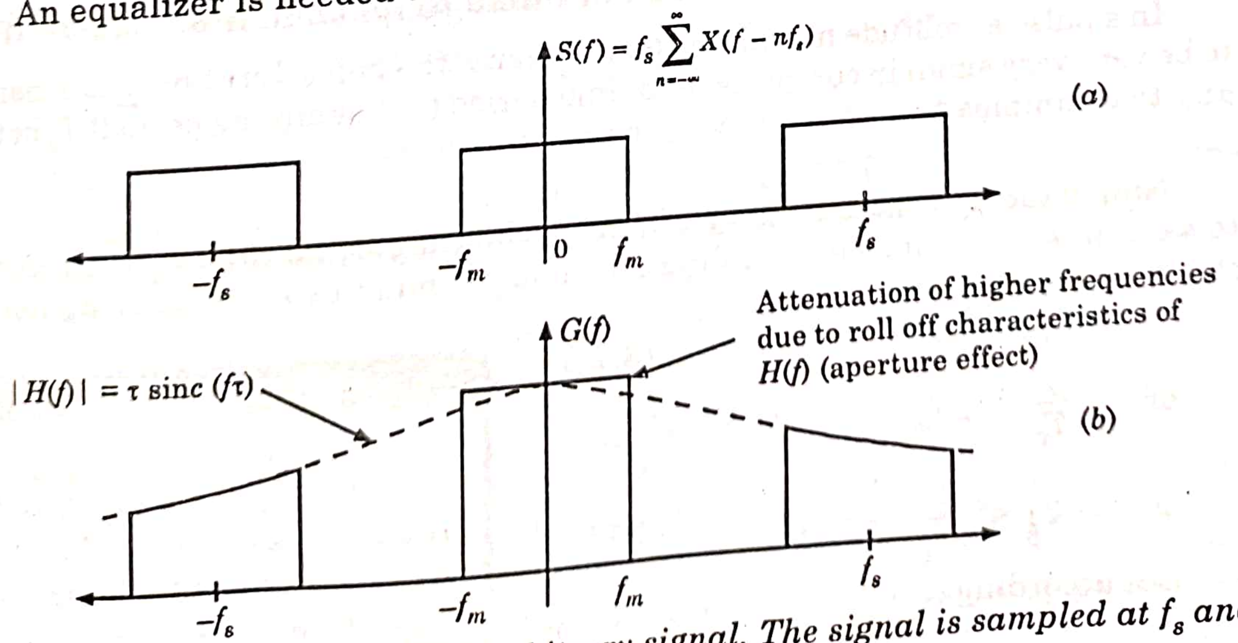


Fig. 8.25. (a) Spectrum of some arbitrary signal. The signal is sampled at f_s and maximum frequency in the signal is f_m,
(b) Spectrum of flat top signal . The dotted curve is $H(f) = \tau \sin c\, (f\tau)$

8.13.1. Naturally Sampled Pulse Amplitude Modulated (PAM) Signal

We have discussed natural sampling in article 8.9. This natural sampling is basically pulse amplitude modulation (PAM). Therefore, it is called naturally sampled PAM signal.

Thus, time-domain representation of a naturally-sampled PAM signal will be given as

$$g(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t), \text{sinc}(f_n\tau)\, e^{j2\pi n f_s t} \quad ...(8.71)$$

and the frequency-domain representation, *i.e.* frequency-spectrum of a naturally-sampled PAM signal will be given as

$$G(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}\,(nf_s\tau)\cdot X(f - nf_s) \quad ...(8.72)$$

8.13.2. Instantaneous or Idealy Sampled Pulse Amplitude Modulated (PAM) Signal

We have discussed ideal or instantaneous sampling in article 8.9. This instantaneous sampling is basically pulse amplitude modulation (PAM). Therefore, it is called ideally or instantaneously sampled PAM signal.

Thus, time-domain representation of a ideally or instantaneously sampled PAM signal will be given as

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\cdot\delta(t - nT_s) \quad ...(8.73)$$

and the frequency-domain representation *i.e.*, frequency-spectrum of a ideally or instantaneously - sampled PAM sinal will be given as

$$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s). \quad ...(8.74)$$

8.13.3. Transmission Bandwidth in Pulse Amplitude Modulation (PAM)

In a pulse amplitude modulated (PAM) signal the pulse duration 'τ' is considered to be very very small in comparison to time period (*i.e.*, sampling period) T_s between any two samples *i.e.*,

$$\tau << T_s \quad ...(8.75)$$

Now, if the maximum frequency in the modulating signal $x(t)$ is f_m, then according to sampling theorem, the sampling frequency f_s must be equal to or higher than the Nyquist rate, *i.e.*

or $$f_s \geq 2f_m \quad ...(8.76)$$

or $$\frac{1}{T_s} \geq 2f_m \qquad \left(\because f_s = \frac{1}{T_s}\right)$$

or $$T_s \leq \frac{1}{2f_m}$$

But according to equation (8.75), we have

$$\tau << T_s$$

DO YOU KNOW?

Pulse-amplitude modulation signals may be multiplexed by allowing samples of several signals to be interleaved into adjacent time slots.

Therefore

$$\tau << T_s \leq \frac{1}{2f_m} \quad ...(8.77)$$

Now, if the 'ON' and 'OFF' time of the pulse amplitude modulated (PAM) pulse is same as shown in figure 8.26(*a*) then maximum frequency of the PAM pulse will be

$$f_{max} = \frac{1}{\tau+\tau} = \frac{1}{2\tau} \quad ...(8.78)$$

Therefore, the bandwidth required for the transmission of a PAM signal would be equal to the maximum frequency f_{max} given by the equation (8.78).

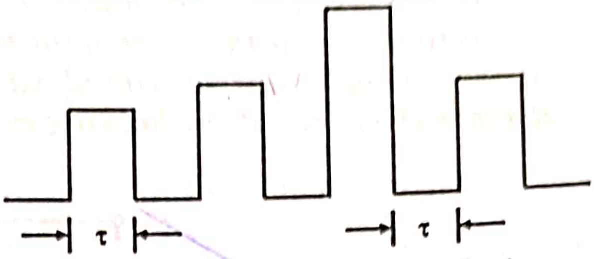


Fig. 8.26. (a) Illustration of maximum frequency in PAM signal.

