JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem. – B. Tech II year, Sem.-IV

Subject –Discrete Mathematics Structure, Unit – 2

Topic – Proportional Logic

Presented by – Dr. Kashish Parwani

Designation - Associate Professor

Department - Mathematics

VISION OF INSTITUTE

To become a renowned centre of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

MISSION OF INSTITUTE

❖**Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.**

❖**Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.**

❖**Offer opportunities for interaction between academia and industry.**

❖**Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge in a range of professions.**

Engineering Mathematics: Course Outcomes

Students will be able to:

Upon successful completion of this course students will be able to:

CO1. Understand the concepts of Sets, Relations, Functions and their Operations.

CO2. Learn the concept of Propositional Logic and Finite State Machines.

CO3. Discuss and develop the Posets, Hasse Diagram, Lattices and Combinatorics.

CO4. Use and apply the concept of Algebraic Structures, Groups, Rings and Graph Theory.

Vision and Mission of the Institute

Vision of the Institute

To become a renowned centre of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

Mission of the Institute

- Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.
- Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academia and industry.
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

RTU Scheme

4IT2-01: **Discrete Mathematics Structure**
Class: IV^{th} Sem. B.Tech.
Branch: Information Technology
Schedule per Week- Lectures: 3
Examination Time = Three (3) Hours
Maximum Marks = 150
Evaluation: [Mid-terms (24), Assignments (06), External (120)]

4IT2-01: Discrete Mathematics Structure

Credit: 3 **Max. Marks: 150(IA:30, ETE:120)**
3L+0T+0P **End Term Exam: 3 Hours**

SN	Contents	Hours
1	**Introduction:** Objective, scope and outcome of the course.	1
2	Set Theory: Definition of sets, countable and uncountable sets, Set operations, Partition of set, Cardinality (Inclusion-Exclusion & Addition Principles) Venn Diagrams, proofs of some general identities on sets. Relation: Definition, types of relation, composition of relations, Pictorial representation of relation, Equivalence relation, Partial ordering relation, Job-Scheduling problem. Function: Definition, type of functions, one to one, into and onto function, inverse function, composition of functions, recursively defined functions, pigeonhole principle. Theorem proving Techniques: Mathematical induction, Proof by contradiction. Composition of Functions. The Pigeonhole and Generalized Pigeonhole Principles.	7
3	Propositional Logic: Proposition, First order logic, Basic logical operation, truth tables, tautologies, Contradictions, Algebra of Proposition, logical implications, logical equivalence, predicates, Normal Forms, Universal and existential quantifiers. 2 way predicate logic. Introduction to finite state machine Finite state machines as models of physical system equivalence machines, Finite state machines as language recognizers.	8
4	Posets, Hasse Diagram and Lattices: Introduction, ordered set, Hasse diagram of partially, ordered set, isomorphic ordered set, well ordered set, properties of Lattices, bounded and complemented lattices. Combinatorics: Introduction, Permutation and combination, Binomial Theorem, Multimodal Coefficients Recurrence Relation and Generating Function: Introduction to Recurrence Relation and Recursive algorithms, linear recurrence relations with constant coefficients, Homogeneous solutions, Particular solutions, Total solutions, Generating functions, Solution by method of generating functions.	8
5	Algebraic Structures: Definition, Properties, types: Semi Groups, Monoid, Groups, Abelian group, properties of groups, Subgroup, cyclic groups, Cosets, factor group, Permutation groups, Normal subgroup, Homomorphism and isomorphism of Groups, example and standard results, Rings and Fields: definition and standard results.	8
6	Graph Theory: Introduction and basic terminology of graphs, Planer graphs, Multigraphs and weighted graphs, Isomorphic graphs, Paths, Cycles and connectivity, Shortest path in weighted graph, Introduction to Eulerian paths and circuits, Hamiltonian paths and circuits, Graph coloring, chromatic number, Isomorphism and Homomorphism of graphs, matching, vertex/edge covering.	8
	Total	**40**

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COURSE OUTCOMES:

Subject – Discrete Structure Mathematics
Code – 4IT2-01
Branch – Information Technology
Semester- IVth
Upon successful completion of this course students will be able to:

By the end of this course, the students will be able to:

CO1. Understand the concepts of Sets, Relations, Functions and their Operations.

CO2. Learn the concept of Propositional Logic and Finite State Machines.

