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From above Eq^n

$$\oint_L H \cdot dl = I_{enc} = \int_S (\nabla \times H)\, ds$$ (By stoke's theorem.

$\because$ $I_{enc} = \int_S J \cdot dS$

$\therefore$ $$\int_S J \cdot dS = \int_S (\nabla \times H)\, ds$$

$\therefore$ $$\boxed{\nabla \times H = J} \quad -(2)$$

here, time varying component is negliable $\frac{\partial D}{\partial t}$.

It means $(\nabla \times H)$ is not equal to zero, so, magnetostatic field is not conservative.

⇒ Uniform plane Wave :- (TEM wave)

Uniform plane wave is the special case of electromagnetic waves in which electric field Intensity (E) & magnetic field intensity (H) are ⊥ to each other. & also to the dir^n of the propagation.

If the phase of a wave is the same for all points on a plane surface, it is c/d plane wave.

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If amplitude is also const. in a plane wave, it is c/d uniform plane wave.

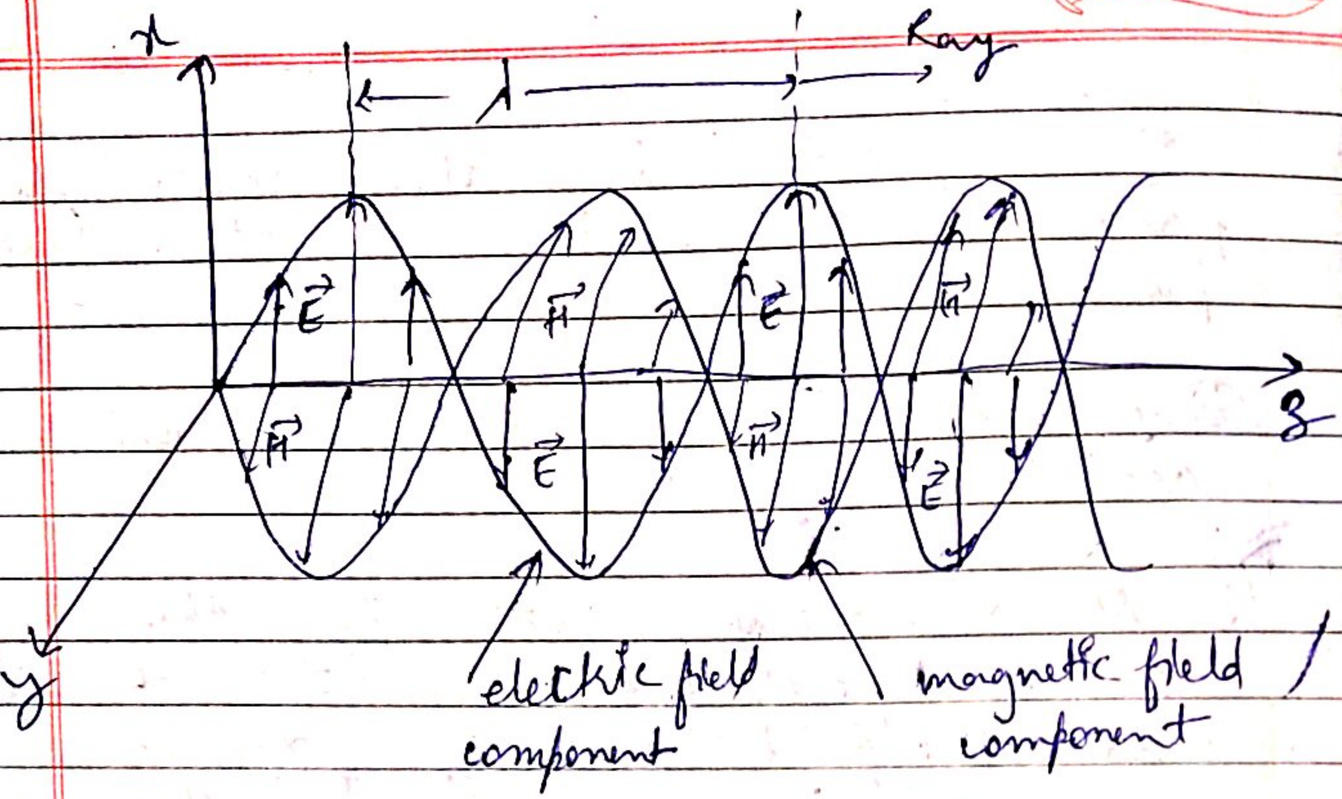
Some properties of uniform plane waves are as :-

(1) At a every point in the space the electric field intensity (E) & (H) magnetic field intensity are ⊥ (transverse or 90°) to each other & to the dir^n of propogation, so such waves are also c/d as transverse electromagnetic waves. (TEM waves)

(2) Both fields $\vec{E}$ & $\vec{H}$ vector have same dir^n, magnitudes & phase at every point in any plane ⊥ to the dir^n of wave propogation.

