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Unit- V :-

Plane wave in Arbitary direction :-
or
Wave vector at Arbitary direction :-

Let the wave is moving to dir^n x, y, & z & making angles ϕ_x, ϕ_y, ϕ_z with ~~ref~~ x, y, z respectively as shown in fig.

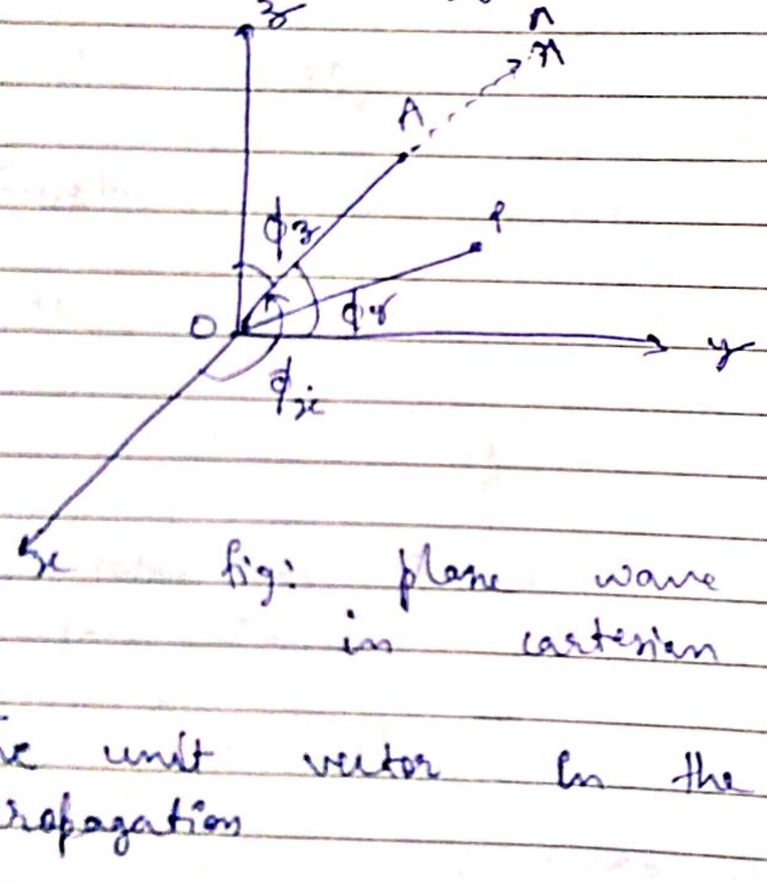

fig: plane wave or wave vector in cartesian plane.

The unit vector in the dir^n of wave propagation

$$\hat{n} = \cos\phi_x \hat{x} + \cos\phi_y \hat{y} + \cos\phi_z \hat{z}$$

where, $\cos\phi_x$, $\cos\phi_y$, $\cos\phi_z$ are c/d the dir^n cosine of the vector $\hat{n}$.

The Eq^n of a const. phase (the

phase front) is g(N as

$$\hat{n} \cdot OP = \hat{n} \cdot r = \text{const.} \qquad (OP = r)$$

So, the phase of this const. phase plane is

$$\boxed{\beta |OA| = \beta \hat{n} \cdot r}$$

⇒ the electric & magnetic field for wave moving in dirⁿ of $\hat{n}$:-

→ The ~~active~~ electric field of a plane wave travelling in dirⁿ $\hat{n}$ can be written as

$$\boxed{\vec{E} = \vec{E_0}\, e^{-j\beta \hat{n} \cdot r}}$$

where, $\vec{E_0}$ is a vector ⊥ to the unit vector $\hat{n}$

→ let us defined wave fo vector as

$$K = \beta \hat{n} = K_x \hat{x} + K_y \hat{y} + K_z \hat{z}$$

then the electric field for this wave vector

$$E' = E_0 e^{-jK \cdot r}$$

$$\cancel{\vec{E} = E_0 e^{-j}(K_x X + K}$$

$$\vec{E} = E_0 e^{-j(K_x X + K_y y + K_z Z)}$$

$\nabla \rho = \cancel{\nabla} r$

$$\frac{\partial}{\partial x} = -jK_x \; ; \; \frac{\partial}{\partial y} = -jK_y \; ; \; \frac{\partial}{\partial z} = -jK_z$$

The magnetic field is obtained as

$$H = \left(\frac{-1}{j\omega\mu}\right) \nabla \times E = \left(\frac{-1}{j\omega\mu}\right)(-jK \times E)$$

$$= \frac{-1}{j\omega\mu} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -jK_x & -jK_y & -jK_z \\ E_x & E_y & E_z \end{vmatrix}$$

$$\boxed{H = H_0 \, e^{-jK \cdot r}}$$

The E_0 & H_0 in $\hat{n}$ vectors are mutually $\perp$ to each other &

$$\frac{|E_0|}{|H_0|} = \text{Intrinsic Impedance of the medium } (\eta)$$

→ Phase velocity & wavelength :-

- The electric field of a uniform plane wave travelling in a dir^n which makes angles ϕ_x, ϕ_y, ϕ_z with 3 - axis x, y, z respectively.

