5.6 Directional Couplers

Directional couplers are the microwave junctions that can sample a small amount of microwave power for measurement purposes. It is a four port device consisting of primary waveguide and secondary auxillary waveguide as shown in figure (5.13).

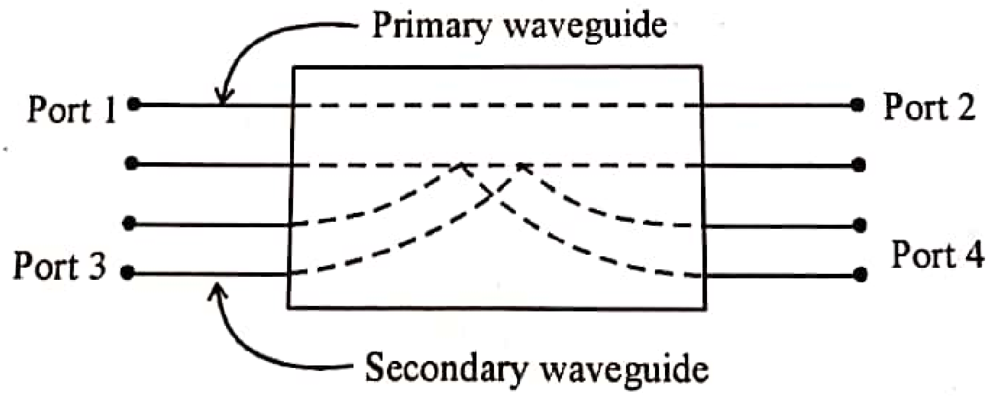


Figure 5.13 Schematic of directional coupler

Properties of Ideal Directional Coupler

1. If the device are terminated in the characteristic impedance then power fed at port (1) flows to port (2) and port (4) but not to port (3)
2. A portion of power travelling from port (2) to port (3) is coupled to port (1) but not to port (4)
3. A portion of power incident on port (3) is coupled to port (2) but not to port (1) and portion of the power incident on port (4) is coupled to port (1) but not to port (2). Also port (1) and (3) are decoupled as are ports (2) & (4).

Figure 5.13 indicates the various powers input/output

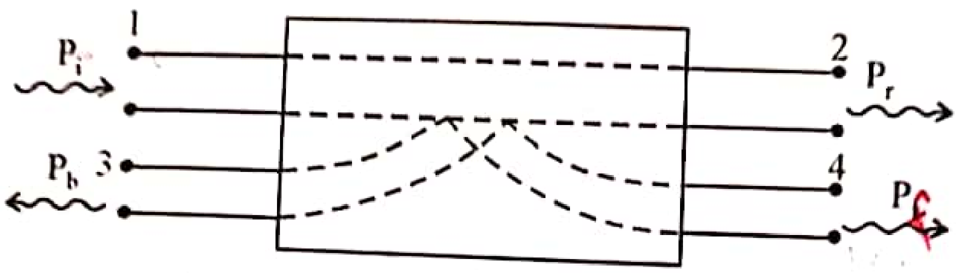


Figure 5.14 Directional coupler indicating power

Here P_i = Incident power at port (1)

P_f = Forward power coupled to port (4)

P_r = received power at port (2)

P_b = back power at port (3)

A directional coupler is specified in terms of two parameters which are defined as follows.

1. Coupling factor (C)

It is defined as the ratio of the incident power 'P_i' to forward power 'P_f' measured in dB.

$$C = 10\log_{10}\frac{P_i}{P_f}dB \qquad .....(5.6.1)$$

2. Directivity (D)

It is defined as ratio of forward power 'P_f' to the back power 'P_b' in dB.

$$D = 10\log_{10}\frac{P_f}{P_b}dB \qquad .....(5.6.2)$$

The directional coupler provides a method of sampling energy from within a waveguide for measurement. Also this power sampled can be used to drive another circuit.

Here the coupling factor is a measure of how much of the incident power is being sampled. Directivity is measure of how well the directional coupler distinguishes between the forward and reverse travelling powers.

3. Isolation (*I*)

It is a parameter that defines the directive properties of directional coupler. It is the ratio of incident power P_i to the back power P_b expressed in dB.

$$I = 10\log_{10}\frac{P_i}{P_b}\, dB \qquad .....(5.6.3)$$

By comparing equations (5.6.1) to (5.6.3) we get

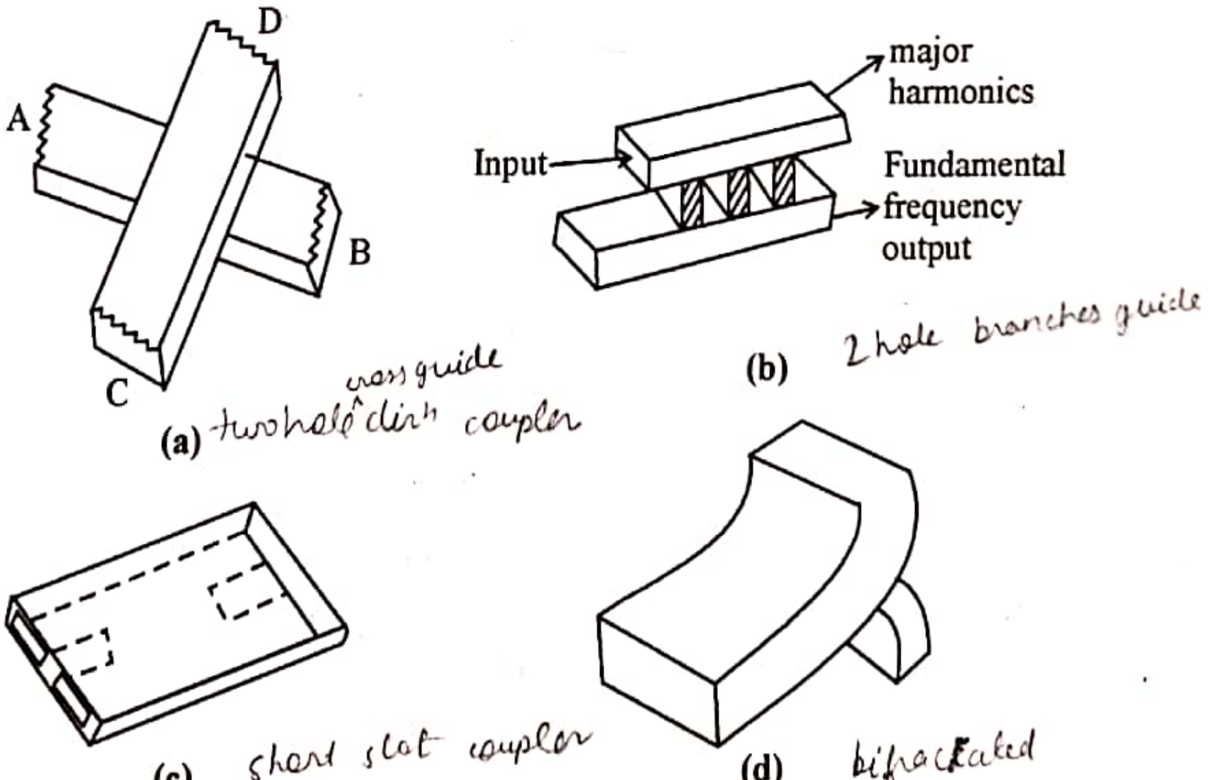
$$I = C + D\ (dB) \qquad .....(5.6.4)$$

4. SWR, Frequency Range, Transmission Loss

Low SWR ensures minimum mismatch errors wide frequency range dominates the need for several octaves band couplers to cover the broad band range and minimum transmission loss for significant power availability for measurement set up.

Types of directional couplers:

The various types available are shown in figure (5.15)



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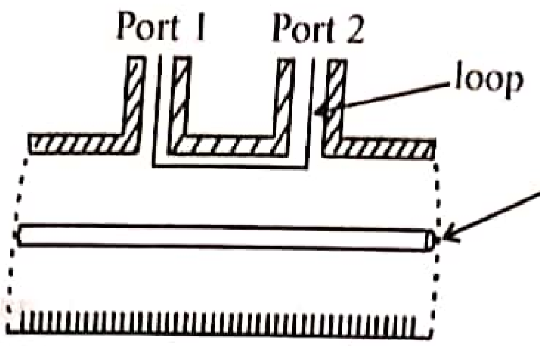


Figure 5.15 (a) Two hole cross guide directional coupler (b) Two hole branches guide couplers. (c) short slot coupler (d) Bifurcated coupler (e) Loop directional coupler.

Most couplers sample energy travelling in one direction only however, directional coupler can be constructed that sample energy in both directions. These are called as directional couplers.

Normally in directional coupler the two waveguides (primary and secondary) share a common wall. This wall has got holes for coupling the energy flowing into main waveguide to the side waveguide. The most commonly used waveguides are two hole directional coupler and single hole (Bethe hole) directional coupler are used.