(3) If uniform plane wave propogate in z-dir^n then electric & magnetic field do not have z-components & E & H independent of x & y components.

$$\boxed{\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0}, \quad \boxed{E_z = H_z = 0}$$



Special cases :—

① Solⁿ of wave eqⁿ in free space :
→ free space is a perfect di-electric containing no charge & no conduction current then the field eqⁿ will become.

→ $\rho_v = 0$

$\Rightarrow \nabla \cdot \vec{D} = 0$ —①

→ $\nabla \cdot \vec{B} = 0$ —②

→ $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $(\because \vec{B} = \mu \vec{H})$

$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$ —③

→ $\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$ $(\because \vec{J} = 0)$

$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$ $(\because D = \epsilon E)$ —④

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Taking curl of eq^n. 3. we get

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{H})$$

$$\nabla \times \nabla \times \vec{E} = -\mu \epsilon \frac{\partial^2}{\partial t^2} \vec{E}$$

Using vector identity

$$\nabla \times \nabla \times \vec{E} = \nabla (\nabla \cdot \vec{E}) - \nabla^2 \cdot \vec{E}$$

$$\nabla \times \nabla \times \vec{E} = 0 - \nabla^2 \cdot \vec{E} \qquad (\text{from eq}^n \cdot 4)$$

$$-\nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2}{\partial t^2} \vec{E}$$

$$\boxed{\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2}{\partial t^2} \vec{E}}$$

Similarly,

$$\boxed{\nabla^2 \vec{H} = \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}}$$

(2) Wave Eq^n for conducting medium:
Conducting media are the one for which $\sigma \neq 0$ & the conduction current exist. The sol^n for maxwell's field eq^n. Can be obtained for conducting media. consider the field eq^n.

$$\nabla \cdot \vec{D} = \rho_v \quad -(1)$$

$$\nabla \cdot \vec{B} = 0 \quad -(2)$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad -(3)$$

$$\nabla \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad -(4)$$

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Maxwell Eq^n for time varying field

point form or differential form — Integral form

① $\nabla \cdot \vec{D} = \rho_v$ — $\oint D \cdot d\vec{S} = \oint_v \rho_v \cdot dv$

② $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ — $\oint \vec{E} \cdot \vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot ds$

③ $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ — $\oint \vec{H} \cdot dl = \vec{J} + \int_s \frac{\partial \vec{D}}{\partial t} \cdot \vec{dS}$

④ $\nabla \cdot \vec{B} = 0$ — $\oint \vec{B} \cdot \vec{dS} = 0$

$$J = \sigma \vec{E}$$

$$\therefore \nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

Taking curl of Eq^n -3

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{H})$$

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} \left[\sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \right]$$

$$\nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

& from Eq^n-1 $\nabla \cdot \vec{D} = \rho_v$ but there is no charge within the conductor as the charges resides on the surface of the conductor, it means,

$$\rho_v = 0$$

$$\nabla \cdot \vec{D} = 0 , \because D = \epsilon E , \therefore \epsilon [\nabla \cdot \vec{E}] = 0$$

$$\therefore \nabla \cdot \vec{E} = 0$$

$$-\nabla^2 \vec{E} = -\mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} - \mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad - (6)$$

which is the uniform plane ~~for~~ in conducting for E-field.

Similarly, for Eq^n-4

$$\nabla \times \nabla \times \vec{H} = \nabla \times \vec{J} + \epsilon \frac{\partial (\nabla \times \vec{E})}{\partial t}$$

$$(\because \vec{J} = \sigma \vec{E})$$

$$\nabla \times \nabla \times \vec{H} = \sigma (\nabla \times \vec{E}) + \epsilon \frac{\partial}{\partial t} (\nabla \times \vec{E})$$

$$\nabla \cdot (\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = \sigma (\nabla \times \vec{E}) + \epsilon \frac{\partial}{\partial t} (\nabla \times \vec{E})$$

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Uniform plane wave propagation :-

Let the uniform plane wave be propagating in z-dirⁿ. For this wave x & y component will be present but z-component will be absent it means

$$E_z = H_z = 0$$

There will not be any variation in x & y dirⁿ. so $\vec{E}$ & $\vec{H}$ must be independent of x & y.

i.e. $$\frac{\partial \vec{E}}{\partial x} = \frac{\partial \vec{E}}{\partial y} = \frac{\partial \vec{H}}{\partial x} = \frac{\partial \vec{H}}{\partial y} = 0$$

The general wave Eqⁿ for free space are

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad - (1) \qquad (\mu = \mu_0, \epsilon = \epsilon_0 \text{ for free space})$$

$$\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} \quad - (2)$$

where $\vec{E} \rightarrow$ electric field intensity
$\vec{H} \rightarrow$ magnetic field intensity
$\epsilon_0 \rightarrow$ permittivity of free space
$\mu_0 \rightarrow$ permeability of free space

Considering Eqⁿ-1 we have

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$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\vec{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z\right)$$
$$= \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\left(E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z\right)$$

