

Advance Engineering Mathematics(AEM)

**Branch :Information Technology,
Sem:IIIrd**



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Vision of the Institute

To become a renowned centre of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities

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Mission of the Institute

- Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.
- Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academia and industry.
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

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Course Outcomes

- **CO2:** To learn the formulation of different mathematical problems into optimization problems.
- **CO3:** Apply the principles of optimization using differential calculus.
- **CO4:** To understand the concepts of Linear Programming
- **CO1:** To learn the concepts and principles of Random variables and Probability distribution.

Linear Programming

- Simplex Method
- Two Phase Method,
- Duality in Linear Programming,
- Assignment Problems,
- Transportation Problems

Simplex Method

- When decision variables are **more than 2**, it is always advisable to use Simplex Method of avoid lengthy graphical procedure.
- The simplex method is not used to examine all the feasible solutions.
- It deals only with a small and unique set of feasible solutions the set of vertex points (i.e. extreme points) of the convex feasible space that contains the optimal solution
- The most popular method used for the solution of Linear programming problems (LPP) is the simplex method. Simplex method is developed by George Dantignin 1946
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When decision variables are *more than 2*, we always use Simplex Method

□ **Slack Variable**: Variable added to a \leq constraint to convert it to an equation (=).

❖ A slack variable represents unused resources .

❖ A slack variable contributes nothing to the objective function value.

□ **Surplus Variable:** Variable subtracted a \geq constraint to convert it to an equation (=).

❖ A surplus variable represents an excess above constraint requirement level.

❖ surplus variables contribute nothing to the calculated value of the objective function.

- ❑ **Basic Solution (BS)** : This solution is obtained by setting any n variables (among $m+n$ variables) equal to zero and solving for remaining m variables, provided the determinant of the coefficients of these variables is non-zero. Such m variables are called **basic variables** and remaining n zero valued variables are called **non basic variables**
- ❑ **Basic Feasible Solution (BFS)** : It is a basic solution which also satisfies the non negativity restrictions.

□ **BFS are of two types :**

- ***Degenerate BFS*** : If once or more basic variables are zero
- ***Non-Degenerate BFS*** : All basic variables are non-zero.

Optimal BFS: BFS which optimizes the objective function

Examples:

$$\text{Max. } Z = 13x_1 + 11x_2$$

Subject to constraints:

$$4x_1 + 5x_2 \leq 1500$$

$$5x_1 + 3x_2 \leq 1575$$

$$x_1 + 2x_2 \leq 420$$

$$x_1 + x_2 \geq 0$$

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□ **Step 1:** Convert all the inequality constraints into equalities by the use of slack variables

Let S_1, S_2, S_3 be three slack variables.

Introducing these slack variables into the inequality constraints and rewriting the objective function such that all variables are on the left-hand side of the equation. Model can be rewritten as :

$$Z - 13x_1 - 11x_2 = 0$$

Subject to Constraints:

$$4X_1 + 5X_2 + S_1 = 1500$$

$$5X_1 + 3X_2 + S_2 = 1575$$

$$X_1 + 2X_2 + S_3 = 420$$

$$X_1, X_2, S_1, S_2, S_3 = 0$$

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❑ **Step II** : Find the Initial BFS.

One Feasible solution that satisfies all the constraints is : $x_1 = 0$, $x_2 = 0$, $S_1 = 1500$,
 $S_2 = 1575$, $S_3 = 420$ and $Z = 0$.

Now, S_1 , S_2 , S_3 are basic variables.

❑ **Step III** : Set up an initial table as:

| Row NO. | Basic Variable | Coefficients of: | | | | | | Sol. | Ratio |
|---------|----------------|------------------|-------|-------|-------|-------|-------|------|-------|
| | | Z | x_1 | x_2 | S_1 | S_2 | S_3 | | |
| A1 | Z | 1 | -13 | -11 | 0 | 0 | 0 | | |
| B1 | S_1 | 0 | 4 | 5 | 1 | 0 | 0 | 1500 | 375 |
| C1 | S_2 | 0 | 5 | 3 | 0 | 1 | 0 | 1575 | 315 |
| D1 | S_3 | 0 | 1 | 2 | 0 | 0 | 1 | 420 | 420 |



Step IV: a) Choose the most negative number from row A1 (i.e. Z row). Therefore, x_1 is a *entering variable*.

b) Calculate Ratio = Sol col. / x_1 col. ($x_1 > 0$)

c) Choose minimum Ratio. That variable (i.e. S_2) is a *departing variable*.

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Step V: x_1 becomes *basic variable* and S_2 becomes *non basic variable*. New table is:

| Row NO. | Basic Variable | Coefficients of: | | | | | | Sol. | Ratio |
|---------|----------------|------------------|-------|---------|-------|--------|-------|------|-------|
| | | Z | x_1 | x_2 | S_1 | S_2 | S_3 | | |
| A1 | Z | 1 | 0 | $-16/5$ | 0 | $13/5$ | 0 | 4095 | |
| B1 | S_1 | 0 | 0 | $13/5$ | 1 | $-4/5$ | 0 | 240 | 92.3 |
| C1 | x_1 | 0 | 1 | $3/5$ | 0 | $1/5$ | 0 | 315 | 525 |
| D1 | S_3 | 0 | 0 | $7/5$ | 0 | $-1/5$ | 1 | 105 | 75 |



Next Table is :

| Row NO. | Basic Variable | Coefficients of: | | | | | | Sol. |
|---------|----------------|------------------|-------|-------|-------|-------|-------|------|
| | | Z | x_1 | x_2 | S_1 | S_2 | S_3 | |
| A1 | Z | 1 | 0 | 0 | 0 | 15/7 | 16/7 | 4335 |
| B1 | S_1 | 0 | 0 | 0 | 1 | -3/7 | -13/7 | 45 |
| C1 | x_1 | 0 | 1 | 0 | 0 | 2/7 | -3/7 | 270 |
| D1 | x_2 | 0 | 0 | 1 | 0 | -1/7 | 5/7 | 75 |

Optimal Solution is : $x_1 = 270$, $x_2 = 75$, $Z = 4335$

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Solve:

Solve the Simplex method

$$\text{Max } z = x_1 - x_2 + 3x_3$$

$$\text{Sub to } x_1 + x_2 + x_3 \leq 10$$

$$2x_1 - x_3 \leq 2$$

$$2x_1 - 2x_2 + 3x_3 \leq 0$$

$$x_1, x_2, x_3 \geq 0$$

Reference:

- <https://www.slideshare.net/sachin.mk/simple-x-method>
- Engineering Mathematics III CS/IT Engineering
Vardhan Publication

Thank You

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