→ Since $|\cos\phi_x|, |\cos\phi_y|, |\cos\phi_z| \leq 1$

the velocities v_{px}, v_{py}, v_{pz} are always greater than or equal to v_0

$$v_0 \leq v_{px}, v_{py}, v_{pz} \leq \infty$$

→ The wavelength of the wave in x, y & z dir^n are given as

$$\lambda_x = \frac{v_{px}}{f} = \frac{\lambda_0}{\cos\phi_x} \qquad \left(\because \lambda_0 = \frac{v_0}{f}\right)$$

$$\lambda_y = \frac{v_{py}}{f} = \frac{\lambda_0}{\cos\phi_y}$$

$$\lambda_z = \frac{v_{pz}}{f} = \frac{\lambda_0}{\cos\phi_z}$$

NOTE

→ If we consider the unbound medium as the free space, the phase velocity of the wave is $v_0 = c$ (velocity of light)

$$c \leq v_{px}, v_{py}, v_{pz} \leq \infty$$

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Topic : Reflection & Refraction at di-electric interface.

An EM wave has to pass through different media while in travelling in space each medium has a different dielectric ~~coef~~ const. & permeability or conductivity. When the EM wave travelling in one medium strikes upon a 2nd medium then the wave will partially transmitted & partially reflected the bending of transmitted wave into other medium is K/N as reflection. Whether the wave will be transmitted ~~or~~ partially ~~refracted~~ reflected or refracted, it depends upon the type of medium & on the type wave incidence.

The 2-type of medium are conductor & di-electric & type of wave incidence are normal incidence or oblique incidence.

Reflection by a perfect conductor :-

→ Normal incidence, when the wave incident normally on the surface of perfect conductor the wave is entirely reflected. ∵ there are be no loss within ~~of~~ a perfect conductor, none of the energy is absorbed.

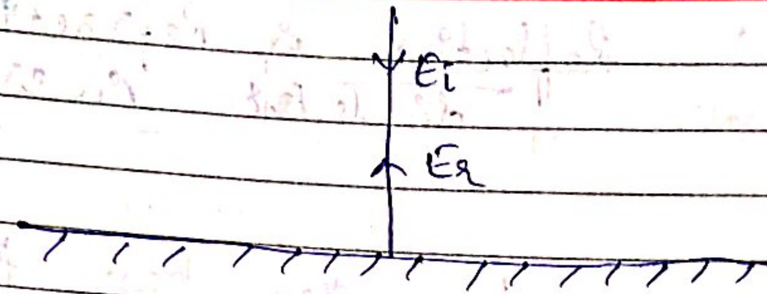

fig - 1 Normal Incidence

$\therefore$, the amplitude of electric field & magnetic field in the reflected wave are the same as in incident wave. the only difference in the dir^n of power ~~over~~flow (out of phase by 180°].

i.e $\quad E_i = -E_r$

$|E_i| = |-E_r|$

let, the electric field of incidence wave

$= E_i e^{-j\beta z}$

& reflected wave will be $= E_r e^{j\beta z}$

The resultant electric field is the sum of electric fields of incidence & refracted wave

$$E_T(z) = E_i e^{-j\beta z} + E_r e^{j\beta z}$$

$$(E_i = -E_r)$$

$$E_T(z) = E_i e^{-j\beta z} - E_i e^{j\beta z}$$

$$E_T(z) = E_i [e^{-j\beta z} - e^{j\beta z}]$$

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$$E_T(z) = -2j\, E_i \sin\beta z$$

$$E_T(z,t) = -2j\, E_i \sin\beta z\, e^{-j\omega t}$$

$$= -2j\, E_i \sin\beta z\,(\cos\omega t - j\sin\omega t)$$

$$= -2j\, E_i \sin\beta z \cdot \cos\omega t - 2E_i \sin\beta z \sin\omega t$$

real part,

$$E_T(z,t) = -2E_i \sin(\beta z)\sin(\omega t)$$

→ Oblique Incidence :- {Perfect conductor}

When a wave is a incident obliquely on a perfect conductor it is necessary to consider separately to polarizations

1st → Horizontal polarization
2nd → vertical "

In 1st case electric vector is parallel to boundary surface or in the other words ⊥ to the plane of incidence. This is K/A as horizontal polarization.

