Advance Engineering Mathematics(AEM)

Branch :Information Technology, Sem:IIIrd

Dr. Kashish Parwani

Associate Professor, Dept. of Mathematics

JECRC, Sitapura Jaipur

Vision of the Institute

To become a renowned centre of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities

Mission of the Institute

- Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.
- Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academia and industry.
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

Course Outcomes

- **CO1:** To learn the concepts and principles of Random variables and Probability distribution.
- **CO2:** To learn the formulation of different mathematical problems into optimization problems.
- **CO3:** Apply the principles of optimization using differential calculus.
- **CO4:** To understand the concepts of Linear Programming.

Probability Density Function

The function f(x) for a continuous random variable X is said to be probability density function (p.d.f.) provided it satisfies the following conditions:

(i) $f(x) \geq 0$; $-\infty < x < \infty$

(ii) $$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Moreover $P(a \leq X \leq b) = \int_{a}^{b} f(x)\,dx$

Dr. Kashish Parwani
Associate Professor (Mathematics) JECRC, Jaipur

Sol: f(x) be the given pdf,

So,

1 $$\int_0^{\infty} f(x) = 1 \Rightarrow \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^3 f(x)dx = 1$$

$$\Rightarrow \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax + 3a)\,dx = 1$$

$$\Rightarrow a\left(\frac{x^2}{2}\right)_0^1 + a(x)_1^2 + \left(\frac{-ax^2}{2} + 3ax\right)_2^3 =$$

$$\Rightarrow a\left(\frac{1}{2}\right) + a(1) + \left(\frac{-a}{2}\right)(5) + 3a(1) = 1$$

$$2a = 1 \quad \Rightarrow \quad a = \frac{1}{2}$$

Dr. Kashish Parwani

Associate Professor (Mathematics) JECRC, Jaipur

(ii)

$$P(x \leq 1.5) = \int_0^1 f(x)dx + \int_1^{1.5} f(x)dx = \int_0^1 ax\, dx$$

$$+\int_1^{1.5} adx = a\left(\frac{x^2}{2}\right)_0^1 + ax_1^{15} = \frac{a}{2} + (0.5)a = a = \frac{1}{2}$$

Dr. Kashish Parwani

Associate Professor (Mathematics) JECRC, Jaipur

(iii) for $x \le 0$ $F(x) = 0$,

3

$$\text{for } 0 \le x \le 1, F(x) = \int_0^x x df(x) dx = \int_0^x ax\, dx = a\left(\frac{x^2}{2}\right)_0^x$$

$$= a\frac{x^2}{2} = \frac{x^2}{4} \quad a = \left(\frac{1}{2}\right)$$

$$a = \left(\frac{1}{2}\right)$$

For

$$1 \leq x \leq 2, \quad F(x) = \int_0^1 f(x)dx + \int_1^x f(x)dx + \int_0^1 ax\,dx + \int_1^x a\,dx$$

$$a\left(\frac{x^2}{2}\right)_0^x + a(x)_1^x = \frac{a}{2} + a(x-1) = \frac{1}{4} + \frac{1}{2}(x-1) = \frac{x}{2} - \frac{1}{4}$$

$$a = \left(\frac{1}{2}\right)$$

$$\text{For } 2 \leq x \leq 3, F(x) \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^x f(x)dx$$

$$= \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^x (-ax + 3a)dx$$

$$\Rightarrow \frac{a}{2} + a(1) + \left(-a\frac{x^2}{2} + 3ax\right)_2^x$$

$$\Rightarrow \frac{3a}{2} - \left(\frac{a}{2}\right)[x^2 - 4] + 3a(x - 2)$$

$$\frac{3a}{2} - \frac{ax^2}{2} + 2a + 3ax - 6a$$

$$-\frac{5a}{2} + 3ax - \frac{ax^2}{2}$$

$$\frac{-5}{4} + \frac{3x}{2} - \frac{x^2}{4}$$

Do it.......

(iv) From the distribution function it is clear that

$$F(3) = P(X \le 3) = \frac{5}{10} = 0.5$$

$$F(4) = P(X \le 4) = \frac{8}{10} = 0.8 > \frac{1}{2}$$

$$F(5) = P(X \le 5) = \frac{81}{100} = 0.81 > \frac{1}{2},$$ and so on.

Hence the minimum value of c for which $P(x \le c) > \frac{1}{2}$ is 4.

Therefore $c = 4$.

(v) $$P\left(\frac{1.5<X<4.5}{X>2}\right)=\frac{P[(1.5<X<4.5)\cap(X>2]}{P(X>2)}$$

$$=\frac{P(2<X<4.5)}{1-P(X\leq 2)}=\frac{P(3)+P(4)}{1-[P(X=0)+P(X=1)+P(X=2)]}$$

$$=\frac{\frac{2}{10}+\frac{3}{10}}{1-\frac{3}{10}}=\frac{\frac{5}{10}}{\frac{7}{10}}=\frac{5}{7}.$$

Dr. Kashish Parwani
Associate Professor (Mathematics, JECRC, Jaipur

Solve it:

Example 4. From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. If the sample is drawn without replacement and the random variable X denotes the number of defective items in the sample, find :

(i) The probability distribution of X.

(ii) $P(X \leq 1)$

(iii) $P(X < 1)$

(iV) $P(0 < X < 2)$

Thank You!

Dr. Kashish Parwani
Associate Professor (Mathematics, JECRC, Jaipur

References:

1. https://www.slideshare.net/lovemucheca/random-variable-and-distribution
2. https://www.youtube.com/watch?v=UftY0e2ilM4
3. https://www.digimat.in/nptel/courses/video/117104117/L01.html