# Jaipur Engineering College and Research Center

- Year & Sem. – B. Tech. I-Year, Semester - I
- Subject – Engineering Physics (1FY2-02)
- Chapter - Electromagnetism (Part-II)
- Department - Applied Science (Physics)

# Vision and Mission

- **Vision:**
- ➢ To become a renowned centre of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

- **Mission:**
- ➢ Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.
- ➢ Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.
- ➢ Offer opportunities for interaction between academia and industry.
- ➢ Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

# Syllabus and Course Outcome of Introduction to Electromagnetism

**Syllabus:** Divergence and curl of electrostatic field, Laplace’s and Poisson’s equations for electrostatic potential, Bio-Savart law, Divergence and curl of static magnetic field, Faraday’s law, Displacement current and magnetic field arising from time dependent electric field, Maxwell’s equations, Flow of energy and Poynting vector.

**CO4:** Students will be able to describe key concepts and acquire basics of electrostatics and electromagnetism (e.g., Maxwell’s equations) to explain electromagnetic waves propagation and generation in free space, dielectrics and conducting media.

# Lecture Plan of Introduction to Electromagnetism

| S. No | Topics | Lectures required | Lect. No. |
|---|---|---|---|
| 1 | General introduction to Electromagnetism. | 1 | 33 |
| 2 | Divergence and curl of electrostatic & magnet field | 1 | 34 |
| 3 | Laplace's and Poisson's equations for electrostatic potential. | 1 | 35 |
| 4 | Bio-Savart law and Faraday's law. | 1 | 36 |
| 5 | Displacement current and magnetic field arising from time dependent electric field | 1 | 37 |
| 6 | Maxwell's Equation first and second | 1 | 38 |
| 7 | Maxwell's Equation third and fourth | 1 | 39 |
| 8 | Flow of energy and Poynting vector. | 1 | 40 |

# Content

- Maxwell’s Equations
- Poission’s and Laplace’s Equation
- Poynting Theorem and Poynting Vector
- Displacement Current
- Suggested reference books & links from NPTEL/IIT/RTU Plateforms
- Important Questions.

# Maxwell's Equations

Faraday showed that the electric field is produced by changing magnetic field. James Clark Maxwell introduced a concept that the magnetic field can be produced by changing electric field. So to explain electric and magnetic fields or electromagnetics, Maxwell gave some basic equations, called Maxwell's equations. These equations are:-

- Maxwell I equation - Gauss's law for electric field
- Maxwell II equation - Gauss's law for magnetic field
- Maxwell III equation - Faraday's law of electromagnetic induction
- Maxwell IV equation - Ampere's circuital law

# Maxwell's First Equation

Maxwell's first equation is based on Gauss's law for electric field. According to this law, total electric flux passing through a closed surface S in vacuum is equal to the product of total charge (Q) contained insite and $1/\epsilon_0$.

i.e.
$$\phi_E = \oint_S \vec{E} \cdot \vec{da} = \frac{Q}{\epsilon_0}$$

for medium,
$$\phi_E = \oint_S \vec{E} \cdot \vec{da} = \frac{Q}{\epsilon} \quad \text{.......(1)}$$

If charge inside the closed surface is continuously distributed and $\rho$ is charge volume density, then we can write

$$Q = \int_v \rho dv \quad \text{.......(2)}$$

from equation (1) and (2)

$$\boxed{\oint_S \vec{E}.\vec{da} = \frac{1}{\epsilon}\int_V \rho dV} \quad \text{......(3)}$$

This is integral form of Maxwell's I equation.

Using Gauss's divergence theorem

$$\oint_S \vec{E}.\vec{da} = \int_V (\nabla \cdot \vec{E}) dV$$

So, equation (3) can be written as

$$\int_V (\nabla \cdot \vec{E}) dV = \frac{1}{\epsilon} \int_V \rho dv$$

$\Rightarrow$ $$\boxed{\nabla \cdot \vec{E} = \rho/\epsilon} \text{ or } \boxed{\text{div}\,\vec{E} = \rho/\epsilon} \quad \ldots\ldots(4)$$

This is differential form of Maxwell's I equation.