In 2nd case the electric vector is parallel to plane of incidence or ⊥ to boundary surface. This is K/A as vertical polarization.

→ Horizontal polarization :-

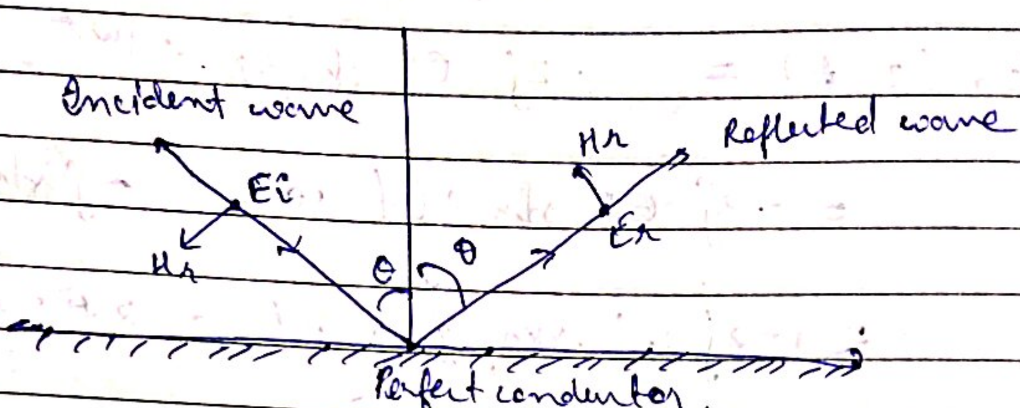

fig : Horizontal polarization.

$$\theta_i = \theta_r = \theta$$

The incident wave is expressed as

$$E_{in} = E_i \, e^{-j\beta \hat{n} \cdot r}$$

where, $\hat{n}$ is the unit vector in the dir. of incident wave.

for the normal of incident wave

$$\hat{n} \cdot r = z \cos\frac{\pi}{2} + y \cos\left(\frac{\pi}{2} - \theta\right) + x \cos(\pi - \theta)$$

$$\boxed{\hat{n} \cdot r = y \sin\theta - x \cos\theta}$$

$$E_{in} = E_i \, e^{-j\beta(y \sin\theta - x\cos\theta)}$$

Similarly the reflected wave is expressed as

$$E_{ref} = E_r \, e^{-j\beta \hat{n} \cdot r}$$

~~$2j\, E_i \sin(\beta x)$~~

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for normal reflected wave

$$\hat{n}\cdot r = z\cos\frac{\pi}{2} + y\cos\left(\frac{\pi}{2}-\theta\right) + x\cos\theta$$

$$\hat{n}\cdot r = y\sin\theta + x\cos\theta$$

$$E_{ref} = E_r \, e^{-j\beta(y\sin\theta + x\cos\theta)}$$

for the boundary condi.

$$E_r = -E_i$$

The total electric field is

$$E_T = E_{in} + E_{ref}$$

$$= E_i\, e^{-j\beta(y\sin\theta - x\cos\theta)} + E_r\, e^{-j\beta(y\sin\theta + x\cos\theta)}$$

$$= E_i\left[e^{-j\beta(y\sin\theta - x\cos\theta)} - e^{-j\beta(y\sin\theta + x\cos\theta)}\right]$$

$$= E_i\left[e^{-j\beta y\sin\theta}\cdot e^{j\beta x\cos\theta} - e^{-j\beta y\sin\theta}\cdot e^{-j\beta x\cos\theta}\right)$$

$$= E_i\, e^{-j\beta y\sin\theta}\left[e^{j\beta x\cos\theta} - e^{-j\beta x\cos\theta}\right)$$

