Microwave Transmission Lines:-

Introduction:-

Microwaves are electromagnetic waves of wavelengths ranging from about 30 cm down to about 0.3 mm corresponding frequency range of 10^9 Hz to 10^{12} Hz. The wavelength (λ) are very short, typically from a few tens of cm to a fraction of a mm.

→ Applications of microwaves:-

① **Telecommunication:-** Inter continental telephone and T.V. space communication (earth to space and space to earth) telementry communication link for railways etc.

② **Radars:-** Detect aircraft, track/guide supersonic missiles, observe and track weather patterns, air traffic control (ATC), burglar alarms, garage door openers, police speed detectors etc.

③ **Commercial and Industrial Application use Heat property of microwaves:-**

(a.) Microwave oven (2.45 GHz, 600W)
(b.) Drying machines textile, food and paper industry for drying clothes, potato chips, printed matter etc.
(c.) Food processing industry precooled / cooking. Heat frozen / refrigerated precooled meals roasting of food grains / beans.
(d.) Rubber industry / Plastic / Chemical / forest Product industries.

(e.) Mining / Public works, breaking rock, tunnel boring drying / breaking up concrete, breaking up coal seems, curing of cement.

(f.) Drying inks, drying grains, drying pharmaceuticals, drying textiles, leather, tobacco; power transmission.

(g.) Bio medical applications (diagnostic / therapeutic) diathermy for localized superficial heating, deep electromagnetic heating for treatment of cancer, hyperthermia (local, regional or whole body for cancer therapy), electromagnetic transmission through human body has been used for monitoring of heart beat, lung water detection etc.

(4.) Electronic warfare :- ECM (Electronic Counter measure) systems, spread spectrum system.

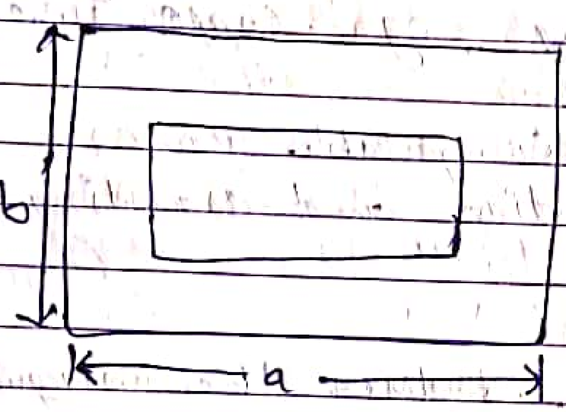
(5.) Light generated charge carries in a microwave semiconductor makes it possible to create a whole new world of microwave device, fast fitter-free switches, phase shift less. HF generation, tuning elements etc.

Waveguides:-

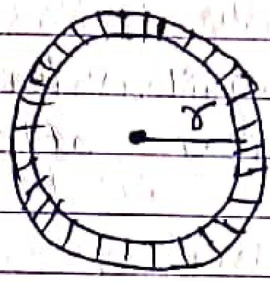
At microwave frequencies, wave guides are used for the transmission of electromagnetic waves with minimum loss. Practical waveguides are usually in the form of rectangular or circular cylinders. This section gives the detailed mathematical analysis of guided wave propagation.

~~The term k~~ At frequencies higher than 3 GHz, transmission of EM waves along transmission line cables becomes difficult due to ~~some~~ losses that occurs both in solid dielectric needed to support the conductor and in the conductor themselves. So a metallic tube can be used to transmit electromagnetic waves. ~~In~~ In general, a waveguide consists of a ~~hol~~ hollow metallic tube of rectangular or circular shape.