If medium is homogenous and isotopic, then

$$\nabla \cdot (\epsilon \vec{E}) = \rho$$

$\Rightarrow$ $$\boxed{\nabla . \vec{D} = \rho}$$

Where $\vec{D} = \epsilon \vec{E}$, is called displacement vector.

For free space $\rho$ = J = 0

then Maxwell's I equation becomes $\oint_S \vec{E} . \vec{da} = 0$

$\Rightarrow$ $$\nabla . \vec{E} = 0$$

or $$\nabla . \vec{D} = 0$$

# Maxwell's Second Equation

Maxwell' second equation is based on Gauss's law for magnetic field. According to the law, net outgoing magnetic flux from any surface is always zero. This is due to fact that magnetic field lines always form closed loop.

i.e. $\phi_B = 0$

$$\Rightarrow \quad \boxed{\int_S \vec{B}.\vec{da} = 0} \quad \ldots.(5)$$

This is integral form of Maxwell's II equation.

Using Gauss's divergence theorem

$$\int_S \vec{B}.\vec{da} = \int_V (\mathrm{div}\vec{B})dV$$

$$\Rightarrow \quad \boxed{\mathrm{div}\vec{B} = 0} \text{ or } \boxed{\nabla \cdot \vec{B} = 0} \quad \ldots.(6)$$

This is differential form of Maxwell's II equation.

This equation prove theoretically that magnetic monopole does not exist in nature.

# Maxwell's Third Equation

Maxwell's third equation is based on Faraday's law of electromagnetic induction. According to this law, whenever magnetic flux linked with a circuit changes, an e.m.f. is induced in the circuit and this induced e.m.f. is directly proportional to the rate of change of magnetic flux linked with the circuit.

According to Lenz's law, it's direction is always opposite to the changes responsible for it's production.

i.e. $$e = -\frac{\partial \phi_B}{\partial t}$$

Since, line integral of electric field is also equal to the e.m.f.

i.e. $$e = \oint_c \vec{E}.\vec{dr}$$

and we know that magnetic flux $\phi_B = \oint_S \vec{B}.\vec{da}$

So, $$\oint_c \vec{E}.\vec{dr} = -\frac{\partial}{\partial t}\oint_S \vec{B}.\vec{da}$$

or $$\boxed{\oint_c \vec{E}.\vec{dr} = -\oint_S \frac{\partial \vec{B}}{\partial t}\cdot\vec{da}} \qquad .....(7)$$

This is integral form of Maxwell's III equation.

Using Stoke's curl theorem

$$\oint_C \vec{E}.\vec{dr} = \oint_S (\nabla \times \vec{E}) \cdot \vec{da}$$

So, $$\oint_S (\nabla \times \vec{E}) \cdot \vec{da} = -\oint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{da}$$

$$\Rightarrow \quad \boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \text{ or } \boxed{\text{curl}\vec{E} = -\frac{\partial \vec{B}}{\partial t}} \qquad .....(8)$$

This is differential form of Maxwell's III equation.

For static fields (w.r.t. time) i.e. for $\frac{\partial \vec{B}}{\partial t} = 0 = \frac{\partial \vec{E}}{\partial t}$

$$\Rightarrow \quad \nabla \times \vec{E} = 0$$

or $$\oint_S \vec{E} \cdot \vec{dr} = 0$$

# Maxwell's Fourth Equation

Maxwell's fourth equation is based on Ampere's circuital law. According to this law, 'line integral of magnetic field over a closed path is equal to μ times the net current passes through the closed path.

