Chapter 6

Radiation

Outline

7	Radiation-Solution for potential function, Radiation from the Hertz dipole, Power radiated by hertz dipole, Radiation Parameters of antenna, receiving antenna, Monopole and Dipole antenna	07

Text Books

Elements of electromagnetics by matthew n.o. sadiku

Introduction

- Radiation may be thought of as the process of transmitting electric energy. The radiation or launching of the waves into space is efficiently accomplished with the aid of conducting or dielectric structures called ***antennas***.

- Theoretically, any structure has capability to radiate but only few structures can radiate efficiently

- **Antenna** – a structure which behaves as a transducer matching a transmission line or a waveguide to the surrounding medium or vice versa

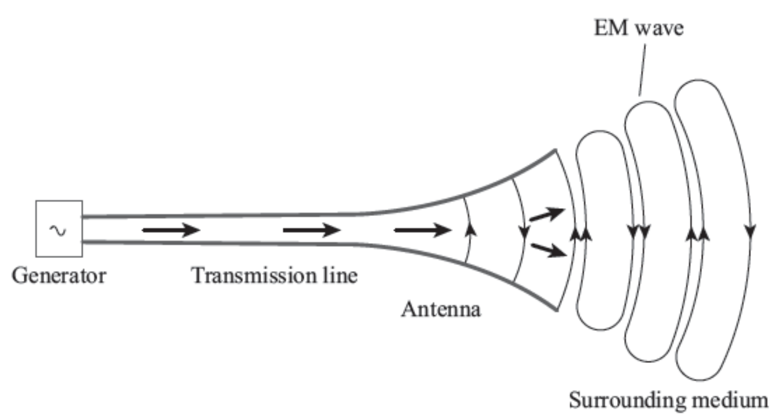


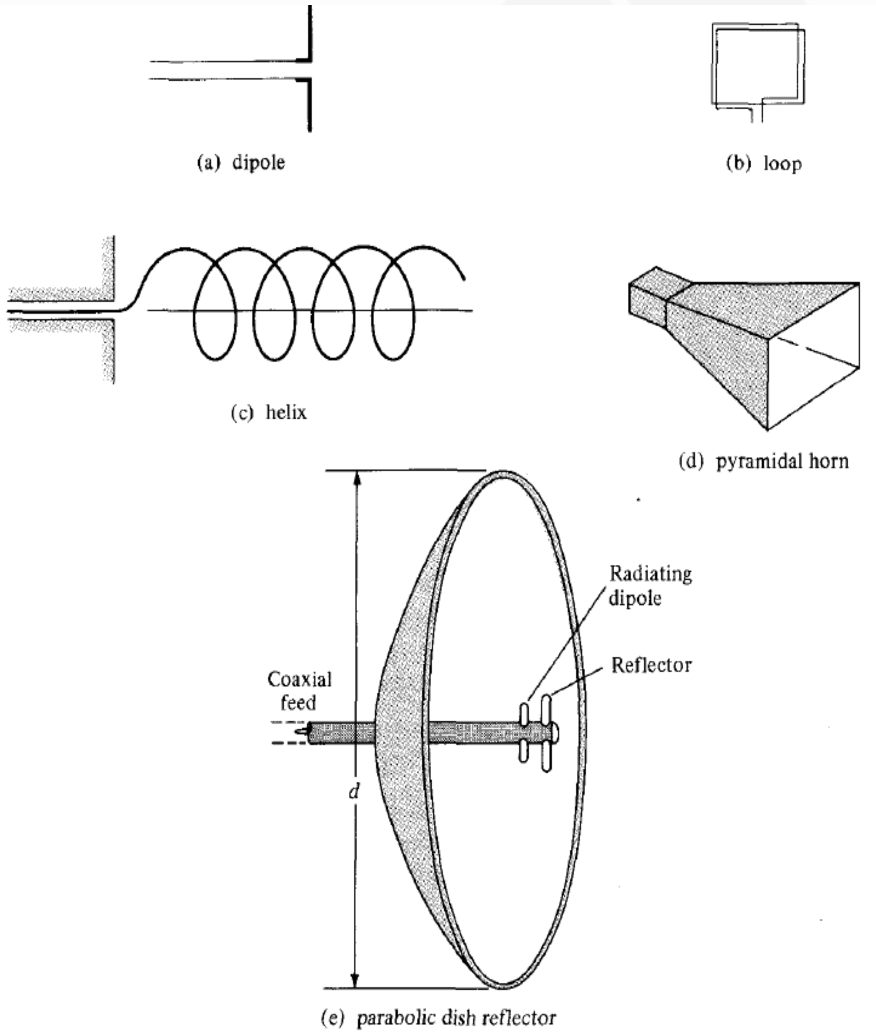
Figure 13.1 An antenna as a matching device between the guiding structure and the surrounding medium.

- The antenna is needed for two main reasons: efficient radiation and matching wave impedances in order to minimize reflection.

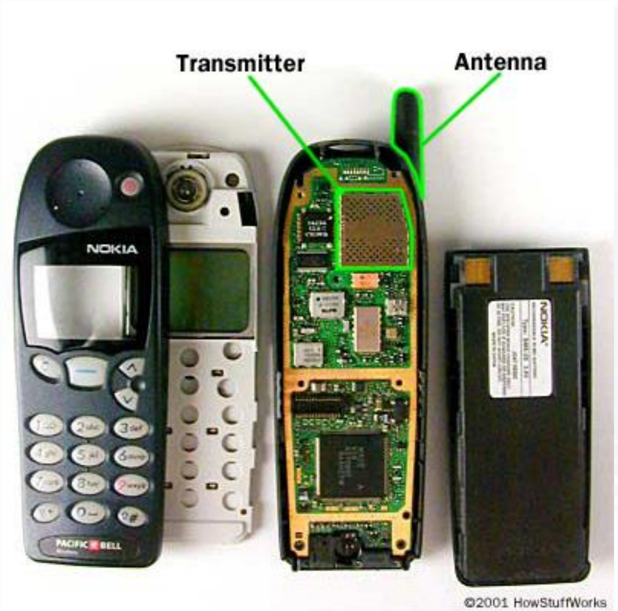
- The antenna uses voltage and current from the transmission line (or the EM fields from the waveguide) to launch an EM wave into the medium.