Two Hole Directional Coupler

It consists of two guides the main and auxillary with two thin holes as shown in figure 5.16.

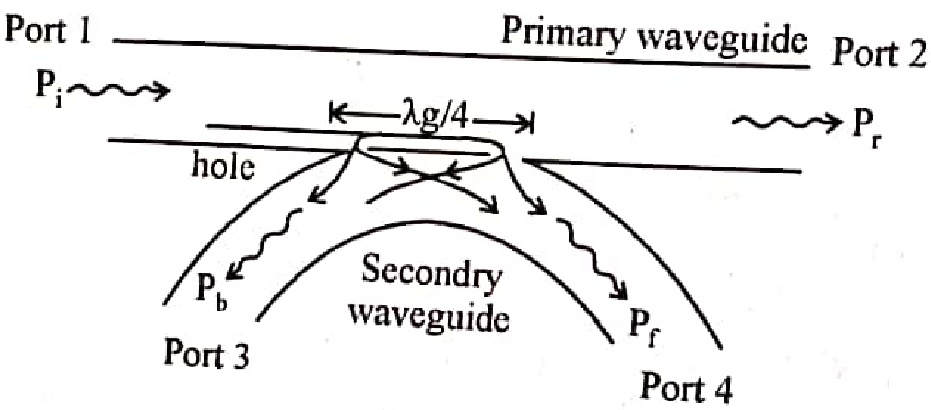


Figure 5.16 Two hole directional coupler

Here the two holes are separated by distance of $\lambda_g/4$ where λ_g is guide wavelength. The two waves coming out of hole 2 and hole (1) meets in phase at port (4) contributing to P_f. But the two waves coming out of hole (1) & (2) meets out of phase at port (3) since wave coming from hole (2) travel distance of $\lambda_g/4 + \lambda_g/4 = \lambda_g/2$ distance when it comes back to port (3) resulting in 180° phase shift So $P_b \simeq 0$.

Bethe Hole Directional Coupler/Single Hole Directional Coupler

In this type of directional coupler the main waveguide is fixed and the secondary guide is rotated about a hole in the centre of the common broad walls by rotation, as shown in figure (5.17).

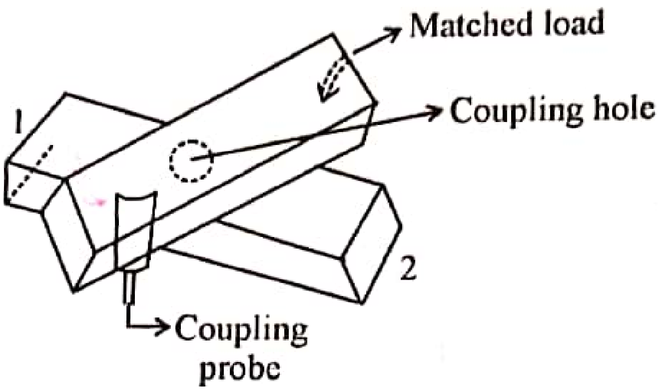


Figure 5.17 Bethe hole coupler

Here the auxiliary guide is placed at such an angle that the magnitude of the magnetically excited wave is equal to that of electrically excited wave for improved directivity. In this coupler, the waves in auxiliary guide are generated through a single hole which include both electric field and magnetic field. Here also the phase relationship involved in the coupling process all set through angle θ in such a way the signal generated by two types of coupling cancel in the forward direction and reinforce in the reverse direction.

Scattering matrix of Directional coupler

1. It is a four port device so $[S]$ is a 4×4 matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

2. All the four ports are perfectly matched to the junction

So,
$$S_{11} = S_{22} = S_{23} = S_{44} = 0 \quad .....(5.6.5)$$

3. From symmetric property, we know $S_{ij} = S_{ji}$

$$S_{23} = S_{32} \; ; \; S_{13} = S_{31} \; ; \; S_{41} = S_{14}$$
$$S_{24} = S_{42} \; ; \; S_{34} = S_{43} \quad .....(5.6.6)$$

4. Ideally we set, back power is zero ($P_b = 0$)

So there is no coupling from port 1 to 3 and port 2 to 4 [figure 5.14]

So $$S_{13} = S_{31} = 0$$

$$S_{24} = S_{42} = 0 \qquad \text{.....(5.6.}$$

on substituting equations (5.6.5) to (5.6.7) in $[S]$ we get,

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \qquad \text{.....(5.6.8}$$

5. Also by unitary property

$$[S]\ [S]^* = [I]$$

or
$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}\begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12} & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

On multiplying

$$R_1C_1 : |S_{12}|^2 + |S_{14}|^2 = 1 \qquad \text{.....(5.6.59)}$$

$$R_2C_2 : |S_{12}|^2 + |S_{23}|^2 = 1 \qquad \text{.....(5.6.10)}$$

$$R_3C_3 : |S_{23}|^2 + |S_{34}|^2 = 1 \qquad \text{.....(5.6.11)}$$

$$R_4C_4 : |S_{14}|^2 + |S_{34}|^2 = 1 \qquad \text{.....(5.6.12)}$$

Comparing equations (5.6.9) and (5.6.10) we get

$$S_{14} = S_{23} \qquad \text{.....(5.6.13)}$$

Comparing equation (5.6.10) and (5.6.11)

$$S_{12} = S_{34}$$

Let S_{12} be real and positive = 'p'

So
$$S_{12} = S_{34} = P = S_{34}^*$$

$$R_1C_3 : S_{12}S_{23}^* + S_{14}S_{34}^* = 0 \qquad \text{.....(5.6.14)}$$

From (5.6.14) and (5.6.15) we gets, ...(5.6.15)

$$P\,S_{23}^{*} + S_{14}P = 0$$

$$P\,(S_{23}^{*} + S_{14}) = 0$$

From equation (5.6.13) we gets–

$$P\,(S_{23}^{*} + S_{23}) = 0$$

Since $P \neq 0$ So $S_{23} = -S_{23}^{*}$

So $S_{23} = jy$

$S_{23}^{*} = -jy$

It means S_{23} is imaginary

Let $S_{23} = jq$ .....(5.6.16)

Substituting these values of parameters in (5.6.8) we get,

$$[S] = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix} \quad .....(5.6.17)$$

Applications of Directional Couplers

Directional coupler is generally used as a power monitoring and thus is used to check the performance of microwave equipment.

Various application includes

1. Power measurements.
2. Reflectometers
3. Fixed attenuators and direction power dividers
4. Variable impedance or matching device
5. Balance duplexer.

5.7 Microwave Attenuators

For perfect matching sometimes we require that the microwave power in a waveguide be absorbed completely without any reflection and also insensitive to frequency. For this we make use of attenuators.

Attenuators are commonly used for measuring power gain or loss in dBs, for providing isolation between instruments, for reducing the power input to a particular stage to prevent overloading and also for providing the signal generators with a means of calibrating their outputs accurately so that precise measurement could be made. Attenuators can be classified as fixed or variable (continuous or step variation) types.

Fixed Attenuators are used where fixed amount of attenuation is to be provided. If such a fixed attenuator absorbs all the energy entering into it, we call it as a waveguide terminator. This normally consists of a short section of waveguide with tapered plug of absorbing material at the end. The tapering is done for providing a gradual transition from the waveguide medium to the absorbing medium thus reducing the reflection occurring at the media interface. Figure 5.18 shows such a fixed attenuator where a dielectric slab consisting of glass slab coated with aquadog or carbon film has been used as a plug.

Here the lossy dielectric or vane shown is V-shaped and can occupy the whole of the waveguide.

Variable attenuators provide continuous or step wise variable attenuation. For rectangular waveguides, these attenuators can be flap type or vane type. For circular waveguides rotary type is used.

The flap type attenuator shown in figure 5.19 consists of a resistive element or disc inserted into a longitudinal slot cut along the centre of the wider dimension of the guide. The flap is mounted on the hinged into the centre of the waveguide. The degree of the flap.