CO3. Discuss and develop the Posets, Hasse Diagram, Lattices and Combinatorics.

CO4. Use and apply the concept of Algebraic Structures, Groups, Rings and Graph Theory.

CO/PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	H	L	-	-	-	-	-	-	L	-	-	M
CO2	H	L	-	-	-	-	-	-	L	-	-	M
CO3	H	L	-	-	-	-	-	-	L	-	-	M
CO4	H	L	-	-	-	-	-	-	L	-	-	M

Proposition

Definition:

- A declarative sentence that is either true (T) or false (F), but not both.
- A proposition (*Simple statement*) may be denoted by a variable like p q, r,…., called a proposition (statement) variable.

Introduction to Proposition

Definition: The value of a proposition is called its <u>truth value</u>; denoted by

- *T* or 1 if it is true or
- *F* or 0 if it is false

Opinions, interrogative, and imperative are not propositions

Truth table

P
0
1

Logical connectives

Connectives are used to create a compound proposition from two or more propositions

- **Negation** or Not (e.g., $\neg a$ or $!a$ or $\bar{a}$)
- **Conjunction** or And (denoted $\wedge$)
- **Disjunction** or Or (denoted $\vee$)
- **Implication** (denoted $\Rightarrow$ or $\rightarrow$)
- **Biconditional** (denoted $\Leftrightarrow$ or $\leftrightarrow$)

We define the meaning (semantics) of the logical connectives using truth tables

Logical Connectives

Formal Name	Nickname		Symbol
Negation operator	NOT	Unary	$\neg$
Conjunction operator	AND	Binary	$\wedge$
Disjunction operator	OR	Binary	$\vee$
Exclusive-OR operator	XOR	Binary	$\oplus$
Implication operator	IMPLIES	Binary	$\rightarrow$
Biconditional operator	"if & only if"	Binary	$\leftrightarrow$

Precedence of Logical Operators

By convention…

1. Negation ($\neg$)
2. Conjunction ($\wedge$)
3. Disjunction ($\vee$)
4. Implication ($\rightarrow$)
5. Biconditional ($\leftrightarrow$)

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Logical Connective: Negation

Definition 1

- Let p be a proposition. The **negation** of p, denoted by $\neg p$, is the statement "It is not the case that p."
- The proposition $\neg p$ is read "not p."
- The truth value of the negation of p, $\neg p$ is the opposite of the truth value of p.

Logical Connective: Negation

The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

Logical Connective: Negation

Examples

- Find the negation of the proposition "Today is Friday." and express this in simple English.
- **Solution:** The negation is
- "It is not the case that *today is Friday*." In simple English, "Today is not Friday." or "It is not Friday today."

Your Task

Let p = "It is hot", and
q = "It is sunny"

Find negation of p ($\neg p$) and also Negation of q ($\neg q$).

Solution:

It is not hot.

It is not sunny.

Logical Connective: Logical And

Definition 2

- Let *p* and *q* be propositions. The *conjunction* of *p* and *q*, denoted by $p \wedge q$, is the proposition "*p* and *q*".

- The conjunction $p \wedge q$ is true when both *p* and *q* are true and is false otherwise.

Logical Connective: Logical And

The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Logical Connective: Logical And

Examples

- Find the conjunction of the propositions *p* and *q* where *p* is the proposition "Today is Friday." and *q* is the proposition "It is raining today.", and the truth value of the conjunction.
- **Solution:** The conjunction is the proposition "Today is Friday and it is raining today." The proposition is true on rainy Fridays.

Your task

Let p = "It is hot", and

q = "It is sunny"

Find *p* and *q* ($p \wedge q$).

Solution:

It is hot and it is sunny.

It is hot **and** sunny.

Logical Connective: Logical OR

Example 1

p: "Today is Friday"
q: "It is raining today"

$p \vee q$: "Today is Friday or it is raining today"

Logical Connective: Exclusive Or

The Truth Table for the Exclusive Or (XOR) of Two Propositions.

p	q	p q
T	T	F
T	F	T
F	T	T
F	F	F

Logical Connective: Implication

- Let p and q be propositions. The *conditional statement* $p \rightarrow q$, is the proposition "if p, then q." The conditional statement is false when p is true and q is false, and true otherwise.
- In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

Logical Connective: Implication

Example 1

let

p = "You study hard."

q = "You will get a good grade."