- An antenna may be used for either transmitting or receiving EM energy.

Types of Antennas



- The dipole antenna in Figure (a) consists of two straight wires lying along the same axis.
- The loop antenna in Figure (b) consists of one or more turns of wire.
- The helical antenna in Figure (c) consists of a wire in the form of a helix backed by a ground plane.
- Antennas in Figure (a-c) are called wire antennas; they are used in automobiles, buildings, aircraft, ships, and so on.
- The horn antenna in Figure (d), an example of an aperture antenna, is a tapered section of waveguide providing a transition between a waveguide and the surroundings. Since it is conveniently flush mounted, it is useful in various applications such as aircraft.
- The parabolic dish reflector in Figure (e) utilizes the fact that EM waves are reflected by a conducting sheet. When used as a transmitting antenna, a feed antenna such as a dipole or horn, is placed at the focal point. The radiation from the source is reflected by the dish (acting like a mirror) and a parallel beam results. Parabolic dish antennas are used in communications, radar, and astronomy.


Transmitter
Antenna
NOKIA
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Wi-Fi antenna
5G antenna
Diversity antennas
Near-field communication (NFC) antenna
SAMSUNG
5G antenna
Three antennas (low to midband, midband, carrier aggregation)

Steps to determine the radiation fields by an Antenna

1. Select an appropriate coordinate system and determine the magnetic vector potential $\mathbf{A}$.
2. Find $\mathbf{H}$ from $\mathbf{B} = \mu\mathbf{H} = \nabla \times \mathbf{A}$.
3. Determine $\mathbf{E}$ from $\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t}$ or $\mathbf{E} = \eta \mathbf{H} \times \mathbf{a}_k$ assuming a lossless medium ($\sigma = 0$).
4. Find the far field and determine the time-average power radiated using

$$P_{\text{rad}} = \int \mathcal{P}_{\text{ave}} \cdot d\mathbf{S}, \qquad \text{where} \qquad \mathcal{P}_{\text{ave}} = \frac{1}{2} \operatorname{Re} (\mathbf{E}_s \times \mathbf{H}_s^*)$$

Hertzian dipole

- It is an infinitesimal current element whose length is less than 0.1λ
- Consider the Hertzian dipole.
- It is located at the origin of a coordinate system and carries a uniform current (constant throughout the dipole), $I = Io \cos \omega t$

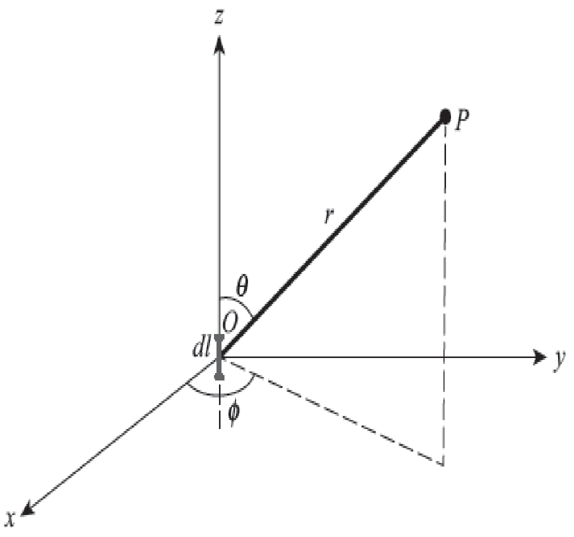


Figure 13.3 A Hertzian dipole carrying current $I = I_o \cos \omega t$.

The retarded magnetic vector potential at the field point *P*, due to the dipole, is given by

$$\mathbf{A} = \frac{\mu[I]\,dl}{4\pi r}\,\mathbf{a}_z$$

[I] is the retarded current given by

$$[I] = I_o \cos \omega\left(t - \frac{r}{u}\right) = I_o \cos(\omega t - \beta r)$$
$$= \mathrm{Re}\,[I_o e^{j(\omega t - \beta r)}]$$

$$\beta = \omega/u = 2\pi/\lambda, \text{ and } u = 1/\sqrt{\mu\varepsilon}.$$

The current is said to be *retarded* at point *P*,because there is a propagation time delay *r/u* or phase delay βr from *O* to *P*

Rewrite A in phasor form as

$$A_{zs} = \frac{\mu I_o dl}{4\pi r} e^{-j\beta r}$$

Transforming this vector in Cartesian to spherical coordinates

$$\mathbf{A}_s = (A_{rs}, A_{\theta s}, A_{\phi s})$$

where

$$A_{rs} = A_{zs} \cos\theta, \qquad A_{\theta s} = -A_{zs} \sin\theta, \qquad A_{\phi s} = 0$$

But $\mathbf{B}_s = \mu \mathbf{H}_s = \nabla \times \mathbf{A}_s$; hence, we obtain the **H** field as

$$H_{\phi s} = \frac{I_o dl}{4\pi} \sin\theta \left[\frac{j\beta}{r} + \frac{1}{r^2}\right] e^{-j\beta r}$$

$$H_{rs} = 0 = H_{\theta s}$$

We find the **E** field using $\nabla \times \mathbf{H} = \varepsilon\, \partial \mathbf{E}/\partial t$ or $\nabla \times \mathbf{H}_s = j\omega\varepsilon \mathbf{E}_s$,

$$E_{rs} = \frac{\eta I_o dl}{2\pi} \cos\theta \left[\frac{1}{r^2} - \frac{j}{\beta r^3}\right] e^{-j\beta r}$$

$$E_{\theta s} = \frac{\eta I_o dl}{4\pi} \sin\theta \left[\frac{j\beta}{r} + \frac{1}{r^2} - \frac{j}{\beta r^3}\right] e^{-j\beta r}$$

$$E_{\phi s} = 0$$

where

$$\eta = \frac{\beta}{\omega e} = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_{\theta s} = \frac{\eta I_o dl}{4\pi} \sin\theta \left[\frac{j\beta}{r} + \frac{1}{r^2} - \frac{j}{\beta r^3}\right] e^{-j\beta r}$$

$1/r^3$, $1/r^2$, and $1/r$

This term is called the *electrostatic field* since it corresponds to the field of an electric dipole. This term dominates over other terms in a region very close to the Hertzian dipole.