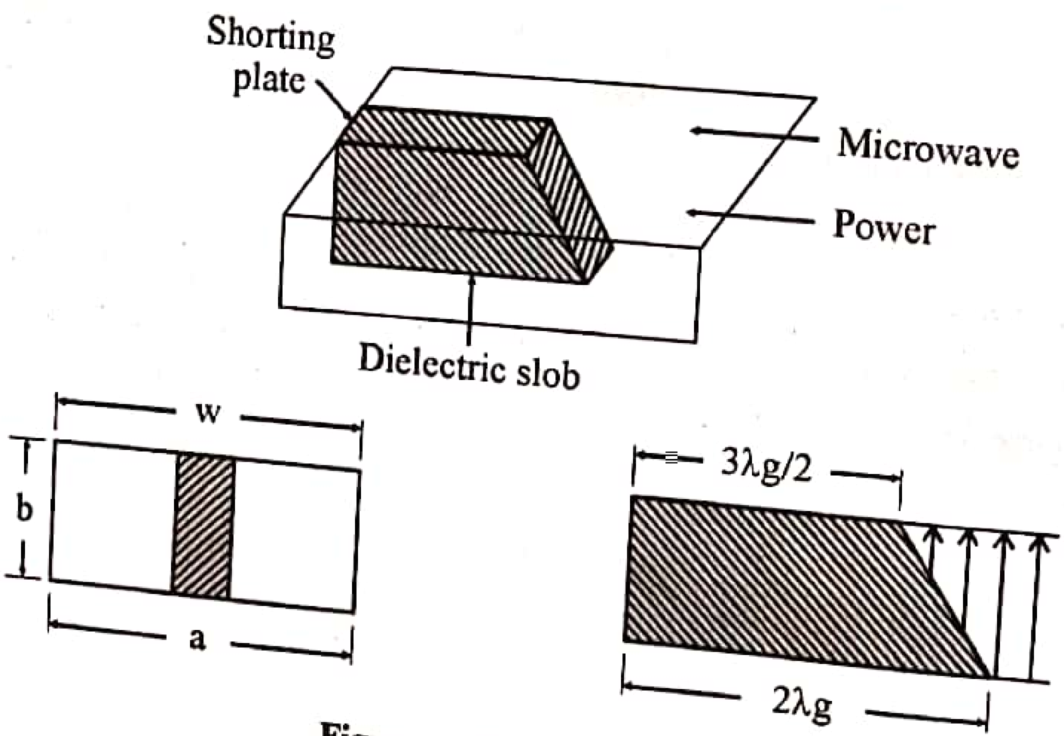


Figure 5.18 Fixed attenuator

A resistive rotary vane attenuator provides precision attenuation with an accuracy of ± 2.1% of the indicated attenuation over the operating frequency range. It consists of three vanes. The central vane rotating type placed in the central section of a circular waveguide arrangement tapered at both ends. The other two vanes are in the rectangular sections as shown in figure 5.20.

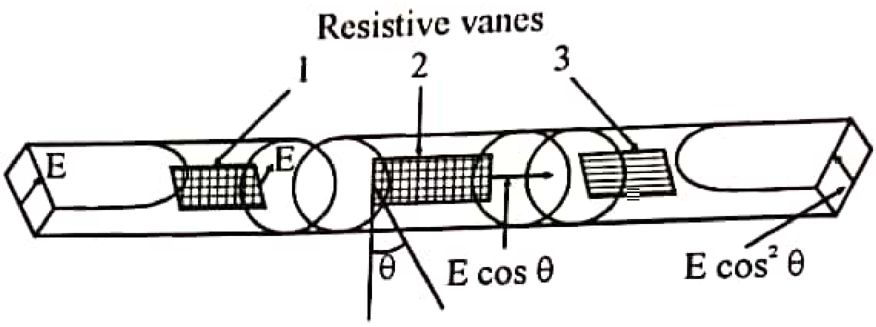


Figure 5.21 Rotary wave precision attenuator

When all the three vanes are aligned their planes are at 90° to the direction of electric field. Hence there is no (or zero) attenuation. Vane 1 prevents any horizontal polarisation and hence electric field at the output of vane 1 is vertically polarised. The centre vane 2 is rotating type and if it as rotated by an angle θ, the E sinθ sinθ component is attenuated and E cosθ component is present at the output of vane 2 and the final output of the attenuator becomes $E \cos^2\theta$, which has the same polarisation as the input wave. The attenuation due to this rotary vane attenuator is then equal to $20 \log \cos^2\theta = 40 \log \cos\theta$ that is independent of frequency and is precise.

5.4 Microwave Components

In microwave, energy is transmitted by EM waves rather than by voltages and current. So, special components are designed to transmit this energy. Such components are the coupling components like E-plane Tee, H plane *Tee*. Magic tee, hybrid rings, corners, bends and twists. The junction does not simply transmit the waves as such, instead types of junctions affect the energy in different ways. So we need to understand the basic operating principles of the most commonly used junctions.

The *T* junction is the most simple of the various commonly used waveguide junctions. There are several type of *T* junctions

1. H plane *Tee* junction
2. E plane *Tee* junction
3. E-H plane *Tee* (Hybrid T junction)
4. Magic *Tee* junction
5. Rat race junction

5.4.1 H Plane Tee Junction

To make *H* plane *tee*, cut a rectangular slot along the width of main waveguide and attach to the height of the collinear arm as shown in figure (5.2). The side arm thus formed is called as *H* arm.

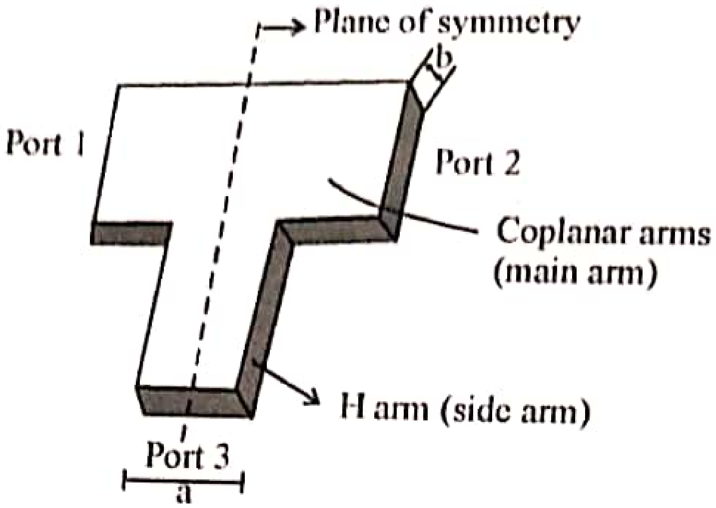


Figure 5.2 Structure of H plane *Tee*

H-Plane *Tee* is so called because the axis of the side arm is placed parallel to the planes of the main transmission line. As all the three arm of H-plans *Tee* lies in the plane of magnetic field, the magnetic field divides itself into the arms. So this is also called as current junction.

Scattering Matrix for H Plane Tee

The properties of H plane *Tee* can be best defined by [*S*] matrix.

Since there are 3 ports, so it has 3 possible inputs and 3 outputs so a scattering matrix of order 3 × 3 is formed

$$\therefore \qquad [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \qquad .....(5.4.1)$$

Now applying the properties of [*S*] matrix to determine *S*-Parameter.

1. Since there is a plane of symmetry the junction scattering coefficient must be equal. It means that if input is fed from port 3, the wave will split equally into port 1, and port 2, in phase in the same magnitude. As shown in figure (5.3) the current division indicates that three waveguides are connected in shunt.

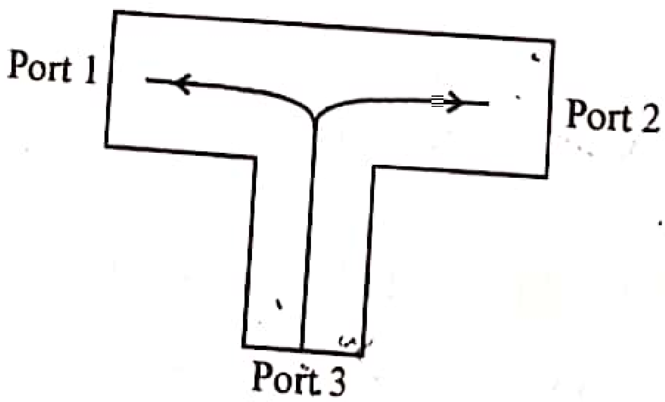


Figure 5.3 Top view of H plane waveguide tee

So, $S_{13} = S_{23}$

2. From symmetry property

$$S_{ij} = S_{ji}$$

So $S_{12} = S_{21},\ S_{13} = S_{31},\ S_{23} = S_{32} = S_{13}$

3. The port is perfectly matched so reflection coefficient is zero

$$S_{33} = 0$$

Putting these values of Sparameters in (5.4.1) we get

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \quad .....(5.4.2)$$

4. From unitary property

$$[S]\ [S]^* = I \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On solving these matrix

$$R_1C_1:\ |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad .....(5.4.3)$$

$$R_2:C_2:\ |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad .....(5.4.4)$$

$$R_3C_3:\ |S_{13}|^2 + |S_{13}|^2 = 1 \quad .....(5.4.5)$$

$$R_3C_1:\ S_{13}S_{11}^* + S_{13}S_{12}^* = 0 \quad .....(5.4.6)$$

from equation (5.4.5)

$$S_{13} = \frac{1}{\sqrt{2}} \quad .....(5.4.7)$$

from equation (5.4.3) and (5.4.4)

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2$$

$$S_{11} = S_{22} \quad .....(5.4.8)$$

from equation (5.4.6)

$$S_{13}\left[S_{11}^* + S_{12}^*\right] = 0$$

Since $\quad S_{13} \neq 0$

So $\quad S_{11}^* + S_{12}^* = 0$

or $\quad S_{11}^* = -S_{12}^*$

or $\quad S_{11} = -S_{12}$ .....(5.4.9)

Substituting equation (5.4.9) and (5.4.8) in equation (5.4.3)

$$S_{11}^2 + S_{11}^2 + \frac{1}{2} = 1$$

$$2S_{11}^2 = \frac{1}{2}$$

or $$S_{11} = \frac{1}{2} \quad .....(5.4.10)$$

from equation (5.4.9) and (5.4.8)

$$S_{12} = -\frac{1}{2} \quad .....(5.4.11)$$

$$S_{22} = \frac{1}{2}$$

Putting there values of S_{11}, S_{12}, S_{22}, S_{13} in equation (5.4.2) we get

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \quad .....(5.4.12)$$

Now we know

$$[b] = [S]\ [a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

i.e.
$$b_1 = \frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \quad .....(5.4.13)$$

$$b_2 = -\frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \quad .....(5.4.14)$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{2}}a_2 \quad .....(5.4.15)$$

Now by applying inputs to various ports we can check the working of H plane *tee* output from each port.