$$= 2j\, E_i\, e^{-j\beta y\sin\theta}\sin(\beta x\cos\theta)$$

$$(\because)\ (\cdot\ \beta_x = \beta\cos\theta\ )$$

$$= 2j\, E_i\, e^{-j\beta y\sin\theta}\sin(\beta_x \times x)$$

$$\beta = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{2\pi}{\lambda}$$

$\beta_x = \beta \cos\theta$ = phase shift in x-dir^n

$\beta_y = \beta \sin\theta$ = phase shift in y-dir^n

Vertical polarization :-

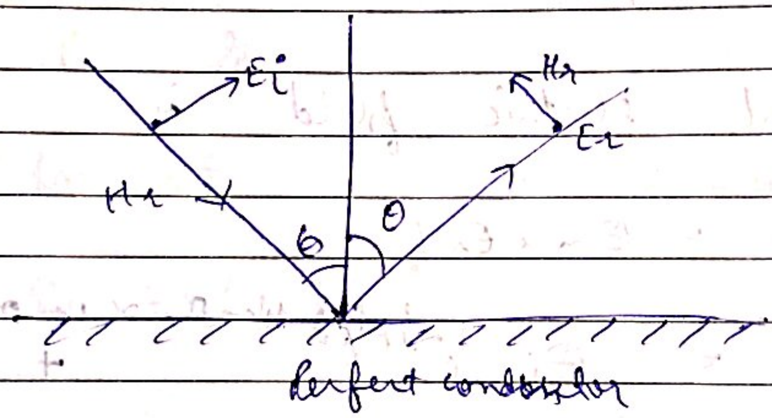

fig:

Let, the incident & reflected wave makes angle of H_i & H_r

$$\theta_i = \theta_r = \theta$$

The incident wave is expressed as

$$H_{in} = H_i e^{-j\beta(y\sin\theta - x\cos\theta)}$$

$$H_{ref} = H_r e^{-j\beta(y\sin\theta + x\cos\theta)}$$

$H_i = H_r$ [No phase reversal]

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& $H_T = H_{in} + H_{ref}$

$$H_T = H_i \left[e^{-j\beta(y\sin\theta - x\cos\theta)} + e^{-j\beta(y\sin\theta + x\cos\theta)} \right]$$

$$= 2H_i \, e^{-j\beta y\sin\theta} \cos j\beta x\cos\theta$$

$$H_T = 2H_i \, e^{-j(\beta_y \cdot y)} \cos j(\beta_x \cdot x)$$

$$\frac{E}{H} = \eta$$

$$\frac{E_i}{H_i} = \eta \; ; \quad \boxed{E_i = \eta H_i}$$

x & y components of electric field for incidence wave is g/n by ~~$E_x = \eta\sin\theta H_i$~~

$$E_x = \eta \sin\theta H_i$$
$$E_y = \eta \cos\theta H_i$$

Similarly, x & y component of electric field for reflected wave is G/N by

$$E_x = \eta \sin\theta H_r$$
$$E_y = -\eta \sin\theta H_r$$

Total x component of electric field intensity is GIN by

$$E_x = \eta \sin\theta H_i + \eta \sin\theta H_r$$

$$E_x = \eta \sin\theta \, (H_i + H_r)$$

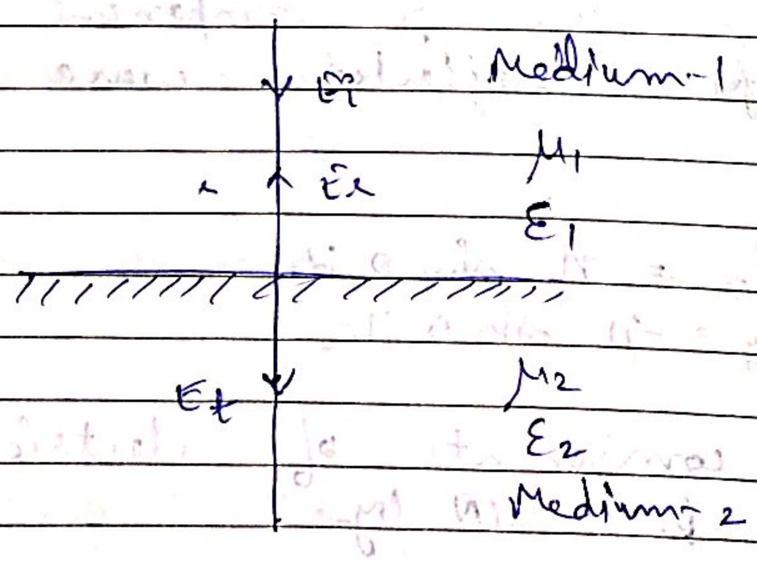
$$\boxed{E_x = \eta \sin\theta \, H_T}$$

⇒ **Reflection by perfect dielectric**

When a plane EM wave is incident of on the surface of perfect dielectric then some part of energy is transmitted & some part of it reflected.

A perfect dielectric is one with zero conductivity so there is no loss on absorption a power in propagation through perfect dielectric medium.

→ **Normal incidence :-**



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Consider 2 perfect dielectric medium separated by 2 medium as shown in fig.

Let, ε_1 & μ_1 are the permittivity & permeability of medium-1, respectively.

Similarly, ε_2 & μ_2 of medium-2.

Let E_i be the electric field of incident wave, E_r be the electric field of reflected wave. & E_t electric field of transmitted wave.