But, $\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$ & $E_z = 0$

$$\frac{\partial^2}{\partial z^2}\left(E_x \hat{a}_x + E_y \hat{a}_y\right) = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\left(E_x \hat{a}_x + E_y \hat{a}_y\right)$$

$$\frac{\partial^2}{\partial z^2} E_x \hat{a}_x + \frac{\partial^2}{\partial z^2} E_y \hat{a}_y = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} E_x \hat{a}_x + \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} E_y \hat{a}_y$$

On comparing,

$$\frac{\partial^2}{\partial z^2} E_x = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} E_x \quad -(3)$$

$$\frac{\partial^2}{\partial z^2} E_y = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} E_y \quad -(4)$$

Similarly for Eqn-2. (Solve it).

$$\frac{\partial^2}{\partial z^2} H_x = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} H_x \quad -(5)$$

$$\frac{\partial^2}{\partial z^2} H_y = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} H_y \quad -(6)$$

The Eq^n 3, 4, 5 & 6 are the differential Eq^n of 2nd order. The general sol^n of such differential Eq^n can be obtained in the form

$$E = f_1(z - ct) + f_2(z + ct) \quad - (7)$$

where, f_1 & f_2 = f^n of $(z-ct)$ & $(z+ct)$ respectively.

c = velocity of electromagnetic wave, which is equal to light velocity.

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \, m/s$$

$f_1(z - ct)$ = represents a wave travelling in (+)ve z-dir^n, &

$f_2(z + ct)$ = represents a wave travelling in (−)ve z-dir^n.

If there is no reflecting surface present then the wave travelling in (−)ve z-dir^n will be zero. So, the 2nd term in Eq^n 7 becomes zero. So the general sol^n of wave Eq^n is

$$E = f_1(z - ct) \quad - (8)$$

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Relationship b/w $\vec{E}$ & $\vec{H}$ in uniform plane wave :-

Here, we will show that $\vec{E}$ & $\vec{H}$ are mutually $\perp$ to each other. According to maxwell's eqn we have

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{— (1)}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$\vec{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$$

$$-\frac{\partial}{\partial t}[\vec{B}] = -\frac{\partial}{\partial t}[B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z]$$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = -\frac{\partial}{\partial t}[B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z] \quad \text{— (2)}$$

Let the uniform plane wave is travelling in z- dirn.

then, $\boxed{\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0}$ and $\boxed{E_z = 0}$

$$\therefore \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 0 & \partial/\partial z \\ E_x & E_y & 0 \end{vmatrix} = -\frac{\partial}{\partial t}[B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z] \quad \text{— (3)}$$

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$$\hat{a}_x\left[0-\frac{\partial}{\partial z}E_y\right]-\hat{a}_y\left[0-\frac{\partial}{\partial z}E_x\right]=-\frac{\partial}{\partial t}B_x\hat{a}_x-\frac{\partial}{\partial t}B_y\hat{a}_y$$
$$-\frac{\partial}{\partial t}B_z\hat{a}_z$$

$$-\frac{\partial}{\partial z}E_y\hat{a}_x+\frac{\partial}{\partial z}E_x\hat{a}_y=-\frac{\partial}{\partial t}B_x\hat{a}_x-\frac{\partial}{\partial t}B_y\hat{a}_y$$

∴ On comparing

$$\boxed{\frac{\partial}{\partial z}E_y=\frac{\partial}{\partial t}B_x} \quad -(4)$$

&

$$\boxed{\frac{\partial}{\partial z}E_x=-\frac{\partial}{\partial t}B_y} \quad -(5)$$

or $$-\frac{\partial}{\partial t}B_z=0$$

$$\therefore \boxed{B_z=0}$$

from $Eq^n - 4$

$$B=\mu H$$
$$B_x=\mu H_x$$

$$\boxed{\frac{\partial}{\partial z}E_y=\mu\frac{\partial}{\partial t}H_x} \quad -(6)$$

$$\boxed{\frac{\partial}{\partial z}E_x=-\mu\frac{\partial}{\partial t}H_y} \quad -(7)$$

Similarly from Maxwell 2nd Eq^n,

$$\nabla\times\vec{H}=\frac{\partial\vec{D}}{\partial t}$$

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$$\boxed{\frac{\partial H_y}{\partial z} = -\epsilon \frac{\partial E_x}{\partial t}} \quad (8) \quad \& \quad \boxed{\frac{\partial H_x}{\partial z} = \epsilon \frac{\partial E_y}{\partial t}} \quad (9)$$

(1) Now consider the Eqn. of uniform plane wave will E having component y of as

$$E_y = f_1(z - ct) = f \quad - (10)$$

where c is the velocity of the propagated wave.

$$c = \frac{1}{\sqrt{\mu \epsilon}}$$

Differentiate Eqn 10 w.r.t (t).

$$\frac{d}{dt} E_y = f'(z - ct) \frac{d}{dt}(z - ct)$$

$$= f'(z - ct)(-c)$$

$$= -c f_1'(z - ct)$$

$$\frac{d}{dt} E_y = -c f'$$

Substituting this value in Eqn -9.

$$\frac{\partial}{\partial z} H_x = \epsilon(-c f')$$

$$\int \partial H_x = -\epsilon c \int f' \, dz$$

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$$H_x = -K \epsilon c f$$

where, K = constant of integration.