i.e. $$\oint_c \vec{B}\cdot\vec{dr} = \mu\Sigma I$$

If current density is $\vec{J}$, then $\Sigma I = \oint_S \vec{J}\cdot\vec{da}$

So, $$\oint_c \vec{B}\cdot\vec{dr} = \mu\oint_S \vec{J}\cdot\vec{da} \qquad ....(9)$$

From Stoke's curl theorem $$\oint_c \vec{B}\cdot\vec{dr} = \oint_S (\nabla\times\vec{B})\cdot\vec{da} \qquad ....(10)$$

On comparing equation (9) and (10), we get

$$\nabla \times \vec{B} = \mu \vec{J} \qquad .....(11)$$

But, there is an inconsistency in this law, as taking divergence both side of this equation, we get

$$\nabla \cdot (\nabla \times \vec{B}) = \mu(\nabla \cdot \vec{J})$$

But, we know that divergence of curl of a vector is always zero, i.e. $\nabla \cdot (\nabla \times \vec{B}) = 0$

$$\Rightarrow \qquad \nabla \cdot \vec{J} = 0 \qquad .....(12)$$

But, from continuity equation

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \qquad ......(13)$$

On comparing equation (12) and (13), we can say that Ampere's law is valid only for steady currents, i.e. for which $\rho = 0$ or constant such that $\frac{\partial \rho}{\partial t} = 0$.

Maxwell removed this inconsistancy by giving a suggestion that total current density $\vec{J}$ can be considered as made of two type of currents; free current density $\vec{J}_f$, which comes from motion of free charges and displacement current density $\vec{J}_d$, which comes from the change in electric field w.r.t time.

i.e. $$\vec{J} = \vec{J}_f + \vec{J}_d$$

equation (6.27) now, becomes

$$\nabla \times \vec{B} = \mu(\vec{J}_f + \vec{J}_d)$$

Again, taking divergence both sides, we get

$$\nabla \cdot (\nabla \times \vec{B}) = \mu(\nabla \cdot \vec{J}_f + \nabla \cdot \vec{J}_d)$$

$$\Rightarrow \quad \nabla \cdot \vec{J}_f + \nabla \cdot \vec{J}_d = 0$$

$$\Rightarrow \quad \nabla \cdot \vec{J}_d = -\nabla \cdot \vec{J}_f \qquad \text{......(14)}$$

But, from continuity equation

$$\nabla \cdot \vec{J}_f + \frac{\partial \rho}{\partial t} = 0$$

or

$$\nabla \cdot \vec{J}_f = -\frac{\partial \rho}{\partial t} \qquad \ldots\ldots(15)$$

On comparing equation (14) and (15) , we have

$$\nabla \cdot \vec{J}_d = \frac{\partial \rho}{\partial t}$$

$\Rightarrow$

$$\nabla \cdot \vec{J}_d = \frac{\partial}{\partial t}(\nabla \cdot \vec{D}) \qquad \text{(using Maxwell's I equation)}$$

$\Rightarrow$

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

Thus, after Maxwell's modification, equation (9) becomes

$$\oint_C \vec{B} \cdot \vec{dr} = \mu \oint_S \vec{J}_f + \vec{J}_d) \cdot d\vec{a}$$

$$\Rightarrow \quad \boxed{\oint_c \vec{B} \cdot \vec{dr} = \mu \oint_S \left( \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{a}} \quad \text{......(16)} \cdots$$

This is integral form of Maxwell's IV equation.

using Stoke's curl theorem

$$\oint_c \vec{B} \cdot \vec{dr} = \oint_S (\nabla \times \vec{B}) \cdot \vec{da}$$

So, equation (16) becomes

$$\oint_S (\nabla \times \vec{B}) \cdot \vec{da} = \mu \int_S \left( \vec{J}_f + \frac{\partial D}{\partial t} \right) \cdot \vec{da}$$

$$\Rightarrow \quad \boxed{\nabla \times \vec{B} = \mu \left( \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \right)} \text{ or } \boxed{\nabla \times \vec{B} = \mu \vec{J}_f + \mu \in \frac{\partial \vec{E}}{\partial t}} \quad \text{......(17)} \cdots$$

This is differential form of Maxwell's IV equation.