Similarly, H_i, H_r, H_t, magnetic field of incident wave, reflected wave & transmitted wave, respectively.

The intrinsic medium (η) of medium-1

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \quad ; \quad \eta = \frac{E}{H}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \quad ; \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

$E_i = \eta_1 H_i \quad ; \quad E_r = -\eta_1 H_r$

$E_t = \eta_2 H_t$

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According to boundary condⁿ, the tangential component of E & H is continuous across the boundary

$$H_t = H_i + H_r$$
$$E_t = E_i + E_r$$

from above Eqⁿ

$$H_i = \frac{E_i}{\eta_1} \; ; \quad H_r = -\frac{E_r}{\eta_1} \; ; \quad H_t = \frac{E_t}{\eta_2}$$

$$\boxed{H_t = H_i + H_r = \frac{1}{\eta_1}[E_i - E_r]} \quad -①$$

$$E_t = [E_i - E_r]$$

$$\boxed{H_t = \frac{E_t}{\eta_2} = \frac{1}{\eta_2}(E_i + E_r)} \quad -②$$

from Eqⁿ 1 & 2

$$\frac{1}{\eta_1}[E_i - E_r] = \frac{1}{\eta_2}[E_i + E_r]$$

$$\eta_2(E_i - E_r) = \eta_1(E_i + E_r)$$

$$E_i(\eta_2 - \eta_1) = E_r(\eta_1 + \eta_2)$$

$$\boxed{\text{Reflection coefficient} = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}}$$

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Transmission coefficient :-

$$\frac{E_t}{E_i} = \frac{E_i + E_r}{E_i} = 1 + \frac{E_r}{E_i} = 1 + \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$$

$$\boxed{\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}}$$

Similarly for analysis of magnetic field

$$\boxed{\frac{H_r}{H_i} = \frac{-\eta_2 + \eta_1}{\eta_1 + \eta_2}}$$ (Reflection coefficient)

$$\boxed{\frac{H_t}{H_i} = \frac{2\eta_1}{\eta_1 + \eta_2}}$$ (Tx coefficient)

Since, the permeability of perfect di-electrics do not differ for the free space it means $\boxed{\mu_1 = \mu_2}$

$$\mu_1 = \mu_2 = \mu_0$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \; ; \; \eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}} \; ; \; \eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_0}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_0}}$$

$$\frac{E_t}{E_i} = 2\frac{\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_0}}$$

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$$\frac{H_r}{H_i} = \frac{-\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \; ; \; \frac{H_t}{H_i} = \frac{2\sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

⇒ **Total Internal reflection :-**

TIR is a scenario where a travelling wave of light strikes a boundary of medium but at an angle which is larger than the critical angle.

So the concept behind that if the refrative index on the other side of the boundary of medium is lower, & the light wave strikes & greater angle, so the wave is reflected internally entirely. It is used in various application for fast travelling of light wave

Critical Angle :-

It is a specific angle where the light travells from a denser to rarer medium at this angle it goes in straight line across the boundary.

