Electromagnetics Waves
5EC4-02

Chapter 5 Waveguide

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5EC4-02: Electromagnetics Waves

Credit: 3 | **Max. Marks: 150(IA:30, ETE:120)**
3L+0T+0P | **End Term Exam: 3 Hours**

SN	Contents	Hours
1	Introduction: Objective, scope and outcome of the course.	01
2	Transmission Lines-Equations of Voltage and Current on TX line, Propagation constant and characteristic impedance, and reflection coefficient and VSWR, Impedance Transformation on Loss-less and Low loss Transmission line, Power transfer on TX line, Smith Chart, Admittance Smith Chart, Applications of transmission lines: Impedance Matching, use transmission line sections as circuit elements.	08
3	Maxwell's Equations-Basics of Vectors, Vector calculus, Basic laws of Electromagnetics,Maxwell's Equations, Boundary conditions at Media Interface.	03
4	Uniform Plane Wave-Uniform plane wave, Propagation of wave, Wave polarization, Poincare's Sphere, Wave propagation in conducting medium, phase and group velocity, Power flow and Poynting vector, Surface current and power loss in a conductor.	08
5	Plane Waves at a Media Interface-Plane wave in arbitrary direction, Reflection and refraction at dielectric interface, Total internal reflection, wave polarization at media interface, Reflection from a conducting boundary.	07
6	Waveguides- Wave propagation in parallel plate waveguide, Analysis of waveguide general approach, Rectangular waveguide, Modal propagation in rectangular waveguide, Surface currents on the waveguide walls, Field visualization, Attenuation in waveguide.	08
7	Radiation-Solution for potential function, Radiation from the Hertz dipole, Power radiated by hertz dipole, Radiation Parameters of antenna, receiving antenna, Monopole and Dipole antenna	07
	Total	**42**

Text Books

Elements of electromagnetics by matthew n.o. sadiku
Microwave Devices and Circuits by Samuel Y. **Liao**.

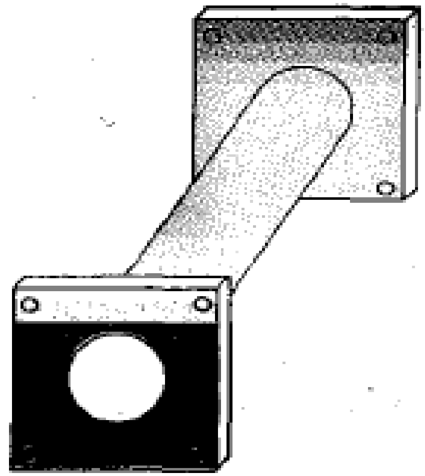
WAVEGUIDES

- Transmission line can be used to guide EM energy from one point (generator) to another (load).
- A waveguide is another means of achieving the same goal.

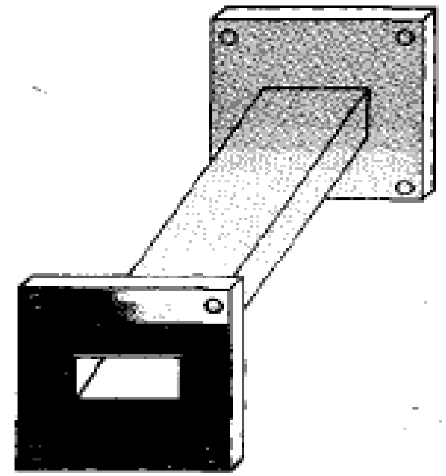
However, a waveguide differs from a transmission line in following aspects.

  - A transmission line can support only a transverse electromagnetic (TEM) wave, whereas a waveguide can support many possible field configurations.
  - At microwave frequencies (roughly 3-300 GHz), transmission lines become inefficient due to skin effect and dielectric losses; waveguides are used at that range of frequencies to obtain larger bandwidth and lower signal attenuation. Moreover, a transmission line may operate from dc to a very high frequency.

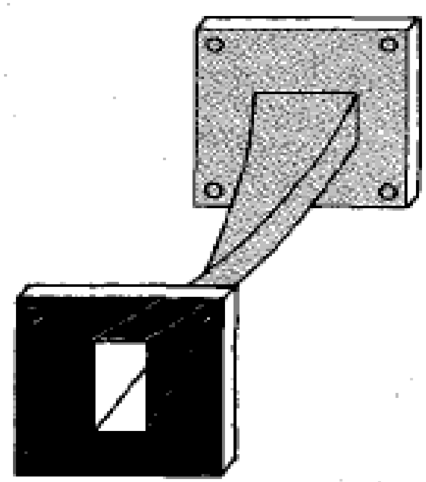
- A waveguide can operate only above a certain frequency called the *cutoff frequency* and therefore acts as a high-pass filter. Thus, waveguides cannot transmit dc, and they become excessively large at frequencies below microwave frequencies.
- Although a waveguide may assume any arbitrary but uniform cross section, common
- waveguides are either rectangular or circular.