$p \rightarrow q$ = "If you study hard, then you will get a good grade." (else, it could go either way)

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Propositional Logic

Other conditional statements:

Converse of $p \to q$: $q \to p$

- $p \to q$: "If it is noon, then I am hungry."
- $q \to p$: "If I am hungry, then it is noon."

Contrapositive of $p \to q$: $\neg q \to \neg p$

- $p \to q$: "If it is noon, then I am hungry."

$\neg q \to \neg p$: "If I am not hungry, then it is not noon."

Inverse of $p \to q$: $\neg p \to \neg q$

$p \to q$: "If it is noon, then I am hungry."

$\neg p \to \neg q$: "If it is not noon, then I am not hungry."

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Logical Connective: Biconditional

- Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q."
- The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

Logical Connective: Biconditional

Example 1

Let

p: "You can take the flight"

q: "You buy a ticket"

$p \leftrightarrow q$: "You can take the flight if and only if you buy a ticket"

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Logical Connective: Biconditional

p	q	p?q	p?q	p?q	p?q	p?q
0	0	0	0	0	1	1
0	1	0	1	1	1	0
1	0	0	1	1	0	0
1	1	1	1	0	1	1

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Truth table for $\sim p \wedge (q \vee \sim r)$

p	q	r	$\sim r$	$q \vee \sim r$	$\sim p$	$\sim p \wedge (q \vee \sim r)$
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	F	T	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	T	F	T	T	T	T
F	F	T	F	F	T	F
F	F	F	T	T	T	T

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Your Homework

Draw Truth Tables for:

1. $\sim p \wedge q$
2. $\sim p \wedge (q \vee \sim r)$
3. $(p \vee q) \wedge \sim (p \wedge q)$

Tautology

Definition:

- A tautology is a proposition which is always true
- Example

$p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

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Contradiction

Definition:

- A contradiction is a proposition which is always false.
- Example

$p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

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Contingency

Definition:

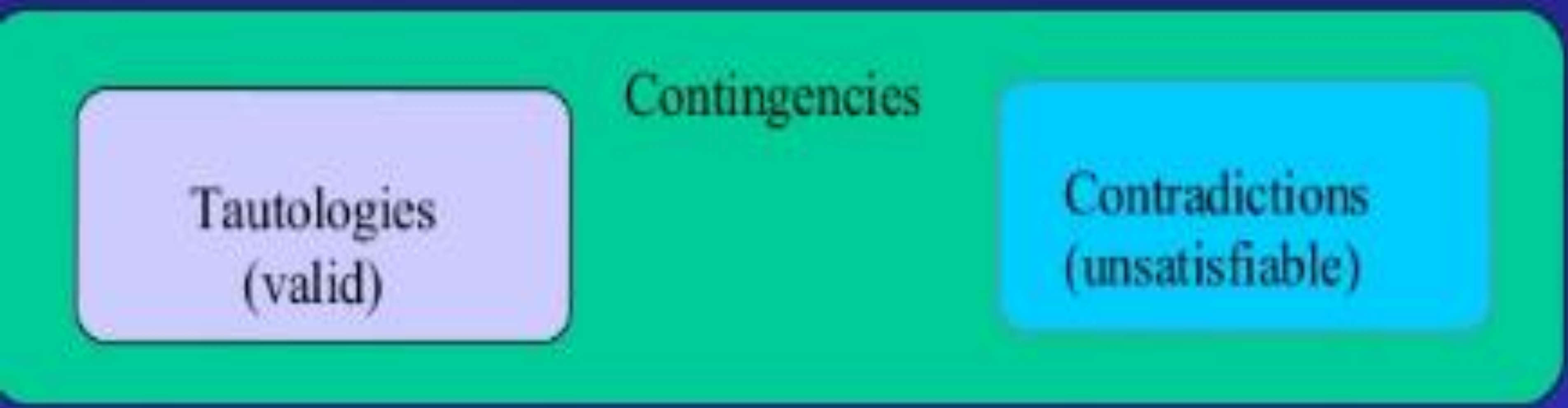
- A contingency is a proposition which neither a tautology nor a contradiction.
- Example:

$(p \vee q) \rightarrow \neg p$

$p \rightarrow \neg p$

p	$\neg p$	$p \rightarrow \neg p$
T	F	F
F	T	T

Propositional Equivalences



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Your task

- Use truth table to show that $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$ is a tautology.

