

Advance Engineering Mathematics(AEM)

**Branch :Information Technology,
Sem:IIIrd**



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Vision of the Institute

To become a renowned centre of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities

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Mission of the Institute

- Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.
- Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academia and industry.
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

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Course Outcomes

- **CO2:** To learn the formulation of different mathematical problems into optimization problems.
- **CO3:** Apply the principles of optimization using differential calculus.
- **CO4:** To understand the concepts of Linear Programming
- **CO1:** To learn the concepts and principles of Random variables and Probability distribution.

Duality Theory

- ⌘ The notion of duality within linear programming asserts that every linear program has associated with it a related linear program called its **dual**. The original problem in relation to its dual is termed the **primal**.
- ⌘ it is the **relationship** between the primal and its dual, both on a mathematical and economic level, that is truly the essence of duality theory.

Q1. Write the dual of the problem

$$\text{Max } z_p = 2x_1 + 4x_2$$

$$\text{S.to } 2x_1 + 3x_2 \leq 48$$

$$x_1 + 3x_2 \leq 42$$

$$x_1 + x_2 \leq 21$$

$$x_1, x_2 \geq 0$$

Solution : It is a maximization problem with all constraints having \leq sign.

$$\begin{array}{ll} \text{Max} & z_p = 2x_1 + 4x_2 \\ \text{s.to} & 2x_1 + 3x_2 \leq 48 \quad w_1 \\ & x_1 + 3x_2 \leq 42 \quad w_2 \\ & x_1 + x_2 \leq 21 \quad w_3 \\ & x_1, x_2 \geq 0 \end{array}$$

The dual is

$$\begin{array}{ll} \text{Min} & z_D = 48w_1 + 42w_2 + 21w_3 \\ \text{s.to} & 2w_1 + w_2 + w_3 \geq 2 \\ & 3w_1 + 3w_2 + w_3 \geq 4 \\ & w_1, w_2, w_3 \geq 0 \end{array}$$

Q2. Write the dual of the problem

$$\text{Max } z = x_1 + 2x_2 - x_3$$

$$\text{s.to } 2x_1 + 3x_2 + 4x_3 \leq 5$$

$$2x_1 - 2x_2 \leq 6$$

$$3x_1 - 3x_3 \geq 4$$

$$x_1, x_2, x_3 \geq 0$$

Solution: The above problem is

$$\begin{array}{ll} \text{Max} & z_p = x_1 + 2x_2 - x_3 \\ \text{s.to} & 2x_1 + 3x_2 + 4x_3 \leq 5 \quad w_1 \\ & 2x_1 - 2x_2 \leq 6 \quad w_2 \\ & -3x_1 + 3x_3 \leq -4 \quad w_3 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Dual:

$$\begin{array}{ll} \text{Min} & z_D = 5w_1 + 6w_2 - 4w_3 \\ \text{s.to} & 2w_1 + 2w_2 - 3w_3 \geq 1 \\ & 3w_1 - 2w_2 \geq 2 \\ & 4w_1 + 3w_3 \geq -1 \\ & w_1, w_2, w_3 \geq 0 \end{array}$$

Q3. Write the dual of the problem

$$\text{Max } z_p = x_1 + 3x_2$$

$$\text{s.to } 3x_1 + 2x_2 \leq 6$$

$$3x_1 + x_2 = 4$$

$$x_1, x_2 \geq 0$$

Solution: Given problem can be written as s

$$\begin{array}{ll} \text{Max} & z_p = x_1 + 3x_2 \\ \text{s.to} & 3x_1 + 2x_2 \leq 6 \quad w_1 \\ & 3x_1 + x_2 \leq 4 \quad w_2 \\ & -3x_1 - x_2 \leq -4 \quad w_3 \\ & x_1, x_2 \geq 0 \end{array}$$

Dual:

$$\begin{array}{ll} \text{Min} & z_D = 6w_1 + 4w_2 - 4w_3 \\ \text{s.to} & 3w_1 + 3w_2 - 3w_3 \geq 1 \\ & 2w_1 + w_2 - w_3 \geq 3 \\ & w_1, w_2, w_3 \geq 0 \end{array}$$

Dual:

$$\text{Min } z_D = 6w_1 + 4w_2 - 4w_3$$

$$\text{s.to } 3w_1 + 3w_2 - 3w_3 \geq 1$$

$$2w_1 + 3w_2 - w_3 \geq 3$$

$$w_1, w_2, w_3 \geq 0$$

Replace $(w_2 - w_3)$ by w'_2

$$\text{Min. } z_D = 6w_1 + 4w'_2$$

$$\text{s.to } 3w_1 + 3w_2 \geq 1$$

$$2w_1 + w_2 \geq 3$$

$w_1 \geq 0$ and w'_2 unrestricted in sign

$$\begin{array}{ll}
 \text{Q4. Min} & z_p = x_1 - 3x_2 - 2x_3 \\
 \text{s.to} & -3x_1 + x_2 - 2x_3 \geq -7 \quad w_1 \\
 & -2x_1 - 4x_2 \geq 12 \quad w_2 \\
 & -4x_1 + 3w_2 + 8x_3 = 10 \quad w_3 \\
 & x_1, x_2 \geq 0, x_3 \text{ unrestricted}
 \end{array}$$

$$\begin{array}{ll}
 \text{MaX} & z_D = -7w_1 + 12w_2 + 10w_3 \\
 \text{s.to} & -3w_1 - 2w_2 - 4w_3 \leq 1 \\
 & w_1 - 4w_2 + 3w_3 \leq -3 \\
 & -2w_1 + 8w_3 = -2, \\
 & w_1, w_2, w_3 \geq 0 \text{ unrestricted}
 \end{array}$$

Thank You

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