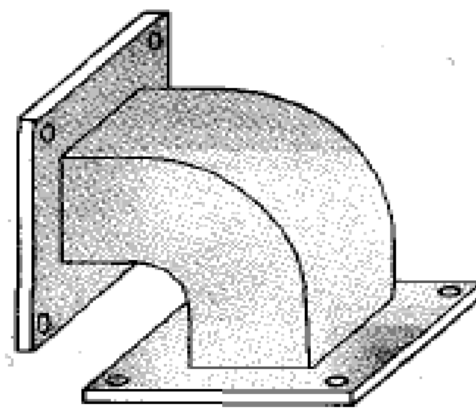
Circular



Rectangular



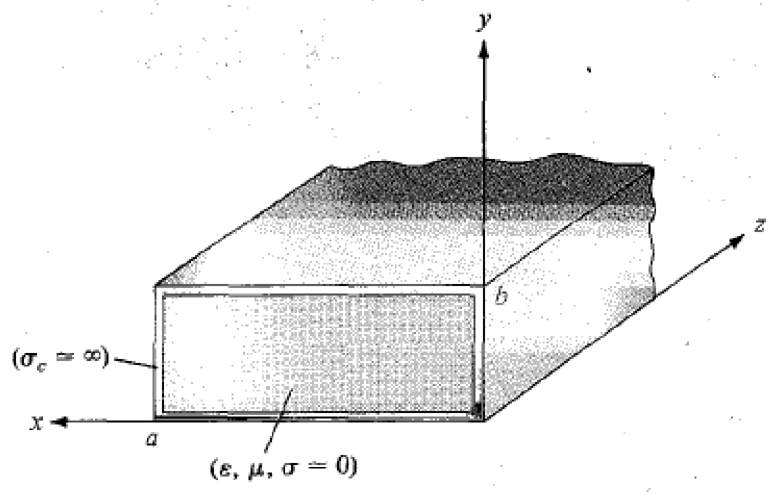
Twist

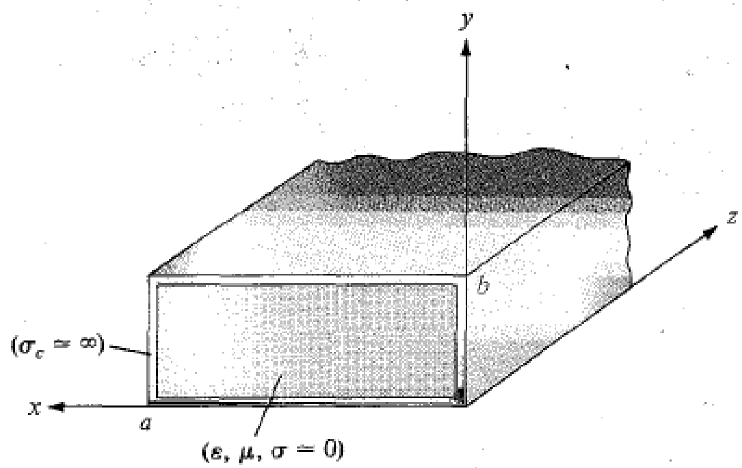


90° elbow

RECTANGULAR WAVEGUIDES

Assume that the waveguide is filled with a source-free lossless dielectric material and its walls are perfectly conducting.





$$\nabla^2 \mathbf{E}_s + k^2 \mathbf{E}_s = 0 \qquad (12.1)$$

$$\nabla^2 \mathbf{H}_s + k^2 \mathbf{H}_s = 0 \qquad (12.2)$$

$$\boxed{k = \omega\sqrt{\mu\varepsilon}}$$

$$\mathbf{E}_s = (E_{xs}, E_{ys}, E_{zs}) \qquad \text{and} \qquad \mathbf{H}_s = (H_{xs}, H_{ys}, H_{zs})$$

$$\frac{\partial^2 E_{zs}}{\partial x^2} + \frac{\partial^2 E_{zs}}{\partial y^2} + \frac{\partial^2 E_{zs}}{\partial z^2} + k^2 E_{zs} = 0 \qquad (12.4)$$

$\nabla \times H = (\sigma + j\omega\epsilon)E$ — ①

$\nabla \times E = -j\omega\mu H$ — ②

$\nabla \times \nabla \times E = -j\omega\mu \, \nabla \times H$

$-\nabla^2 E + \nabla(\nabla \cdot E) = -j\omega\mu(\sigma + j\omega\epsilon)E$

∵ for perfect cond. $\rho = 0 \Rightarrow \nabla \cdot D = \nabla \cdot E = 0$

$\Rightarrow \nabla^2 E = j\omega\mu(\sigma + j\omega\epsilon)E \Rightarrow \nabla^2 E = \gamma^2 E$ — ③ where $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$= \alpha + j\beta$

similarly for H. $\nabla^2 H = \gamma^2 H$ — ④

eqn ③ and ④ satisfy $\nabla^2 \psi = \gamma^2 \psi$ ∴ Helmholtz eqn

$$E_{zs}(x, y, z) = X(x)\, Y(y)\, Z(z) \tag{12.5}$$

where $X(x)$, $Y(y)$, and $Z(z)$ are functions of x, y, and z, respectively. Substituting eq. (12.5) into eq. (12.4) and dividing by XYZ gives