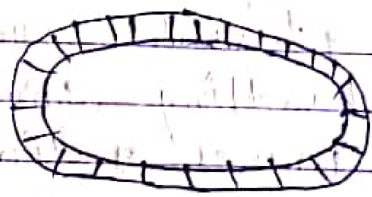
→ Types of wave guides:-



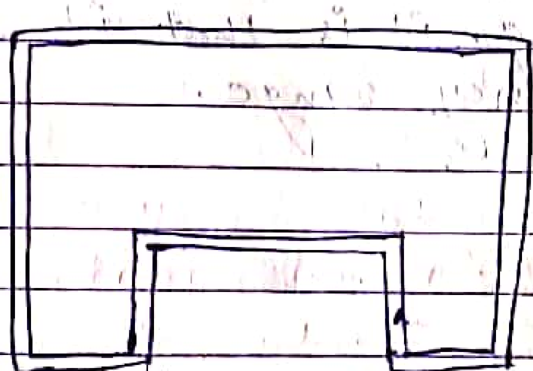
(a.) Rectangular wave guide



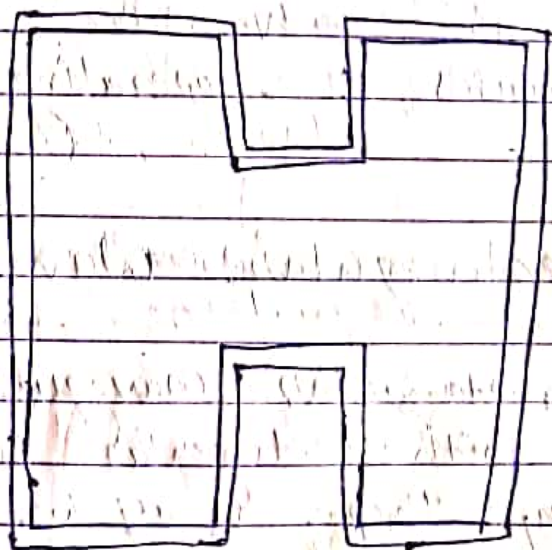
(b.) circular wave guide



(c.) elliptical waveguide



(d.) single Ridge



(e.) Double Ridge

→ Any shapes can be used shown below in the diagram but rectangular and circular waveguides are generally used because of ease of analysing them.

→ Rectangular waveguides are generally used over circular because later tends to twist the waves as these travel along them.

→ Circular waveguides are used for rotating antennas as in radars.

→ Elliptical shape is often preferred in flexible waveguides where there is need of bending stretching or twisting, eg. - copper tube.

→ Flexible waveguides have comparable power handling capability " attenuation and standing wave ratio.

→ By using ridging we can reduce the waveguide dimension and thereby increases the critical wavelength. But simultaneously Ridges increases the attenuation, reduces power handling capability and increases distortions. But the advantage of using it is that it increases the operating frequency range.

→ Rectangular Waveguides :-

A rectangular waveguides is a hollow metallic tube with rectangular cross section. In a typical setup there may be an antenna at one end of the waveguide and load at the other. Cross section of Rectangular waveguide is shown in fig.

