JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – I Year & II Sem
Subject –Engineering Mathematics-II
Unit – III
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VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

MISSION OF INSTITUTE

- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the area of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

CONTENTS (TO BE COVERED)

Particular Integral case -2

P.I when $X = \sin ax$ or $\cos ax$

$$P.I = \frac{1}{f(D)} \sin ax \text{ or } \frac{1}{f(D)} \cos ax$$

Considering only even Powers of D in $f(D)$, we have

$$\frac{1}{f(D^2)} \sin ax = \frac{1}{f(-a^2)} \sin ax \quad ; f(-a^2) \neq 0$$

$$\& \quad \frac{1}{f(D^2)} \cos ax = \frac{1}{f(-a^2)} \cos ax \quad ; f(-a^2) \neq 0$$

If $f(-a^2)=0$ i.e $f(D)=(D^2+a^2)$, then

$$\frac{1}{(D^2+a^2)}\sin ax = -\frac{x}{2a}\cos ax$$

& $$\frac{1}{D^2+a^2}\cos ax = \frac{x}{2a}\sin ax$$

Ex: $(D^3+D^2+D+1)y = \sin^2 x$

Sol: Auxilliary eqn is $m^3+m^2+m+1=0$

$\Rightarrow m^2(m+1)+1(m+1)=0$

$\Rightarrow (m+1)(m^2+1)=0 \Rightarrow m=-1, \pm i$

$C.F = c_1 e^{-x} + c_2 \cos x + c_3 \sin x$

$$P.I = \frac{1}{D^3+D^2+D+1}\sin^2 x = \frac{1}{D^3+D^2+D+1}\left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{2}\left[\frac{1}{D^3+D^2+D+1}e^{0x} - \frac{1}{D^3+D^2+D+1}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{D(-4)+(-4)+D+1}\cos 2x\right] = \frac{1}{2}\left[1 - \frac{1}{(-3D-3)}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{3(D+1)}\cos 2x\right] = \frac{1}{2}\left[1 + \frac{1}{3}\frac{(D-1)}{D^2-1}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{3}\frac{(D-1)}{(-4-1)}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{15}(D-1)\cos 2x\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{15}(-2\sin 2x - \cos 2x)\right]$$

$$= \frac{1}{30}\left[15 + 2\sin 2x + \cos 2x\right]$$

Complete sol is $y = C.F + P.I$

$$y = c_1 e^{-x} + c_2 \cos x + c_3 \sin x + \frac{1}{30}\left[15 + 2\sin 2x + \cos 2x\right]$$

Ex: $(D^4 + 2D^3 - 3D^2)y = 3e^{2x} + 4\sin x$

Sol: Auxilliary eqn is:

$m^4 + 2m^3 - 3m^2 = 0 \Rightarrow m^2(m^2 + 2m - 3) = 0$

$\Rightarrow m^2(m-1)(m+3) = 0 \Rightarrow m = 0, 0, 1, -3.$

$C.F = (C_1 + C_2 x)e^{0x} + C_3 e^{x} + C_4 e^{-3x}$

$$P.I = \frac{1}{D^4+2D^3-3D^2}\left(3e^{2x}+4\sin x\right)$$

$$= \frac{1}{D^4+2D^3-3D^2}\left(3e^{2x}\right) + \frac{1}{D^4+2D^3-3D^2}\left(4\sin x\right)$$

$$= 3 \cdot \frac{1}{16+16-12} e^{2x} + 4 \cdot \frac{1}{(-1)^2+2D(-1)-3(-1)} \sin x$$

$$= \frac{3}{20} e^{2x} - 4 \cdot \frac{1}{(2D-4)} \sin x$$

$$= \frac{3}{20} e^{2x} - \frac{2}{(D-2)} \sin x$$

$$= \frac{3}{20} e^{2x} - \frac{2(D+2)}{(D^2-4)} \sin x = \frac{3}{20} e^{2x} - \frac{2(D+2)}{-1-4} \sin x$$

$$= \frac{3}{20} e^{2x} + \frac{2}{5} (D+2) \sin x$$

$$= \frac{3}{20} e^{2x} + \frac{2}{5} (\cos x + 2 \sin x).$$

Complete soln is $y = C.F + P.I$

$$y = c_1 + c_2 x + c_3 e^{x} + c_4 e^{-3x} + \frac{3}{20} e^{2x} + \frac{1}{5}(\cos x + 2\sin x).$$

