JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – I Year & II Sem
Subject –Engineering Mathematics-II
Unit – III
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VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

MISSION OF INSTITUTE

- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the area of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

CONTENTS (TO BE COVERED)

Differential Equations with Constant Coefficient (Introduction and Complementary function)

Linear differential equations of higher order with Constant Coefficients.

The general form of the linear diff. eqn. of second order is

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

where P and Q are constants & R is a function of x or constants.

here we use

$$Dy = \frac{dy}{dx}, \quad D^2y = \frac{d^2y}{dx^2}, \quad D^3y = \frac{d^3y}{dx^3}$$

& $\frac{1}{D}$ stands for integration.

Then the above eqn can be written as

$$D^2y + PDy + Qy = R$$

or $(D^2 + PD + Q)y = R$

Its complete sol. is given by

Complete sol = Complementary function + Particular Integral

or Complete Sol = C.F + P.I

To find, C.F first we put the R.H.S of

$$f(D)y = X$$

is equal to equal to zero., we have

$$f(D)y = 0. \quad \cdots (1)$$

Let $y = e^{mx}$ be a solution of it.

Put $D = m$, then $f(D)$ is called auxilliary equ. (A.E) of it which is algebraic equ of degree n which has n roots. Depending upon the nature of roots of A.E, C.F is given as:

Case. (1) If all the roots of equ (1) are real & distinct. say $m = m_1, m_2, \dots m_n$

then $C.F = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + \dots + c_n e^{m_n x}$

Case. (2) If two roots are real & equal & remaining are real & distinct.

say $m = m_1, m_1, m_3, m_4 \dots$ then

$C.F = (c_1 + c_2 x) e^{m_1 x} + c_3 e^{m_3 x} + \dots + c_n e^{m_n x}$

If 3 roots are real & equal & others are distinct then

$$C.F = (c_1 + c_2 x + c_3 x^2) e^{m_1 x} + c_4 e^{m_4 x} + \ldots$$

Case 3. If roots are complex $(\alpha \pm i\beta)$ then

$$C.F = e^{\alpha x}\left[c_1 \cos\beta x + c_2 \sin\beta x\right]$$

If two pairs of imaginary roots are equal then say $m = \alpha \pm i\beta,\ \alpha \pm i\beta$.

$$C.F = e^{\alpha x}\left[(c_1 + c_2 x)\cos\beta x + (c_3 + c_4 x)\sin\beta x\right]$$

Case 4. If two roots of A.E are real & surds say $m = \alpha \pm \sqrt{\beta}$ & others are real & distinct.

$$C.F = e^{\alpha x}\left[C_1 \cosh\sqrt{\beta}\,x + C_2 \sinh\sqrt{\beta}\,x\right] + C_3 e^{m_3 x} + \ldots$$

Ex: Solve $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$

Sol: given eqn can be written as

$$(D^2 - 8D + 15)y = 0 \quad ; \quad D = \frac{d}{dx}$$

Its auxilliary eqn is $m^2 - 8m + 15 = 0$

$\Rightarrow \quad (m-3)(m-5) = 0 \qquad \therefore \quad m = 3, 5$

so the sol. is

$$y = c_1 e^{3x} + c_2 e^{5x}.$$

Ex: $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

Sol: given eqn can be written as

$$(D^2 - 6D + 9)y = 0$$

A.E is $m^2 - 6m + 9 = 0$

$\Rightarrow (m-3)^2 = 0 \Rightarrow m = 3, 3$

then C.F $= (C_1 + C_2 x)e^{3x}$.

Ex: $\dfrac{d^5y}{dx^5} - \dfrac{d^3y}{dx^3} = 0$

Sol: given eqn can be written as

$(D^5 - D^3)\,y = 0, \quad D = \dfrac{d}{dx}$

A.E is $m^5 - m^3 = 0 \Rightarrow m^3(m^2-1) = 0$

$\Rightarrow m^3(m+1)(m-1) = 0$

$\Rightarrow m = 0, 0, 0, 1, -1.$

Sol is: $y = (c_1 + c_2x + c_3x^2)e^{0x} + c_4e^{x} + c_5e^{-x}.$

Ex: $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$

Sol: given equ can be written as

$$(D^3 - 6D^2 + 11D - 6)y = 0, \quad D = \frac{d}{dx}$$

A. E is: $m^3 - 6m^2 + 11m - 6 = 0$

$$\Rightarrow (m-1)(m^2 - 5m + 6) = 0$$

$$\Rightarrow (m-1)(m-2)(m-3) = 0$$

$$\Rightarrow m = 1, 2, 3.$$

C. F is: $C_1 e^x + C_2 e^{2x} + C_3 e^{3x}$.

Ex: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$

Sol: $(D^2 - 4D + 1)y = 0$; $D = \frac{d}{dx}$

A.E is: $m^2 - 4m + 1 = 0$

$$m = \frac{4 \pm \sqrt{16-4}}{2} \Rightarrow \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$$

C.F is: $e^{2x}(C_1 \cosh\sqrt{3}x + C_2 \sinh\sqrt{3}x)$.

Ex: $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 5y = 0$

Sol: $(D^2 + 4D + 5)y = 0$; $D = \dfrac{d}{dx}$

A.E is: $m^2 + 4m + 5 = 0$

$$m = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm i$$

$$C.F = e^{-2x}[C_1 \cos x + C_2 \sin x].$$

Ex: $(D^2+D+1)^2 y = 0$; $D = \frac{d}{dx}$

Sol: A.E is $(m^2+m+1)^2 = 0$

Now $m^2+m+1=0$, $m = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$

So $m = \frac{-1 \pm \sqrt{3}\, i}{2}, \frac{-1 \pm \sqrt{3}\, i}{2}$

$$C.F = e^{-\frac{x}{2}}\left[(c_1+c_2x)\cos\left(\frac{\sqrt{3}x}{2}\right) + (c_3+c_4x)\sin\left(\frac{\sqrt{3}}{2}x\right)\right].$$

Practice Problems:

1. $$\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 8y = 0$$

Ans: $y = (C_1 + C_2 x + C_3 x^2) e^{-2x}$

2. $$\frac{d^4y}{dx^4} + 32\frac{d^2y}{dx^2} + 256 = 0$$

Ans: $y = (C_1 + C_2 x) \cos 4x + (C_3 + C_4 x) \sin 4x$

3. $\dfrac{d^4y}{dx^4} + 4\dfrac{d^3y}{dx^3} - 5\dfrac{d^2y}{dx^2} - 36\dfrac{dy}{dx} - 36y = 0$

Ans: $y = c_1 e^{-3x} + c_2 e^{3x} + (c_3 + c_4 x)e^{-2x}$

4. $\dfrac{d^2y}{dx^4} - a^4 y = 0$

Ans: $y = c_1 e^{-ax} + c_2 e^{ax} + c_3 \cos ax + c_4 \sin ax$

5. $\dfrac{d^4y}{dx^4} + 8\dfrac{d^2y}{dx^2} + 16y = 0$

Ans: $y = (C_1 + C_2x)\cos 2x + (C_3 + C_4x)\sin 2x$

6. $\dfrac{d^3y}{dx^3} - y = 0$

Ans: $C_1e^x + e^{-\frac{x}{2}}\left[C_2\cos\left(\dfrac{\sqrt{3}x}{2}\right) + C_3\sin\left(\dfrac{\sqrt{3}x}{2}\right)\right]$

7. $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0$

Ans: $(C_1 + C_2x)e^{-x}$.

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Thank you!