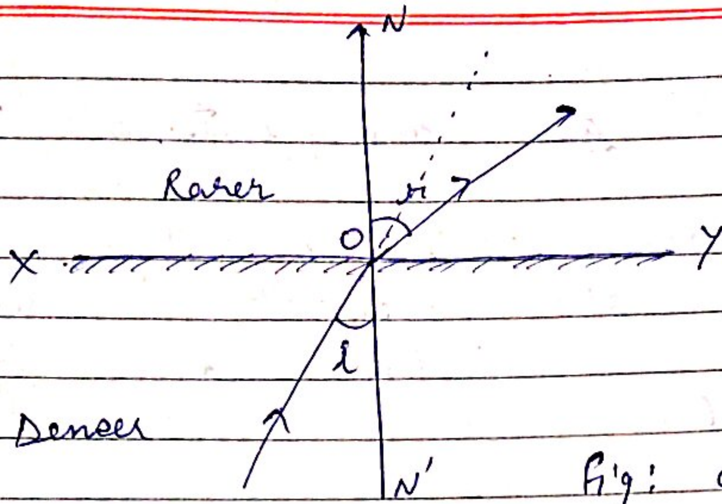

Fig: away from normal

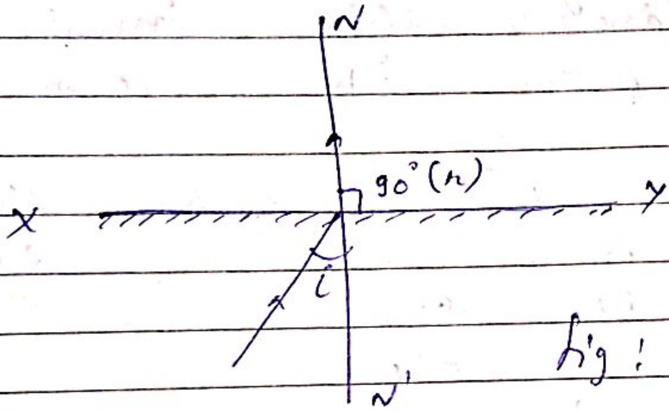

Fig: Critical angle

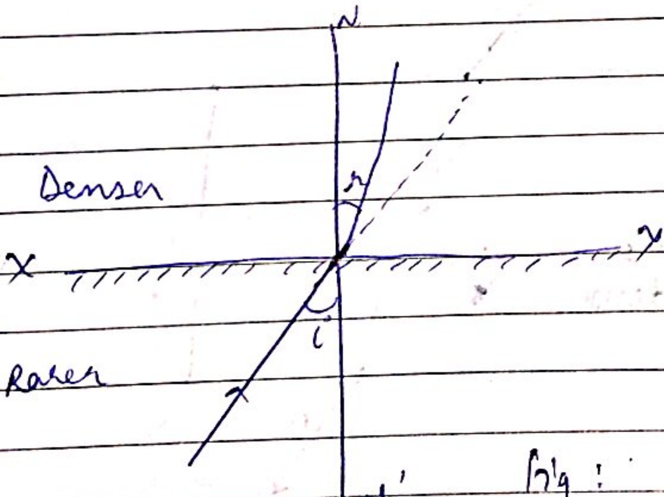

Fig: towards normal

The critical angle for a medium is thus defined as the angle of incident in ~~a~~ medium for refraction angle in air is 90°.

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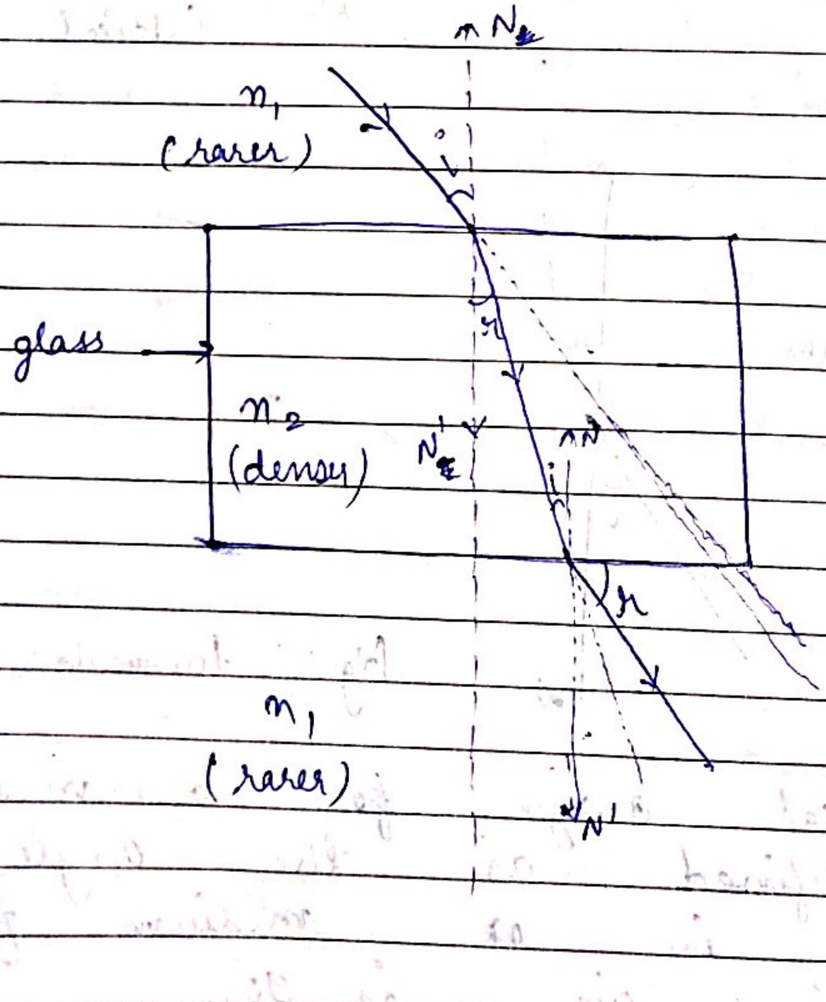
Refraction at an interface :-

→ Snell's law tells us light bends towards the normal when going from low index to high index material.

→ When going from high index to low index light must bend away from the normal.