Ex: $\frac{d^3y}{dx^3} + y = \sin 3x - \cos^2\frac{x}{2}$

Sol: Auxilliary eqn is

$$m^3 + 1 = 0$$

$$\Rightarrow (m+1)(m^2+m+1) = 0$$

$$\Rightarrow m = -1, \frac{1 \pm i\sqrt{3}}{2}$$

$$C.F = c_1 e^{-x} + e^{\frac{x}{2}}\left[c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 \sin\left(\frac{\sqrt{3}x}{2}\right)\right]$$

$$P.I = \frac{1}{(D^3+1)}\left(\sin 3x - \cos^2\frac{x}{2}\right)$$

$$= \frac{1}{(D^3+1)}\sin 3x - \frac{1}{(D^3+1)}\cos^2\frac{x}{2}$$

$$= \frac{1}{(-9D+1)} \sin 3x - \frac{1}{(D^3+1)} \left(\frac{1+\cos x}{2} \right)$$

$$= \frac{(1+9D)}{1-81D^2} \sin 3x - \frac{1}{2} \left[\frac{1}{(D^3+1)} \cdot e^{0x} + \frac{1}{(D^3+1)} \cos x \right]$$

$$= \frac{(1+9D)}{[1-81(-9)]} \sin 3x - \frac{1}{2} \left[e^{0x} + \frac{1}{(-D+1)} \cos x \right]$$

$$= \frac{(1+9D)}{730} \sin 3x - \frac{1}{2} \left(1 + \frac{(1+D)}{(1-D^2)} \cos x \right)$$

$$= \frac{1}{730}\left(\sin 3x + 27\cos 3x\right) - \frac{1}{2}\left(1 + \frac{(1+D)}{2}\cos x\right)$$

$$= \frac{1}{730}\left(\sin 3x + 27\cos 3x\right) - \frac{1}{2}\left(1 + \frac{1}{2}\cos x - \frac{1}{2}\sin x\right)$$

$$= \frac{1}{730}\left(\sin 3x + 27\cos 3x\right) - \frac{1}{2} - \frac{1}{4}\cos x + \frac{1}{4}\sin x$$

hence the Complete soln is

$$y = C.F + P.I$$

$$y = c_1 e^{-x} + e^{x/2}\left[c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 \sin\left(\frac{\sqrt{3}x}{2}\right)\right]$$

$$+ \frac{1}{730}\left(\sin 3x + 27\cos 3x\right) - \frac{1}{2} + \frac{1}{4}\left(\sin x - \cos x\right).$$

Ex: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = \sin(3x+1)$

Sol: The auxilliary eqn. is

$m^2 - 2m + 2 = 0$

$\Rightarrow m = \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$

$C.F = e^x(c_1 \cos x + c_2 \sin x)$

$$P.I = \frac{1}{(D^2-2D+2)} \sin(3x+1)$$

$$= \frac{1}{(-9-2D+2)} \sin(3x+1) = \frac{1}{(-7-2D)} \sin(3x+1)$$

$$= \frac{-1}{(7+2D)} \sin(3x+1) = \frac{-(7-2D)}{49-4D^2} \sin(3x+1)$$

$$= \frac{-(7-2D)}{49-4(-9)} \sin(3x+1) = \frac{-(7-2D)}{85} \sin(3x+1)$$

$$= -\frac{1}{85}\left[(7\sin(3x+1) - 6\cos(3x+1)\right]$$

$$= \frac{6\cos(3x+1) - 7\sin(3x+1)}{85}$$

hence the complete sol. is

$$y = C\cdot F + P\cdot I$$

$$y = e^{x}(C_1 \cos x + C_2 \sin x) + \frac{6\cos(3x+1) - 7\sin(3x+1)}{85}$$

Ex: $(D^2 + 2n\cos\alpha \cdot D + n^2)x = a\cos nt$; $D = \frac{d}{dt}$

and $x = \frac{dx}{dt} = 0$ at $t = 0$.