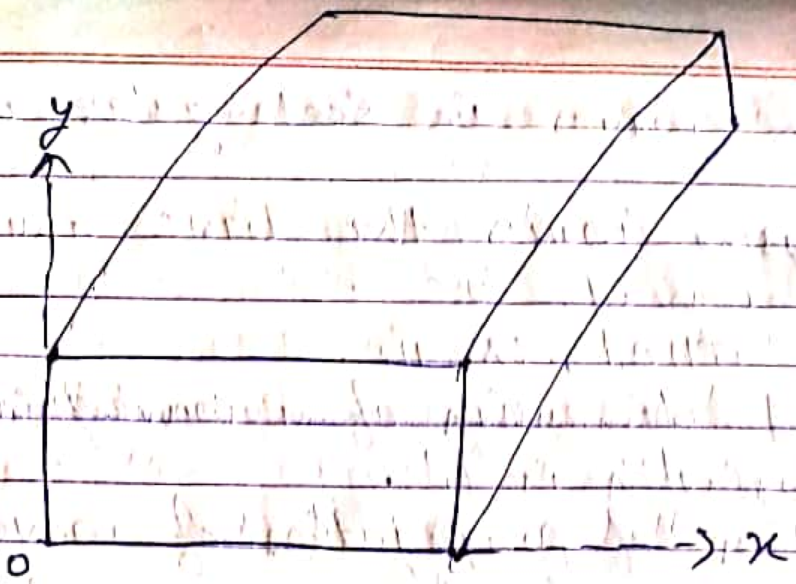


fig. → cross section of Rectangular waveguide

→ Here the conduction of energy takes place not through the walls but the function of the walls is just to confine the energy through the dielectric medium which is usually air. In waveguides we speak of electric and magnetic field instead of current and voltages.

→ Since the cross section dimension of waveguides must be of order of wavelength, use at frequencies below 1 GHz is not normally considered. It is generally seen that waveguide dimensions are conveniently designed in the range of 3 to 100 GHz. Within this range the waveguides are superior to coaxial cables.

→ Both waveguides and transmission lines can pass several signals simultaneously but in waveguides it is sufficient for them to be propagated in different modes to be separated. They do not have to be of different frequencies.

→ Analysis of waves in Rectangular waveguides:-

Before analysing waves the basic assumptions which are made are -

1. Dielectric filled is air.
2. The broad dimension of a waveguide is 'a' and the breadth is 'b'.
3. The direction of propagation of wave is positive z-direction.

TEM / TE / TM wave -

1. TEM :- Both electric and magnetic fields are perpendicular to direction of propagation.

2. TM :- magnetic field is perpendicular to direction of propagation and electric field is not purely transverse.

$$E_z \neq 0 \text{ and } H_z = 0$$

3. TE :- Here only electric field is perpendicular to direction of propagation and magnetic field is not purely transverse.

$$E_z = 0 \text{ and } H_z \neq 0$$

For wave travelling in +z direction then wave equations takes the form

$$\nabla^2 E_z = -\omega^2 \mu \epsilon E_z \text{ for TM wave} \quad —(1)$$

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z \text{ for TE wave} \quad —(2)$$

then $E_z = E_{0z} e^{-\gamma z}$ (in phasor form) — (3)
or $H_z = H_{0z} e^{-\gamma z}$ — (4)

E_{0z}, H_{0z} = maximum value of electric field / magnetic field along +z-direction.

$\gamma = \alpha + j\beta$ = propagation constant
α = attenuation constant
β = phase constant = $2\pi/\lambda$

Now, if the wave is propagating without attenuation that is $\alpha = 0$.
It implies that γ should be imaginary.

Differentiating equation (3) w.r.t. z

$$\frac{\partial E_z}{\partial z} = -\gamma E_{0z} e^{-\gamma z} \quad \text{— (5)}$$

$$\frac{\partial E_z}{\partial z} = -\gamma E_z \quad \left[\because E_{0z} e^{-\gamma z} = E_z\right]$$

So we define operator $\frac{\partial}{\partial z} = -\gamma$

Again differentiating (5) w.r.t. z again.

$$\frac{\partial^2 E_z}{\partial z^2} = \gamma^2 E_{0z} e^{-\gamma z}$$

$$\frac{\partial^2 E_z}{\partial z^2} = \gamma^2 E_z \quad \text{— (6)}$$

Thus we define another operator

$$\frac{\partial^2}{\partial z^2} = \gamma^2 \quad \text{--- (7.)}$$

Now expanding (1) in rectangular coordinates

$$\nabla^2 E_z = -\omega^2 \mu \epsilon E_z$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \epsilon E_z \quad \text{--- (8.)}$$

from eq^n (6)

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \epsilon E_z \quad \text{--- (9.)}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \epsilon) E_z = 0$$

Let $\gamma^2 + \omega^2 \mu \epsilon = h^2$ be a Constant

So $\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0$ for TM mode --- (10.)

and $\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0$ for TE mode --- (11.)