Thus, we have

Transmission bandwidth

$$BW \geq f_{max} \quad ...(8.79)$$

But $$f_{max} = \frac{1}{2\tau}$$

Hence $$BW \geq \frac{1}{2\tau}$$

Again, since $$\tau << \frac{1}{2f_m}$$

Therefore $$BW \geq \frac{1}{2\tau} >> f_m \quad ...(8.80)$$

or $$BW >> f_m \quad ...(8.81)$$

8.13.4. Demodulation of PAM Signals

As discussed earlier, demodulation is the reverse process of modulation in which the modulating signal is recovered back from a modulated signal. For pulse-amplitude modulated (PAM) signals, the demodulation is done using a Holding circuit. Figure 8.26(*b*) shows the block diagram of a PAM demodulator.

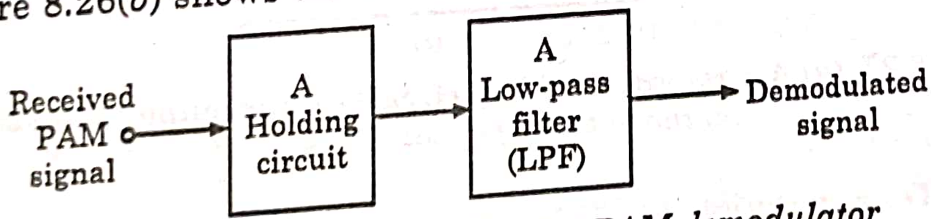


Fig. 8.26(b). A block diagram of PAM demodulator

In this method, the received PAM signal is allowed to pass through a Holding circuit and a low pass filter (LPF) as shown in above figure. Now, figure 8.27(*a*) illustrates a very simple holding circuit. Here the switch '*S*' is closed after the arrival of the pulse and it is opened at the end of the pulse. In this way, the capacitor *C* is charged to the pulse amplitude value and it holds this value during the interval between the two pulses. Hence, the sampled values are held as shown in figure 8.27(*c*). After this the holding circuit output is smoothened in Low Pass filter as shown in figure 8.27(*c*). It may be observed that some kind of distortion is introduced due to the holding circuit. In fact the circuit of figure

DO YOU KNOW?

Special clock recovery circuits use the PAM signal itself to derive the clock signal at the receiver rather than generating it independently. This ensures perfect frequency and phase relationships.

8.27 (b) is known as **zero-order Holding circuit.** This zero-order Holding circuit considers only the previous sample to decide the value between the two pulses.

Note :

It may be noted the first order hold circuit considers the previous two samples whereas a second order holding circuit considers the previous three samples and so on. However, as the order of the holding circuit increases, the distortion decreases at the cost of the circuit complexity. In fact, the amount of permissible distortion decides the order of the holding circuit.

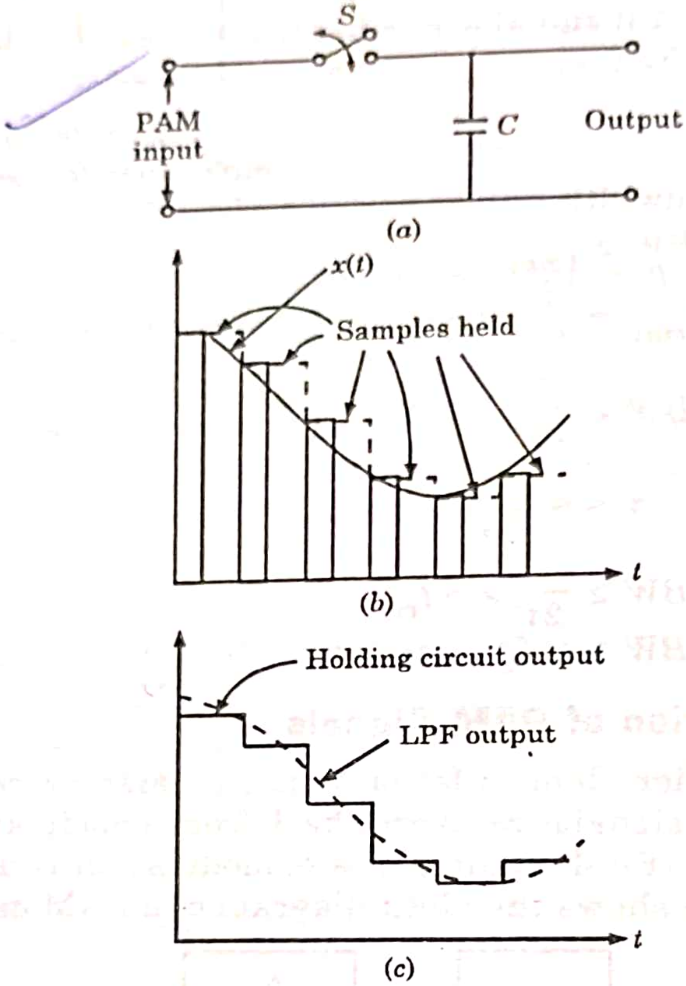


Fig. 8.27. (a) A zero-order holding circuit (b) the output of holding circuit (c) the output of a Low Pass filter (LPF)

8.13.5. Transmission of PAM signals

If the PAM signals are to be transmitted directly *i.e.*, over a pair of wires then no further signal processing is necessary. However, if they are to be transmitted through the space using an antenna, they must first be amplitude or frequency or phase modulated by a high frequency carrier and only then they can be transmitted. Thus, the overall system will be then known as PAM-AM or PAM-FM or PAM-PM respectively. At the receiving end, AM or FM or PM detection is first employed to get the PAM signal and then the message signal is recovered from it.

EXAMPLE 8.8. For a pulse-amplitude modulated (PAM) transmission of voice signal having maximum frequency equal to f_m = 3 kHz, calculate the transmission bandwidth. It is given that the sampling frequency f_s = 8 kHz and the pulse duration $\tau = 0.1T_s$.

Solution : We know that the sampling period T_s is expressed as

$$T_s = \frac{1}{f_s} = \frac{1}{8 \times 10^3} \text{ seconds}$$

$$T_s = 0.125 \times 10^{-3} \text{ seconds}$$

or

$$T_s = 125 \ \mu \text{ seconds} \quad ...(i)$$

Also, τ is given that

$$\tau = 0.1 \ T_s$$

Using (*i*), we get

$$\tau = 0.1 \times 125 = 15.5 \ \mu \text{ seconds} \quad ...(ii)$$

Now, we know that the transmission bandwidth for PAM signal is expressed as

$$BW \geq \frac{1}{2\tau}$$

Using equation (*ii*), we get

$$BW \geq \frac{1}{2 \times 12.5 \times 10^{-6}} \geq \frac{1 \times 10^6}{25}$$

$$BW \geq 40 \text{ kHz}$$ **Ans.**

8.13.6. Drawbacks of Pulse-Amplitude Modulated (PAM) Signal

Following are the drawbacks of a PAM signal :

(*i*) The bandwidth required for the transmission of a PAM signal is very very large in comparison to the maximum frequency present in the modulating signal.