Case 1. If input is given at port (3) and no input at ports (1) and port (2)

or in other words

$$a_1 = 0 = a_2\ ;\ a_3 \neq 0$$

substituting these values in equation (5.4.13) to (5.4.15) we get

$$b_1 = \frac{a_3}{\sqrt{2}}, b_2 = \frac{a_3}{\sqrt{2}}, b_3 = 0$$

It shows that power coming out of port (1) and port (2) are equal in phase and magnitude.

Let P_3 be the power from port 3. So this power divides equally in two ports P_1 and P_2.

We can write $\quad P_1 = P_2$

Also $\quad P_3 = P_1 + P_2$

So $\quad P_3 = 2P_1$

The amount of power coming out of port (1) or port (2) due to input at port (3) can be given as :

$$= 10\log_{10}\frac{P_1}{P_3}$$

$$= 10\log_{10}\frac{P_1}{2P_1}$$

$$= 10\log_{10}\left(\frac{1}{2}\right)$$

$$= -\ 3dB$$

So we conclude the power coming out of port (1) & (2) is 3dB down from input power. Thus this is (H plane tee) is also called as 3dB splitter.

Case 2. Equal inputs are given at port (1) & (2) and no input from port (3)

i.e. $a_1 = a_2 = a,\ a_3 = 0$

on again solving (5.4.13) to (5.4.15) putting the values of a_1, a_2, a_3 we get

$$b_1 = 0\ ;\ b_2 = 0\ ;\ b_3 = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}}$$

So output at port (3) is addition of the two inputs at port (1) port (2) and also these are in phase.

5.4.2 E-Plane Tee

To design a E plane tee a rectangular slot is cut along the broader dimension of a long waveguide and a side arm is attached as shown in figure (5.4). In E plane tee the electric field is parallel to the side or tee's arm axis i.e; the width of all three waveguides is same.

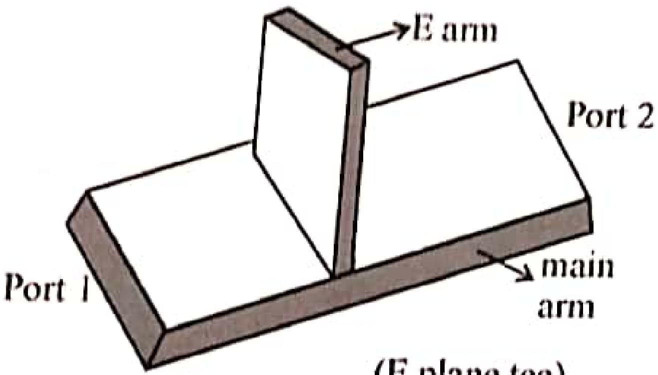


Figure 5.4 E-plane tee

In E tee if we feed TE_{10} mod. to the side arm placed at the centre of collinear arm the output at port (1)and port (2) will be equal but in opposite phase as shown in figure 5.5.

Since the electric field coming out of port (1) & (2) change their direction when fed from port (2), this is called a E-plane tee.

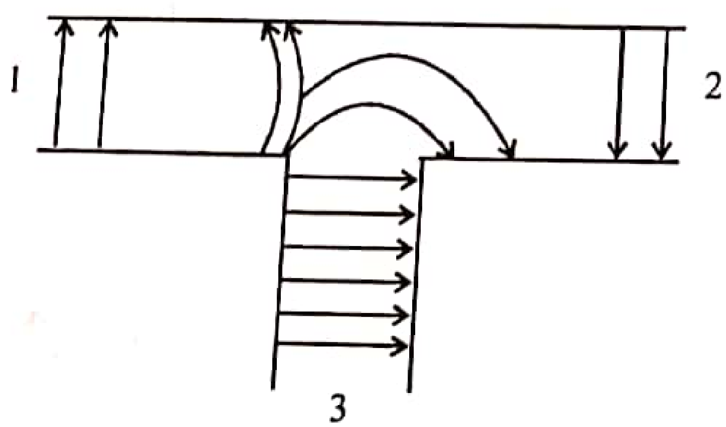


Figure 5.5

Conversely if power is fed through port (1) and port (2) the output from port (3) is difference between the two input. It means that if the power from ports (1) & (2) are entering in phase and are equal in amplitude then output is zero and if input power is out of phase then output is doubled as shown in figure 5.6

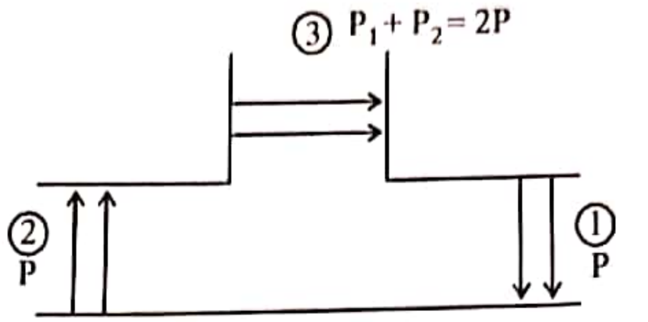


Figure 5.6 (a) Output is doubled at port (3) if power fed from port (1) & (2) are opposite in phase and same in amplitude.

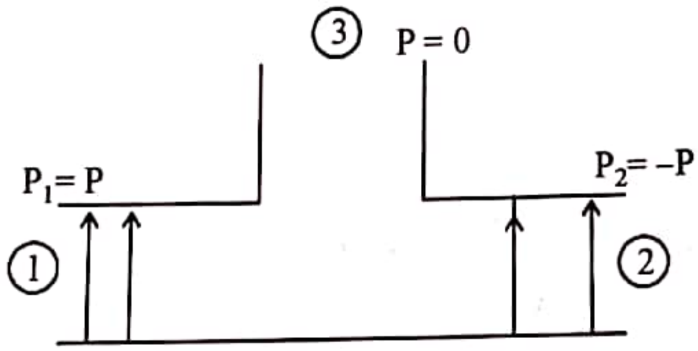


Figure 5.6 (b) Zero output at port (3) if power fed from port (1) & (2) are in same phase and same in amplitude

Scattering Matrix for E-plane tee

1. This is a 3-port device, so matrix of order 3 × 3 is formed.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

2. Since we have seen that when power is fed from port (3) then output from port (1) & (2) are opposite in phase so

$$S_{23} = -S_{13} \quad .....(5.4.16)$$

3. For port (3) to be perfectly matched to the junction

$$S_{33} = 0 \quad .....(5.4.17)$$

4. From symmetric property

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21},\ S_{23} = S_{32},\ S_{13} = S_{31} \quad .....(5.4.18)$$

Putting the above values in $[S]$ we get

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \quad .....(5.4.19)$$

(5) From unitary property

$$[S]\ [S]^* = [I]$$

So
$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying

$$R_1C_1 \Rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad .....(5.4.20)$$

$$R_2C_2 \Rightarrow |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad .....(5.4.21)$$

$$R_3C_3 \Rightarrow |S_{13}|^2 + |S_{13}|^2 = 1 \quad .....(5.4.22)$$

or
$$|S_{13}|^2 = \frac{1}{2}$$

$$S_{13} = \frac{1}{\sqrt{2}} \quad .....(5.4.23)$$

$$R_3C_1 \Rightarrow S_{13}S_{11}^* - S_{13}S_{12}^* = 0$$

$$S_{13}(S_{11}^* - S_{12}^*) = 0$$

Since
$$S_{13} \neq 0$$

So
$$S_{11}^* = S_{12}^*$$

or
$$S_{11} = S_{12} \quad .....(5.4.24)$$

Comparing equation (5.4.20) & (5.4.21)

$$|S_{11}|^2 \quad |S_{12}|^2+|\cancel{S_{13}}|^2 = |S_{12}|^2 \quad |S_{22}|^2+|\cancel{S_{13}}|^2$$

$$|S_{11}|^2 + |S_{12}|^2 = |S_{12}|^2 + |S_{22}|^2$$

From equation (5.4.24)

$$|S_{11}|^2+|S_{11}|^2 = |S_{11}|^2+|S_{22}|^2$$

$$S_{22} = S_{11} \qquad .....(5.4.25)$$

Substituting values from (5.4.23) to (5.4.25) in (5.4.20) we get,

$$|S_{11}|^2+|S_{11}|^2+\frac{1}{2} = 1$$

$$2|S_{11}|^2 = \frac{1}{2}$$

$$S_{11} = \frac{1}{2}$$

So $$S_{11} = \frac{1}{2} = S_{22} = S_{12}$$

Substituting these values of these S parameters in (5.4.19) we get,

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \qquad .....(5.4.26)$$

We know $[b] = [S]\ [a]$

So we can write

$$b_1 = \frac{1}{2}a_1 + \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \qquad .....(5.4.27)$$

$$b_2 = \frac{1}{2}a_1 + \frac{1}{2}a_2 - \frac{1}{\sqrt{2}}a_3 \qquad .....(5.4.28)$$