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2 \tag{12.6}$$

Since the variables are independent, each term in eq. (12.6) must be constant, so the equation can be written as

$$-k_x^2 - k_y^2 + \gamma^2 = -k^2 \tag{12.7}$$

where $-k_x^2$, $-k_y^2$, and γ^2 are separation constants. Thus, eq. (12.6) is separated as

$$X'' + k_x^2 X = 0 \tag{12.8a}$$

$$Y'' + k_y^2 Y = 0 \tag{12.8b}$$

$$Z'' - \gamma^2 Z = 0 \tag{12.8c}$$

$$X(x) = c_1 \cos k_x x + c_2 \sin k_x x \tag{12.9a}$$

$$Y(y) = c_3 \cos k_y y + c_4 \sin k_y y \tag{12.9b}$$

$$Z(z) = c_5 e^{\gamma z} + c_6 e^{-\gamma z} \tag{12.9c}$$

Substituting eq. (12.9) into eq. (12.5) gives

$$\begin{aligned} E_{zs}(x, y, z) = {} & (c_1 \cos k_x x + c_2 \sin k_x x)(c_3 \cos k_y y \\ & + c_4 \sin k_y y)\,(c_5 e^{\gamma z} + c_6 e^{-\gamma z}) \end{aligned} \tag{12.10}$$

$$E_{zs}(x, y, z) = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)e^{-\gamma z} \qquad (12.11)$$

where $A_1 = c_1 c_6$, $A_2 = c_2 c_6$, and so on.

constant $c_5 = 0$ because the wave has to be finite at infinity

$$H_{zs}(x, y, z) = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y)e^{-\gamma z} \qquad (12.12)$$

$$\nabla \times \mathbf{E}_s = -j\omega\mu \mathbf{H}_s$$

$$\nabla \times \mathbf{H}_s = j\omega\varepsilon \mathbf{E}_s$$

$$\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z} = -j\omega\mu H_{xs} \qquad (12.13a)$$

$$\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} = j\omega\varepsilon E_{xs} \qquad (12.13b)$$

$$\frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x} = j\omega\mu H_{ys} \qquad (12.13c)$$

$$\frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x} = j\omega\varepsilon E_{ys} \qquad (12.13d)$$

$$\frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y} = -j\omega\mu H_{zs} \qquad (12.13e)$$

$$\frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y} = j\omega\varepsilon E_{zs} \qquad (12.13f)$$

$$\nabla \times E = -j\omega\mu H \quad — (1)$$

$$\nabla \times H = j\omega\epsilon E \quad — (2)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu(H_x \hat{i} + H_y \hat{j} + H_z \hat{k})$$

$$\hat{i}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \hat{j}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \hat{k}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -j\omega\mu(H_x \hat{i} + H_y \hat{j} + H_z \hat{k})$$

$$E_{xs} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y} \quad (12.15a)$$

$$E_{ys} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x} \quad (12.15b)$$

$$H_{xs} = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial x} \quad (12.15c)$$

$$H_{ys} = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial y} \quad (12.15d)$$

where

$$h^2 = \gamma^2 + k^2 = k_x^2 + k_y^2 \quad (12.16)$$

From eqs. (12.11), (12.12), and (12.15), we notice that there are different types of field patterns or configurations. Each of these distinct field patterns is called a *mode*. Four different mode categories can exist, namely:

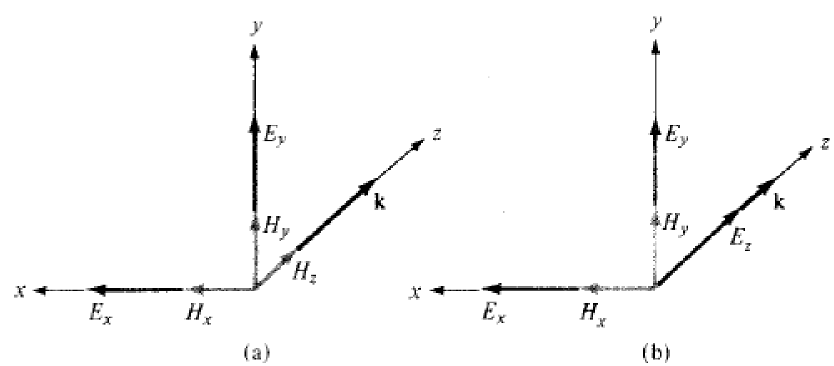


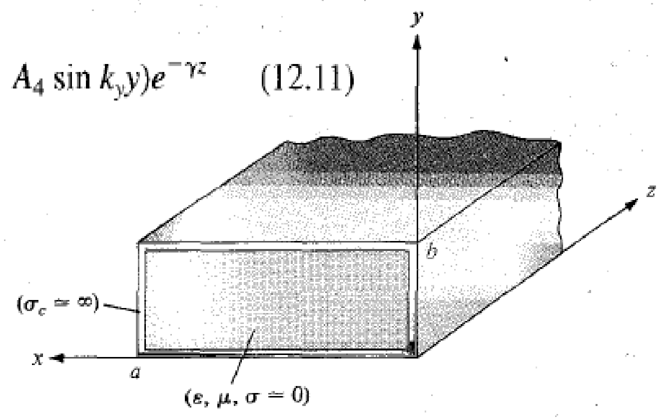
Figure 12.3 Components of EM fields in a rectangular waveguide: **(a)** TE mode $E_z = 0$, **(b)** TM mode, $H_z = 0$.

1. $E_{zs} = 0 = H_{zs}$ (TEM mode): This is the *transverse electromagnetic* (TEM) mode, in which both the **E** and **H** fields are transverse to the direction of wave propagation. From eq. (12.15), all field components vanish for $E_{zs} = 0 = H_{zs}$. Consequently, we conclude that a rectangular waveguide cannot support TEM mode.

2. $E_{zs} = 0$, $H_{zs} \neq 0$ (TE modes): For this case, the remaining components (E_{xs} and E_{ys}) of the electric field are transverse to the direction of propagation $\mathbf{a}_z$. Under this condition, fields are said to be in *transverse electric* (TE) modes. See Figure 12.3(a).
3. $E_{zs} \neq 0$, $H_{zs} = 0$ (TM modes): In this case, the **H** field is transverse to the direction of wave propagation. Thus we have *transverse magnetic* (TM) modes. See Figure 12.3(b).
4. $E_{zs} \neq 0$, $H_{zs} \neq 0$ (HE modes): This is the case when neither **E** nor **H** field is transverse to the direction of wave propagation. They are sometimes referred to as *hybrid* modes.

TRANSVERSE MAGNETIC (TM) MODES

Boundary conditions: $E_{zs}(x, y, z) = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)e^{-\gamma z}$ (12.11)



At the walls of the waveguide, the tangential components of the E field must be continuous; that is,

$$E_{zs} = 0 \quad \text{at} \quad y = 0 \tag{12.17a}$$

$$E_{zs} = 0 \quad \text{at} \quad y = b \tag{12.17b}$$

$$E_{zs} = 0 \quad \text{at} \quad x = 0 \tag{12.17c}$$

$$E_{zs} = 0 \quad \text{at} \quad x = a \tag{12.17d}$$

Equations (12.17a) and (12.17c) require that $A_1 = 0 = A_3$ in eq. (12.11), so eq. (12.11) becomes

$$E_{zs} = E_o \sin k_x x \sin k_y y \, e^{-\gamma z} \tag{12.18}$$

where $E_o = A_2 A_4$.

Also eqs. (12.17b) and (12.17d) when applied to eq. (12.18) require that

$$\sin k_x a = 0, \qquad \sin k_y b = 0 \tag{12.19}$$

This implies that

$$k_x a = m\pi, \qquad m = 1, 2, 3, \ldots \tag{12.20a}$$

$$k_y b = n\pi, \qquad n = 1, 2, 3, \ldots \tag{12.20b}$$

or

$$k_x = \frac{m\pi}{a}, \qquad k_y = \frac{n\pi}{b} \tag{12.21}$$

Substituting eq. (12.21) into eq. (12.18) gives

$$E_{zs} = E_o \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

We obtain other field components from eqs. (12.22) and (12.15)

$$E_{xs} = -\frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_{ys} = -\frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) E_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_{xs} = \frac{j\omega\varepsilon}{h^2}\left(\frac{n\pi}{b}\right) E_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_{ys} = -\frac{j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

where

$$h^2 = k_x^2 + k_y^2 = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

$$h^2 = \gamma^2 + k^2 = k_x^2 + k_y^2$$

- Integer m equals the number of half-cycle variations in the x direction, and integer n is the number of half-cycle variations in the y-direction.

- Also notice from eqs. (12.22) and (12.23) that if (m, n) is (0, 0), (0, n), or (m, 0), all field components vanish. Thus neither m nor n can be zero. Consequently, TM_{11} is the lowest-order mode of all the TM_{mn} modes.