(*ii*) Since the amplitude of the PAM pulses varies in accordance with the modulating signal therefore the interference of noise is maximum in a PAM signal. This noise cannot be removed easily.

(*iii*) Since the amplitude of the PAM signal varies, therefore, this also varies the peak power required by the transmitter with modulating signal.

8.14. PULSE TIME MODULATION

In pulse time modulation, the signal to be transmitted is sampled as in pulse amplitude modulation (PAM). In pulse time modulation, amplitude of pulse is held constant, whereas position of pulse or width of pulse is made proportional to the amplitude of signal at the sampling instant. There are two types of pulse time modulation, viz. Pulse Width Modulation (PWM) and Pulse Position Modulation (PPM). Because in both PWM and PPM, amplitude is held constant and does not carry any information, therefore amplitude limiters can be used. The amplitude limiters, similar to those used in FM, will clip off the portion of the signal corrupted by noise and hence provide a good degree of noise immunity.

DO YOU KNOW?

In PDM, long pulses expend considerable power while bearing no additional information. If this unused power is subtracted from PDM so that only time transitions are preserved, we obtain PPM. Accordingly, PPM is a more efficient form of pulse modulation than PDM.

8.14.1. Pulse Width Modulation

Let us first discuss Pulse Width Modulation (PWM). This is also known as Pulse Duration Modulation (PDM). Three variations of pulse width modulation possible. In one variation, the leading edge of the pulse is held constant and change in pulse width with signal is measured with respect to the leading edge. In other variation, the tail edge is held constant and with respect to it, pulse width is measured. In the third variation, centre of the pulse is held constant and pulse width changes on either side of the centre of the pulse. This has been illustrated in figure 8.28.

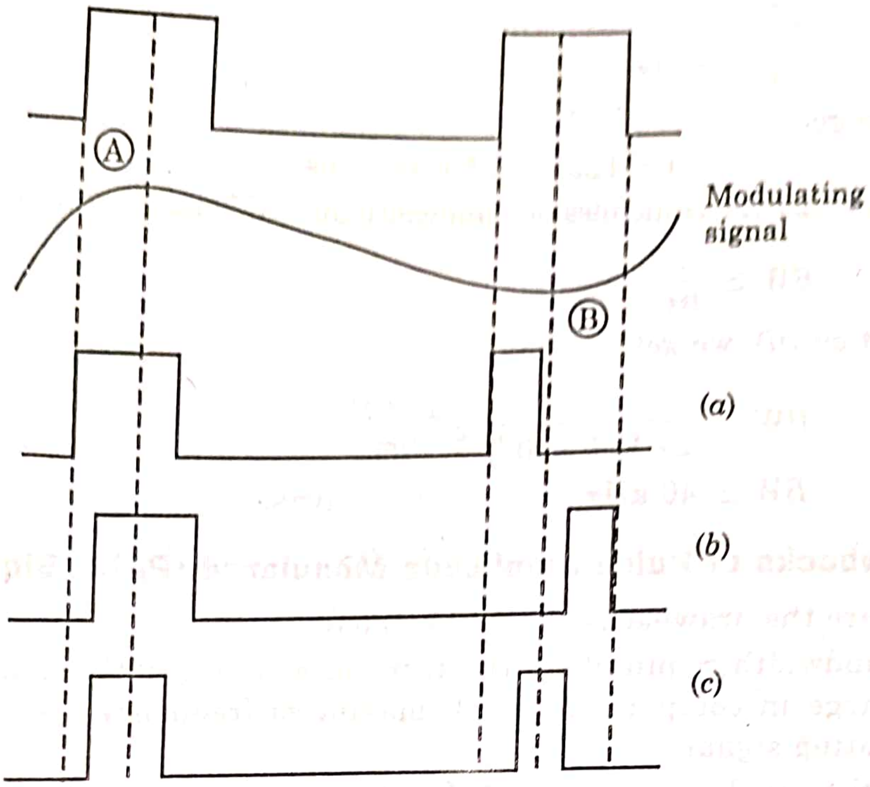


Fig. 8.28. PWM waveforms

The modulating signal is at its positive peak at point (A) and at its negative peak at (B). In figure 8.28 (a), the leading edge of pulse is kept constant and pulse width is measured from the lead edge. As shown, pulse width is maximum corresponding to point (A), while it is minimum at point (B).

In figure 8.28 (*b*), the tail edge of the pulse is kept constant and pulse width is measured from the tail end of the pulse. As before, pulse width is maximum corresponding to positive peak of the modulating signal and minimum at the negative peak.

As shown in figure 8.28 (*c*), the center of the pulse is kept constant and pulse extends on either side of the center of the pulse, depending upon the modulating signal.

8.14.2. Frequency Spectrum for PWM Wave

With a sinusoidal modulating signal at frequency f_m, the spectrum of PWM signal consists the modulating signal frequency f_m along with several harmonics. This is shown in figure 8.29.

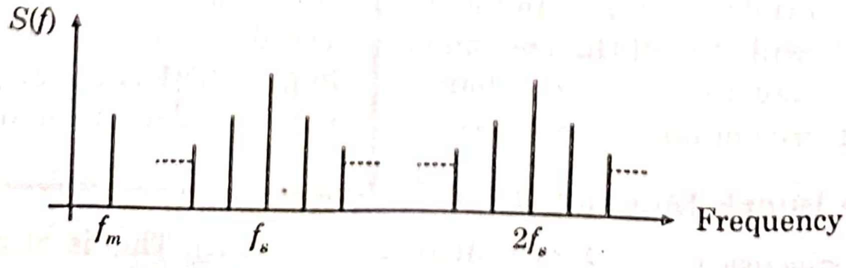


Fig. 8.29. Spectrum of PWM signal

To have a better separation with respect to frequency, between highest frequency of baseband signal [in Fig. 8.29, f_m] and lower sidebands of f_s (sampling frequency), a higher sampling frequency which is more than Nyquist rate is used; and pulse width deviation is kept small.