3

$$b_3 = \frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{2}}a_2 \qquad .....(5.4.29)$$

Case 1. Let input be applied from port (3) and no inputs from port (1) & (2)

i.e. $a_1 = a_2 = 0 \; ; \; a_3 \neq 0$

Substituting in equations (5.4.27) to (5.4.29) we get,

$$b_1 = \frac{1}{\sqrt{2}}a_3, b_2 = -\frac{1}{\sqrt{2}}a_3, b_3 = 0$$

we conclude that if input at port (3) is applied then it gets equally divided in port (1) & port (2) but are 180° out of phase. Hence E-plane tee is also called as *3dB* splitter.

Case 2. If power is fed from ports (1), (2) and no power from port (3)

$$a_1 = a_2 = a \; : \; a_3 = 0$$

Substituting again these values in equations (5.4.27) to (5.4.29) we get–

$$b_1 = \frac{a}{2} + \frac{a}{2}, b_2 = \frac{a}{2} + \frac{a}{2}$$

$$b_3 = \frac{a}{\sqrt{2}} - \frac{a}{\sqrt{2}} = 0$$

This implies that power at port (3) is zero if equal power of same amplitude and phase is fed from ports (1), (2).

Case 3. If power is fed from port (1) and no power from port (2), (3) then

$$a_1 \neq 0, \; a_2 = 0, \; a_3 = 0$$

$$b_1 = \frac{a_1}{2}, b_2 = \frac{a_1}{2}, b_3 = \frac{-a}{\sqrt{2}}$$

Similarly by applying inputs at different ports we can determine outputs at different ports.

5.4.3 E-H Plane tee/(Hybrid or Magic tee)

A magic tee is a combination of the E plane tee and H plane tee see figure 5.7. Rectangular slots one cut from main waveguide and side arms attached as shown in figure Ports (1) & (2) are collinear arms. Port (3) is the *H* arm and port (4) is the E arm.

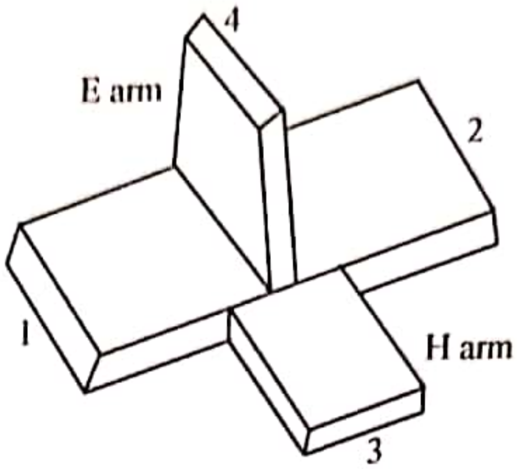


Figure 5.7 Magic tee

The basic property is that arms (3) & (4) are connected to arms (1) & (2) but not to each other. Such a junction is symmetrical about an imaginary plane bisecting arms (3) & (4) and has some very useful and interesting properties. Since it is made up of E and H plane tee, using both the properties we can find out its scattering matrix.

1. $[S]$ is a four port device so its $[S]$ is a 4 × 4 matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad .....(5.4.30)$$

2. using property of H plane tee for H section

$$S_{23} = S_{13} \quad .....(5.4.31)$$

3. using property of E plane tee for E sections

$$S_{24} = -S_{14} \quad .....(5.4.32)$$

4. Since the E-H plane tee is designed such that ports (3) & (4) are isolated to each other

So $$S_{34} = S_{43} = 0 \quad .....(5.4.33)$$

5. From symmetric property $S_{ij} = S_{ji}$

$$S_{12} = S_{21},\ S_{13} = S_{31},\ S_{23} = S_{32} \quad .....(5.4.34)$$

$$S_{34} = S_{43},\ S_{24} = S_{42},\ S_{41} = S_{14}$$

6. For perfectly matched ports i.e. for port (3) & port (4)

$$S_{33} = 0, \qquad S_{44} = 0 \quad .....(5.4.35)$$

Substituting properties from (5.4.31) to (5.4.35) in (5.4.30) we get–

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad .....(5.4.36)$$

7. Again from unitary property

$$[S]\ [S^*] = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

on multiplying

$$R_1C_1 : |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad .....(5.4.37)$$

$$R_2C_2 : |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad .....(5.4.38)$$

$$R_3C_3 : |S_{13}|^2 + |S_{13}|^2 = 1 \quad .....(5.4.39)$$

$$R_4C_4 : |S_{14}|^2 + |S_{14}|^2 = 1 \quad .....(5.4.40)$$

From equation (5.4.39) and (5.4.40) we get

$$S_{13} = \frac{1}{\sqrt{2}} \quad .....(5.4.41)$$

$$S_{14} = \frac{1}{\sqrt{2}} \quad .....(5.4.42)$$

From equation (5.4.37) & (5.4.38) we get

$$|S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2} = \cancel{|S_{12}|^2} + |S_{22}|^2 + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2}$$

$$S_{11} = S_{22} \quad .....(5.4.43)$$

Substituting equation (5.4.41) to (5.4.43) in (5.4.37) we get

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0 \quad .....(5.4.44)$$

Again from (5.4.38)

$$|S_{12}|^2 + |S_{22}|^2 = 0$$

or $$|S_{12}|^2 + |S_{11}|^2 = 0 \qquad .....(5.4.45)$$

From (5.4.44) and (5.4.45) we get

$$S_{12} = S_{11} = 0 = S_{22}$$

Now since S_{11} and S_{22} are also equal to zero, ports (1) & (2) are also perfectly matched

Thus we conclude that in any four port function if any two ports are perfectly matched then the remaining two ports are automatically matched to the junction. This type of junction where all four ports are perfectly matched to the junction is called as magic tee.

Substituting these values of S parameters we get

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \qquad .....(5.4.46)$$

Since $$[b] = [S]\ [a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

i.e. $$b_1 = \frac{1}{\sqrt{2}}(a_3 + a_4), b_2 = \frac{1}{\sqrt{2}}(a_3 - a_4)$$

$$b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2), b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2) \quad .....(5.4.47)$$

Case 1. If power is fed from port (3) only

$$a_3 \neq 0,\ a_1 = a_2 = a_4 = 0$$

then
$$b_1 = \frac{a_3}{\sqrt{2}}, b_2 = \frac{a_3}{\sqrt{2}}, b_3 = b_4 = 0$$

This is property of H plane tee

Case 2. If power is fed from port (4) only

$$a_4 \neq 0,\ a_1 = a_2 = a_3 = 0$$

$$b_1 = \frac{a_4}{\sqrt{2}}, b_2 = -\frac{a_4}{\sqrt{2}}, b_3 = 0, b_4 = 0$$

So this is property of E plane tee

Case 3. Equal powers fed from port (1) & (2) only

$$a_1 = a_2 = a,\ a_3 = a_4 = 0$$

$$b_1 = 0,\ b_2 = 0,\ b_3 = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}},\ b_4 = 0$$

Case 4. Power fed from port (1) only

$$a_1 \neq 0,\ a_2 = a_3 = a_4 = 0$$

then
$$b_1 = 0,\ b_2 = 0,\ b_3 = \frac{a_1}{\sqrt{2}}, b_4 = \frac{a_1}{\sqrt{2}}$$

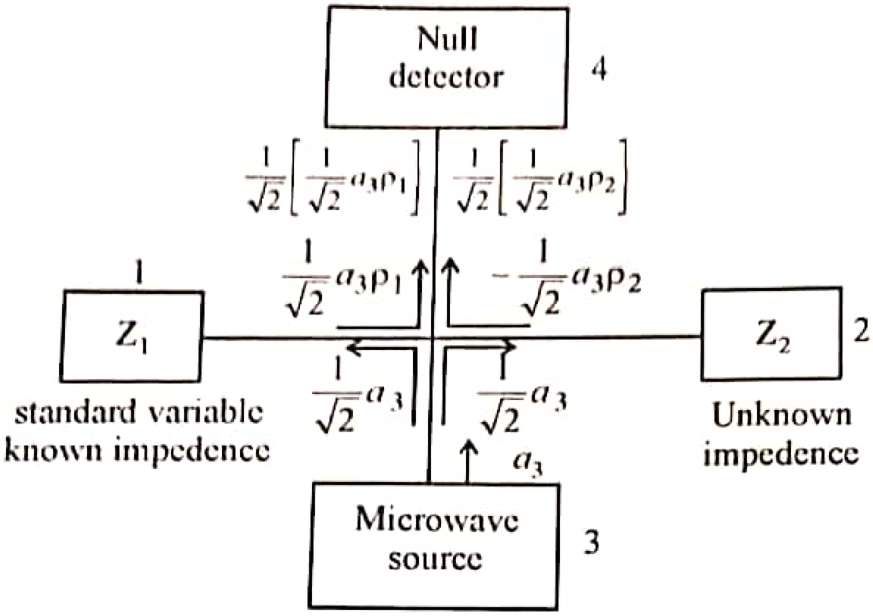
Here we see that when power is fed at port (1) then no power comes out of port (2) even when they are collinear ports (magic !!). This is the reason this type of Tee is also called as magic Tee. Ports (1) & (2) are here called as isolated ports similarly E and H ports are isolated ports.