By substituting eq. (12.21) into eq. (12.16), we obtain the propagation constant

$$\gamma = \sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2 - k^2} \qquad (12.25)$$

where $k = \omega\sqrt{\mu\varepsilon}$ $\qquad \gamma = \alpha + j\beta.$

CASE A (cutoff):

$$h^2 = k_x^2 + k_y^2 = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

$$h^2 = \gamma^2 + k^2 = k_x^2 + k_y^2$$

$$k^2 = \omega^2\mu\varepsilon = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

$$\gamma = 0 \qquad \text{or} \qquad \alpha = 0 = \beta$$

The value of ω that causes this is called the *cutoff angular frequency* ω_c; that is,

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2} \qquad (12.26)$$

CASE B (evanescent):

$$k^2 = \omega^2\mu\varepsilon < \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

$$\gamma = \alpha, \qquad \beta = 0$$

In this case, we have no wave propagation at all. These nonpropagating or attenuating modes are said to be *evanescent.*

CASE C (propagation):

$$k^2 = \omega^2\mu\varepsilon > \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

$$\gamma = j\beta, \qquad \alpha = 0$$

from eq. (12.25) the phase constant becomes

$$\beta = \sqrt{k^2 - \left[\frac{m\pi}{a}\right]^2 - \left[\frac{n\pi}{b}\right]^2} \tag{12.27}$$

This is the only case when propagation takes place because all field components will have the factor $e^{-\gamma z} = e^{-j\beta z}$.

Thus for each mode, characterized by a set of integers m and n, there is a corresponding *cutoff frequency* f_c

The **cutoff frequency** is the operating frequency below which attenuation occurs and above which propagation takes place.

The waveguide therefore operates as a high-pass filter. The cutoff frequency is obtained from eq. (12.26) as

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2}$$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u' = \frac{1}{\sqrt{\mu\varepsilon}}$$

phase velocity of uniform plane wave in the lossless dielectric medium filling the waveguide.

The *cutoff wave length* is given by

$$\lambda_c = \frac{u'}{f_c}$$

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

The phase constant in eq. (12.27) can be written in terms of f_c as

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

The phase velocity u_p and the wavelength in the guide are, respectively, given by

$$u_p = \frac{\omega}{\beta}, \qquad \lambda = \frac{2\pi}{\beta} = \frac{u_p}{f}$$

The intrinsic wave impedance of the mode is obtained from eq. (12.23) as

$$\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\beta}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

$$\eta_{TM} = \eta'\sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

where $\eta' = \sqrt{\mu/\varepsilon}$ = intrinsic impedance of uniform plane wave in the medium.

Field configuration

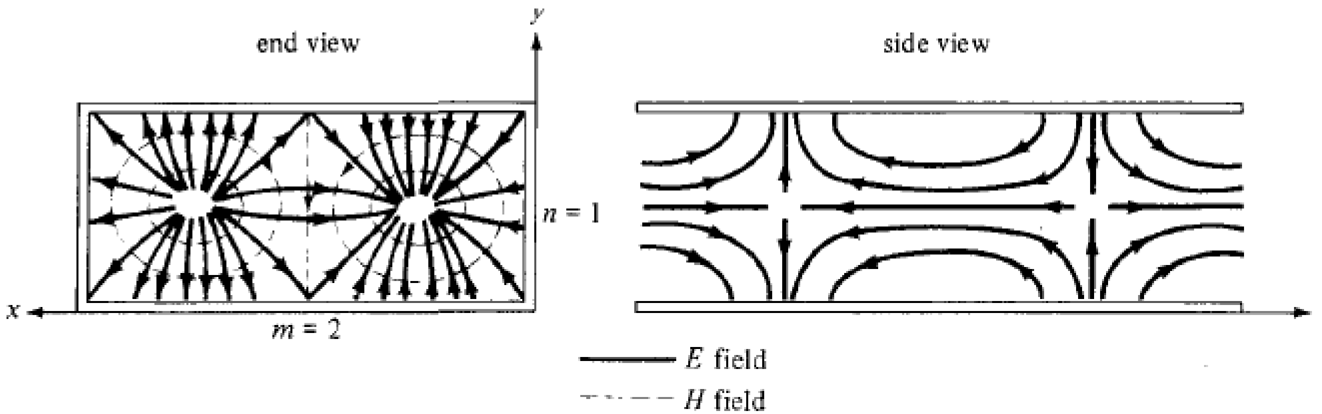
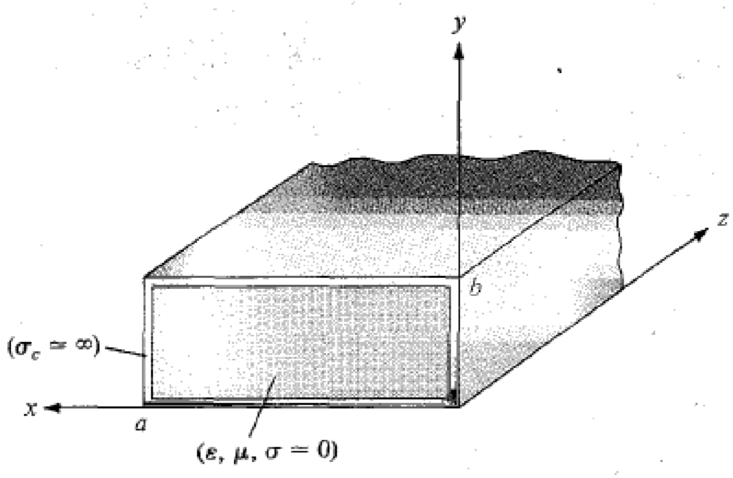


Figure 12.4 Field configuration for TM_{21} mode.