∵ field is static so that K can be neglected.

$$\boxed{H_x = -\epsilon c f}$$

Put $f = f_1(z - ct) = E_y$

$$H_x = -\frac{\epsilon \cdot 1}{\sqrt{\mu\epsilon}} \cdot E_y \qquad \left(\because c = \frac{1}{\sqrt{\mu\epsilon}}\right)$$

$$H_x = -\sqrt{\frac{\epsilon}{\mu}}\, E_y$$

$$\boxed{\frac{E_y}{H_x} = -\sqrt{\frac{\mu}{\epsilon}}} \quad -(11)$$

(2) Now similarly consider the Eq^n of uniform plane wave with E having component x as

$$E_x = f_1(z - ct) = F \quad -(12)$$

Differentiating both sides w.r.t (t) we get,

$$\frac{\partial}{\partial t} E_x = F' = f_1'(z - ct)(-c) = -cF'$$

from eq^n - 8

$$\frac{\partial}{\partial z} H_y = -\epsilon \frac{\partial}{\partial t} E_x = -\epsilon(-CF')$$

$$\frac{\partial}{\partial z} H_y = \epsilon C F'$$

$$\int \partial H_y = \epsilon C \cdot \int F' \, dz$$

$$H_y = K \epsilon C F$$

(K is neglected)

$$H_y = \epsilon C F$$

$$H_y = \frac{\epsilon}{\sqrt{\mu \epsilon}} \cdot E_x \qquad \left(\text{From } Eq^n - 12 \ \& \ C = \frac{1}{\sqrt{\mu \epsilon}} \right)$$

$$\boxed{\frac{E_x}{H_y} = \sqrt{\frac{\mu}{\epsilon}}} \quad \text{--- (13)}$$

The mag^n of electric field intensity is given as

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

$$\because E_z = 0$$

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2}$$

Similarly,

$$|\vec{H}| = \sqrt{H_x^2 + H_y^2}$$

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$\therefore$, $|\vec{E}| = \sqrt{\frac{\mu}{\epsilon}(H_y)^2 + \frac{\mu}{\epsilon}(H_x)^2}$ (from eq^n 11 & 13)

$|\vec{E}| = \sqrt{\frac{\mu}{\epsilon}} \cdot \sqrt{H_y^2 + H_x^2}$

$|\vec{E}| = \sqrt{\frac{\mu}{\epsilon}} \cdot |\vec{H}|$

$$\boxed{\frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu}{\epsilon}}}$$

(from NOTES)

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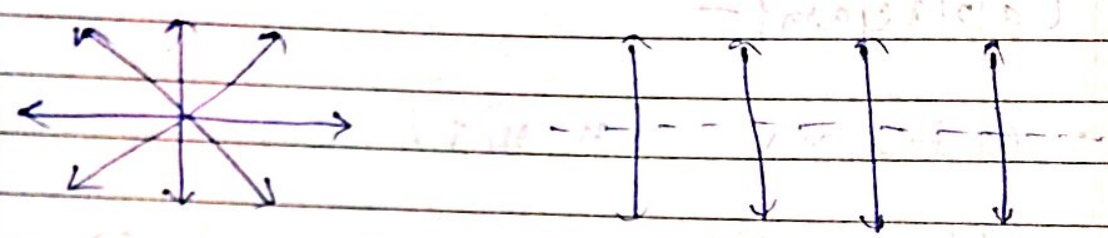
Topic : Wave polarization

As we know that, we can propagate in different dir^n, x, y, z, respectively.

When we allow the wave to propagate in a single dir^n. & restrict in the other dir^n is K/N as propagation of wave.



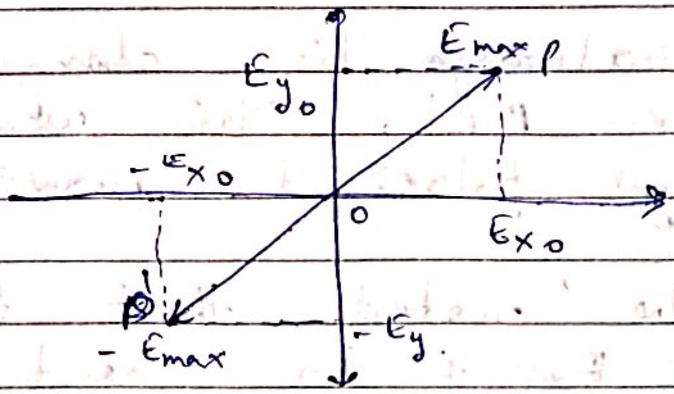
(a) Unpolarized wave dir^n (b) Polarized wave dir^n.

In optics we talk about the wave of polarization while in electromagnetics, we will talk about the dir^n of polarization.