# Generalized Forms of Maxwell's Equations

| S.No. | Differential form | Integral form | Ramarks |
|---|---|---|---|
| 1. | $\nabla \cdot \vec{E} = \rho/\in$ | $\oint_s \vec{E} \cdot d\vec{a} = \frac{1}{\in}\int_v \rho\, dv$ | Gauss's law for electrostatics |
| 2. | $\nabla \cdot \vec{B} = 0$ | $\oint_s \vec{B}.d\vec{a} = 0$ | Gauss's law for magnetostatics |
| 3. | $\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$ | $\oint_C \vec{E}.d\vec{r} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{a}$ | Faraday's law |
| 4. | $\nabla \times \vec{B} = \mu\left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t}\right)$ | $\oint_C \vec{B}.d\vec{r} = \mu\int_S \left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t}\right).d\vec{a}$ | Ampere's law |

# Poisson's and Laplace's Equation

We know that the point form of the Gauss's Law

$$\vec{\nabla} \cdot \vec{D} = \rho_v \qquad \dots\dots(1)$$

Where $\vec{D}$ →Electric flex density

$\rho_v$ →Volume charge density

Now, the relation between $\vec{D}$ and electric field intensity $\vec{E}$,

$$\vec{D} = \epsilon \vec{E} \qquad \dots\dots\dots(2)$$

Where $\epsilon$→permittivity of medium

Substituting equation (2) into eq$^n$(1)

$$\vec{\nabla} \cdot (\epsilon \vec{E}) = \rho_v \qquad \dots\dots\dots(3)$$

But the gradient relationship says

$$\vec{E} = -\vec{\nabla} V \qquad \dots\dots\dots(4)$$

Then from eqn (3)

$$-\vec{\nabla}\cdot(\epsilon\,\vec{\nabla}V) = \rho_v \qquad \ldots\ldots\ldots..(5)$$

$$\vec{\nabla}\cdot\vec{\nabla}V = -\frac{\rho_v}{\epsilon}$$

$$\boxed{\vec{\nabla}^2 V = -\frac{\rho_v}{\epsilon}} \qquad \because\ \vec{\nabla}\cdot\vec{\nabla} = \vec{\nabla}^2$$

Eqn (6) is poisson's equation

If $\rho_v = 0$ indicating zero volume charge density but allowing point charges, line charges and surface charge density to exist at singular locations as sources of the field, then

$$\boxed{\vec{\nabla}^2 V = 0} \qquad \ldots\ldots\ldots..(7)$$

This is the Laplace's equation and $\vec{\nabla}^2$ Operator is called Laplacian operator
Laplace's equation in Cartesian coordinates

$$\vec{\nabla}^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad \dots\dots (8)$$

# Applications of Laplace's and Poissions Equations

1. Using Laplace and Poissions equations we can obtain potential at any point in between two surface when potential at two surface are given.

2. We can also obtain capacitance between these two surface.

# Poynting Vector

When electromagnetic wave travels in space it carries energy. The energy density is always associated with electric and magnetic fields.

The vector is defined as: The rate of flow of energy per unit area per second that is called vector i.e.

$$\text{Poynting vector} = \frac{Energy}{Area \times Time}$$

$$\text{Poynting vector} = \frac{Power}{Area} = \text{Power density}$$

Mathematically it can be represented as

$$\boxed{\vec{P} = \vec{E} \times \vec{H} = \frac{\vec{E} \times \vec{B}}{\mu}}$$

# Poynting Theorem

The poynting theorem state that "power flowing out of a given volume is equal to the time rate of decrease is the energy stored with volume minus the ohmic losses. i.e.