→ At some critical angle the transmitted beam in the low index material will be 90°.



here, $n_1 < n_2$

Total internal reflection will happen when the light will travel from water towards air but the scenerio when it travel from air towards water will not occur.

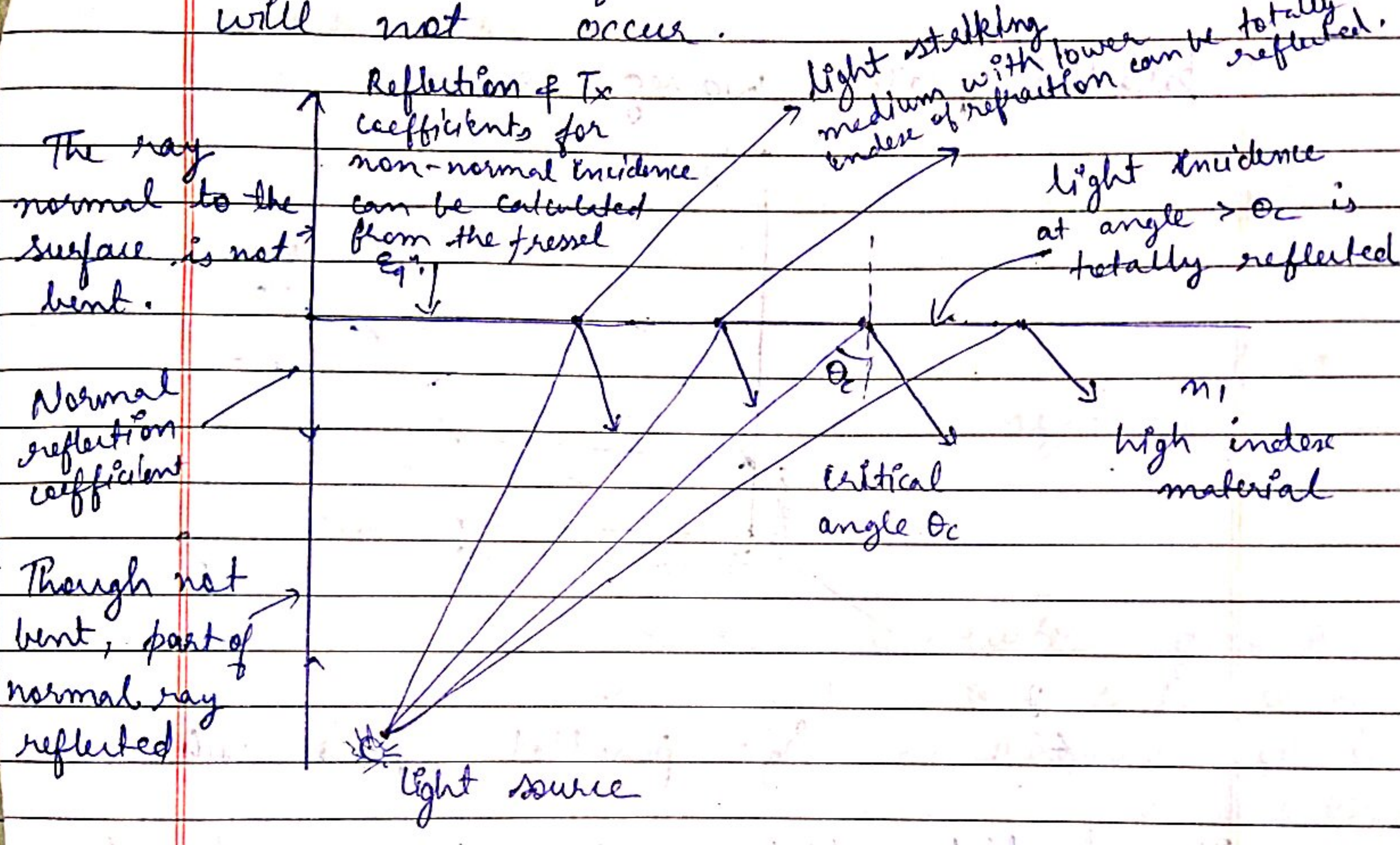


Fig: ~~Source~~ Scenerio of total internal reflection.

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Topic : Wave polarization or media Interface

Media Interface can be changed by changing the state of polarization of electromagnetic wave.