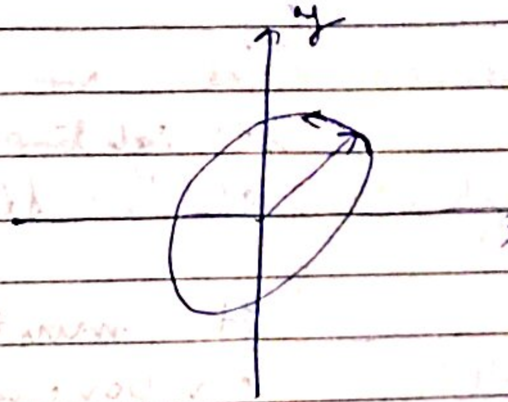
For any instance for a plane wave propagating in z-dir^n. then $E_z = 0$

$$\vec{E} = E_x \hat{a}_x + E_y \hat{a}_y$$

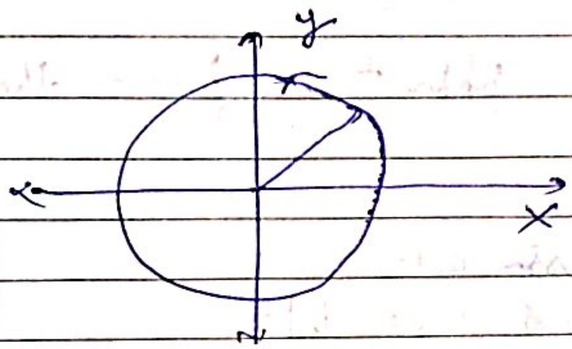
And the wave is set to be polarised in the dir^n of this $\vec{E}$.



a) Linear



(b) Elliptical



(c) Circular

fig! Polarization of Wave

If $E_x = E_{x_0} \sin \omega t$
$E_y = E_{y_0} \sin \omega t$

at any point in a plane

$$|E| = \sqrt{(E_x)^2 + (E_y)^2}$$

The length of $\vec{E}$ vary from 0 to E_{max} & back to zero in one dirⁿ & then from 0 to $-E_{max}$ & back to zero in reverse dirⁿ. (shown dotted in fig (a).)

The ~~ve~~ $\vec{E}$ always lie along the straight line PP'. This wave is c/d linearly polarized wave.

It must be noted that the component E_x & E_y need not always be in time phase, Even a plane wave.

Thus, in any point from the plane wave

$$X = E_x = E_{x_0} \sin \omega t$$
$$Y = E_y = E_{y_0} \sin(\omega t + \phi)$$

then by eliminating (ωt) from these Eq^n we get an Eq^n representing the locus of the tip of $\vec{E}$.

It is found to be an Eq^n to an ellipse with it's axis inclined to x-axis and y-axis. , thus, the $\vec{E}$ constantly changes both it's magnitude & dir^n the tip of the vector $\vec{E}$ describe the ellipse.

The magnitude attains it's max^m value E_{max} in the dir^n of major axis of the ellipse & it's min^m value E_{min} in the dir^n of minor axis

The rate at which the $\vec{E}$ rotates around the origin is ω radian/sec, such a wave is said to be an elliptically polarized plane wave.

In the very special case of elliptically polarized in which :

$\boxed{E_{x_0} = E_{y_0} = E_0}$ & the phase difference, $\phi = \frac{\pi}{2}$

We get,

$$x = E_x = E_{x_0} \sin \omega t$$
$$y = E_y = E_{y_0} \sin(\omega t + \frac{\pi}{2})$$
$$= E_0 \cos \omega t$$

& eliminating (ωt).

$$\boxed{x^2 + y^2 = E_0^2} \quad - (1)$$

The Eq^n - 1 represents a circle of a const. radius E_0 describing by the tip of $\vec{E}$ rotating at ratio of (ω) rad/sec as shown in fig (c). This wave is also said to be a circularly polarized plane wave.

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Topic :- Poincare sphere :-

The polarization ellipse is an excellent way to visualize polarized light

But for the de-generate polarization state it is practically impossible to determine the orientation & ellipticity angles for the polarization ellipse.

To overcome these difficulties poincare (1892) suggested using a sphere now K/N as poincare sphere to represent polarized light.