TRANSVERSE ELECTRIC (TE) MODES

In the TE modes, the electric field is transverse (or normal) to the direction of wave propagation.

set $E_z = 0$ and determine other field components E_x, E_y, H_x, H_y, and H_z

$$\boxed{H_{zs} = H_o \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}} \tag{12.35}$$

$$E_{xs} = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \tag{12.36a}$$

$$E_{ys} = -\frac{j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right) H_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \tag{12.36b}$$

$$H_{xs} = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) H_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \tag{12.36c}$$

$$H_{ys} = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) H_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \tag{12.36d}$$

For TE modes, (m, n) may be (0, 1) or (1, 0) but not (0, 0). Both m and n cannot be zero at the same time because this will force the field components in eq. (12.36) to vanish.

This implies that the lowest mode can be TE_{10} or TE_{01} depending on the values of a and b, the dimensions of the guide.

Standard practice to have a > b

Thus TE_{10} is the lowest mode because. This mode is called the dominant mode of **the** waveguide and is of practical importance. The cutoff frequency for the TE10 mode is obtained from eq. (12.28) as (m = 1, n= 0)

$$f_{c_{10}} = \frac{u'}{2a}$$

The **dominant mode** is the mode with the lowest cutoff frequency (or longest cutoff wavelength).

The intrinsic impedance for the TE mode is not the same as for TM modes. From eq. (12.36),

$$\eta_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{\sqrt{1 - \left[\frac{f_c}{f}\right]^2}}$$

$$\boxed{\eta_{TE} = \frac{\eta'}{\sqrt{1 - \left[\frac{f_c}{f}\right]^2}}}$$

$$\eta_{TE}\,\eta_{TM} = \eta'^2$$

Field configuration

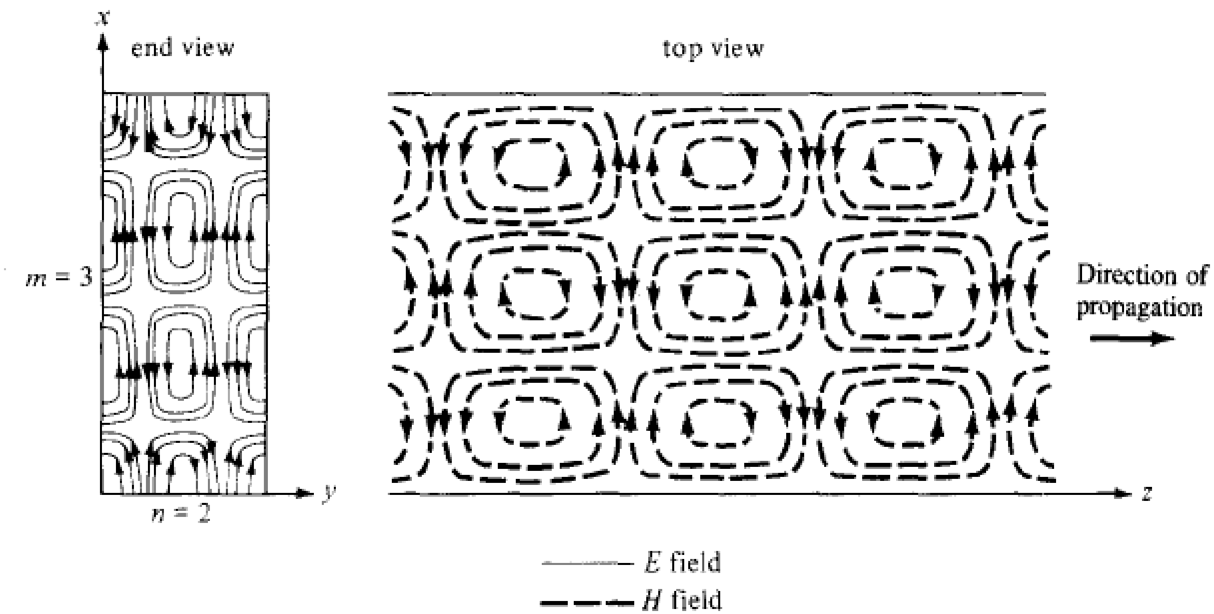


Figure 12.5 Field configuration for TE_{32} mode.