- Use truth table to show that $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction.

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Propositional Equivalences

Equivalence: The compound propositions p and q are *logically equivalent* if $p \leftrightarrow q$ is a tautology.

In other words, p and q are logically equivalent if their truth tables are the same. We write $p \equiv q$.

Example: $\neg(p \vee q) \equiv \neg p \wedge \neg q$.

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are not logically equivalent

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

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Homework

- Are the statements $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$ logically equivalent

- Are the statements $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$ logically equivalent

Propositional Equivalences

Use De Morgan's law to express the negation of "Ahmed has a mobile and he has a laptop"

p: "Ahmed has a mobile"

q: "Ahmed has a laptop"

$p \wedge q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg$ p: "Ahmed has not a mobile"

$\neg$ q: "Ahmed has not a laptop"

"Ahmed has not a mobile or he has not a laptop"

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Propositional Equivalences

Simplify

$p \vee [\sim(\sim p \wedge q)]$

Solution

$p \vee [\sim(\sim p \wedge q)]$

$\equiv p \vee [\sim(\sim p) \vee (\sim q)]$ DeMorgan's Law

$\equiv p \vee [p \vee (\sim q)]$ Double Negative Law

$\equiv [p \vee p] \vee (\sim q)$ Associative Law for $\vee$

$\equiv p \vee (\sim q)$ Indempotent Law

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Propositional Equivalences

Verify logical Equivalence

$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$

Solution

$\sim(\sim p \wedge q) \wedge (p \vee q)$	
$\equiv (\sim(\sim p) \vee \sim q) \wedge (p \vee q)$	DeMorgan's Law
$\equiv (p \vee \sim q) \wedge (p \vee q)$	Double Negative Law
$\equiv p \vee (\sim q \wedge q)$	Distributive Law in Reverse
$\equiv p \vee c$	Negation Law
$\equiv p$	Identity Law

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Propositional Equivalences

Example: Show that $\neg(p \vee (\neg p \wedge q))$ is logically equivalent to $\neg p \wedge \neg q$

Solution:

$\neg(p \vee (\neg p \wedge q))$	$\equiv$	$\neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
	$\equiv$	$\neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
	$\equiv$	$\neg p \wedge (p \vee \neg q)$	by the double negation law
	$\equiv$	$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by the second distributive law
	$\equiv$	$F \vee (\neg p \wedge \neg q)$	because $\neg p \wedge p \equiv F$
	$\equiv$	$(\neg p \wedge \neg q) \vee F$	by the commutative law for disjunction
	$\equiv$	$(\neg p \wedge \neg q)$	by the identity law for **F**

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Propositional Equivalences

Example: Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Solution:

$(p \wedge q) \rightarrow (p \vee q)$	$\equiv$	$\neg(p \wedge q) \vee (p \vee q)$	by truth table for $\rightarrow$
	$\equiv$	$(\neg p \vee \neg q) \vee (p \vee q)$	by the first De Morgan law
	$\equiv$	$(\neg p \vee p) \vee (\neg q \vee q)$	by associative and commutative laws laws for disjunction
	$\equiv$	$T \vee T$	by truth tables
	$\equiv$	T	by the domination law

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Your Task

Show that $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

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Practice Question:

1. Show that $(p \wedge q) \wedge \sim (p \vee q)$ is a contradiction or fallacy.

2. Write the contraposition, converse and inverse of the statement
   "The home team wins whenever it is raining ".
   Also construct the truth table for each statement.

3. Show that $p \wedge q \rightarrow p \vee q$ is a tautology. [by truth table]

4. State the converse, inverse and contra positive of the statement "If today is Easter, then tomorrow is Monday" Also construct truth table.

5. Show that $(p \vee q) \leftrightarrow (p \vee q)$ is an equivalent [By truth table].

6. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ [Distributive Properties by truth table].

Suggested links from NPTEL & other Platforms:

1. https://nptel.ac.in/courses/111/106/111106086/
2. https://nptel.ac.in/courses/111/107/111107058/
3. https://nptel.ac.in/courses/106/106/106106183/#
4. https://nptel.ac.in/content/storage2/courses/111106086/Lecture3.pdf

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Thank you!

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