This term is called the *inductive field*, and it is predictable from the Biot-Savart law. The term is important only at near field, that is, at distances close to the current element.

This term is called the *far field or radiation field* because it is the only term that remains at the far zone,that is, at a point very far from the current element. Here, we are mainly concerned with the far field or radiation zone

($\beta r \gg 1$ or $2\pi r \gg \lambda$).

the terms in $1/r^3$ and $1/r^2$ can be neglected in favor of the $1/r$ term. Thus at far field,

$$H_{\phi s} = \frac{jI_o\beta dl}{4\pi r} \sin\theta \, e^{-j\beta r}, \qquad E_{\theta s} = \eta H_{\phi s}$$

$$H_{rs} = H_{\theta s} = E_{rs} = E_{\phi s} = 0$$

we define the boundary between the near and the far zones by the value of r given by $r = \frac{2d^2}{\lambda}$

where *d* is the largest dimension of the antenna.

The time-average power density is obtained as

$$\mathcal{P}_{ave} = \frac{1}{2}\,\text{Re}\,(\mathbf{E}_S \times \mathbf{H}_s^*) = \frac{1}{2}\,\text{Re}\,(E_{\theta s} H_{\phi s}^* \, \mathbf{a}_r) = \frac{1}{2}\,\eta |H_{\phi s}|^2\, \mathbf{a}_r$$

$$P_{rad} = \int \mathcal{P}_{ave} \cdot d\mathbf{S} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{I_o^2 \eta \beta^2\, dl^2}{32\pi^2 r^2} \sin^2\theta \, r^2 \sin\theta \, d\theta \, d\phi = \frac{I_o^2 \eta \beta^2\, dl^2}{32\pi^2}\, 2\pi \int_0^{\pi} \sin^3\theta \, d\theta$$

$$\int_0^{\pi} \sin^3\theta \, d\theta = \int_0^{\pi} (1 - \cos^2\theta)\, d(-\cos\theta) = \left.\frac{\cos^3\theta}{3} - \cos\theta \right|_0^{\pi} = \frac{4}{3}$$

$$\beta^2 = 4\pi^2/\lambda^2.$$

$$P_{rad} = \frac{I_o^2 \pi \eta}{3} \left[\frac{dl}{\lambda}\right]^2$$

If free space is the medium of propagation, $\eta = 120\pi$ and

$$P_{rad} = 40\pi^2 \left[\frac{dl}{\lambda}\right]^2 I_o^2$$

This power is equivalent to the power dissipated in a fictitious resistance R_{rad} by current $I = I_o \cos \omega t$ that is

$$P_{rad} = I_{rms}^2 R_{rad}$$

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$$

where I_{rms} is the root-mean-square value of I.

$$R_{rad} = \frac{2P_{rad}}{I_o^2}$$

$$R_{rad} = 80\pi^2 \left[\frac{dl}{\lambda}\right]^2$$

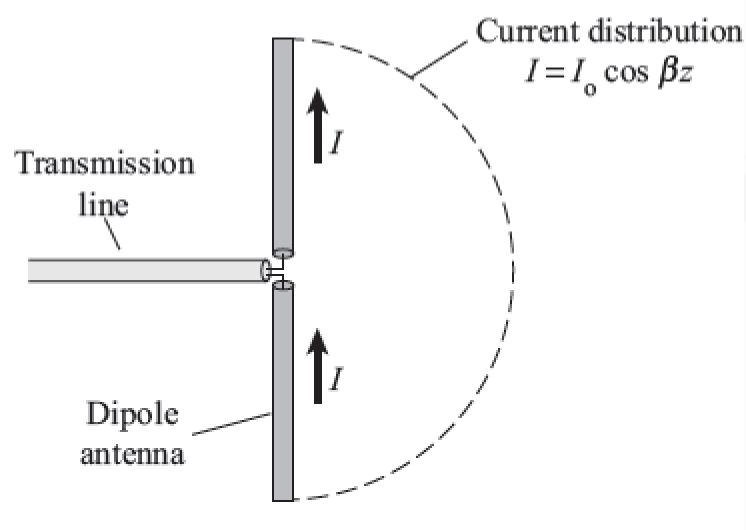
The resistance R_{rad} is a characteristic property of the Hertzian dipole antenna and is called its *radiation resistance.*

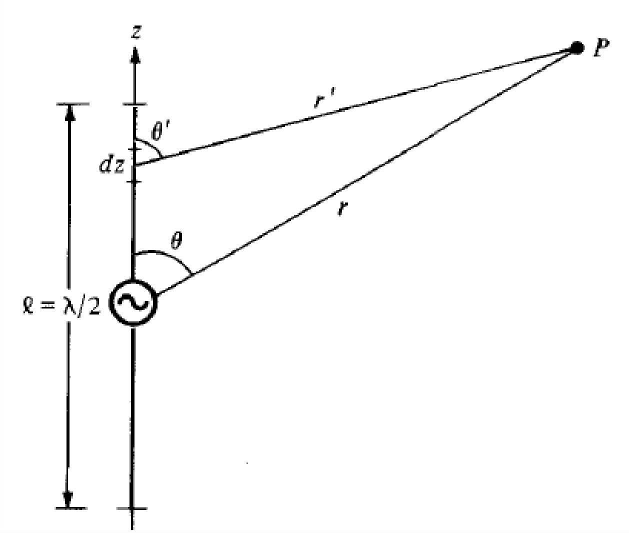
- Antennas with large radiation resistance can deliver high amounts of power
- An Hertzian dipole has a small radiation resistance (approx. 2 ohms) and hence it cannot be easily matched to a real transmission line
- A more useful antenna is a half wave dipole