Important Equations for TM and TE Modes

TM Modes	TE Modes
$E_{xs} = -\frac{j\beta}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$E_{xs} = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
$E_{ys} = -\frac{j\beta}{h^2}\left(\frac{n\pi}{b}\right) E_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$E_{ys} = -\frac{j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right) H_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
$E_{zs} = E_o \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$E_{zs} = 0$
$H_{xs} = \frac{j\omega\varepsilon}{h^2}\left(\frac{n\pi}{b}\right) E_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$H_{xs} = \frac{j\beta}{h^2}\left(\frac{m\pi}{a}\right) H_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
$H_{ys} = -\frac{j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$H_{ys} = \frac{j\beta}{h^2}\left(\frac{n\pi}{b}\right) H_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
$H_{zs} = 0$	$H_{zs} = H_o \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
$\eta = \eta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$	$\eta = \frac{\eta'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\lambda_c = \frac{u'}{f_c}$$

$$\beta = \beta'\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$u_p = \frac{\omega}{\beta} = f\lambda$$

$$\text{where } h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2,\ u' = \frac{1}{\sqrt{\mu\varepsilon}},\ \beta' = \frac{\omega}{u'},\ \eta' = \sqrt{\frac{\mu}{\varepsilon}}$$

EXAMPLE PROBLEMS

In a rectangular waveguide for which $a = 1.5$ cm, $b = 0.8$ cm, $\sigma = 0$, $\mu = \mu_o$, and $\varepsilon = 4\varepsilon_o$,

$$H_x = 2 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \sin\left(\pi \times 10^{11} t - \beta z\right) \text{ A/m}$$

Determine

(a) The mode of operation

(b) The cutoff frequency

(c) The phase constant β

(d) The propagation constant γ

(e) The intrinsic wave impedance η.

In a rectangular waveguide for which $a = 1.5$ cm, $b = 0.8$ cm, $\sigma = 0$, $\mu = \mu_o$, and $\varepsilon = 4\varepsilon_o$,

$$H_x = 2 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \sin(\pi \times 10^{11} t - \beta z) \text{ A/m}$$

Determine

(a) The mode of operation

(b) The cutoff frequency

(c) The phase constant β

(d) The propagation constant γ

(e) The intrinsic wave impedance η.

Solution: The guide is operating at TM_{13} or TE_{13}

(b)

$$f_{c_{mn}} = \frac{u'}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$$u' = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{2}$$

Hence

$$f_{c_{13}} = \frac{c}{4}\sqrt{\frac{1}{[1.5 \times 10^{-2}]^2} + \frac{9}{[0.8 \times 10^{-2}]^2}}$$

$$= \frac{3 \times 10^8}{4}(\sqrt{0.444 + 14.06}) \times 10^2 = 28.57 \text{ GHz}$$

(c)

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left[\frac{f_c}{f}\right]^2} = \frac{\omega\sqrt{\varepsilon_r}}{c}\sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

$$\omega = 2\pi f = \pi \times 10^{11} \quad \text{or} \quad f = \frac{100}{2} = 50 \text{ GHz}$$

$$\beta = \frac{\pi \times 10^{11}(2)}{3 \times 10^8}\sqrt{1 - \left[\frac{28.57}{50}\right]^2} = 1718.81 \text{ rad/m}$$

(d) $\gamma = j\beta = j1718.81$ /m

(e)

$$\eta_{TM_{13}} = \eta'\sqrt{1 - \left[\frac{f_c}{f}\right]^2} = \frac{377}{\sqrt{\varepsilon_r}}\sqrt{1 - \left[\frac{28.57}{50}\right]^2}$$

$$= 154.7 \ \Omega$$

Repeat Previous Example Problem if TE_{13} mode is assumed. Determine other field components for this mode.

In a rectangular waveguide for which $a = 1.5$ cm, $b = 0.8$ cm, $\sigma = 0$, $\mu = \mu_o$, and $\varepsilon = 4\varepsilon_o$,

$$H_x = 2 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \sin(\pi \times 10^{11} t - \beta z) \text{ A/m}$$

Determine

(a) The mode of operation

(b) The cutoff frequency

(c) The phase constant β

(d) The propagation constant γ

(e) The intrinsic wave impedance η.

Answer:

$$f_c = 28.57 \text{ GHz},\ \beta = 1718.81 \text{ rad/m},\ \mu = j\beta,\ \eta_{TE_{13}} = 229.69\ \Omega$$

$$E_x = 2584.1 \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) \sin(\omega t - \beta z) \text{ V/m}$$

$$E_y = -459.4 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \sin(\omega t - \beta z) \text{ V/m}, \qquad E_z = 0$$

$$H_y = 11.25 \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) \sin(\omega t - \beta z) \text{ A/m}$$

$$H_z = -7.96 \cos\left(\frac{\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \cos(\omega t - \beta z) \text{ A/m}$$