8.14.3. Modulation of PWM Signal or PWM Generation

(Anna University, Chennai)(10 marks)

Figure 8.30 shows pulse width modulator. It is basically a monostable multivibrator with a modulating input signal applied at the control voltage input. Internally, the control voltage is adjusted to the 2/3 V_{CC}. Externally applied modulating signal changes the control voltage, and hence the threshold voltage level. As a result, the time period required to charge the capacitor up to threshold voltage level changes, giving pulse modulated signal at the output, as shown in the figure 8.30(*b*).

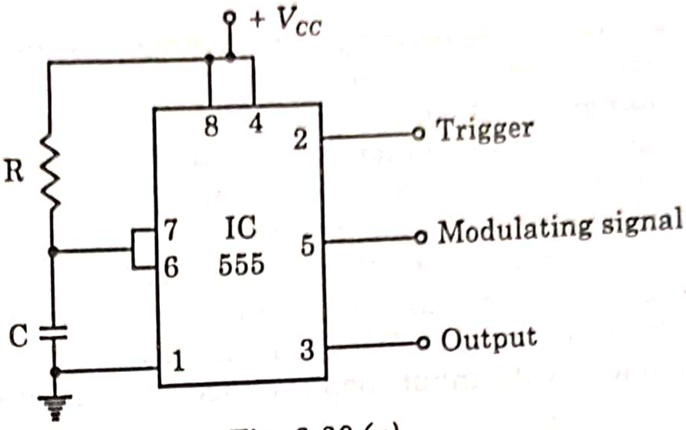


Fig. 8.30 (a)

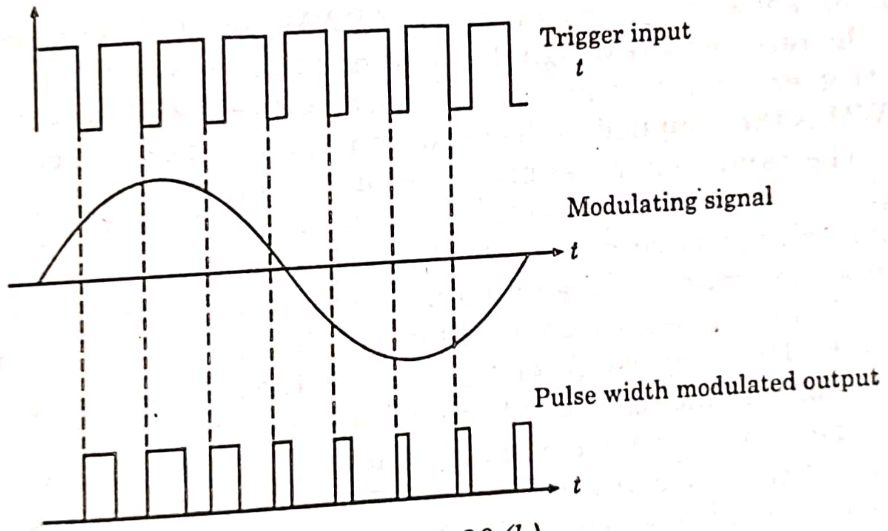


Fig. 8.30 (b)

Figure 8.31 illustrates another monostable multivibrator circuit to generate pulse width modulation (PWM).

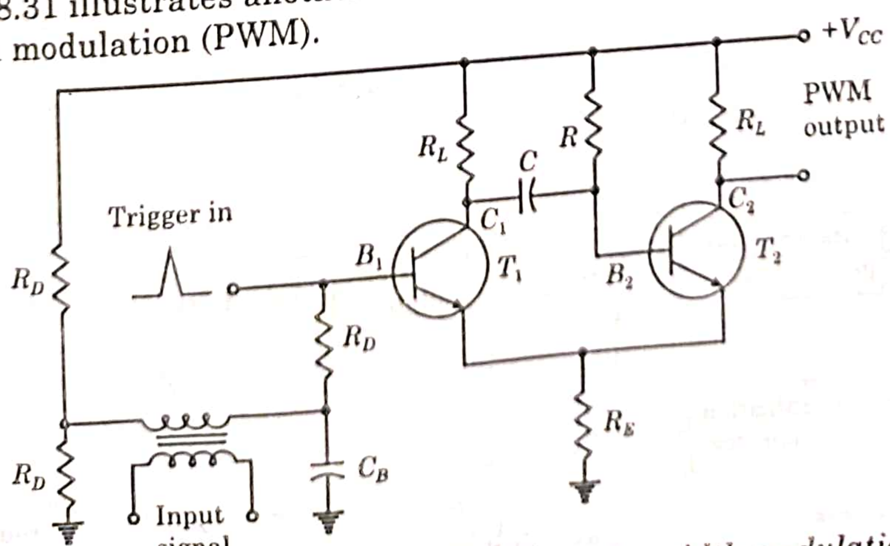


Fig. 8.31. Monostable multivibrator generating pulse width modulation (PWM).

The stable state for above circuit is achieved when T_1 is OFF and T_2 is ON. The positive going trigger pulse at B_1 switches T_1 ON. Because of this, the voltage at C_1 falls as T_1 now begins to draw the collector current. As a result, voltage at B_2 also falls and T_2 is switched OFF, C begins to charge up to the collector supply voltage (V_{CC}) through resistor R. After a time determined by the supply voltage and the RC time constant of the charging network, the base of the T_2 becomes sufficiently positive to switch T_2 ON. The transistor T_1 is simultaneously switched OFF by regenerative action and stays OFF until the arrival of the next trigger pulse. To make T_2 ON, the base of the T_2 must be slightly more positive than the voltage across resistor R_E. This voltage depends on the emitter curent I_E which is controlled by the signal voltage applied at the base of transistor T_1. Therefore, the changing voltage necessary to turn OFF transistor T_2 is decided by the signal voltage. If signal voltage is maximum, the voltage that capacitor should charge to turn ON T_2 is also maximum. Therefore, at maximum signal voltage, capacitor has to charge to maximum voltage requiring maximum time to charge. This gives us maximum pulse width at maximum input signal voltage. At minimum signal voltage, capacitor has to charge for minimum voltage and we get minimum pulse width at the output. With this discussion, it can be noted that pulse width is controlled by the input signal voltage, and we get pulse width modulated waveform at the output.