Total power leaving the volume = Time rate of decrease in energy - ohmic power losses

**Proof.** The energy density carried by the electromagnetic wave can be calculated using maxwell's eq$^n$s.

We know the maxwell's eq$^n$s for electric & magnetic fields.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \ldots\ldots\ldots\ldots(1)$$

&

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \ldots\ldots\ldots\ldots(2)$$

Taking dot proluct of eq$^n$ (1) with $\vec{H}$ and eq$^n$ (2) with $\vec{E}$

we get

$$\vec{H} \cdot (\vec{\nabla} \times \vec{E}) = -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} \qquad \ldots\ldots\ldots\ldots(3)$$

and

$$\vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \qquad \ldots\ldots(4)$$

now from eq$^n$ (3) - (4)

$$\vec{H}\cdot(\vec{\nabla}\times\vec{E})-\vec{E}\cdot(\vec{\nabla}\times\vec{H})=-\vec{H}\cdot\frac{\partial\vec{B}}{\partial t}-\vec{E}\cdot\vec{J}-\vec{E}\cdot\frac{\partial\vec{D}}{\partial t}$$

$$=-\vec{H}\cdot\frac{\partial\vec{B}}{\partial t}-\vec{E}\cdot\frac{\partial\vec{D}}{\partial t}-\vec{E}\cdot\vec{J} \qquad \ldots\ldots\ldots(5)$$

as vector identity

$$\vec{\nabla}\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\vec{\nabla}\times\vec{A})-\vec{A}\cdot(\vec{\nabla}\times\vec{B})$$

So, from eq$^n$ (5)

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H})=-\left[\vec{H}\cdot\frac{\partial\vec{B}}{\partial t}+\vec{E}\cdot\frac{\partial\vec{D}}{\partial t}\right]-\vec{E}\cdot\vec{J} \qquad \ldots\ldots\ldots\ldots(6)$$

now $\vec{H}\cdot\frac{\partial\vec{B}}{\partial t}=\vec{H}\cdot\frac{\partial}{\partial t}(\mu\vec{H})=\frac{1}{2}\mu\frac{\partial}{\partial t}(H^2)=\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right)$

& $\vec{E}\cdot\frac{\partial\vec{D}}{\partial t}=\vec{E}\cdot\frac{\partial}{\partial t}(\in\vec{E})=\frac{1}{2}\in\frac{\partial}{\partial t}(E^2)=\frac{\partial}{\partial t}\left(\frac{1}{2}\in E^2\right)$

From eq$^n$ (6)

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H})=-\frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2+\frac{1}{2}\in E^2\right]-\vec{E}\cdot\vec{J}$$

Or $\vec{E}\cdot\vec{J}=-\frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2+\frac{1}{2}\in E^2\right]-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(7)$

Taking integral of $eq^n$ (7) over a volume 'V' enclosed by a surface 's'

$$\oint_V \vec{E}\cdot\vec{J}dV = -\oint_V \frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2 + \frac{1}{2}\in E^2\right]dV - \oint_V \vec{\nabla}\cdot\left(\vec{E}\times\vec{H}\right)dV \qquad \ldots\ldots(8)$$

By Gauss's divergen'e theorem

$$\oint_s \left(\vec{E}\times\vec{H}\right)\cdot d\vec{s} = \oint_V \vec{\nabla}\cdot\left(\vec{E}\times\vec{H}\right)dV$$

$$\oint_V \vec{E}\cdot\vec{J}dV = -\frac{\partial}{\partial t}\oint_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\in E^2\right]dv - \oint_s \left(\vec{E}\times\vec{H}\right)ds$$

$$\oint_s \left(\vec{E}\times\vec{H}\right)\cdot d\vec{s} = -\frac{\partial}{\partial t}\oint_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\in E^2\right]dV - \oint_V \vec{E}\cdot\vec{J}dV \qquad \ldots\ldots(9)$$