Applications of magic Tee

1. Measurement of Impedance.
2. Magic Tee as duplexer
3. Magic Tee as mixer.

1. Measurement of Impedance

To measure impedance the set up is shown in fig (5.8)



ρ_1 = reflection coefficient of Z_1

ρ_2 = reflection coefficient of Z_2

Figure 5.8 Set up for measurement of unknown impedance from magic Tee

As we can see from the figure microwave source is connected to port (3) and a null detector is connected to port (4). The unknown impedance is connected to port (2) and variable known impedance in port (1).

Now applying properties of magic Tee, the power fed from port (3) gets equally divided in port (1) & (2) i.e. $\left(\frac{a_3}{\sqrt{2}}\right)$. The impedance in port (1) & (2) are not characteristic Impedance so reflection takes place from these ports. If ρ_1 and ρ_2 are reflection coefficients of, power $\rho_1 \frac{a_3}{\sqrt{2}}$ and $\rho_2 \frac{a_3}{\sqrt{2}}$ are powers from ports (1) & (2) respectively as shown in figure 5.8.

These powers enters in port 4. The wave reaching null detector is :

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}a_3\rho_1\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}a_3\rho_2\right)$$

$$= \frac{1}{2}a_3(\rho_1 - \rho_2) \quad .....(5.4.48)$$

For perfect balancing of bridge (null detector)

$$a_3(\rho_1 - \rho_2) = 0$$

or

$$\rho_1 - \rho_2 = 0$$

or

$$\rho_1 = \rho_2$$

$$\frac{z_1 - z_0}{z_1 + z_0} = \frac{z_2 - z_0}{z_2 + z_0}$$

where

z_0 = characteristic impedance

z_1 = known impedance

z_2 = unknown impedance

So

$$z_1 = z_2$$

$$R_1 + j\omega X_1 = R_2 + j\omega X_2$$

equating real and imaginary part

$$R_1 = R_2$$

$$X_1 = X_2 \qquad .....(5.4.49)$$

Thus unknown impedance can be known by balancing the variable known impedance till the bridge is balanced and both impedances become equal.

2. Magic Tee as Duplexer

To design duplexer the transmitter and receiver are connected in ports (2) & (1) respectively.

Further the Antenna is connected to E (port (4)) arm and arm H is terminated port (3) as shown in figure 5.9.

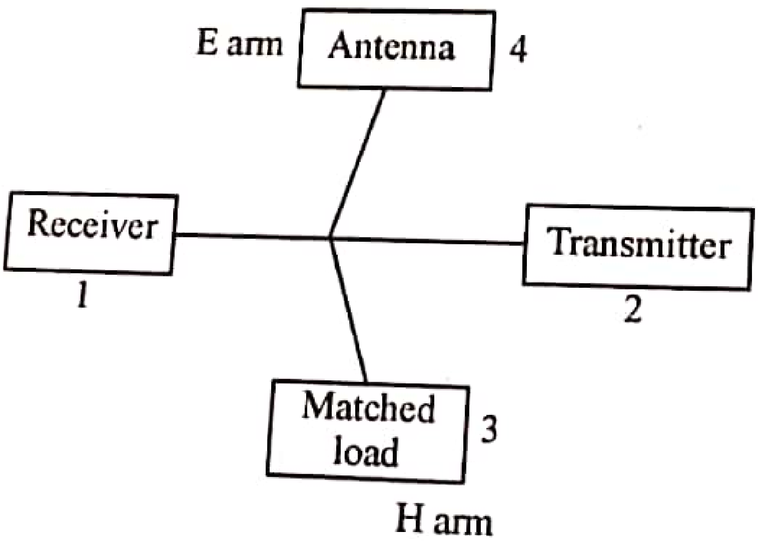


Figure 5.9 Magic Tee in Antenna duplexer

In a duplexer the same Antenna is used as a transmitting and receiving. But these two are isolated since here port (1) and port (2) are isolated to each other in magic Tee. In other words we can say that *a* duplexer system couples two circuits to the same load but avoids mutual coupling.

3. Magic tee as Mixer

As shown in figure 5.10 half of the local oscillator power and half of the received power from antenna goes to the mixer where they are mixed to generate the intermediate frequency.

$$f_f = f_{in} - f_0$$

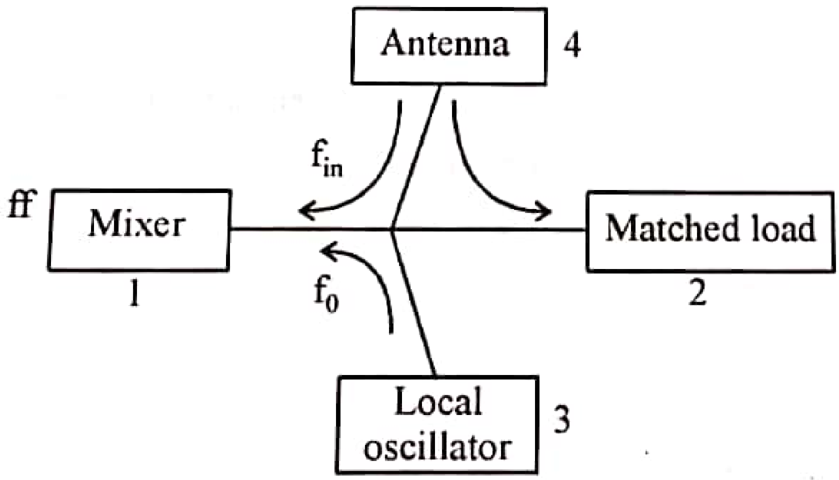


Figure 5.10 Magic Tee as mixer

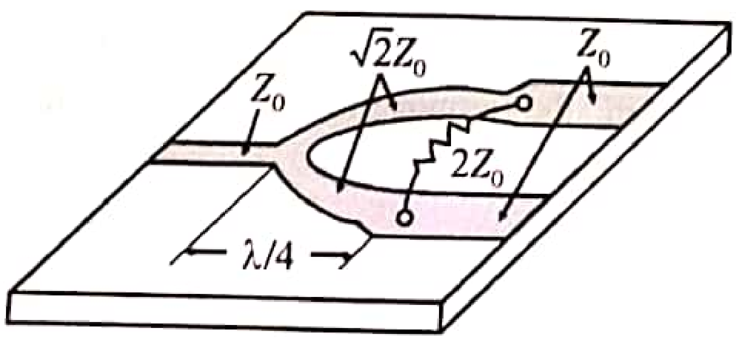
6.9 The Wilkinson Power Divider

The lossless T-junction divider suffers from the disadvantage of not being matched at all ports, and it does not have isolation between output ports. The resistive divider can be matched at all ports, but even though it is not lossless, isolation is still not achieved. We know that a lossy three-port network can be made having all ports matched, with isolation between output ports. The Wilkinson power divider is such a network, with the useful property of appearing lossless when the output ports are matched, that is, only reflected power from the output ports is dissipated.

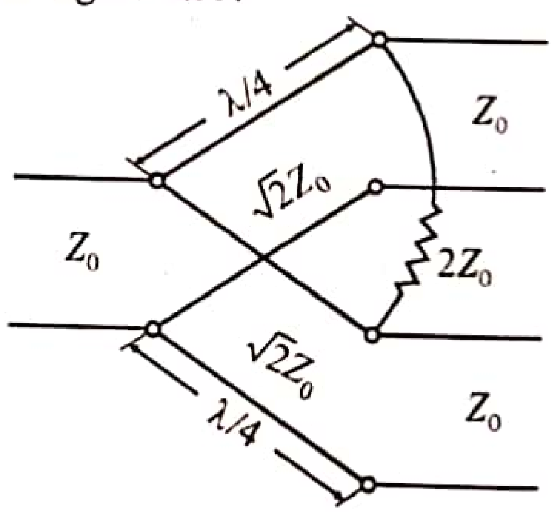
The Wilkinson power divider can be made with arbitrary power division, but we will first consider the equal-split (3 dB) case. This divider is often made in microstrip line or stripline form, as depicted in figure 6.33(a), the corresponding transmission line circuit is given in figure 6.33(b). We will analyze this circuit by reducing it to two simpler circuits driven by symmetric and antisymmetric sources at the output ports. The 'even-odd' mode analysis technique will also be useful for other networks that we will study in later sections.