By solving the above two partial differential equation we can derive the components along x and y directions i.e. E_x, E_y, H_x, H_y.

From maxwell's 1st equation we have

$$\nabla \times H = j\omega\epsilon E \quad \text{--- (12)}$$

on expanding eq^n (12) we get

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} = j\omega\epsilon\left[\hat{i}E_x + \hat{j}E_y + \hat{k}E_z\right]$$

putting $\frac{\partial}{\partial z} = -\gamma$

or

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = j\omega\epsilon\left[\hat{i}E_x + \hat{j}E_y + \hat{k}E_z\right]$$

Equating coefficients of $\hat{i}, \hat{j}, \hat{k}$ after expanding we get

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\epsilon E_x \quad \text{--- (13)}$$

$$+\hat{j}\left[\frac{\partial H_z}{\partial x} + \gamma H_x\right] = -j\omega\epsilon E_y \quad \text{--- (14)}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \quad \text{--- (15)}$$

Now according to maxwell's 2nd equation

$$\nabla \times E = -j\omega\mu H \quad \text{--- (16)}$$

on expanding similarly we get,

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \qquad \text{--- (17)}$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = j\omega\mu H_y \qquad \text{--- (18)}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \qquad \text{--- (19)}$$

from eqn. (18) $H_y = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\mu}E_x$

Putting in (13)

$$\frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma^2}{j\omega\mu}E_x = j\omega\epsilon E_x$$

$$E_x\left(\frac{-\gamma^2}{j\omega\mu} + j\omega\epsilon\right) = \frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x}$$

$$E_x\left(-\gamma^2 - \omega^2\mu\epsilon\right) = j\omega\mu\frac{\partial H_z}{\partial y} + \gamma\frac{\partial H_z}{\partial x}$$

Putting $h^2 = \gamma^2 + \omega^2\mu\epsilon$

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \qquad \text{--- (20)}$$

similarly solving for E_y, H_x, H_y we get

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \qquad \text{--- (21)}$$

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$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y} \quad \text{--- (22)}$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial x} \quad \text{--- (23)}$$

⇒ <u>TM mode in waveguide:-</u>

For TM mode $H_z = 0$ and $E_z \neq 0$

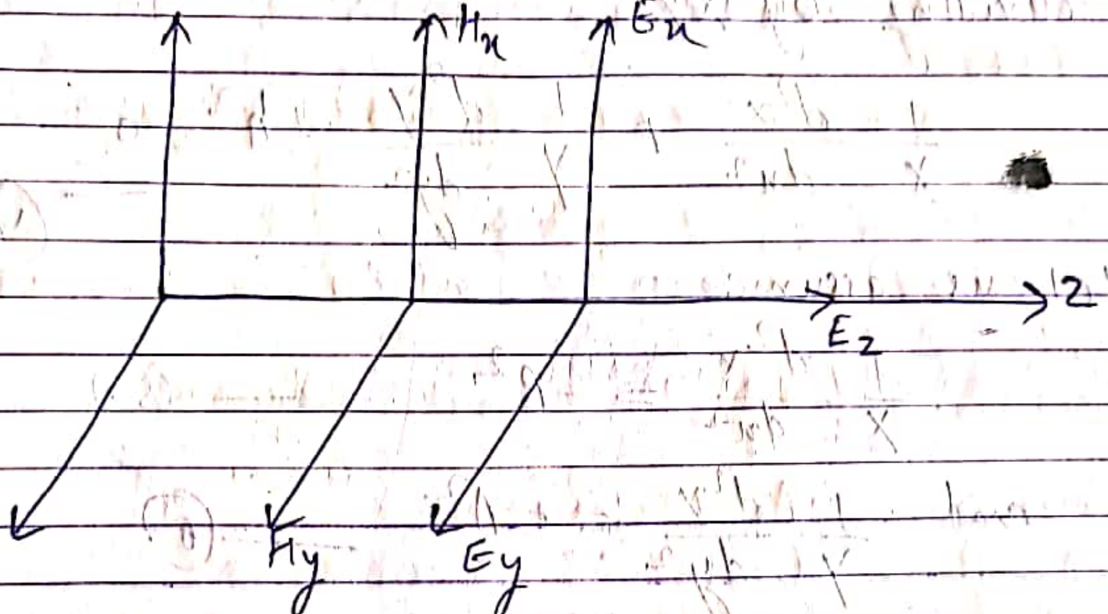

Fig:- field distribution in TM wave.