Half-wave dipole antenna

- The half-wave dipole derives its name from the fact that its length is half a wavelength $(\ell = \lambda/2)$.
- It consists of a thin wire fed or excited at the midpoint by a voltage source connected to the antenna via a transmission line (e.g., a two-wire line).
- The field due to the dipole can be easily obtained if we consider it as consisting of a chain of Hertzian dipoles.
- The magnetic vector potential at *P* due to a differential length *dl(= dz)* of the dipole carrying a phasor current $I_s = I_o \cos \beta z$ is

$$dA_{zs} = \frac{\mu I_o \cos \beta z \, dz}{4\pi r'} e^{-j\beta r'}$$





$r \gg \ell,$

$r' \simeq r$

For the phase term in the numerator replace

r' by $r - z\cos\theta$

$$A_{zs} = \frac{\mu I_o}{4\pi r}\int_{-\lambda/4}^{\lambda/4} e^{-j\beta(r - z\cos\theta)}\cos\beta z\, dz$$

$$= \frac{\mu I_o}{4\pi r}e^{-j\beta r}\int_{-\lambda/4}^{\lambda/4} e^{j\beta z\cos\theta}\cos\beta z\, dz$$

From the integral tables of Appendix A.8,

$$\int e^{az}\cos bz\, dz = \frac{e^{az}(a\cos bz + b\sin bz)}{a^2 + b^2}$$

Applying this to eq. (13.15) gives

$$A_{zs} = \frac{\mu I_o e^{-j\beta r} e^{j\beta z\cos\theta}}{4\pi r}\,\frac{(j\beta\cos\theta\cos\beta z + \beta\sin\beta z)}{-\beta^2\cos^2\theta + \beta^2}\Bigg|_{-\lambda/4}^{\lambda/4}$$

Since $\beta = 2\pi/\lambda$ or $\beta\,\lambda/4 = \pi/2$ and $-\cos^2\theta + 1 = \sin^2\theta$, eq. (13.16) becomes

$$A_{zs} = \frac{\mu I_o e^{-j\beta r}}{4\pi r\beta^2\sin^2\theta}\left[e^{j(\pi/2)\cos\theta}(0 + \beta) - e^{-j(\pi/2)\cos\theta}(0 - \beta)\right]$$

Using the identity $e^{jx} + e^{-jx} = 2\cos x$, we obtain

$$A_{zs} = \frac{\mu I_o e^{-j\beta r}\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{2\pi r\beta\sin^2\theta}$$

We use eq. (13.4) in conjunction with the fact that $\mathbf{B}_s = \mu \mathbf{H}_s = \nabla \times \mathbf{A}_s$ and $\nabla \times \mathbf{H}_s = j\omega\varepsilon \mathbf{E}_s$ to obtain the magnetic and electric fields at far zone (discarding the $1/r^3$ and $1/r^2$ terms) as

$$\boxed{H_{\phi s} = \frac{jI_o e^{-j\beta r} \cos\left(\dfrac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}, \qquad E_{\theta s} = \eta H_{\phi s}} \tag{13.19}$$

Notice again that the radiation term of $H_{\phi s}$ and $E_{\theta s}$ are in time phase and orthogonal.

Using eqs. (13.9) and (13.19), we obtain the time-average power density as

$$\begin{aligned} \mathcal{P}_{\text{ave}} &= \frac{1}{2}\eta \, |H_{\phi s}|^2 \, \mathbf{a}_r \\ &= \frac{\eta I_o^2 \cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{8\pi^2 r^2 \sin^2\theta} \mathbf{a}_r \end{aligned} \tag{13.20}$$

The time-average radiated power can be determined as

$$\begin{aligned} P_{\text{rad}} &= \int \mathcal{P}_{\text{ave}} \cdot d\mathbf{S} \\ &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{\eta I_o^2 \cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{8\pi^2 r^2 \sin^2\theta} r^2 \sin\theta \, d\theta \, d\phi \\ &= \frac{\eta I_o^2}{8\pi^2} 2\pi \int_0^{\pi} \frac{\cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta \\ &= 30 \, I_o^2 \int_0^{\pi} \frac{\cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta \end{aligned} \tag{13.21}$$

where $\eta = 120\pi$ has been substituted assuming free space as the medium of propagation. Due to the nature of the integrand in eq. (13.21),

$$\int_0^{\pi/2} \frac{\cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}\,d\theta = \int_{\pi/2}^{\pi} \frac{\cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}\,d\theta$$

This is easily illustrated by a rough sketch of the variation of the integrand with θ. Hence

$$P_{\text{rad}} = 60 I_o^2 \int_0^{\pi/2} \frac{\cos^2\left(\dfrac{\pi}{2}\cos\theta\right)d\theta}{\sin\theta} \qquad (13.22)$$

Changing variables, $u = \cos\theta$, and using partial fraction reduces eq. (13.22) to

$$\begin{aligned} P_{\text{rad}} &= 60I_o^2 \int_0^1 \frac{\cos^2 \frac{1}{2}\pi u}{1-u^2}\, du \\ &= 30I_o^2 \left[\int_0^1 \frac{\cos^2 \frac{1}{2}\pi u}{1+u}\, du + \int_0^1 \frac{\cos^2 \frac{1}{2}\pi u}{1-u}\, du \right] \end{aligned} \tag{13.23}$$