8.14.4. Demodulation of PWM Signal

Figure 8.32 (*a*) shows the block diagram of PWM detector. As shown in the figure 8.32 (*a*), the received PWM signal is applied to the Schmitt trigger circuit. This Schmitt trigger circuit removes the noise in the PWM waveform. The regenerated PWM is then applied to the ramp generator and the synchronization pulse detector. The ramp generator produces ramps for the duration of pulses such that height of ramps are proportional to the widths of PWM pulses. The maximum ramp voltage is retained till the next pulse. On the other hand, synchronous pulse detector produces reference pulses with constant amplitude and pulse width. These pulses are delayed by specific amount of delay as shown in the figure 8.32 (*b*). The delayed reference pulses and the output of ramp generator is added with the help of adder. The output of adder is given to the level shifter. Here, negative offset shifts the waveform as shown in the figure 8.32 (*b*). Then the negative part of the waveform is clipped by rectifier. Finally, the output of rectifier is passed through low-pass filter to recover the modulating signal, as shown in the figure 8.32 (*b*).

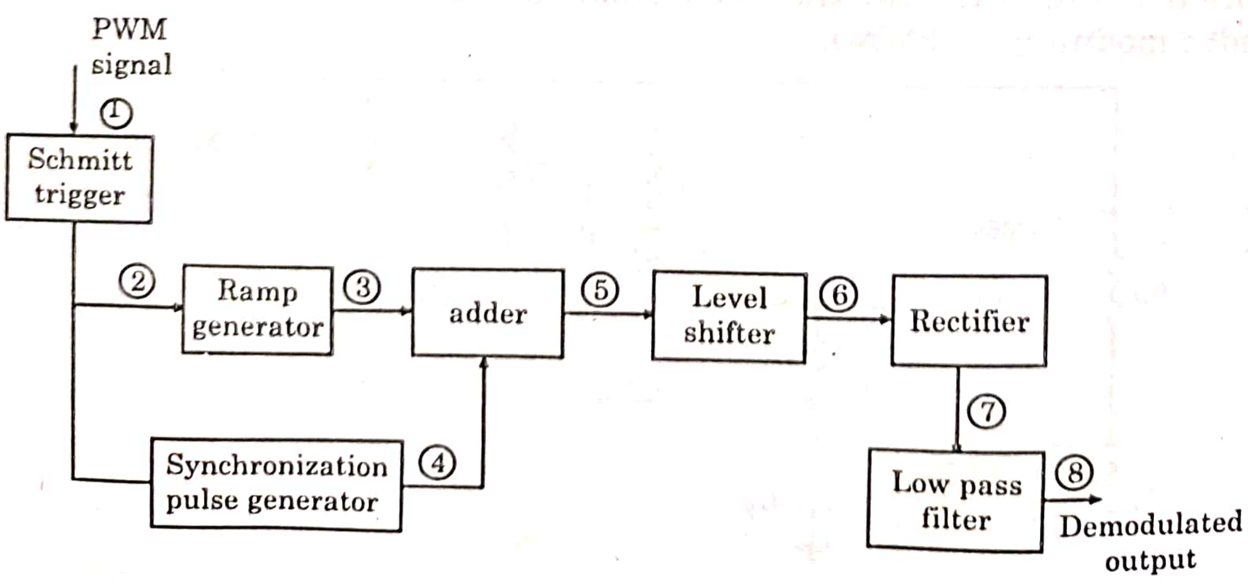


Fig. 8.32. (a) PWM detector

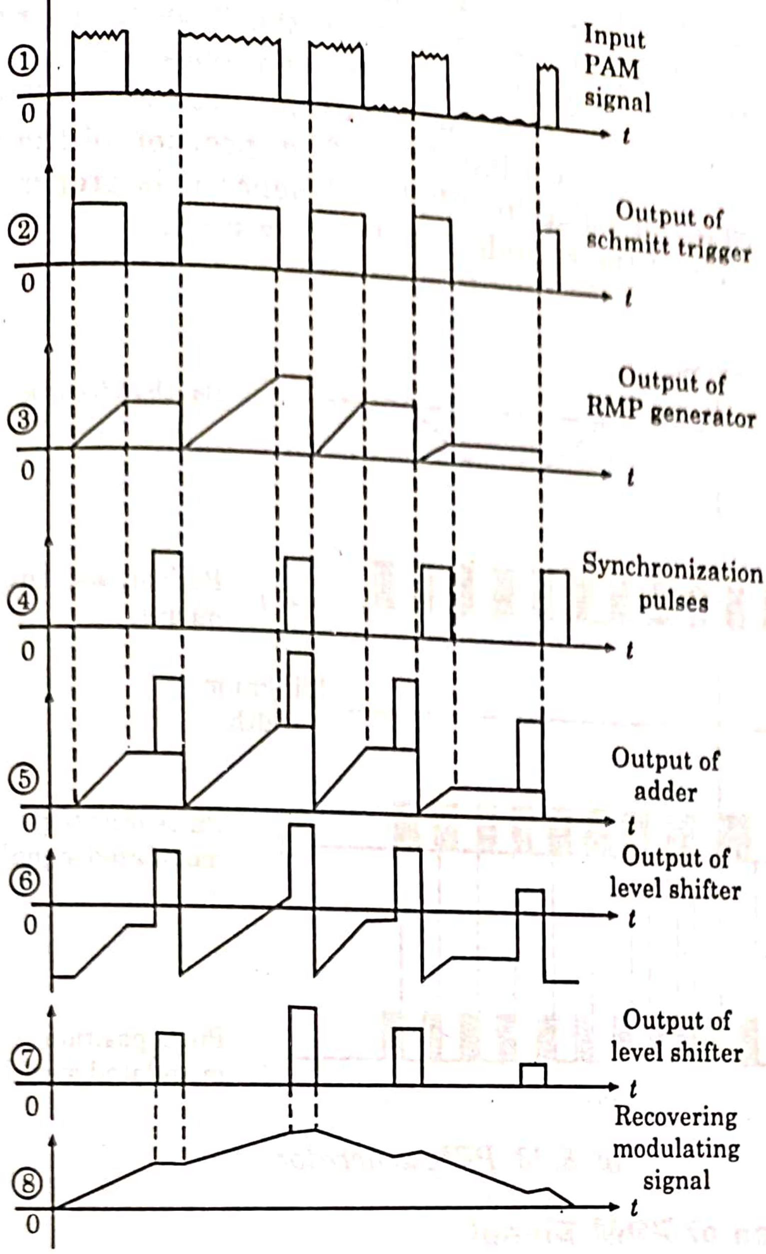


Fig. 8.32. (b) Waveforms for PWM detection circuit.

8.14.5 Advantages of PWM

(i) Unlike, PAM, noise is less, since in PWM, amplitude is held constant.