Even-Odd Mode Analysis

For simplicity, we can normalize all impedances to the characteristic impedance Z_0 and redraw the circuit of figure 6.33(b) with voltage generators at the output ports as shown in figure 6.34. This network has been drawn in a form that is symmetric across the midplane, the two source resistors of normalized value 2 combine in parallel to give a resistor of normalized value 1, representing the impedance of a matched source. The quarter-wave lines have a normalized characteristic impedance Z, and the shunt resistor has a normalized value of r, we shall show that, for the equal-split power divider, these values should be $Z = \sqrt{2}$ and $r = 2$ as given in figure 6.33.



(a)



(b)

Figure 6.33 The Wilkinson power divider (a) An equal-split Wilkinson power divider in microstrip line from (b) Equivalent transmission line circuit

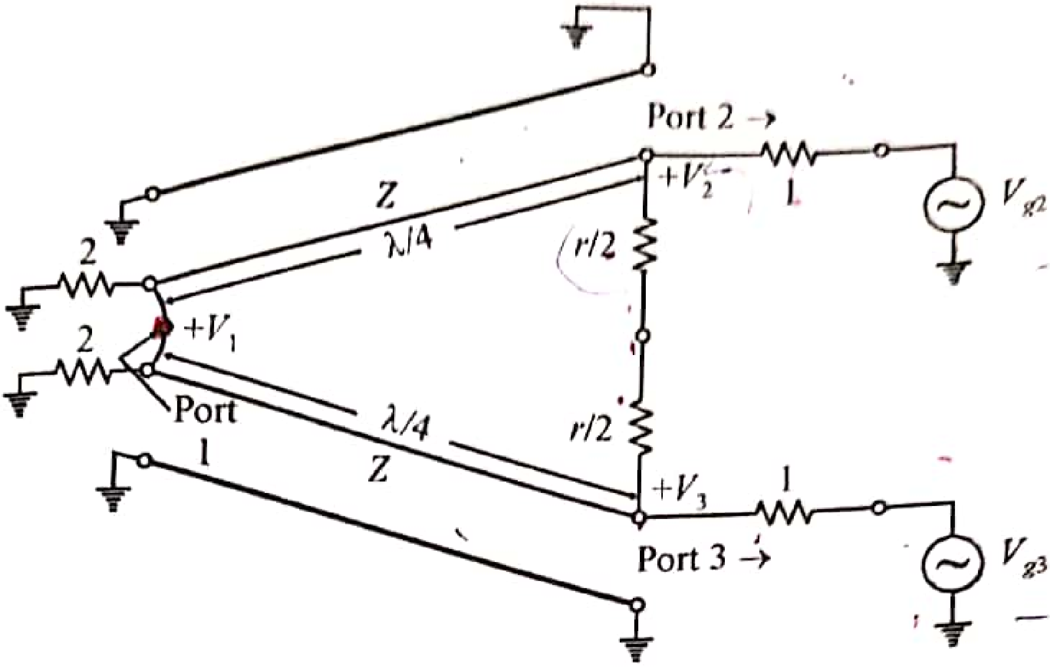
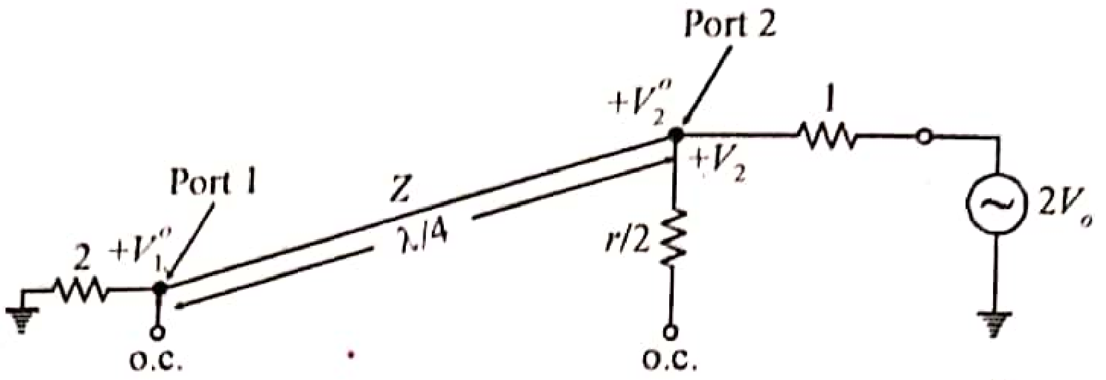


Figure 6.34 The Wilkinson power divider circuit in normalized and symmetric form

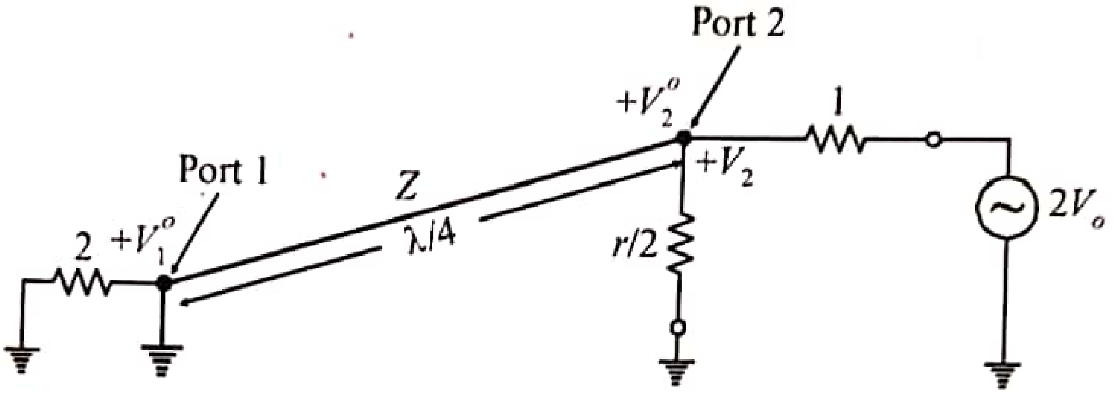
Now define two separate modes of excitation for the circuit of figure 6.34 the even mode, where $V_{g2} = V_{g3} = 2V_0$ and the odd-mode, where $V_{g2} = -V_{g3} = 2V_0$. Superposition of these two modes effectively produces an excitation produces an excitation of $V_{g2} = 4V_0$ and $V_{g3} = 0$, from which we can find the scattering parameters of the network. We now treat these two modes separately.

Even mode: For even-mode excitation, $V_{g2} = V_{g3} = 2V_0$, so $V_2^e = V_3^e$ and therefore no current flows through the $r/2$ resistors or the short circuit between the inputs of the two transmission lines at port 1. We can then bisect the network of figure 6.34 with open circuits at these points to obtain the network of figure 6.35(a) (the grounded side of the λ/4 line is not shown). Then, looking into port 2, we see an impedance

$$Z_{in}^e = \frac{Z^2}{2} \quad \text{....(6.9.1)}$$



(a)



(b)

Figure 6.35 Bisection of the circuit of figure 6.34 (a) Even-mode excitation (b) Odd-mode excitation

Since the transmission line looks like a quarter-wave transformer. Thus, if $Z = \sqrt{2}$, port 2 will be matched for even-mode excitation: then $V_2^e = V_0$ since $Z_{in}^e = 1$. The $r/2$ resistor is superfluous in this case since one end is open-circuited. Next, we find V_1^e from the transmission line equations. If we let $x = 0$ at port 1 and $x = -\lambda/4$ at port 2, we can write the voltage on the transmission line section as

$$V(x) = V^+(e^{-j\beta x} + \Gamma e^{j\beta x})$$

Then

$$V_2^e = V(-\lambda/4) = jV^+(1 - \Gamma) = V_0 \qquad \text{.....(6.9.2(a))}$$

$$V_1^e = V(0) = V^+(1 + \Gamma) = jV_0 \frac{\Gamma + 1}{\Gamma - 1} \qquad \text{.....(6.9.2(b))}$$

The reflection coefficient Γ is that seen at port 1 looking toward the resistor of normalized value 2, so

$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}}$$

and

$$V_1^e = -jV_0\sqrt{2} \quad .....(6.9.3)$$

Odd mode: For odd-mode excitation, $V_{g2} = -V_{g3} = 2V_0$ and so $V_2^o = -V_3^o$, and there is a voltage null along the middle of the circuit in figure 6.34. We can then bisect this circuit by grounding it at two points on its midplane to give the network of figure 6.35(b). Looking into port 2, we see an impedance of $r/2$ since the parallel-connected transmission line is $\lambda/4$ long and shorted at port 1, and so looks like an open circuit at port 2. Thus, port 2 will be matched for odd-mode excitation if we select $r = 2$. Then $V_2^o = V_0$ and $V_1^o = 0$, for this mode of excitation all power is delivered to the $r/2$ resistors, with none going to port 1.