Replacing $1 + u$ with v in the first integrand and $1 - u$ with v in the second results in

$$\begin{aligned} P_{\text{rad}} &= 30I_o^2 \left[\int_0^1 \frac{\sin^2 \frac{1}{2}\pi v}{v}\, dv + \int_1^2 \frac{\sin^2 \frac{1}{2}\pi v}{v}\, dv \right] \\ &= 30I_o^2 \int_0^2 \frac{\sin^2 \frac{1}{2}\pi v}{v}\, dv \end{aligned} \tag{13.24}$$

Changing variables, $w = \pi v$, yields

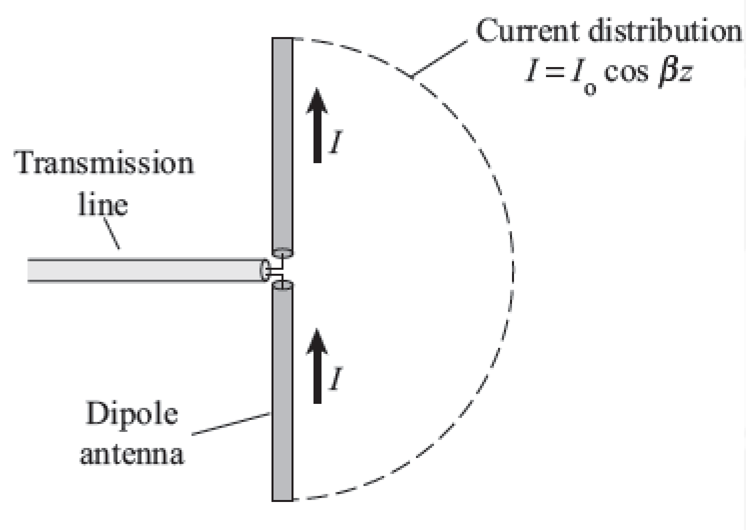
$$\begin{aligned} P_{\text{rad}} &= 30I_o^2 \int_0^{2\pi} \frac{\sin^2 \frac{1}{2} w}{w}\, dw \\ &= 15I_o^2 \int_0^{2\pi} \frac{(1 - \cos w)}{w}\, dw \\ &= 15I_o^2 \int_0^{2\pi} \left[\frac{w}{2!} - \frac{w^3}{4!} + \frac{w^5}{6!} - \frac{w^7}{8!} + \cdots \right] dw \end{aligned} \tag{13.25}$$

since $\cos w = 1 - \dfrac{w^2}{2!} + \dfrac{w^4}{4!} - \dfrac{w^6}{6!} + \dfrac{w^8}{8!} - \cdots$. Integrating eq. (13.25) term by term and evaluating at the limit leads to

$$\begin{aligned} P_{\text{rad}} &= 15I_o^2 \left[\frac{(2\pi)^2}{2(2!)} - \frac{(2\pi)^4}{4(4!)} + \frac{(2\pi)^6}{6(6!)} - \frac{(2\pi)^8}{8(8!)} + \cdots \right] \\ &\simeq 36.56\, I_o^2 \end{aligned} \tag{13.26}$$

Half-wave dipole antenna

The half-wave dipole derives its name from the fact that its length is half a wavelength. it consists of a thin wire fed or excited at the midpoint by a voltage source connected to the antenna via a transmission line (e.g., a two-wire line).



$$H_{\phi s} = \frac{j I_o e^{-j\beta r} \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}, \quad E_{\theta s} = \eta H_{\phi s}$$

$$R_{rad} = \frac{2P_{rad}}{I_o^2} \simeq 73\ \Omega$$

The half-wave dipole antenna has much high resistance and capability to radiate greater amounts of power than the Hertzian dipole

Quarter-wave monopole antenna

The quarter-wave monopole antenna consists of one-half of a half-wave dipole antenna located on a conducting ground plane as in Figure.

The monopole antenna is perpendicular to the plane, which is usually assumed to be infinite and perfectly conducting.

It is fed by a coaxial cable connected to its base.

$$P_{rad} \simeq 18.28\, I_o^2$$

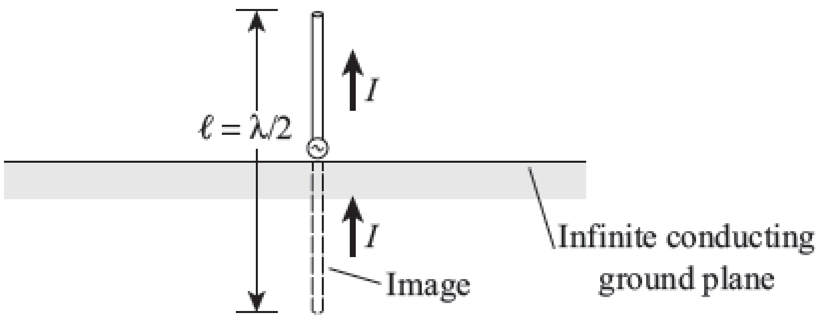


Figure 13.5 The monopole antenna.

$$R_{rad} = \frac{2P_{rad}}{I_o^2}$$

$$\boxed{R_{rad} \simeq 36.5\,\Omega}$$

A magnetic field strength of 5 μA/m is required at a point on $\theta = \pi/2$, 2 km from an antenna in air. Neglecting ohmic loss, how much power must the antenna transmit if it is

(a) A Hertzian dipole of length $\lambda/25$?

(b) A half-wave dipole?

(c) A quarter-wave monopole?

(d) A 10-turn loop antenna of radius $\rho_o = \lambda/20$?