$$\oint_s \left(\vec{E}\times\vec{H}\right)\cdot d\vec{s} = -\frac{\partial}{\partial t}\oint_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\in E^2\right]dv - \oint_V \sigma\vec{E}dV \qquad \therefore \vec{J} = \sigma\vec{E}$$

$$\boxed{\oint_s \vec{P}\cdot d\vec{s} = -\oint_V \frac{\partial Uem}{\partial t}dV - \oint_V \sigma\vec{E}dV} \qquad \ldots\ldots\ldots(10)$$

as $\vec{P} = \vec{E}\times\vec{H}$

This $eq^n$ 10 represents pointing theorem

Term, $\oint_S \vec{P} \cdot d\vec{s} \rightarrow$ Total power leaving the volume

Term, $-\oint_V \frac{\partial Uem}{\partial t} dV$ →denotes the rate of decrease in energy stored in electromagnetic field.

And

Term $\oint_V \sigma E^2 dV$ →the ohmic power dissipated or ohmic losses

# Displacement Current

The current in virtue of flow of charge is known as Conduction current. As there is no electric conductor between two plates of a capacitor, there is no possibility of charge transfer, hence no conduction current.

Displacement current is one of the very important concept introduced by Maxwell. In empty free space the conduction current is zero but magnetic field is present. It is due to displacement current, Maxwell stated.

Consider a parallel combination of a resistance and a capacitor. If we apply voltage V to such combination, current flowing through resistance is given by

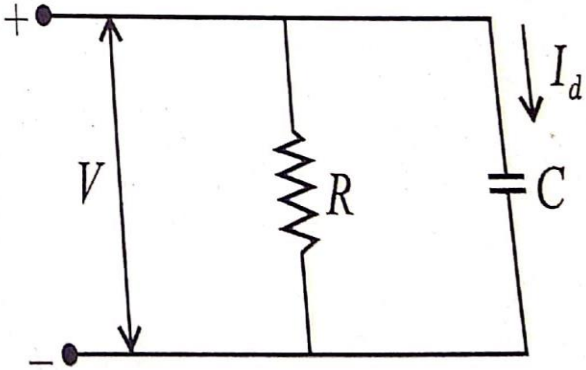


Fig. : Displacement Current

$$I_R = \frac{V}{R} \quad \text{......(1)}$$

This current is because of actual motion of charges hence it is called *conduction current.* Density of this current (conduction current per unit area) is referred to as *conduction current density* ($J_C$). The current flowing through the capacitor is given by

$$I_C = \frac{dQ}{dt}$$

But $\quad Q = CV$

$$\therefore \quad I_C = C\frac{dV}{dt} \quad \text{.....(2)}$$

The current $I_C$ is not because of actual motion of charges. The current flows through the capacitor only when the voltage is changing. The current flowing out from one plate is equal to the current flowing in the other plate. This current is referred to as *displacement current* ($I_d$). The displacement current per unit area is called *displacement current density* ($J_d$).

$$\therefore \quad I_d = I_C = C\frac{dV}{dt} \quad \text{....(3)}$$

Now let

$C$ = Capacitance of parallel plate capacitor $\left(C = \dfrac{\varepsilon A}{d}\right)$

$A$ = Common Area of plates

$\varepsilon$ = Permittivity of medium

$d$ = Spacing between the plates or thickness of dielectric

$E$ = Electric field intensity across the capacitor $\left(E = \dfrac{V}{d}\right)$

$$I_d = C\frac{dV}{dt}$$

$$= \frac{\varepsilon A}{d}\frac{dV}{dt} = \frac{\varepsilon A}{d}\left[d\frac{dE}{dt}\right]$$