The electric field for any state of polarization can be resolved into 2 component orthogonal components one parallel to the plane of incidence and other perpendicular to it. ∴, the incident electric field can be written as

$$E_i = E_{i\parallel} + E_{i\perp} e^{j\phi}.$$

The reflected & transmitted fields can be written as

$$E_r = E_{r\parallel} + E_{r\perp} = \Gamma_{\parallel} E_{i\parallel} + \Gamma_{\perp} E_{i\perp} e^{j\phi}$$

$$E_t = E_{t\parallel} + E_{t\perp} = z_{\parallel} E_{i\parallel} + z_{\perp} E_{i\perp} e^{j\phi}$$

where the reflection coefficient $\Gamma_{\parallel}$ & $\Gamma_{\perp}$ are real for ordinary reflection & complex for total internal reflection.

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Linearly Polarized Incident wave —
for a linearly polarized wave $\phi = 0$
then,

a) for ordinary reflection since Γ &
Z are real the reflected and
transmitted fields also remain linearly
polarized.
However, since in general $\Gamma_{\parallel} \neq \Gamma_{\perp}$
and $Z_{\parallel} \neq Z_{\perp}$ the plane of polarization
changes.

b) If the reflection is total internal
then $\Gamma_{\parallel}$ & $\Gamma_{\perp}$ are complex & so the
reflected & transmitted field component
are not in phase consequently the
transmitted and reflected waves
are elliptically polarized.

Circularly polarized waves :—

In this case $|E_{i\parallel}| = |E_{i\perp}|$ &
$\phi = \pm \frac{\pi}{2}$

In general since $\Gamma_{\parallel} \neq \Gamma_{\perp}$ & $Z_{\parallel} \neq Z_{\perp}$ —
the reflected & transmitted both
waves will become elliptically polarized

NOTE :- A linearly polarized waves

remains linearly polarized at TIR.

- Brewester Angle :-

Brewester angle is the angle of incidence for which there is no reflection from the media interface i.e it is the angle of incidence for which the reflection coefficient is zero.

- Brewester angle for di-electric Interface

For a dielectric interface the permeability of both the media's are same as that of the free space.

- There is no brewster angle for $\perp$ polarization.

- For $\parallel$ polarization equaling the reflection coefficient to zero we get the Brewster angle as

$$\theta_{B\parallel} = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)$$

- $\because$ tangent of an angle can attain any value b/w zero & infinity for any value of ϵ_1. ϵ_2. The Brewster angle exist

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Topic : Reflection at conducting Boundary

We can analyze reflection of an electro-magnetic wave at a non-conductor - conductor interface in a similar way to that used for a non- conductor interface. In a similar way to that used for a non- conductor - non conductor Interface.
In the case of normal incidence using Maxwell eq^n

$$\epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_f \quad -(1)$$

$$B_1^{\perp} - B_2^{\perp} = 0 \quad -(2)$$

$$E_1^{\parallel} - E_2^{\parallel} = 0 \quad -(3)$$

$$\frac{1}{\mu_1} B_1^{\parallel} - \frac{1}{\mu_2} B_2^{\parallel} = K_f \times \hat{n} \quad -(4)$$

We will take medium 1 as the non-conductor (air ...) & medium 2 as conductor. We are allowing for the presence of free surface charge density σ_f & free current density K_f as the boundary

If we are dealing with a conductor that obey's ohm's law, the volume current density is proportional to the electric field.

$$J_f = \sigma E$$

where here 'σ' is the conductivity, not a charge density. Recall that J_f is the amount of current flowing through a unit area in the conductor. If we had a surface current density 'K_f' this current flows along the boundary as a sheet of moving charge with infinite thickness, so that the cross-sectional area occupied by K_f is essentially zero, making the volume charge density infinite for a finite conductivity 'σ' It would take an infinite electric field to produce this surface current, so we can safety assume that $K_f = 0$ in what follows.

The incident & reflected waves are both in medium-1, so, if we polarize the wave in the x dir^n, we have for the incident wave :-

$$\tilde{E}_I = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{x} \quad - (6)$$

$$\tilde{B}_I = \frac{1}{v_1} \tilde{E}_{01} e^{i(k_1 z - \omega t)} \hat{y} \quad - (7)$$

where, v_1 the speed of the wave in medium

The reflected wave is travelling is in the z-dirⁿ. and has eqⁿ. -

$$\tilde{E}_R = \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{x} \quad - (8)$$

$$\tilde{B}_R = -\frac{1}{v_1} \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{y} \quad - (9)$$

The transmitted wave is inside the conductor, so its equation can be written as -

$$\tilde{E}_T(z, t) = \tilde{E}_{0T} e^{i(\tilde{k}_2 z - \omega t)} \hat{x} \quad - (10)$$

$$\tilde{B}_T(z, t) = \frac{\tilde{k}_2}{\omega} \tilde{E}_{0T} e^{i(\tilde{k}_2 z - \omega t)} \hat{y}$$