Solution:

(a) For a Hertzian dipole,

$$|H_{\phi s}| = \frac{I_o \beta\, dl \sin\theta}{4\pi r}$$

where $dl = \lambda/25$ or $\beta\, dl = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{25} = \frac{2\pi}{25}$. Hence,

$$5 \times 10^{-6} = \frac{I_o \cdot \frac{2\pi}{25}(1)}{4\pi\,(2 \times 10^3)} = \frac{I_o}{10^5}$$

or

$$I_o = 0.5 \text{ A}$$

$$P_{rad} = 40\pi^2 \left[\frac{dl}{\lambda}\right]^2 I_o^2 = \frac{40\pi^2(0.5)^2}{(25)^2}$$
$$= 158 \text{ mW}$$

(b) For a $\lambda/2$ dipole,

$$|H_{\phi s}| = \frac{I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

$$5 \times 10^{-6} = \frac{I_o \cdot 1}{2\pi\,(2 \times 10^3) \cdot (1)}$$

or

$$I_o = 20\pi \text{ mA}$$

$$P_{rad} = 1/2 I_o^2 R_{rad} = 1/2(20\pi)^2 \times 10^{-6}(73)$$
$$= 144 \text{ mW}$$

(c) For a λ/4 monopole,

$$I_o = 20\pi \text{ mA}$$

as in part (b).

$$P_{rad} = 1/2 I_o^2 R_{rad} = 1/2(20\pi)^2 \times 10^{-6}(36.56)$$
$$= 72 \text{ mW}$$

(d) For a loop antenna,

$$|H_{\theta s}| = \frac{\pi I_o}{r} \frac{S}{\lambda^2} \sin\theta$$

For a single turn, $S = \pi\rho_o^2$. For N-turn, $S = N\pi\rho_o^2$. Hence,

$$5 \times 10^{-6} = \frac{\pi I_o 10\pi}{2 \times 10^3} \left[\frac{\rho_o}{\lambda}\right]^2$$

or

$$I_o = \frac{10}{10\pi^2} \left[\frac{\lambda}{\rho_o}\right]^2 \times 10^{-3} = \frac{20^2}{\pi^2} \times 10^{-3}$$
$$= 40.53 \text{ mA}$$

$$R_{rad} = \frac{320\ \pi^4 S^2}{\lambda^4} = 320\ \pi^6 N^2 \left[\frac{\rho_o}{\lambda}\right]^4$$
$$= 320\ \pi^6 \times 100 \left[\frac{1}{20}\right]^4 = 192.3\ \Omega$$

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad} = \frac{1}{2} (40.53)^2 \times 10^{-6} (192.3)$$
$$= 158 \text{ mW}$$

A Hertzian dipole of length $\lambda/100$ is located at the origin and fed with a current of $0.25 \sin 10^8 t$ A. Determine the magnetic field at

(a) $r = \lambda/5, \theta = 30°$

(b) $r = 200\lambda, \theta = 60°$

Answer: (a) $0.2119 \sin(10^8 t - 20.5°)\, \mathbf{a}_\phi$ mA/m, (b) $0.2871 \sin(10^8 t + 90°)\, \mathbf{a}_\phi$ μA/m.

An electric field strength of 10 μV/m is to be measured at an observation point $\theta = \pi/2$, 500 km from a half-wave (resonant) dipole antenna operating in air at 50 MHz.

(a) What is the length of the dipole?

(b) Calculate the current that must be fed to the antenna.

(c) Find the average power radiated by the antenna.

(d) If a transmission line with $Z_o = 75\ \Omega$ is connected to the antenna, determine the standing wave ratio.

Solution:

(a) The wavelength $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{50 \times 10^6} = 6$ m.

Hence, the length of the half-dipole is $\ell = \frac{\lambda}{2} = 3$ m.

(b) From eq. (13.19),

$$|E_{\phi s}| = \frac{\eta_o I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

or

$$\begin{aligned} I_o &= \frac{|E_{\theta s}|\, 2\pi r \sin\theta}{\eta_o \cos\left(\frac{\pi}{2}\cos\theta\right)} \\ &= \frac{10 \times 10^{-6}\, 2\pi\,(500 \times 10^3) \cdot (1)}{120\pi\,(1)} \\ &= 83.33 \text{ mA} \end{aligned}$$

(c)

$$R_{\text{rad}} = 73\ \Omega$$

$$\begin{aligned} P_{\text{rad}} &= \frac{1}{2} I_o^2 R_{\text{rad}} = \frac{1}{2}(83.33)^2 \times 10^{-6} \times 73 \\ &= 253.5 \text{ mW} \end{aligned}$$

(d)

$$\begin{aligned} \Gamma &= \frac{Z_L - Z_o}{Z_L + Z_o} \quad (Z_L = Z_{\text{in}} \text{ in this case}) \\ &= \frac{73 + j42.5 - 75}{73 + j42.5 + 75} = \frac{-2 + j42.5}{148 + j42.5} \\ &= \frac{42.55\angle 92.69^\circ}{153.98\angle 16.02^\circ} = 0.2763\angle 76.67^\circ \end{aligned}$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.2763}{1 - 0.2763} = 1.763$$

Antenna Characteristics

A. Antenna Patterns

An **antenna pattern** (or **radiation pattern**) is a three-dimensional plot of its radiation at far field.

When the amplitude of a specified component of the E field is plotted, it is called the *field pattern* or *voltage pattern*. When the square of the amplitude of E is plotted, it is called the *power pattern*.