(ii) Signal and noise separation is very easy, as shown in figure 8.32 (b).

(iii) PWM communication does not require synchronization between transmitter and receiver.

8.14.6. Disadvantages of PWM

(i) In PWM, pulses are varying in width and therefore their power contents are variable. This requires that the transmitter must be able to handle the power contents of the pulse having maximum pulse width.

(ii) Large bandwidth is required for the PWM communication as compared to PAM.

8.14.7. Pulse Position Modulation

In this system, the amplitude and width of the pulses are kept constant, while the position of each pulse, with reference to the position of a reference pulse, is changed according to the instantaneous sampled value of the modulating signal. Thus, the transmitter has to send synchronizing pulses to keep the transmitter and receiver in synchronism. As the amplitude and width of the pulses are constant, the transmitter handles constant power output, a definite advantage over the PWM. But the disadvantage of the PPM system is the need for transmitter-receiver synchronization. Pulse position modulation is obtained from pulse width modulation, shown in the figure 8.33. Each trailing edge of the PWM pulse is a starting point of the pulse in the PPM. Therefore, position of the pulse is 1:1 proportional to the width of pulse in PWM and hence it is proportional to the instantaneous amplitude of the sampled modulating signal.

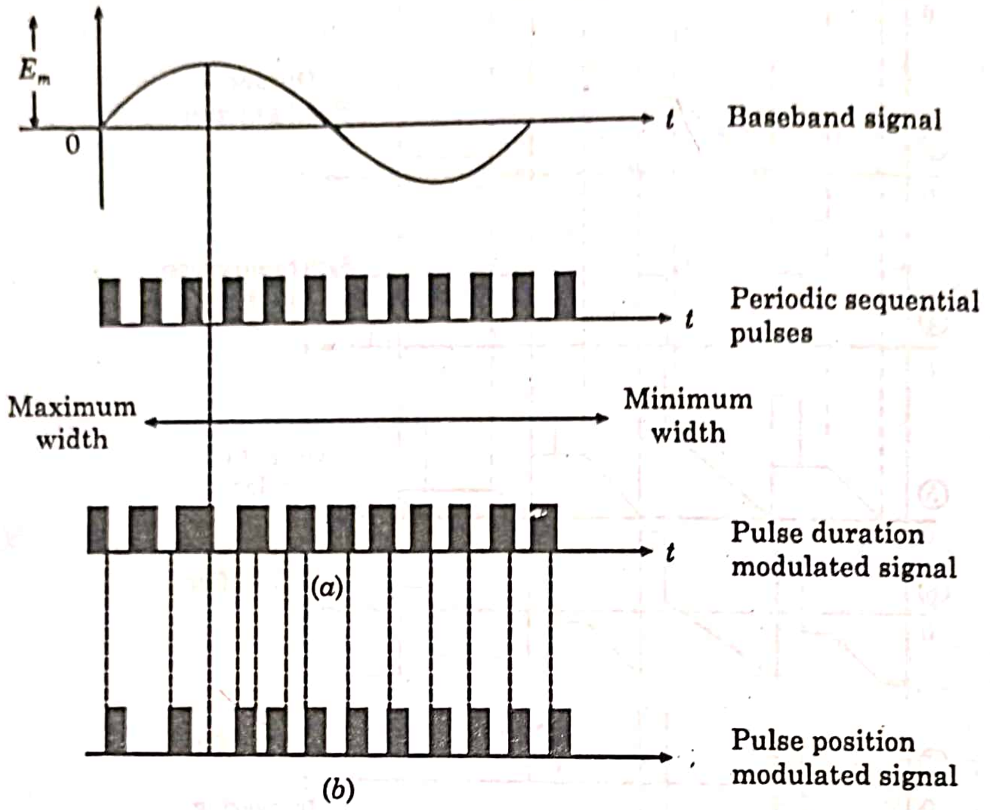


Fig. 8.33. PPM generator

8.14.8. Generation of PPM Signal

Figure 8.34 (*a*) shows the PPM generator. It consists of differentiator and a monostable multivibrator. The input to the differentiator is a PWM waveform. The differen-tiator generates positive and negative spikes corresponding to leading and trailing edges of the PWM waveform. Diode D1 is used to bypass the positive spikes. The negative spikes are used to the trigger monostable multivibrator. The monostable multivibrator then generates the pulses of same width and amplitude with reference to trigger to give pulse position modulated waveform, as shown in figure 8.34 (*b*).

