Satellite Communication

5EC5-14

UNIT-2

Orbital Mechanics

2.1 Orbital equations
2.2 Kepler's laws
2.3 Apogee and Perigee for an elliptical orbit
2.4 Evaluation of velocity, orbital period, angular velocity of a satellite.
2.5 Concepts of Solar day and Sidereal day.

2.1 Orbital equation:

It describe the orbit of satellite around the earth

Gravitational force F acting on satellite and balance by centrifugal force

$$\boxed{\text{Fgav} = \frac{kMm}{r^2}\ \text{Newton}}$$

k = gravity constant = 6.67×10^{-11} m^3/kg Nm^2/kg

M = mass of earth = 5.98×10^{24} kg

m = mass of satellite from the earth

r = radius of circular orbital

$$\boxed{\text{Fcenti} = \frac{mv^2}{r}\ \text{Newton}}$$

v = orbital vector of satellite

Fgav = Fcenti $\qquad \dfrac{kMm}{r^2} = \dfrac{mv^2}{r}$

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$$v^2 = \frac{KM}{r} \qquad v = \sqrt{\frac{KM}{r}} \text{ m/sec}$$

Time take to complete one orbit round the earth .

$$T = \frac{2\pi r}{v} \qquad T = 2\pi\sqrt{\frac{r^3}{KM}} \text{ sec}$$

$T^2 \alpha\, r^3$ Kepler's third law

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k = gravity constant = 6.67×10^{-11} m^3/kg Nm^2/kg

M = mass of earth = 5.98×10^{24} kg

T = 11.66×10^{-6} $r^{3/2}$km minutes

r = radius of circular orbital in km , T in minutes

T = 0.0099527 $\sqrt{r^3 km}$ seconds

Height of orbit above the earth

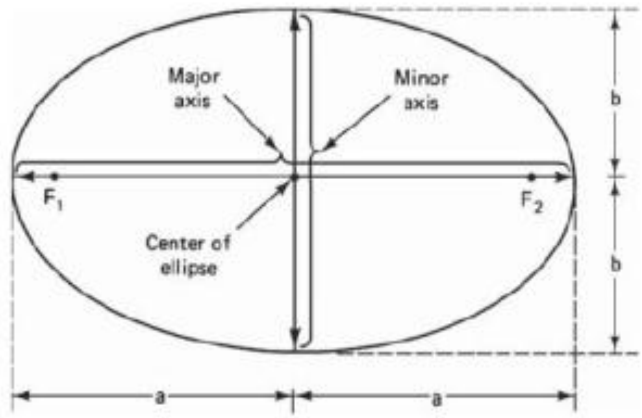
h = (r_{km} - a) = (r_{km} - 6372) km

2.2.Kepler's laws:

2.2.1 Kepler’s First Law:

Keplers first law states that the path followed by a satellite around the primary will be an ellipse. An ellipse has two focal points shown as F1 and F2 in Fig. The center of mass of the two-body system is always centered on one of the foci

- Focus – one of two special points on the major axis of an ellipse.
- Foci – plural of focus
  F1+F2 is always
  the same on
  any point on
  the ellipse
- The semimajor axis of the ellipse is denoted by a,



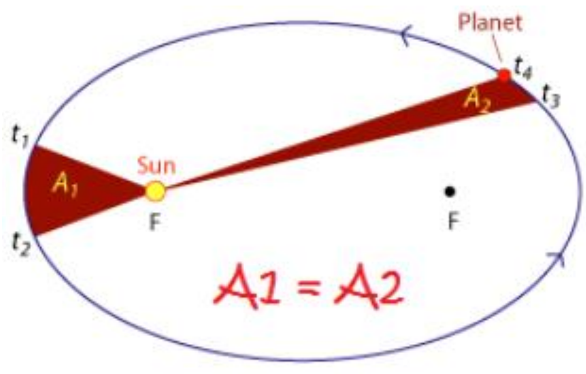
and the semiminor axis, by b. The eccentricity e is given by