The wave equation is—

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \quad \text{--- (24)}$$

This is a partial differential equation which can be solved to get the different field component E_x, E_y, H_x and H_y.

Assuming $E_z = XY$ — (2)

where X is a pure function of x and Y is a pure function of y.

So we get

$$\frac{d^2XY}{dx^2} + \frac{d^2XY}{dy^2} + h^2XY = 0 \quad \text{— (3)}$$

dividing by XY.

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + h^2 = 0 \quad \text{— (4)}$$

Let us assume

$$\frac{1}{X}\frac{d^2X}{dx^2} = -B^2 \quad \text{— (5)}$$

and $$\frac{1}{Y}\frac{d^2Y}{dy^2} = -A^2 \quad \text{— (6)}$$

putting (5) & (6) in (4)

$$A^2 + B^2 = h^2 \quad \text{— (7)}$$

Now equations (5) & (6) are 2nd order differential equations whose solution is

$$X = C_1 \cos Bx + C_2 \sin Bx$$
$$Y = C_3 \cos Ay + C_4 \sin Ay$$

where C_1, C_2, C_3 and C_4 are constant which can be evaluated by applying the boundary conditions.

So $E_z = XY = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]$ —— (8)

Boundary conditions:-

The entire surface of rectangular wave guides acts as a short circuit or ground, $E_z = 0$ all along the boundary walls

→ (1st boundary Condition): (bottom plane or bottom wall)
i.e. $E_z = 0$ at $y = 0$ $\forall$ $x \to 0$ to a,

→ (2nd boundary Condition): (left side or left wall)
i.e. $E_z = 0$ at $x = 0$ $\forall$ $y \Rightarrow 0$ to b.

→ (3rd boundary condition): (top wall/plane)
i.e. $E_z = 0$ at $y = b$ $\forall$ $x \to 0$ to a

→ (4th boundary condition): (right plane or wall)
i.e. $E_z = 0$ at $x = a$, $\forall$ $y \to 0$ to b

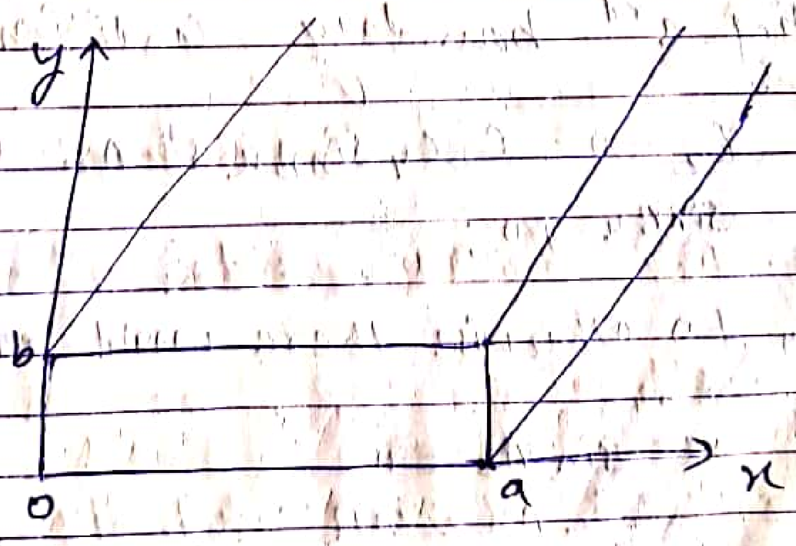

fig.