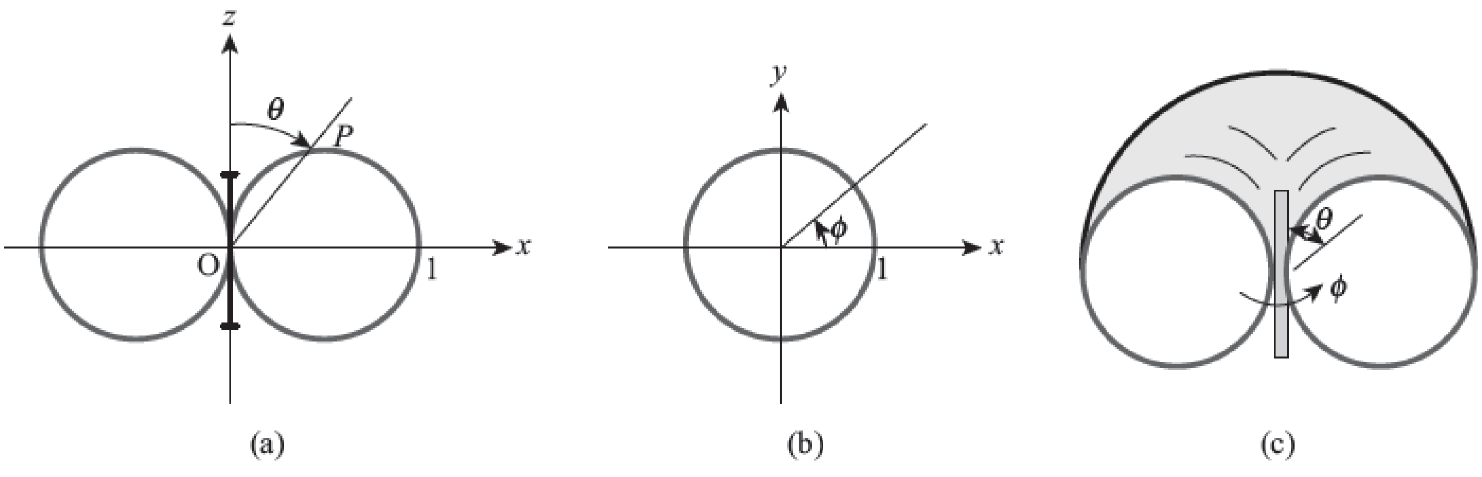


Figure 13.7 Field patterns of the Hertzian dipole: (**a**) normalized *E*-plane or vertical pattern (ϕ = constant = 0), (**b**) normalized *H*-plane or horizontal pattern ($\theta = \pi/2$), (**c**) three-dimensional pattern.

Antenna characteristics

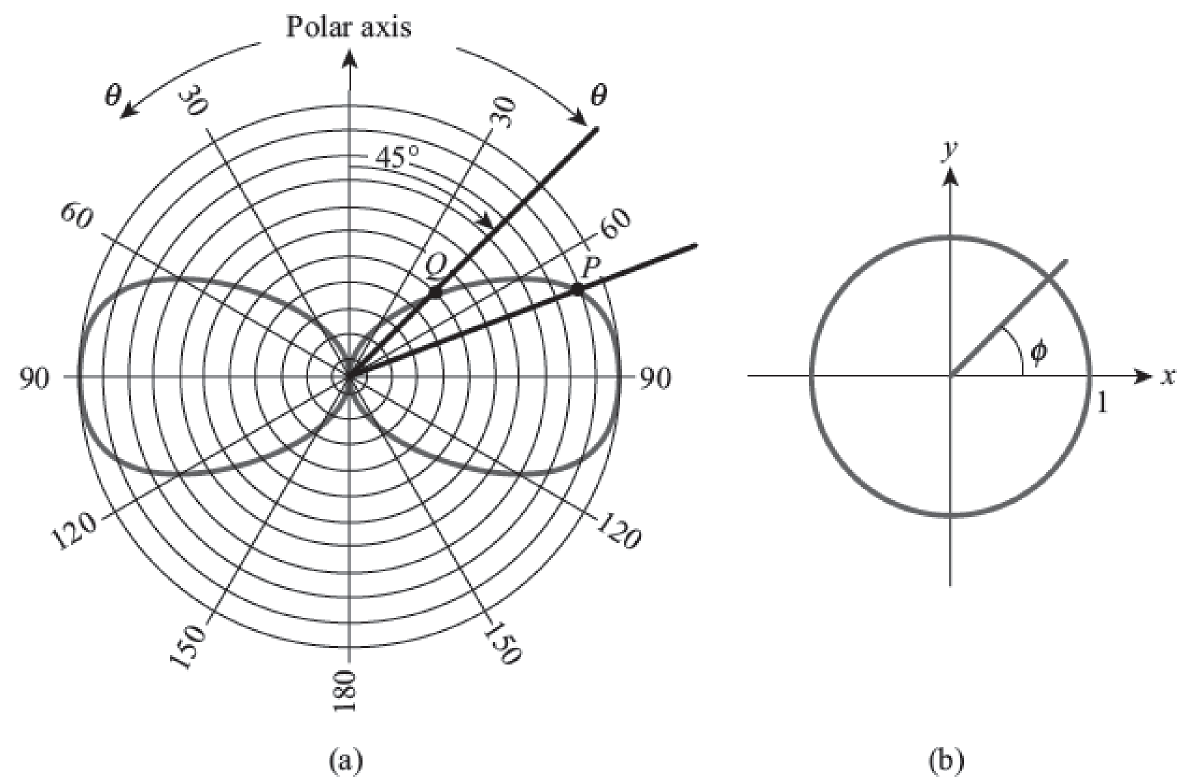


Figure 13.8 Power patterns of the Hertzian dipole: (**a**) (ϕ = constant = 0), (**b**) θ = constant = $\pi/2$.

An **antenna pattern** (or **radiation pattern**) is a three-dimensional plot of its radiation at far field.