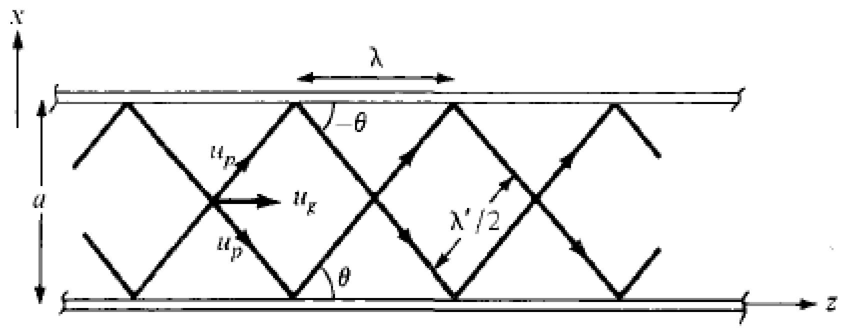
Group velocity

The group velocity u_g is the velocity with which the resultant repeated reflected waves are traveling down the guide and is given by

$$u_g = u' \cos\theta = u' \sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

a group velocity is essentially the velocity of propagation of the wave-packet envelope of a group of frequencies. It is the energy propagation velocity in the guide and is always less than or equal to *u'*.

$$u_p u_g = u'^2$$



POWER TRANSMISSION AND ATTENUATION

To determine power flow in the waveguide, we first find the average Poynting vector

$$\mathcal{P}_{ave} = \frac{1}{2} \operatorname{Re} (\mathbf{E}_s \times \mathbf{H}_s^*)$$

Poynting vector is along the z-direction so that

$$\mathcal{P}_{ave} = \frac{1}{2} \operatorname{Re} (E_{xs} H_{ys}^* - E_{ys} H_{xs}^*) \mathbf{a}_z$$

$$= \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta} \mathbf{a}_z$$

where $\eta = \eta_{TE}$ for TE modes or $\eta = \eta_{TM}$ for TM modes.

The total average power transmitted across the cross section of the waveguide is

$$P_{ave} = \int \mathcal{P}_{ave} \cdot d\mathbf{S}$$

$$= \int_{x=0}^{a} \int_{y=0}^{b} \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta} \, dy \, dx$$

When the dielectric medium is lossy and the guide walls are not perfectly conducting there is a continuous loss of power as a wave propagates along the guide. The power flow in the guide is of the form $P_{ave} = P_o e^{-2\alpha z}$

In order that energy be conserved, the rate of decrease in P_{ave} must equal the time average power loss P_L per unit length, that is,

$$P_L = -\frac{dP_{ave}}{dz} = 2\alpha P_{ave}$$

$$\alpha = \frac{P_L}{2P_{ave}}$$

$$\alpha = \alpha_c + \alpha_d$$

where a_c and a_d are attenuation constants due to ohmic or conduction losses

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

where $\eta' = \sqrt{\mu/\varepsilon}$.

$$\alpha_c \big|_{\mathrm{TE}} = \frac{2R_s}{b\eta' \sqrt{1 - \left[\frac{f_c}{f}\right]^2}} \left[\left(1 + \frac{b}{a}\right)\left[\frac{f_c}{f}\right]^2 + \frac{\frac{b}{a}\left(\frac{b}{a}m^2 + n^2\right)}{\frac{b^2}{a^2}m^2 + n^2}\left(1 - \left[\frac{f_c}{f}\right]^2\right) \right]$$

and for the TM_{mn} modes as

$$\alpha_c \big|_{\mathrm{TM}} = \frac{2R_s}{b\eta' \sqrt{1 - \left[\frac{f_c}{f}\right]^2}} \frac{(b/a)^3 m^2 + n^2}{(b/a)^2 m^2 + n^2}$$

R_s is the skin resistance of the wall $\quad R_s = \frac{1}{\sigma_c \delta} = \sqrt{\frac{\pi f \mu}{\sigma_c}}$

where δ is the skin depth.

PRACTICE EXERCISE

A brass waveguide ($\sigma_c = 1.1 \times 10^7$ mhos/m) of dimensions $a = 4.2$ cm, $b = 1.5$ cm is filled with Teflon ($\varepsilon_r = 2.6$, $\sigma = 10^{-15}$ mhos/m). The operating frequency is 9 GHz. For the TE_{10} mode:

(a) Calculate α_d and α_c.

(b) What is the loss in decibels in the guide if it is 40 cm long?

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1 - \left[\frac{f_c}{f}\right]^2}}$$

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1 - \left[\frac{f_c}{f}\right]^2}}\left(0.5 + \frac{b}{a}\left[\frac{f_c}{f}\right]^2\right)$$

$$\alpha = \alpha_c + \alpha_d$$

$$P_d = P_a\,(e^{2\alpha z} - 1)$$

$$\eta' \simeq \sqrt{\frac{\mu}{\varepsilon}} = \frac{377}{\sqrt{\varepsilon_r}}$$

$$R_s = \frac{1}{\sigma_c\delta} = \sqrt{\frac{\pi f\mu}{\sigma_c}}$$

$$f_c = \frac{u'}{2a}$$

$$u' = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r}}$$

Answer: (a) 1.206×10^{-13} Np/m, 1.744×10^{-2} Np/m, (b) 0.0606 dB.