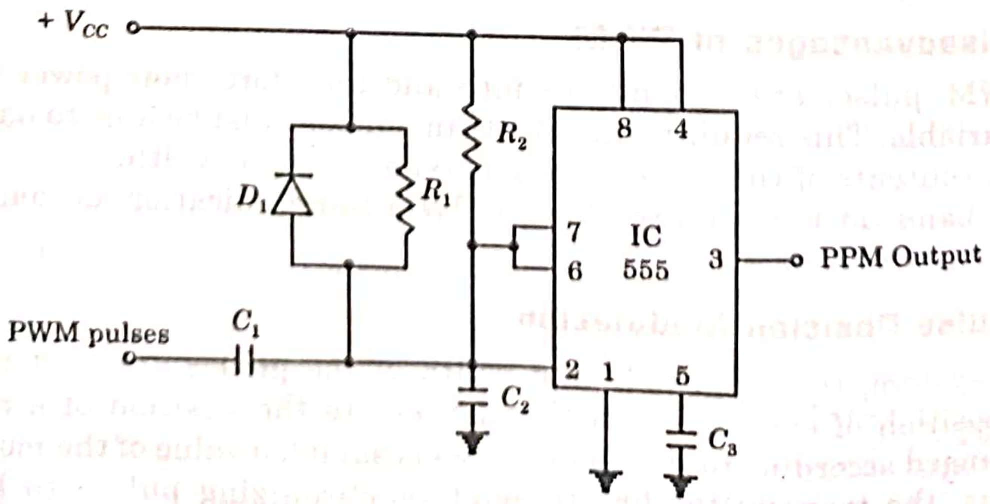


Fig. 8.34. (a) PPM generator

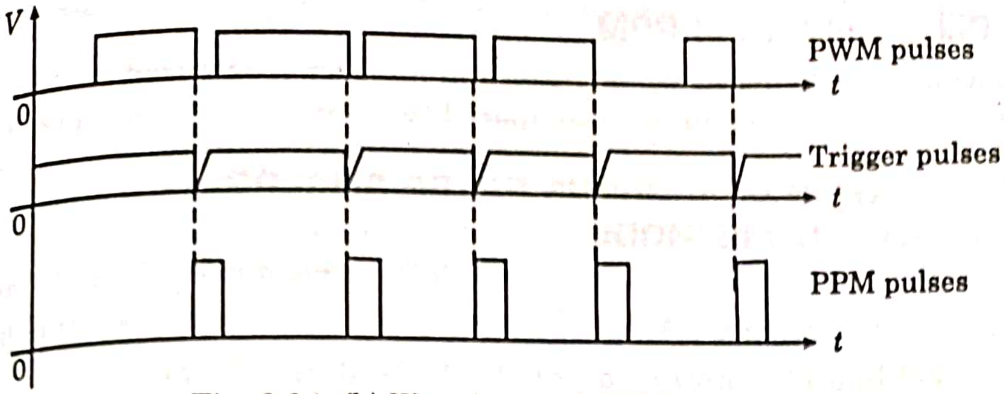


Fig. 8.34. (b) Waveforms of PPM generator

8.14.9. Demodulation of PPM

In case of pulse-position modulation, it is customary to convert the received pulses that vary in position to pulses that vary in length. One way to achieve this is illustrated in figure 8.35.

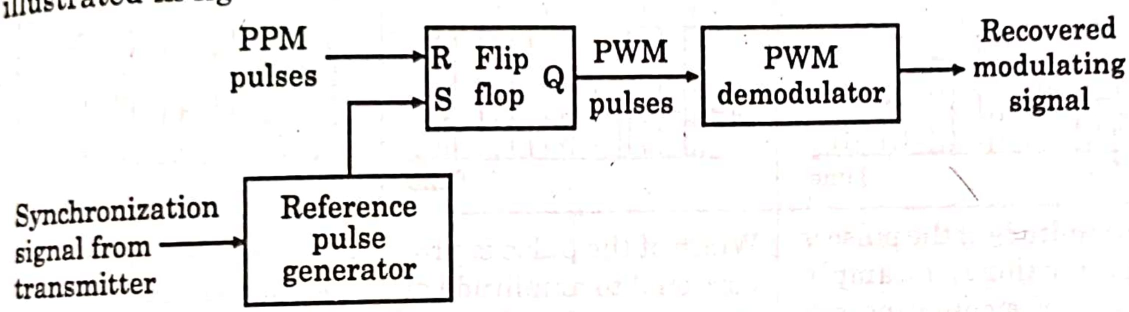


Fig. 8.35. PPM demodulator

As shown in figure 8.35, flip-flop circuit is set or turned 'ON' (giving high output) when the reference pulse arrives. This reference pulse is generated by reference pulse generator of the receiver with the synchronization signal from the transmitter. The flip-flop circuit is reset or turned 'OFF' (giving low output) at the leading edge of the position modulated pulse. This repeats and we get PWM pulses at the output of the flip-flop.

The PWM pulses are then demodulated by PWM demodulator to get original modulating signal.

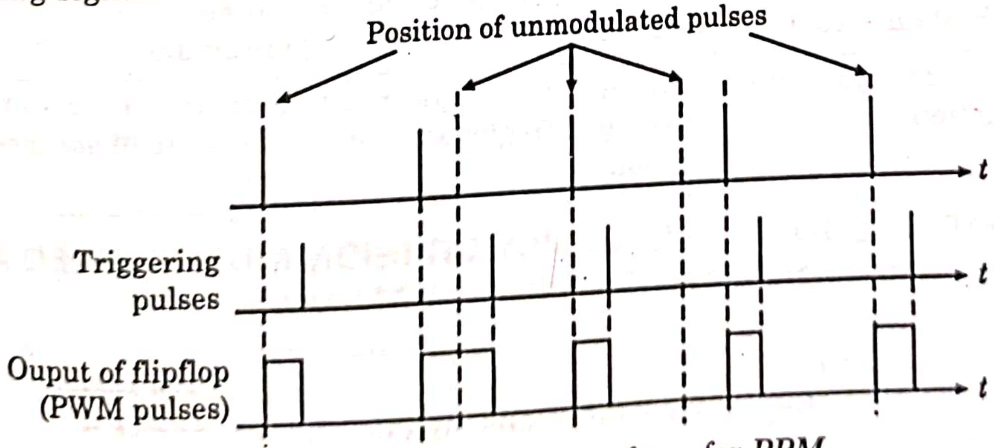


Fig. 8.36. Demodulation waveform for PPM

8.14.10. Advantages of PPM

(*i*) Like PWM, in PPM, amplitude is held constant thus less noise interference.
(*ii*) Like PPM, signal and noise separation is very easy.
(*iii*) Because of constant pulse widths and amplitudes, transmission power for each pulse is same.

8.14.11. Disadvantages of PPM

(i) Synchronization between transmitter and receiver is required.

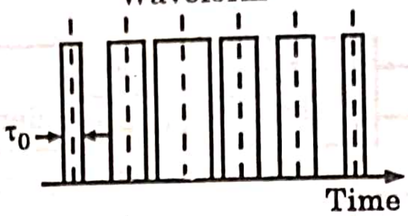
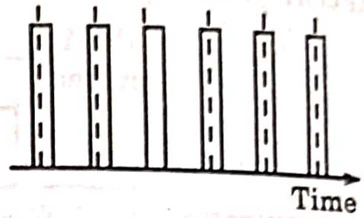
(ii) Large bandwidth is required as compared to PAM.

8.15. COMPARISON OF VARIOUS PULSE ANALOG MODULATION METHODS

(JNTU, Hyderanad, 2003)(04 marks)