B. Radiation Intensity

The radiation intensity of an antenna is defined as

$$U(\theta, \phi) = r^2 \mathcal{P}_{\text{ave}}$$

the total average power radiated can be expressed as

$$\begin{aligned} P_{\text{rad}} &= \oint_S \mathcal{P}_{\text{ave}} \, dS = \oint_S \mathcal{P}_{\text{ave}} \, r^2 \sin\theta \, d\theta \, d\phi \\ &= \int_S U(\theta, \phi) \sin\theta \, d\theta \, d\phi \\ &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta, \phi) \, d\Omega \end{aligned}$$

where $d\Omega = \sin\theta \, d\theta \, d\phi$ is the *differential solid angle* in steradian (sr). Hence the radiation intensity $U(\theta, \phi)$ is measured in watts per steradian (W/sr). The average value of $U(\theta, \phi)$ is the total radiated power divided by 4π sr; that is,

$$U_{\text{ave}} = \frac{P_{\text{rad}}}{4\pi}$$

C. Directive Gain

The **directive gain** $G_d(\theta, \phi)$ of an antenna is a measure of the concentration of the radiated power in a particular direction (θ, ϕ).

$$G_d(\theta, \phi) = \frac{U(\theta, \phi)}{U_{ave}} = \frac{4\pi\, U(\theta, \phi)}{P_{rad}}$$

$$\mathcal{P}_{ave} = \frac{G_d}{4\pi r^2} P_{rad}$$

The **directivity** D of an antenna is the ratio of the maximum radiation intensity to the average radiation intensity.

$$D = \frac{U_{max}}{U_{ave}} = G_{d,}\ \text{max}$$

or

$$D = \frac{4\pi\, U_{max}}{P_{rad}}$$

$D = 1$ for an isotropic antenna; this is the smallest value D can have. For the Hertzian dipole,

$$G_d(\theta, \phi) = 1.5 \sin^2 \theta, \qquad D = 1.5. \tag{13.45}$$

For the $\lambda/2$ dipole,

$$G_d(\theta, \phi) = \frac{\eta}{\pi R_{rad}} f^2(\theta), \qquad D = 1.64 \tag{13.46}$$

where $\eta = 120\pi$, $R_{rad} = 73\ \Omega$, and

$$f(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \tag{13.47}$$

D. Power Gain

P_{in} is the power accepted by the antenna at its terminals during the radiation process, and P_{rad} is the power radiated by the antenna; the difference between the two powers is *P(*, the power dissipated within the antenna.

We define the *power gain* $G_p(\theta, \phi)$ of the antenna as

$$G_p(\theta, \phi) = \frac{4\pi\, U(\theta, \phi)}{P_{in}}$$

The ratio of the power gain in any specified direction to the directive gain in that direction is referred to as the *radiation efficiency*

$$\eta_r = \frac{G_P}{G_d} = \frac{P_{rad}}{P_{in}}$$

Show that the directive gain of the Hertzian dipole is

$$G_d(\theta, \phi) = 1.5 \sin^2 \theta$$

and that of the half-wave dipole is

$$G_d(\theta, \phi) = 1.64 \frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin^2 \theta}$$

Solution:

$$G_d(\theta, \phi) = \frac{U(\theta, \phi)}{U_{ave}} = \frac{4\pi\, U(\theta, \phi)}{P_{rad}}$$

$$U(\theta, \phi) = r^2\, \mathcal{P}_{ave}$$

$$G_d(\theta, \phi) = \frac{4\pi f^2(\theta)}{\int f^2(\theta)\, d\Omega}$$

(a) For the Hertzian dipole,

$$G_d(\theta, \phi) = \frac{4\pi \sin^2\theta}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin^3\theta\, d\theta\, d\phi} = \frac{4\pi \sin^2\theta}{2\pi\,(4/3)}$$
$$= 1.5 \sin^2\theta$$

$d\Omega = \sin\theta\, d\theta\, d\phi$

as required.

(b) For the half-wave dipole,

$$G_d(\theta, \phi) = \frac{\dfrac{4\pi \cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}}{\displaystyle\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right) d\theta\, d\phi}{\sin\theta}}$$

From eq. (13.26), the integral in the denominator gives $2\pi(1.2188)$. Hence,

$$G_d(\theta, \phi) = \frac{4\pi \cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \cdot \frac{1}{2\pi\,(1.2188)}$$
$$= 1.64\, \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

The radiation intensity of a certain antenna is

$$U(\theta, \phi) = \begin{cases} 2 \sin\theta \sin^3\phi, & 0 \le \theta \le \pi, 0 \le \phi \le \pi \\ 0, & \text{elsewhere} \end{cases}$$

Determine the directivity of the antenna.

Solution:

The directivity is defined as

$$D = \frac{U_{\max}}{U_{\text{ave}}}$$

From the given U,

$$U_{\max} = 2$$

$$\begin{aligned}
U_{\text{ave}} &= \frac{1}{4\pi}\int U\,d\Omega\ (=P_{\text{rad}}/4\pi) \\
&= \frac{1}{4\pi}\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} 2\sin\theta\sin^3\phi\sin\theta\,d\theta\,d\phi \\
&= \frac{1}{2\pi}\int_0^{\pi}\sin^2\theta\,d\theta\int_0^{\pi}\sin^3\phi\,d\phi \\
&= \frac{1}{2\pi}\int_0^{\pi}\frac{1}{2}(1-\cos 2\theta)\,d\theta\int_0^{\pi}(1-\cos^2\phi)\,d(-\cos\phi) \\
&= \frac{1}{2\pi}\frac{1}{2}\left(\theta-\frac{\sin 2\theta}{2}\right)\bigg|_0^{\pi}\left(\frac{\cos^3\phi}{3}-\cos\phi\right)\bigg|_0^{\pi} \\
&= \frac{1}{2\pi}\left(\frac{\pi}{2}\right)\left(\frac{4}{3}\right) = \frac{1}{3}
\end{aligned}$$

Hence

$$D = \frac{2}{(1/3)} = 6$$

Evaluate the directivity of an antenna with normalized radiation intensity

$$U(\theta, \phi) = \begin{cases} \sin\theta, & 0 \le \theta \le \pi/2, 0 \le \phi \le 2\pi \\ 0, & \text{otherwise} \end{cases}$$

Answer: 2.546.