(i) Now putting 1st boundary conditions in eqn (8.)

$$E_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]$$

$$0 = [C_1 \cos Bx + C_2 \sin Bx] C_3$$

$$[\because E_z = 0 \text{ at } y = 0 \ \forall x \to 0 \text{ to } a]$$

This is true for $\forall x \to 0$ to a.
Since $C_1 \cos Bx + C_2 \sin Bx \neq 0$; $C_3 = 0$

$$\Rightarrow E_z = (C_1 \cos Bx + C_2 \sin Bx) C_4 \sin Ay \quad \text{--- (9.)}$$

(ii) Putting 2nd boundary condition in eqn (9.)

$$E_z = 0 = C_1 (C_4 \sin Ay) \ \forall y \to 0 \text{ to } b$$

Since $\sin Ay \neq 0$; $C_1 = 0$

So $E_z = C_2 C_4 \sin Bx \sin Ay$ ——— (10.)

(iii) Putting 3rd boundary condition in eqn (10)

$$E_z = 0 = C_2 C_4 \sin Bx \sin Ab \ \forall x \to 0 \text{ to } a \text{ at } y = b$$

Since $\sin Bx \neq 0$;

$C_2, C_4 \neq 0$ otherwise there would be no solution

So $\sin Ab = 0$

$$Ab = n\pi$$

$$A = \frac{n\pi}{b}; \ n = 0, 1, 2, \ldots \quad \text{--- (11.)}$$

(iv.) substituting 4th boundary conditions in (10)

$$E_z = 0 = C_2 C_4 \sin a\beta \sin Ay \text{ at } x = a$$
$$\forall y \to 0 \text{ to } b$$

since $\sin Ay \neq 0$, $C_2 \neq 0$, $C_4 \neq 0$

so $\sin a\beta = 0$

$$a\beta = m\pi \quad , \quad m = 0, 1, 2, \ldots$$

$$\beta = \frac{m\pi}{a} \quad \text{—— (12)}$$

from the values we conclude that the complete solution is –

$$E_z = C_2 C_4 \sin\left(\frac{m\pi}{a}\right) x \sin\left(\frac{n\pi}{b}\right) y \, e^{-\gamma z} e^{j\omega t}$$

where $e^{-\gamma z}$ = propagation along positive z direction

$e^{j\omega t}$ = sinusoidal variation w.r.t. t.

Let $C_2 C_4 = C$

so, $$E_z = C \sin\left(\frac{m\pi}{a}\right) x \sin\left(\frac{n\pi}{b}\right) y \, e^{(j\omega t - \gamma z)}$$

—— (13.)

If E_z is known then putting value from (13.) in eqn's (20) to (23) we can determine various components E_x, E_y, H_x, H_y

Deriving relation for E_x

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y} \quad [\because \text{ from eqn. (20)}]$$

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As for TM mode $H_z = 0$ so $\frac{\partial H_z}{\partial y} = 0$

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x} \quad \text{—— (14)}$$

Differentiating eqn. (13) w.r.t. x and putting in (14) we get

$$E_x = \frac{-\gamma}{h^2} C \frac{m\pi}{a} \cos\left(\frac{m\pi}{a}\right) x \sin\left(\frac{n\pi}{b}\right) y e^{(j\omega t - \gamma z)} \quad \text{—— (15)}$$

Similarly solving for E_y, H_x, H_y we get -

$$E_y = \frac{-\gamma}{h^2} C \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right) x \cos\left(\frac{n\pi}{b}\right) y e^{(j\omega t - \gamma z)} \quad \text{—— (16)}$$

$$H_x = \frac{j\omega \epsilon}{h^2} C \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right) x \cos\left(\frac{n\pi}{b}\right) y e^{(j\omega t - \gamma z)} \quad \text{—— (17)}$$

$$H_y = \frac{-j\omega \epsilon}{h^2} C \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}\right) x \sin\left(\frac{n\pi}{b}\right) y e^{(j\omega t - \gamma z)} \quad \text{—— (18)}$$