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

Eccentricity is the degree of flatness of satellite orbit e = c/a (e = 0 for circle)
c = center to focus
a = half of major axis/Semi-major axis

2.2.2 KepIer’s Second Law: (Law of equal Area)

- The line joining the planet to the sun sweeps out equal areas in equal intervals of time.
- The area from one time to another time is equal to another area with the same time interval.
- **Facts:**
- Planet moves faster when closer to the sun.
  Force acting on the planet increases as distance decreases and planet accelerates in its orbit
- Planet moves slower when farther from the sun.



Kepler's Second Law:
Law of Equal Areas

2.2.3 Keplers's Third Law:

The square of the period of any planet is proportional to the cube of the semi-major of its axis.

$T^2 \alpha a^3$

T = orbital period in years

a = semi-major axis in astronomical unit (AU)

It Can calculate how long it take (period) for planets to orbit if semimajor axis is known.

Astronomical unit – AU

AU is the mean distance between Earth and the Sun

1 AU ≈ 1.5 x 108 km ≈ 9.3 x 107 miles

Examples of 3rd Law:

Calculating the orbital period of 1AU

$T^2 = a^3$

$T^2 = (1)^3 = 1$

T = 1 year

Calculating the orbital period of 4AU

$T^2 = a^3$

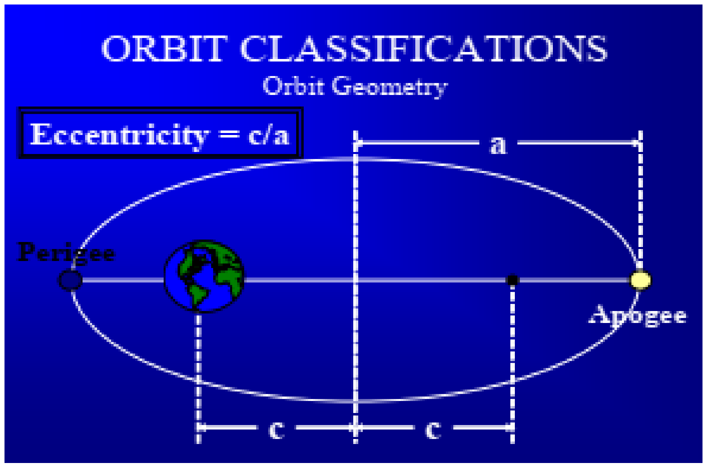
$T^2 = (4)^3 = 64$

T = 8 years

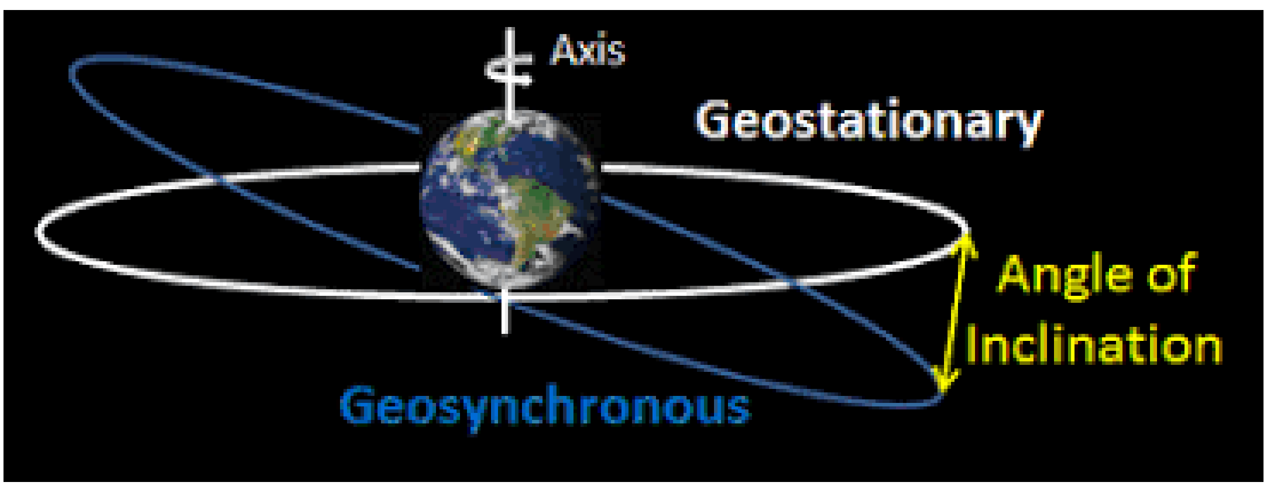
Planet	eccentricity (e)	T (yr)	a (AU)	T^2	a^3
Mercury	0.206	0.24	0.39	0.06	0.06
Venus	0.007	0.62	0.72	0.39	0.37
Earth	0.017	1	1	1	1
Mars	0.093	1.88	1.52	3.53	3.51
Jupiter	0.048	11.9	5.2	142	141
Saturn	0.056	29.5	9.54	870	868