Finally, we must find the input impedance at port 1 of the Wilkinson divider when ports 2 and 3 are terminated in matched loads. The resulting circuit is shown in figure 6.36(a), where it is seen that this is similar to an even mode of excitation since $V_2 = V_3$. No current flows through the resistor of normalized value 2, so it can be removed, leaving the circuit of figure 6.36(b). We have the parallel connection of two quarter-wave transformers terminated in loads of unity (normalized). The input impedance is

$$Z_{in} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1 \quad .....(6.9.4)$$

In summary, we can establish the following scattering parameters for the Wilkinson divider.

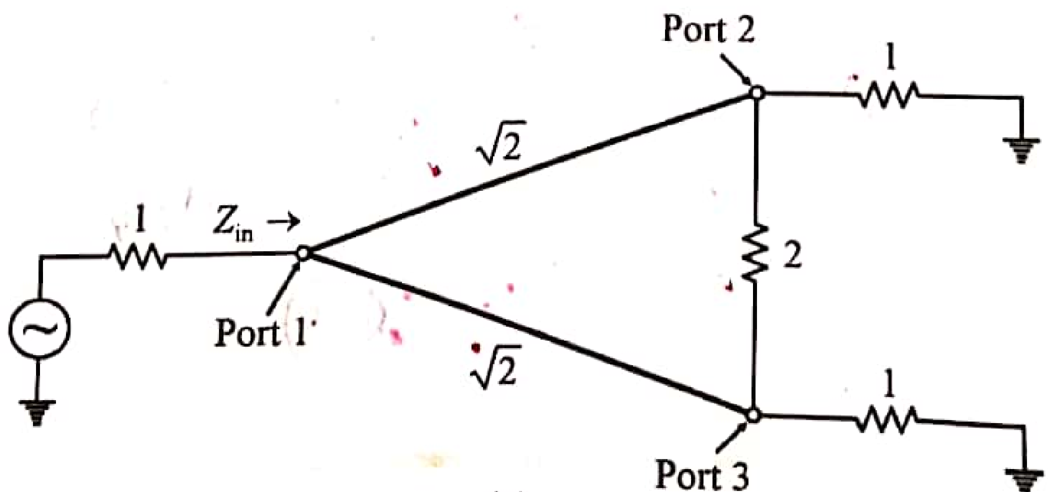
$S_{11} = 0$ ($Z_{in} = 1$ at port 1)

$S_{22} = S_{33} = 0$ (ports 2 and 3 matched for even and odd modes)

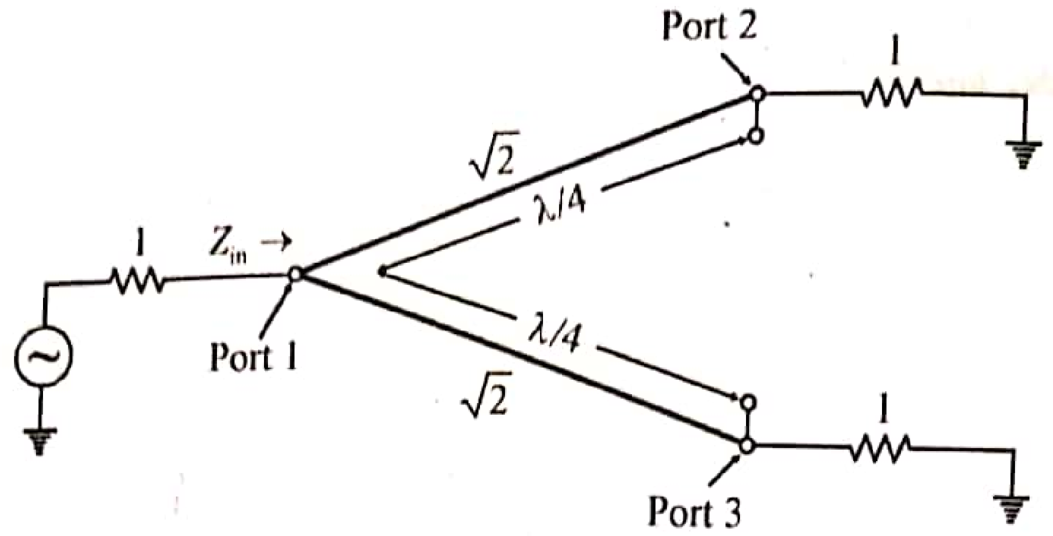
$S_{12} = S_{21} = \dfrac{V_1^e + V_1^o}{V_2^e + V_2^o} = -j/\sqrt{2}$ (Symmetry due to reciprocity)

$S_{13} = S_{31} = -j/\sqrt{2}$ (Symmetry of ports 2 and 3)

$S_{23} = S_{32} = 0$ (due to short or open at bisection)



(a)



(b)

Figure 6.36 Analysis of the Wilkinson divider to find S_{11} (a) The terminated Wilkinson divider (b) Bisection of the circuit in (a)

The preceding formula for S_{12} applies because all ports are matched when terminated with matched loads. Note that when the divider is driven at port 1 and the outputs are matched, no power is dissipated in the resistor. Thus the divider is lossless when the outputs are matched, only reflected power from ports 2 or 3 is dissipated in the resistor.

Because $S_{23} = S_{32} = 0$ port 2 and port 3 are isolated.

6.10 Hybrid Ring (Rat Race Junction)

It is a four port device. This is designed as an annular ring having circumference of 1.5 λg. of as shown in figure 6.37. These arms are connected at proper intervals. These ports are also separated by proper electrical length to sustain standing waves. The distance between each port is kept as $\lambda_g/4$.

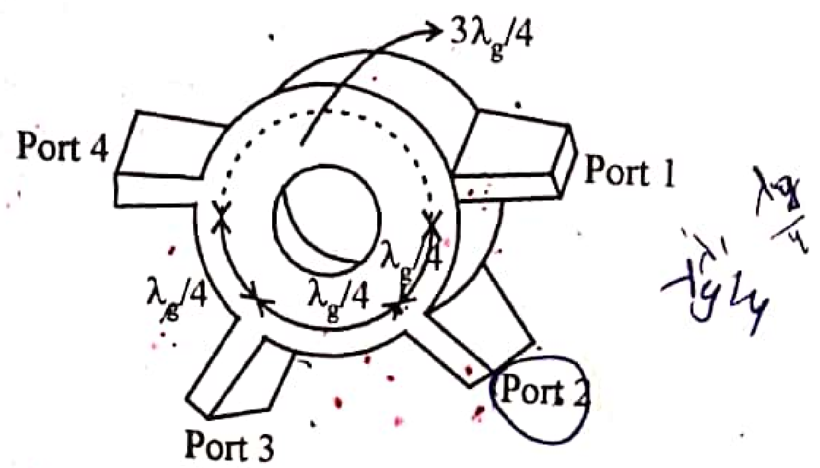


Figure 6.37 Rat race ring

To analysis the working of Rat Race ring let the power be first fed from port (1). It splits equally in both directions (clockwise and anticlock wise).

Case 1.

Power at port 2

Total path travelled by wave 1 (clockwise) is $\lambda_g/4$

Total path travelled by wave 2 (anticlockwise) is $5\lambda_g/4$

So path difference $= \dfrac{5\lambda_g}{4} - \dfrac{\lambda_g}{4} = \dfrac{4\lambda_g}{4} = \lambda_g$

So output at port 2 combine in phase.

Case 2. At port (3).

Path travelled by wave 1 = $2\lambda_g/4$

Path travelled by wave 2 = $4\lambda_g/4$

Path difference $= \dfrac{4\lambda_g}{4} - \dfrac{2\lambda_g}{4} = \dfrac{2\lambda_g}{4} = \dfrac{\lambda_g}{2}$

So output at port (3) is zero.

Case 3. Port (4)

Path travelled by wave 1 = $3\lambda_g/4$

Path travelled by wave 2 = $3\lambda_g/4$

Path difference $= \dfrac{3\lambda_g}{4} - \dfrac{3\lambda_g}{4} = 0$

So output at port (4) combine in phase.

Similarly by applying input at port (3) is equally divided in port (2) & (4) but the output at port (1) is zero.

The application of rat race ring include combining two signals or dividing a single signal into two equal halves. If two unequal signals are applied at port (1). an output proportional to their sum will come from port (2) & (4) and difference from port (3). For local oscillator scattering matrix of rat race ring can be written as :

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{32} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix} \qquad .....(6.10.1)$$