In this section, let us compare PAM, PWM and PPM in the form of a Table 8.2

Table 8.2. Comparison of PAM, PPM and PDM

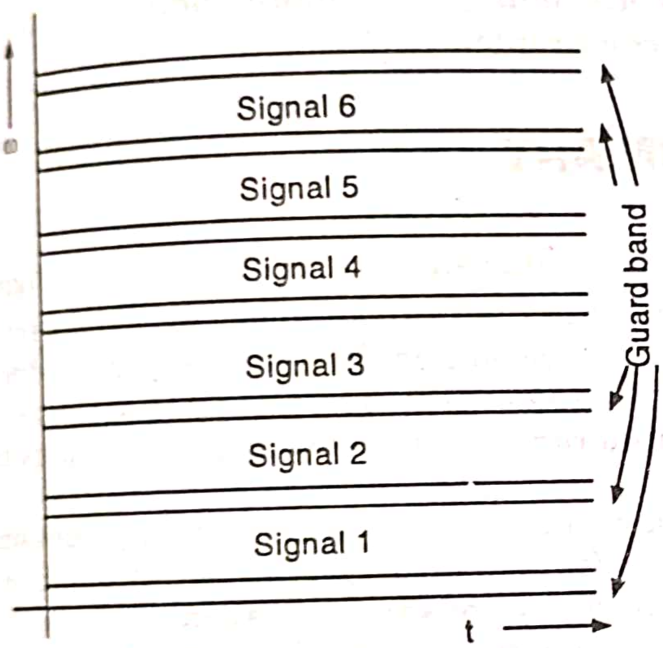
Sr. No.	Pulse Amplitude Modulation (PAM)	Pulse Width/Duration Modulation (PWM) or (PDM)	Pulse Position Modulation (PPM)
1	Waveform 	Waveform 	Waveform 
2	Amplitude of the pulse is proportional to amplitude of modulating signal.	Width of the pulse is proportional to amplitude of modulating signal.	The relative position of the pulse is proportional to the amplitude of modulating signal.
3	The bandwidth of the transmission channel depends on width of the pulse.	Bandwidth of transmission channel depends on rise time of the pulse.	Bandwidth of transmission channel depends on rising time of the pulse.
5	The instantaneous power of the transmitter varies.	The instantaneous power of the transmitter varies.	The instantaneous power of the transmitter remains constant.
6	Noise interference is high. System is complex	Noise, interference is minimum.	Noise, interference is minimum
7	Similar to amplitude modulation.	Simple to implement similar to frequency modulation.	Simple to implement similar to phase modulation.

8.16. COMPARISON OF FREQUENCY DIVISION MULTIPLEXED AND TIME DIVISION MULTIPLEXED SYSTEMS

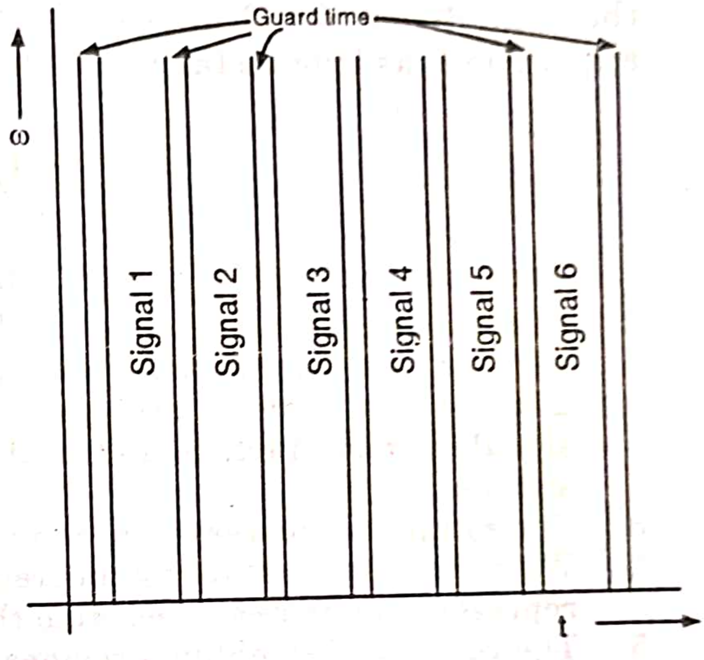
Up to now, we have discussed two methods of simultaneous transmission of several bandlimited signals on a channel. In frequency division multiplexed systems, all of the signals to be transmitted are continuous signals and are mixed in the time domain. However, the spectra of the various modulated signals, occupy different bands in the frequency domain and can be separated by appropriate filters. Hence, the signals are all mixed in the time domain but maintain their identity in the frequency domain.

On the other hand, in case of time division multiplexing, the samples of each signal remain distinct and can be recognized and separated in the time domain. But, the frequency spectra of the various sampled signals occupy the same frequency region and are all mixed beyond recognition. Hence the spectrum identity is maintained in frequency division multiplexed signals, whereas the waveshape identity is maintained in time division multiplexed signals. Since a signal is completely specified either by its time-domain or frequency-domain specification, the multiplexed signals can be separated at the receiver by using appropriate techniques in the respective domains.

The distinction between the two systems can be conveniently represented graphically on a communication space which is used to transmit information. The time frequency communication space is shown in figure 8.37 for frequency division multiplexed systems. For a frequency division multiplexed system, each signals is present on the channel all of the time and all are mixed. But, each of them occupies a finite and distinct frequency interval (not occupied by any other signal). This is shown in figure 8.37(*a*). On the other hand, in a time division multiplexed system, each signal occupies a distinct time interval*. However, the spectra of all the signals have components in the same frequency interval. This is shown in figure 8.37(*b*).



(*a*)



(*b*)

Fig. 8.37. Communication space representation of frequency and time multiplexing.

Quantitatively, we have already shown that the bandwidth requirements for the transmission of a given number of signals either by time division multiplexing of by frequency division multiplexing is the same.** It is therefore evident that for a given channel, the number of bandlimited signals that can be simultaneously transmitted by frequency division multiplexing or by time division multiplexing is the same.

From the practical point of view, the time division multiplexed system proves superior to the frequency division multiplexed system. The first advantage is the simplified circuitry used in time division multiplexed systems compared to that used in frequency division multiple..ed systems. In the latter, one needs to generate different carriers for each channel. Moreover, each channel occupies a different frequency band, and hence needs a different bandpass filter design. On the other hand, time division multiplexed systems require identical circuits for each channel, consisting of relatively simple synchronous switches or gating circuits. The only filters in the detection process are the low-pass filters which are identical for each channel. This circuitry is much simpler compared to the modulators, demodulators, carrier generators, and bandpass filters required in the frequency division multiplexed systems.

The second advantages to the time division multiplexed system is the relative immunity from interference within channels* which arises in frequency division multiplexed systems because of nonlinearities in amplifiers in the path of transmission. The nonlinearities in various amplifiers produce harmonic distortion (due to frequency multiplication) and hence will introduce interference within channels (interchannel crosstalk). Hence the nonlinearity requirements in a frequency division multiplexed system are much more stringent than those for a single channel. On the other hand, for time division multiplexed systems, the signals for different channels are not applied to the system simultaneously but are allotted different time intervals. Hence the nonlinearity requirements in a time division multiplexed system are the same as that for a single channel. For these reasons, the time division multiplexed systems are being used more commonly in such applications as long-distance telephone communication.