$$I_d = \varepsilon A\frac{dE}{dt} \qquad .....(4)$$

But $D = \varepsilon E \left(\text{or } E = \dfrac{D}{\varepsilon}\right)$ then

$$\frac{I_d}{A} = \frac{dD}{dt}$$

Displacement current density

$$J_d = \frac{dD}{dt}$$

In vector form

$$\vec{J}_d = \frac{d\vec{D}}{dt}$$

Vector $\vec{D}$ may vary with space

$$\therefore \quad \vec{J}_d = \frac{\partial \vec{D}}{dt} \qquad ...(2.126)$$

The above equation gives the expression for displacement current density. The .....(5)
density in vector form is given by

$$\vec{J} = \vec{J}_C + \vec{J}_d \qquad .....(6)$$

Let us reconsider curl of magnetic fields (Ampere's circuit law) for time varying electricfields

$$\nabla \times \vec{H} = \vec{J}$$

by using equation (6)

$$\nabla \times \vec{H} = \vec{J}_c + \vec{J}_d = \vec{J}_c + \frac{\partial \vec{D}}{\partial t} \qquad .....(7)$$

Based on the displacement current density, we define the displacement current as

$$I_d = \int_S \vec{J}_d \cdot dS = \int_S \frac{\partial \vec{D}}{\partial t} \cdot dS \qquad .....(8)$$

Using Ampere's circuited law, we know

$$\oint_L \vec{H} \cdot \overrightarrow{dL} = I_{enclosed} = \oint_S \vec{J} \cdot dS = \oint_S (\vec{J} + \vec{J}_d) \cdot dS$$

If there is no conduction current, i.e. $\vec{J} = 0$, then

$$\oint_L \vec{H} \cdot \overrightarrow{dL} = \oint_S \vec{J}_d \cdot dS \qquad .....(9)$$

by using equation (5) we get

$$\oint_L \vec{H} \cdot \overrightarrow{dL} = \int_S \frac{d\vec{D}}{dt} \cdot d\vec{S} = \frac{d}{dt}\int_S \vec{D}.d\vec{S}$$

$$\oint_L \vec{H} \cdot \overrightarrow{dL} = \frac{dQ}{dt} = I \qquad \because \oint_S \vec{D}.d\vec{S} = \oint_S \rho_v dV = Q \qquad .....(10)$$

Hence magnetic field arises through the time varying electric field due to generation of displacement current $(\vec{J}_d)$.

# Suggested reference books & links from NPTEL/IIT/RTU Plateforms

**Reference Books:**

1. Introduction to Electrodynamics: David J Griffiths (Prentice-Hall of India)
2. Engineering Physics: Malik and Singh (Tata McGraw Hill)
3. Engineering Physics: S. Mani Naidu (Pearson Education)
4. Concept of Modern Phyiscs: A. Baiser (Tata McGraw Hill)
5. Engineering Physics : Y. C. Bhatt (Ashirwad Publications)
6. Engineering Physics : S. K. Sharma (Genius Publication)
7. Engineering Physics: D. K. Bhattacharya (Oxford Higher Education)

**Suggested Links:**

1. https://nptel.ac.in/courses/115/101/115101004/ (a series of lectures by Prof. Amol Dighe, IIT Bombay)
2. https://nptel.ac.in/courses.html (a series of video lectures by Prof. Nirmal Ganguli IISER Bhopal)
3. https://youtu.be/ 4nmWFt6fhM (a video lecture prepared by Prof. R. K. Mangal, JECRC-Jaipur)
4. https://nptel.ac.in/courses/115/104/115104088/ ( a series of video lectures by Prof. Manoj Harbola IIT Kanpur)

# Important Question

1. Explain and derive Maxwell's equations for the time varying electric fields in differential and integral form. Also give the physical significance.
2. State Ampere's circuital law.
3. State Poynting vector.
4. Differentiate conduction current and displacement current?
5. State and prove Poynting theorem
6. Explain and derive Poisson's equation and Laplace equation. Give two applications also.
7. For a medium, conductivity = 5 mho/m and dielectric constant is 1. If an electric field E = 250 sin (1010 t) is applied then find the conducting current density and displacement current density.

# Thank You

JECRC Foundation