⇒ **Propagating of TE Modes in Rectangular waveguides :-**

For TE mode $E_z = 0$ and $H_z \neq 0$ According to Helmotz equation -

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z$$

On expanding, we get

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \epsilon H_z$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z + \omega^2 \mu \epsilon H_z = 0$$

$$\left[\because \frac{\partial^2}{\partial z^2} = \gamma^2\right]$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \epsilon) H_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \quad \text{--- (1)}$$

$$\left[\because h^2 = \omega^2 \mu \epsilon + \gamma^2\right]$$

This is partial differential equation and its solution is given by—

$$H_z = XY \quad \text{--- (2)}$$

Where X is a pure function of x only
Y is a pure function of y only

Putting it in eq^n (1)

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} + h^2 XY = 0$$

Dividing L.H.S. and R.H.S. by XY

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + h^2 = 0$$

Let $\frac{1}{X}\frac{d^2X}{dx^2} = -B^2$ and $\frac{1}{Y}\frac{d^2Y}{dy^2} = -A^2$

So $h^2 = A^2 + B^2$ — (3)

Solving for X and Y by separation of variable method we get -

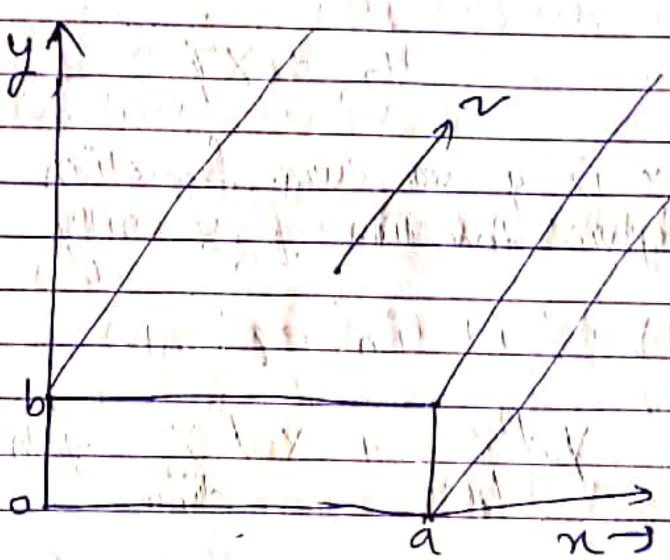
$$X = C_1 \cos Bx + C_2 \sin Bx$$
$$Y = C_3 \cos Ay + C_4 \sin Ay$$

Putting value of X and Y in (2)

$$H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay) \quad \text{— (4)}$$

Values of C_1, C_2, C_3 and C_4 can be evaluated by applying boundary conditions.

Boundary Conditions :-



→ $E_z = 0$ but has component along x and y dir.

→ $E_x = 0$ all along top & bottom walls of waveguide

→ $E_y = 0$ all along left & right walls of waveguides.

1) 1st Boundary condition (bottom wall)

$E_x = 0$ at $y = 0 \; \forall \; x \to 0$ to a

writing E_x in terms of H_z. from eq^n (20)

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \qquad [\because E_z = 0]$$

Differentiating eq^n (4) w.r.t. y we get.

$$E_x = \frac{-j\omega\mu}{h^2}\left[(C_1 \cos Bx + C_2 \sin Bx)(-AC_3 \sin Ay + AC_4 \cos Ay)\right]$$

Putting boundary condition.

$$0 = \frac{-j\omega\mu}{h^2}\left[(C_1 \cos Bx + C_2 \sin Bx)(0 + AC_4)\right]$$

since $C_1 \cos Bx + C_2 \sin Bx \neq 0$ and $A \neq 0$

So $C_4 = 0$

Putting value of C_4 in (4) we get

$$H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay) \quad \text{(5)}$$

(2) 2nd Boundary condition (left wall)