2.3 Apogee and Perigee for an elliptical orbit:

Apogee: The point of satellite farthest from earth.
Perigee: The point of satellite closest approach to earth.



Different Types of Orbit:



Difference between Geostationary, Geosynchronous and polar satellite:

Geostationary	Geosynchronous	Polar
Orbit is circular	Orbit is NOT circular	Orbit is NOT circular
Inclination to equator is zero	Inclination to equator is not zero	Inclination to equator is not zero **(90 degree)**
The angular velocity of the satellite is equal to angular velocity of earth	The angular velocity of the satellite is equal to angular velocity of earth	The angular velocity of the satellite is not equal to angular velocity of earth
Period of revolution is equal to period of rotation of earth	Period of revolution is equal to period of rotation of earth	Period of revolution is not equal to period of rotation of earth
This satellite would appear stationary from the earth	It looks oscillating but NOT stationary	It looks oscillating but NOT stationary
20000 to 36000 km away from the surface of earth.	20000 to 36000 km away from the surface of earth.	typically at 500–800 Kms
telecommunication, navigation, weather forecast **LOS Communication**	telecommunication, navigation, weather forecast	Earth-mapping, Earth observation
A minimum of three satellites are needed to cover the entire earth	More than three to cover earth	More than three to cover earth

2.4 Evaluation of velocity, orbital period, angular velocity of a satellite:

Satellites are made to revolve in an orbit at a height of few hundred kilometres. At this altitude, the friction due to air is negligible. The satellite is carried by a rocket to the desired height and released horizontally with a high velocity, so that it remains moving in a nearly circular orbit. The horizontal velocity that has to be imparted to a satellite at the determined height so that it makes a circular orbit around the planet is called orbital velocity. Let us assume that a satellite of mass m moves around the Earth in a circular orbit of radius r with uniform speed vo. Let the satellite be at a height h from the surface of the Earth. Hence, r = R+h, where R is the radius of the Earth.

The centripetal force required to keep the satellite in circular orbit is $F = \frac{mv_o^2}{r} = \frac{mv_o^2}{R+h}$

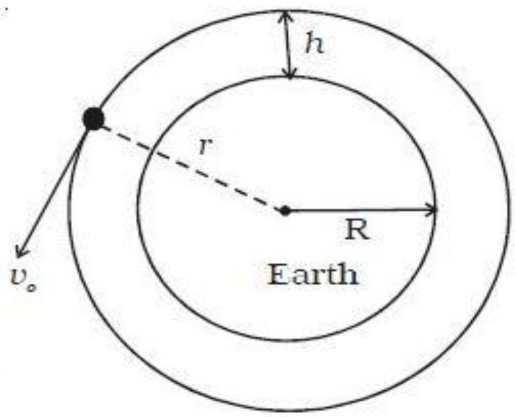
The gravitational force between the Earth and the satellite is