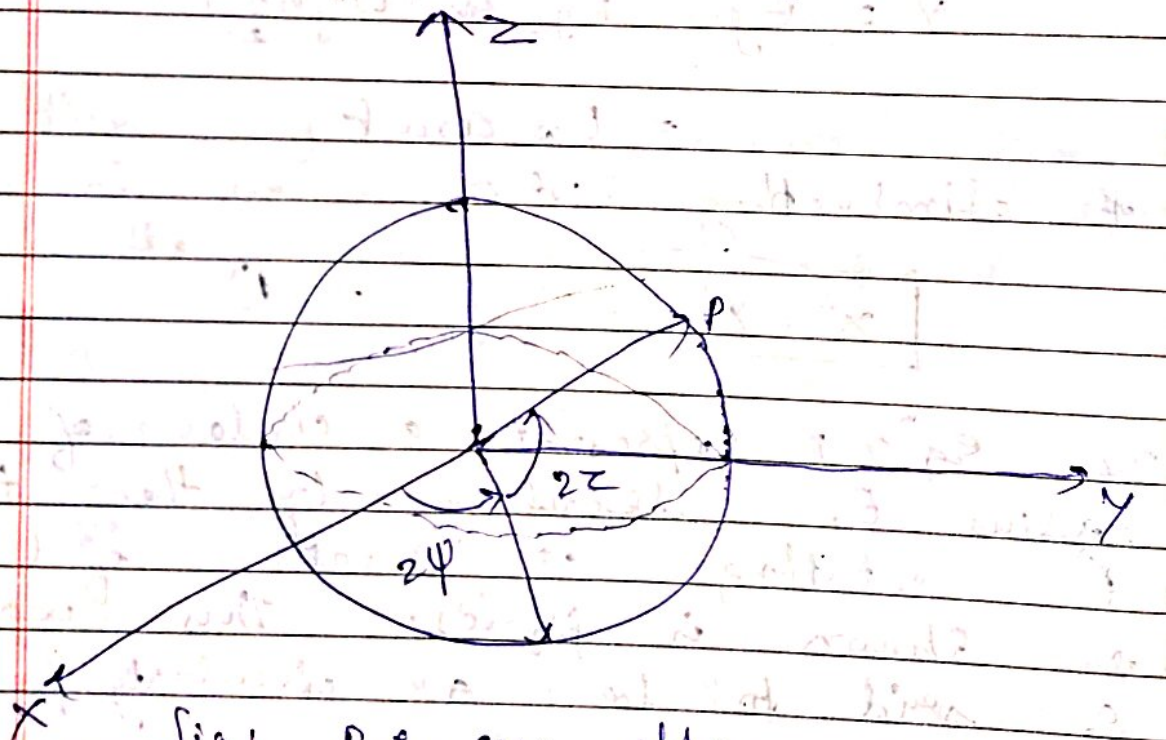

fig: Poincare sphere and its co-ordinate

Here x, y & z are Cartesian co-ordinate axis, ψ, τ are the sperical orientations K/N as ellipitical angles (longitude & latitude).

P is the point on the surface of sphere for a unit sphere the cartesian co-ordinates are related to the spherical co-ordinates by the eqⁿ:

$$X = \cos(2\tau)\cos(2\psi) \qquad \left\{ 0 \le \psi \le \pi \right.$$
$$Y = \cos(2\tau)\sin(2\psi) \qquad \left\{ -\frac{\pi}{4} \le \tau \le \frac{\pi}{4} \right.$$
$$Z = \sin(2\tau).$$

where, $x^2 + y^2 + z^2 = 1$
for a sphere of unit radius.

Phase and group velocity :-

Phase velocity :

The phase velocity of a wave is the rate at which the phase of wave propagates in the space.

this is the velocity at which the phase of any one freqⁿ component of the wave travels.

Phase velocity given terms of the wavelength (λ) & time period (T)

$$\boxed{V_p = \frac{\lambda}{T}}$$

Energy is propagated along the Tx line is in the form of Transverse EM wave (TEM wave).

The phase velocity for TEM wave is given as

$$\boxed{V_p = \frac{1}{\sqrt{\mu \epsilon}}}$$

for a Tx line phase velocity is defined as

$$V_p = \frac{V_0}{\sqrt{\epsilon_r}}$$

where, V_0 = free space velocity.

→ for loss-less & distortion less Tx line

$$\boxed{V_p = \frac{1}{\sqrt{LC}}}$$

Group velocity :-

The group velocity (V_g) is defined by the eq^n.

$$\boxed{V_g = \frac{\partial \omega}{\partial k}} \qquad \left(\because V_p = \frac{\omega}{k} \right)$$

where, ω = waves angular freq^n rad/sec.
k = angular wave no.

The funct^n $\omega(k)$ is K/N as dispersion relation.

If ω proportional to k then $V_p = V_g$ a wave of any shape will travel undistorted at this velocity.

If ω is linear f^n of k but not directly proportional
$$\omega = ak + b$$
Then, V_g & V_p are different.

If ω is not linear f^n of k, then envelope of a wave packet will become distorted as it travels. The V_g of a wave is defined as the velocity of propagation of envelope.

If the phase velocity varies with freq^n then mag^n of V_g & V_p are different.

The V_g & V_p have different dir^n, if the V_p varies with dir^n.

Power flow and poynting vector :—

In order to find the power in a uniform plane wave, it is necessary to develop a power theorem for the EM field. K/N as poynting theorem.

Statement : The vector product of Electric field intensity ($\vec{E}$) & magnetic field intensity ($\vec{H}$) at any point is a measure of the rate of energy flow at that point.

Mathematically poynting theorem is written as

$$\boxed{\vec{P} = \vec{E} \times \vec{H}} \quad (\text{watt}/m^2)$$

where :—

$\vec{P}$ = Power flow

$\vec{E}$ = Electric field intensity

$\vec{H}$ = Magnetic field intensity

The dirⁿ of this energy flow is in the dirⁿ of vector $\vec{E} \times \vec{H}$.