Sol: The auxiliary equ is given as

$$m^2 + 2mn\cos\alpha + n^2 = 0$$

$$\Rightarrow m^2 + 2mn\cos\alpha + n^2\cos^2\alpha + n^2 = n^2\cos^2\alpha$$

$$\Rightarrow (m + n\cos\alpha)^2 = -n^2(1 - \cos^2\alpha) = -n^2\sin^2\alpha$$

$$\Rightarrow \quad m + n\cos\alpha = \pm ni\sin\alpha$$

$$\Rightarrow \quad m = -n\cos\alpha \pm in\sin\alpha$$

$$C.F = e^{(-n\cos\alpha)t}\left[c_1\cos(n\sin\alpha\, t) + c_2\sin(n\sin\alpha)t\right]$$

$$P.I = \frac{1}{(D^2+n^2+2n\cos\alpha . D)} \, a\cos nt$$

$$= a . \frac{1}{(-n^2+n^2+2n\cos\alpha . D)} \cos nt$$

$$= \frac{a}{2n\cos\alpha}\left(\frac{1}{D}\cos nt\right) = \frac{a\sin nt}{2n^2\cos\alpha}$$

hence the complete sol. is

$$x = C.F + P.I$$

$$x = e^{(-n\cos\alpha)t}\left[C_1\cos(n\sin\alpha)t) + C_2\sin(n\sin\alpha)t\right] + \frac{a\sin nt}{2n^2\cos\alpha} \quad \ldots (1)$$

On Putting $x=0, t=0$ in equ ①,

$$0 = c_1 + 0 \Rightarrow c_1 = 0$$

Now differentiate ①, w.r. to t, we get

$$\frac{dx}{dt} = (-n\cos\alpha)e^{-nt\cos\alpha}\left[c_1\cos(n\sin\alpha)t + c_2\sin(n\sin\alpha)t\right]$$

$$+ e^{-nt\cos\alpha}\left[-n\sin\alpha\, c_1\sin(n\sin\alpha)t + n\sin\alpha\, c_2\cos(n\sin\alpha)t\right]$$

$$+ \frac{n\alpha\cos nt}{2n^2\cos\alpha} \quad \cdots ②$$

On Putting $\frac{dx}{dx}=0, t=0$ in ②

$$0 = (-n\cos\alpha)(c_1 + 0) + (n\sin\alpha)c_2 + \frac{\alpha}{2n\cos\alpha}$$

$$\Rightarrow -n\cos\alpha\, c_1 + n\sin\alpha\, c_2 = -\frac{\alpha}{2n\cos\alpha}$$

$$\Rightarrow n\sin\alpha\, c_2 = \frac{-\alpha}{2n\cos\alpha}$$

$$\Rightarrow c_2 = \frac{-\alpha}{2n^2\sin\alpha\cos\alpha} = \frac{-\alpha}{n^2\sin 2\alpha}$$

Put the values of c_1 & c_2 in ①, we have

$$x = \frac{-\alpha}{n^2 \sin 2\alpha} e^{-nt\cos\alpha} \sin(nt \sin\alpha) + \frac{a \sin nt}{2n^2 \cos\alpha}$$

Ex: $(D-1)^2(D^2+1)^2 y = \sin^2\frac{x}{2} + e^x$

Sol: The auxiliary equ is

$(m-1)^2(m^2+1)^2 = 0$

$\Rightarrow (m-1)(m-1)(m^2+1)(m^2+1) = 0$

$\Rightarrow m = 1, 1, \pm i, \pm i$

$C.F = (c_1 + c_2 x)e^x + (c_3 + c_4 x)\cos x + (c_5 + c_6 x)\sin x$

$$P.I = \frac{1}{(D-1)^2(D^2+1)^2}\left(\sin^2\frac{x}{2} + e^x\right)$$

$$= \frac{1}{(D-1)^2(D^2+1)^2}\, e^x + \frac{1}{(D-1)^2(D^2+1)^2}\left(\frac{1-\cos x}{2}\right)$$