$$F = \frac{GMm}{r^2} = \frac{GMm}{(R+h)^2}$$

For the stable orbital motion,

$$\frac{mv_o^2}{R+h} = \frac{GMm}{(R+h)^2}$$

$$v_o = \sqrt{\frac{GM}{R+h}}$$



Orbital Velocity

Since the acceleration due to gravity on Earth's surface is $g = \frac{GM}{R^2}$,

$$v_o = \sqrt{\frac{gR^2}{R+h}}$$

If the satellite is at a height of few hundred kilometres (say 200 km), $(R+h)$ could be replaced by R.

$\therefore$ orbital velocity, $v_o = \sqrt{gR}$

Time period of a satellite:

Time taken by the satellite to complete one revolution round the Earth is called time period.

$$\text{Time period, } T = \frac{\text{circumference of the orbit}}{\text{orbital velocity}}$$

$T = \frac{2\pi r}{v_o} = \frac{2\pi(R+h)}{v_o}$ where r is the radius of the orbit which is equal to $(R+h)$.

$$T = 2\pi\ (R+h)\sqrt{\frac{R+h}{GM}} \qquad \left[\because v_o = \sqrt{\frac{GM}{R+h}}\right]$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

As $GM = gR^2$, $T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

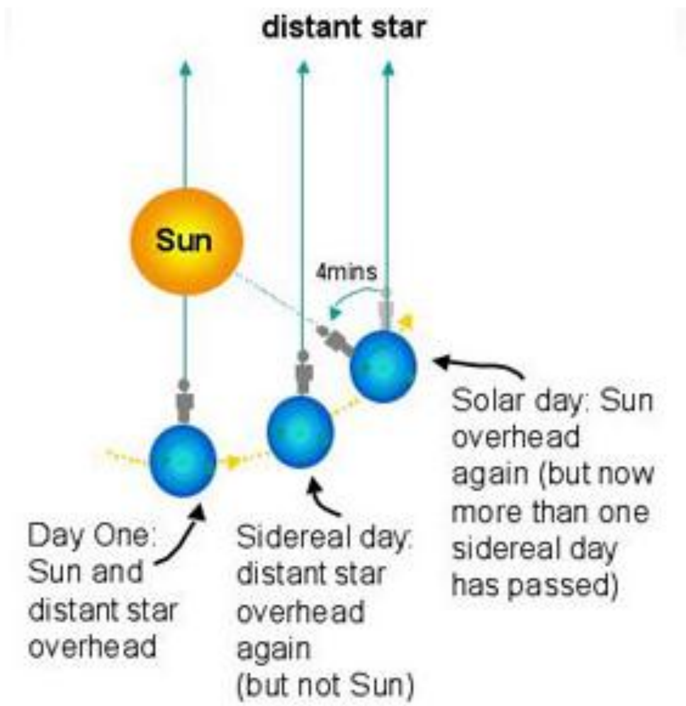
If the satellite orbits very close to the Earth, then $h << R$

$$\therefore T = 2\pi\sqrt{\frac{R}{g}}$$

2.5 Concepts of Solar day and Sidereal day:

A **solar day** is the time it takes for the Earth to rotate about its axis so that the **Sun** appears in the same position in the sky.

A **sidereal day** is the time it takes for the Earth to rotate about its **axis** so that the distant **stars** appear in the same position in the sky. The sidereal day is ~4 minutes shorter than the **solar** day.



The sidereal day is the time it takes for the Earth to complete one rotation about its axis with respect to the 'fixed' stars. By fixed, we mean that we treat the stars as if they were attached to an imaginary **celestial sphere** at a very large **distance** from the Earth.

a sidereal day lasts for 23 hours 56 minutes 4.091 seconds, which is slightly shorter than the solar day measured from noon to noon. Our usual definition of an Earth day is 24 hours, so the sidereal day is 4 minutes faster. This means that a particular star will rise 4 minutes earlier every night.