This dirⁿ is ⊥ to both $\vec{E}$ & $\vec{H}$

Proof :— The energy flow eqⁿ can be obtained by using Maxwell's eqⁿ. According to Maxwell's eqⁿ

$$\Delta \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\boxed{\vec{J} = \nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t}} \quad - (1)$$

Eqⁿ 1 is multiplied dot-product with $\vec{E}$, it will be power per unit volume.

$$\vec{E} \cdot \vec{J} = \vec{E} \cdot \left(\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} \right)$$

$$\vec{E} \cdot \vec{J} = \vec{E} \cdot (\nabla \times \vec{H}) - \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$$

By using vector identity

$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H})$$

$$\therefore \vec{E} \cdot \vec{J} = \vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H}) - \vec{E} \frac{\partial \vec{D}}{\partial t} \quad (\because \vec{D} = \epsilon \vec{E})$$

$$\boxed{\vec{E} \cdot \vec{J} = \vec{H} \cdot (\nabla \times \vec{E}) - \nabla (\vec{E} \times \vec{H}) - \vec{E} \frac{\partial (\epsilon \vec{E})}{\partial t}} \quad - (2)$$

from Maxwell Eqⁿ

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (\vec{B} = \mu \vec{H})$$

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t} (\mu \vec{H}) \quad - (3)$$

from Eqⁿ 3 & 2

$$\vec{E}\cdot\vec{J} = -\vec{H}\cdot\frac{\partial}{\partial t}(\mu\vec{H}) - \nabla(\vec{E}\times\vec{H}) - \epsilon\vec{E}\frac{\partial\vec{E}}{\partial t}$$

We have

$$\boxed{\vec{H}\frac{\partial\vec{H}}{\partial t} = \frac{1}{2}\frac{\partial H^2}{\partial t}}$$

$$\boxed{\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} = \frac{1}{2}\frac{\partial E^2}{\partial t}}$$

$$\boxed{\vec{E}\cdot\vec{J} = -\mu\frac{1}{2}\frac{\partial H^2}{\partial t} - \nabla(\vec{E}\times\vec{H}) - \epsilon\frac{1}{2}\frac{\partial E^2}{\partial t}}$$

$$\vec{E}\cdot\vec{J} = -\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \frac{\epsilon}{2}\frac{\partial E^2}{\partial t} - \nabla(\vec{E}\times\vec{H})$$

$$\vec{E}\cdot\vec{J} = -\frac{\partial}{\partial t}\left[\frac{\mu H^2}{2} + \frac{\epsilon E^2}{2}\right] - \nabla(\vec{E}\times\vec{H}) \quad -(4)$$

Taking the integration over volume,

$$\int_V(\vec{E}\cdot\vec{J})\,dv = -\frac{\partial}{\partial t}\int_V\left[\frac{\mu H^2}{2} + \frac{\epsilon E^2}{2}\right]dV - \int_V \nabla\cdot(\vec{E}\times\vec{H})\,dV$$

By using divergence theorem, we can the last term from volume integration to surface integration.

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$$\boxed{\int_V \nabla \cdot (\vec{E} \times \vec{H})\, dV = \oint_s (\vec{E} \times \vec{H})\, ds}$$

$$\int_V (\vec{E} \cdot \vec{J})\, dV = -\frac{\partial}{\partial t}\int_V \left[\frac{\mu H^2}{2} + \frac{\epsilon E^2}{2}\right] dV - \oint_s (\vec{E} \times \vec{H})\, d\vec{s}$$

Physical interpretation from above expression

1) The term on L.H.S indicates instantaneous power flow in volume (V).

2) The first term of R.H.S indicates the stored energy in volume (V). -ve sign indicates that the energy ~~3)~~ store is rising.

3) The last term indicates the total power flowing in volume.

Topic :- Surface current & power loss in a conductor.

Surface current:-

for a good conductor the propagation const. is given as

$$\gamma = \alpha + j\beta = \sqrt{\frac{\omega\mu_0\sigma}{2}} + j\sqrt{\frac{\omega\mu_0\sigma}{2}}$$