$$= \frac{x^2}{\lfloor 2}\,\frac{e^x}{(1+1)^2} + \frac{1}{2}\cdot\frac{1}{(D-1)^2(D^2+1)^2}\, e^{0x} - \frac{1}{2}\,\frac{1}{(D-1)^2(D^2+1)^2}\cos x$$

$$= \frac{1}{8}x^2e^x + \frac{1}{2}\,\frac{e^{0x}}{(0-1)^2(0+1)^2} - \frac{1}{2}\,\frac{1}{(D^2-2D+1)(D^2+1)^2}\cos x$$

$$= \frac{1}{8}x^2e^x + \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{(-1-2D+1)(D^2+1)^2} \cos x$$

$$= \frac{1}{8}x^2e^x + \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{(D^2+1)^2} \left\{ \frac{1}{D} \cos x \right\}$$

$$= \frac{1}{8}x^2e^x + \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{(D^2+1)^2} \sin x$$

$$= \frac{1}{8}x^2e^x + \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{(D+i)^2(D-i)^2} \text{ Imag. Part of } e^{ix}$$

$$= \frac{1}{8}x^2e^x + \frac{1}{2} + \frac{1}{4(i+i)^2} \cdot \frac{1}{(D-i)^2} \text{ Imag. Part of } e^{ix}$$

$$= \frac{1}{8}x^2e^x + \frac{1}{2} - \frac{1}{16} \cdot \frac{x^2}{\underline{|2}} \text{ Imag. Part of } e^{ix}$$

$$= \frac{1}{8}x^2e^x + \frac{1}{2} - \frac{x^2}{32}\left(\cancel{\cos}\,\sin x\right)$$

hence the Complete sol is

$$y = C.F + P.I$$

$$y = (c_1 + c_2 x)e^x + (c_3 + c_4 x)\cos x + (c_5 + c_6 x)\sin x$$

$$+ \frac{1}{8}x^2 e^x + \frac{1}{2} - \frac{x^2}{32}\sin x.$$

Practice Problems

1. $$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$$

Ans: $y = c_1 e^x + (c_2 + c_3 x)e^{-x} - \frac{2}{25}\sin 2x - \frac{1}{25}\cos 2x$

2. $$(D^3 - 3D^2 + 4D - 2)y = e^x + \cos x$$

Ans: $y = c_1 e^x + e^x(c_2 \cos x + c_3 \sin x) + xe^x + \frac{1}{10}(3\sin x + \cos x)$

3. $(D^2-5D+6)y = \sin 3x$

Ans: $y = C_1 e^{2x} + C_2 e^{3x} + \frac{1}{234}(15\cos 3x - 3\sin 3x)$

4. $\frac{d^3y}{dx^3} + y = \sin 3x - \cos^2\left(\frac{x}{2}\right)$

Ans: $y = C_1 e^{-x} + e^{\frac{x}{2}}\left[C_2 \cos\left(\frac{\sqrt{3}}{2}x\right) + C_3 \sin\left(\frac{\sqrt{3}}{2}x\right)\right]$

$+ \frac{1}{730}(27\cos 3x + \sin 3x) + \frac{1}{4}(\sin x - \cos x) - \frac{1}{2}.$

5. $\dfrac{d^2y}{dx^2} + 4y = \sin^2 x + 4$

Ans: $y = c_1 \cos 2x + c_2 \sin 2x + \dfrac{9}{8} - \dfrac{1}{8} x \sin 2x$

6. $\dfrac{d^4y}{dx^4} - m^4 y = \sin mx$

Ans: $y = c_1 e^{mx} + c_2 e^{-mx} + c_3 \cos mx + c_4 \sin mx + \dfrac{x}{4m^3} \cos mx$

7. $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 4y = 3\cos 2x + \sin 3x$

Ans: $y = c_1 e^x + c_2 \cos 2x + c_3 \sin 2x - \frac{3x}{20}(2\cos 2x + \sin 2x)$
$+ \frac{1}{50}(\sin 3x + 3\cos 3x)$

8. $(D^2+4)y = 8\cos 2x$, ; $D = \frac{d}{dx}$, $y = \frac{dy}{dx} = 0$ at $x = 0$

Ans: $y = 2x \sin 2x$

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Thank you!