Advance Engineering Mathematics(AEM)

Branch :Information Technology, Sem:IIIrd

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Vision of the Institute

To become a renowned centre of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities

Mission of the Institute

- Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.
- Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academia and industry.
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

Course Outcomes

- **CO1:** To learn the concepts and principles of Random variables and Probability distribution.
- **CO2:** To learn the formulation of different mathematical problems into optimization problems.
- **CO3:** Apply the principles of optimization using differential calculus.
- **CO4:** To understand the concepts of Linear Programming.

Probability Mass Function

Let X be a discrete random variable such that $P(X=x_i) = p_i$ is said to be probability mass function (pmf) if it satisfies the following conditions:

(i) $P_i \geq 0$,

(ii) $\Sigma p_i=1$,

The collection of pairs (x_i,p_i) is the probability distribution of the random variable X

Example 1. Check whether the following function serve as probability mass function.

$$P(X = x) = \frac{x-2}{2} \ \forall\ x = 1,2,3,4$$

$$P(X = x) = \frac{x^2}{25} \ \forall\ x = 1,2,3,4$$

Solution

X	:	1	2	3	4
P(X=x)	:	-1/2	0	½	1

As $P(X=1) = -\frac{1}{2} < 0$

Hence P(x) is not a probability mass function

(ii) X :	1	2	3	4
P(X=x) :	1/25	4/25	9/25	16/25

Though $P(X=x) > o$, $x=1,2,3,4$

yet $\Sigma P(X=x) = 30/25$

$= 6/5 > 1$

Hence it also does not serve as a Probability mass function.

Example 2: Four bad oranges are mixed accidentally with 16 good oranges. Find the probability distribution of the number of bad oranges in a draw of two oranges.

Solution: Let the random variable X denote the number of bad oranges in a draw of two oranges. Hence X = 0, 1, 2.

Now P (X =0) = Probability of getting 2 good oranges $\frac{16_{c_2}}{20_{c_2}} = \frac{12}{19}$

P(X = 1) = Probability of getting 1 good orange and 1 bad orange

$$\frac{4_{c_1} X\, 16_{c_1}}{20_{c_2}} = \frac{32}{95}$$

P(X = 2) = Probability of getting 2 bad orange = $\frac{4_{c_2}}{20_{c_2}} = \frac{3}{95}$

Hence the required probability distribution is :

X :	0	1	2
P(X=2) :	12/19	32/95	3/95

Example 4 : A random variable X has the following probability distribution.

X:	0	1	2	3	4	5	6	7
P(X):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find k

(ii) Evaluate $P(X< 6)$, $P(x\geq 6)$, $P(0< X<5)$.

(iii) Determine Distribution Function of X

(iv) If $P(X\leq c)>1/2$ Find the minimum value of c.

(v) Find $P\left(\frac{1.5 < X < 4.5}{X > 2}\right)$

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Solution: (i) Given probability distribution

Hence $$\sum_{x=0}^{7} p(x) = 1$$

$10k^2 + 9k - 1 = 0,$

$k = -1, 1/10,$

K = -1 is not possible as it makes p(x) < 0 which is impossible, as above given is a probability distribution.

Hence $K = \frac{1}{10}$

(ii) $P(X<6) = 1 - P(X \geq 6)$ [$\therefore \sum p(x) = 1$]

$= 1 - [P(X=6) + P(X=7)]$

$= 1 - (9k^2 + k)$

$$= 1 - \frac{1}{10} - \frac{9}{100} = \frac{81}{100}$$

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$$P(x \geq 6) = 1 - P(x < 6) = 1 - \frac{81}{100} = \frac{19}{100}$$

$$P(0 < X < 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

$$= 8k = \frac{8}{10}$$

=4/5,

(iii)

X	$F(X) = p(X \leq x)$
0	0=0
1	k=1/10
2	3k= 3/10
3	5k=5/10
4	8k=8/10
5	$8k+k^2$= 81/100
6	$8k+3k^2$= 83/100
7	$10k^2+ 9k = 1$

(iv) From the distribution function it is clear that

$$F(3) = P(X \le 3) = \frac{5}{10} = 0.5$$

$$F(4) = P(X \le 4) = \frac{8}{10} = 0.8 > \frac{1}{2}$$

$$F(5) = P(X \le 5) = \frac{81}{100} = 0.81 > \frac{1}{2}, \quad \text{and so on.}$$

Hence the minimum value of c for which $P(x \le c) > \frac{1}{2}$ is 4.

Therefore $c = 4$.

(v) $$P\left(\frac{1.5<X<4.5}{X>2}\right)=\frac{P[(1.5<X<4.5)\cap(X>2]}{P(X>2)}$$

$$=\frac{P(2<X<4.5)}{1-P(X\le 2)}=\frac{P(3)+P(4)}{1-[P(X=0)+P(X=1)+P(X=2)]}$$

$$=\frac{\frac{2}{10}+\frac{3}{10}}{1-\frac{3}{10}}=\frac{\frac{5}{10}}{\frac{7}{10}}=\frac{5}{7}.$$

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Solve it:

Example 4. From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. If the sample is drawn without replacement and the random variable X denotes the number of defective items in the sample, find :

(i) The probability distribution of X.

(ii) $P(X \leq 1)$

(iii) $P(X < 1)$

(iV) $P(0 < X < 2)$

Thank You!

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References:

1. https://www.slideshare.net/lovemucheca/random-variable-and-distribution
2. https://www.youtube.com/watch?v=UftY0e2ilM4
3. https://www.digimat.in/nptel/courses/video/117104117/L01.html