The field at any ~~constance~~ distance inside the conductor is g/N as

$$E(z) = E_0 e^{-\gamma z} = E_0 e^{-\alpha z} \cdot e^{-j\beta z}$$

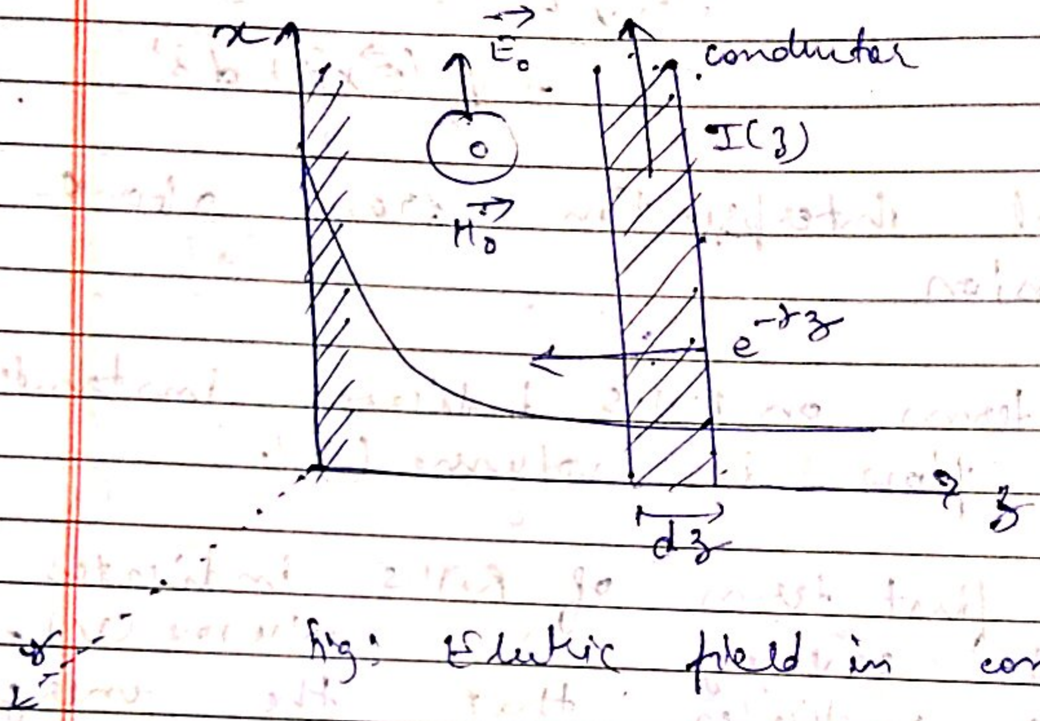

fig: Electric field in conductor.

→ The field amplitude is decreases exponentially inside the conductor.

→ The conduction current density at a depth (z) is g/N as

$$J(z) = \sigma E(z)$$

$$= \sigma \left[E_0 e^{-\alpha z} e^{-j\beta z} \right]$$

The current inside the sheet is shown by the dashed region. is

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$I(z) = J(z)\, dz$

$I(z) = \sigma E_0 e^{-\gamma z} dz\ \hat{x}$

The total current under the unit width of the conductor surface is, σ. ∵ for a good conductor the current is confined to a very thin region below the surface, we may treat the current J_s as the surface current. The true surface current only exist when the conductivity is infinite.

$$J_s = \int_0^\infty E_0 \sigma^{-\gamma z} dz = E_0 \sigma \left[-\frac{e^{-\gamma z}}{\gamma} \right]_0^\infty$$

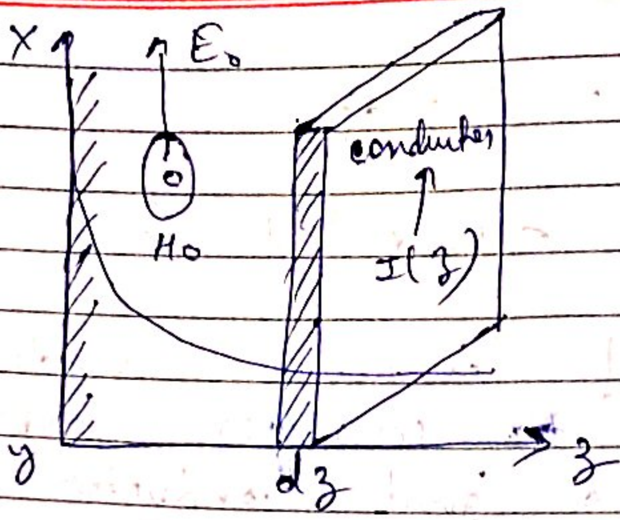
$$= \frac{E_0 \sigma}{\gamma} \hat{x}$$

For non-ideal conductor a parameter c/d surface impedance is defined as.

The real part of $\sqrt{\frac{\omega \mu_0}{2\sigma}}$ is c/d the surface resistance & its value is

Power loss in a Conductor
The resistance of the slab along the dir. of the current I is.

$$dR = \frac{\rho l}{A} = \frac{1}{\sigma dz}$$



The ohmic loss in the slab is

$$dW = |I(z)|^2 \, dR$$

Substituting for $I(z)$ we get

$$dW = |\sigma E_0 e^{-\gamma z} dz|^2 \frac{1}{\sigma dz}$$

$$= |E_0|^2 e^{-2\alpha z} dz$$

The total loss per unit area of the conductor surface $\therefore$ is

$$W = \int_0^\infty \sigma |E_0|^2 e^{-2\alpha z} dz$$

$$= \sigma |E_0|^2 \left[\frac{e^{-2\alpha z}}{-2\alpha} \right]_0^\infty$$

$$W = \frac{\sigma |E_0|^2}{2\alpha} = \frac{\sigma}{2\alpha} \frac{|\gamma|}{\alpha 2} |J_s|^2$$

Substituting for γ and α, the loss per unit area of the conducting surface is

$$W = R_s |J_s|^2 .$$

The power loss is proportional to the surface resistance which ↑s with freq^n f ↓s with conductivity. Higher the conductivity lesser the loss of for ideal conductor when the conductivity is infinite the ohmic loss is zero.