$E_y = 0$ at $x = 0 \; \forall \; y \to 0$ to b

Putting $E_z = 0$ in eq^n (21) we get

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

Again differentiating eq^n (5) w.r.t. x we get

$$E_y = \frac{j\omega\mu}{h^2}\left[(-BC_1 \sin Bx + BC_2 \cos Bx)(C_3 \cos Ay)\right]$$

Putting Boundary condition

$$0 = \frac{j\omega\mu}{h^2}(0 + BC_2)C_3 \cos Ay$$

Now $\cos Ay \neq 0$; $B \neq 0$; $C_3 \neq 0$

So $C_2 = 0$

Putting value of C_2 in eq^n (5) we get.

$$H_2 = C_1 \cos Bx \, C_3 \cos Ay$$

$$H_2 = C_1 C_3 \cos Bx \cos Ay \quad \text{——— (6)}$$

(3) 3rd Boundary condition (top wall)

$$E_x = 0 \text{ at } y = b \quad \forall x \to 0 \text{ to } a$$

Putting $E_x = 0$ in eq^n (20), we get

$$E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

Differentiating eq^n (6) w.r.t. y, we get

$$E_x = \frac{+j\omega\mu}{h^2} C_1 C_3 A \cos Bx \sin Ay$$

Putting boundary condition

$$0 = \frac{j\omega\mu}{h^2} C_1 C_3 A \cos Bx \sin Ab$$

$\cos Bx \neq 0$; $C_1 \neq 0$; $C_3 \neq 0$; $A \neq 0$

So $\sin Ab = 0$

$$Ab = n\pi \quad n = 0, 1, 2, \ldots$$

$$\text{or} \quad A = \frac{n\pi}{b} \quad \text{——— (7)}$$

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(4.) 4^{th} Boundary condition (right wall)

$E_y = 0$ at $x = a$ ∀ $y \to 0$ to b

putting $E_z = 0$ in eqn (21.), we get

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

Differentiating eqn (6.) w.r.t. x, we get

$$E_y = \frac{-j\omega\mu}{h^2} C_1 C_3 B \sin Bx \cos Ay$$

Putting boundary condition

$$0 = \frac{-j\omega\mu}{h^2} C_1 C_3 B \sin Ba \cos Ay$$

since $\cos Ay \neq 0$; $C_1 C_3 \neq 0$

so $\sin Ba = 0$

$$Ba = m\pi \; ; \; m = 0, 1, 2, \ldots$$

$$B = \frac{m\pi}{a} \quad \text{———} \quad (8)$$

Putting value of A & B from eqn (7) & (8) in eqn (6) we get

$$H_z = C_1 C_3 \cos\left(\frac{m\pi}{a}\right) x \cos\left(\frac{n\pi}{b}\right) y$$

Let $C_1 C_3 = C$

So $$H_z = C \cos\left(\frac{m\pi}{a}\right) x \cos\left(\frac{n\pi}{b}\right) y$$

Therefore the complete wave equation as we have seen in TM mode is–

$$H_z = C \cos\left(\frac{m\pi}{a}\right) x \cos\left(\frac{n\pi}{b}\right) y \, e^{(j\omega t - \gamma z)} \quad \text{———} \quad (9)$$

field Components -

Putting $E_z = 0$ and using eq^n. (9) we can find various field components from eq^n. (20) to (23).

so on solving similarly as for TM mode we get.

$$E_x = \frac{j\omega\mu}{h^2} C \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{(j\omega t - \gamma}$$ (10)

$$E_y = \frac{-j\omega\mu}{h^2} C \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\, e^{(j\omega t - \gamma z}$$ (11)

$$H_x = \frac{\gamma}{h^2} C \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\, e^{(j\omega t - \gamma z)}$$ (12)

$$H_y = \frac{-\gamma}{h^2} C \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{(j\omega t - \gamma}$$ (13)