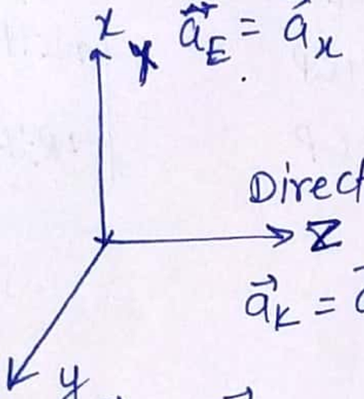
polarization

26/07/20 ①

$E = f(\text{time})\Big|_{z=0}$ space fixed



$\vec{a}_E = \vec{a}_x$

Direction of propogation

$\vec{a}_k = \vec{a}_z$

$\vec{a}_k = \vec{a}_y$

$\vec{E} \quad \vec{H} \quad \vec{P}$

$\vec{E} \times \vec{H} = \vec{P}$

$\boxed{\vec{a}_x \times \vec{a}_y = \vec{a}_k}$

direction of wave propogation

$E(z,t) = E_0 \cos(\omega t - \beta z)\,\vec{a}_k$

$\vec{a}_k = \vec{a}_z$ +z direction

$\beta = \frac{2\pi}{\lambda} = \text{rad/m}$

$\boxed{\eta_0 = \frac{E}{H}}$

Free space medium

$\frac{E}{H} = \eta_0 = 120\pi = 377$ ohm

$H(z,t) = \frac{E_0}{\eta_0} \cos(\omega t - \beta z)\,\vec{a}_y$

→ ω, β → constant for $\vec{E}$ & $\vec{H}$ ① only magnitude changes.
② direction of magnetic field.

$E(z,t) = E_0 \cos(\omega t + \beta z)\,\hat{a}_x$

at $z=0$ fixed space

$\boxed{E(0,t) = E_0 \cos(\omega t)\,\vec{a}_x}$

$\left|\frac{E}{H}\right| = \eta$

$|H| = \frac{|E|}{\eta}$

$\eta > 1 \quad |H| < |E|$

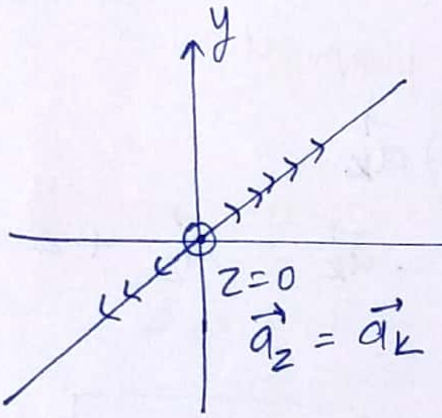
locus of electrical field at fixed space point is called polarization.

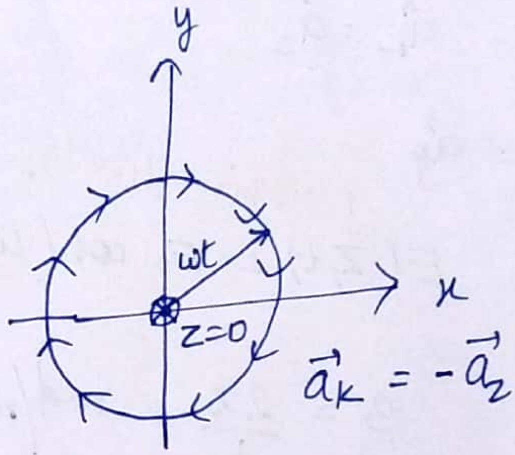
Types of polarization

linear polarization
(i) Horizontal linear polarization
(ii) vertical L. P
(iii) Theta L.P

Circular
- LHCP
- RHCP

Elliptical
- L.H.E.P
- R.H.E.P





linear polarization:

$$E(z,t) = E_{x0}\cos(\omega t - \beta z)\,\vec{a}_x + e^{j\delta} E_{y0}\cos(\omega t - \beta z)\,\vec{a}_y$$

Case-1 $\delta = 2n\pi \quad [n = 0, 1, 2 \ldots]$

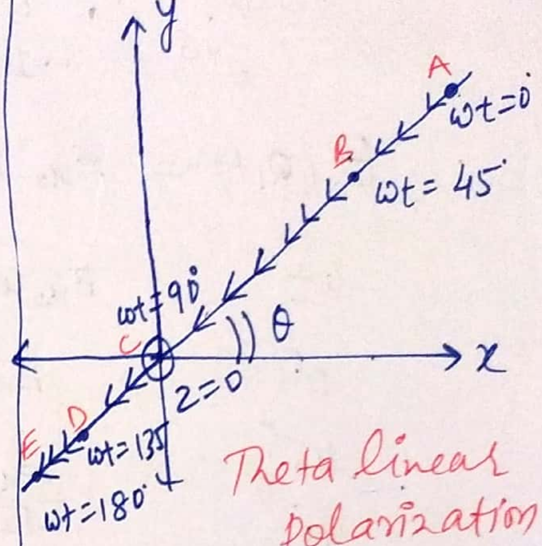
$\delta = 0, 2\pi, 4\pi$

$$\boxed{e^{j\delta} = 1}$$

$$E(z,t) = E_{x0}\cos(\omega t - \beta z)\,\vec{a}_x + E_{y0}\cos(\omega t - \beta z)\,a_y$$

$(z=0)$ $E(0,t) = E_{x0}\cos(\omega t)\,\vec{a}_x + E_{y0}\cos(\omega t)\,a_y$

ωt	$E_{x0}\cos\omega t$	$E_{y0}\cos\omega t$
0°	E_{x0}	E_{y0}
45°	$\frac{E_{x0}}{\sqrt{2}}$	$\frac{E_{y0}}{\sqrt{2}}$
90°	0	0
135°	$-\frac{E_{x0}}{\sqrt{2}}$	$-\frac{E_{y0}}{\sqrt{2}}$
180°	$-E_{x0}$	$-E_{y0}$
225	$-\frac{E_{x0}}{\sqrt{2}}$	$-\frac{E_{y0}}{\sqrt{2}}$
270°	0	0
315	$\frac{E_{x0}}{\sqrt{2}}$	$\frac{E_{y0}}{\sqrt{2}}$
360°	E_{x0}	E_{y0}



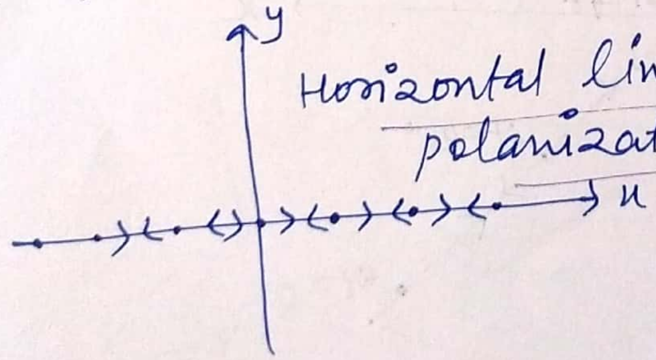
Theta linear polarization

$0 < \theta < 90^\circ$

E_x & E_y	$\tan\theta$	θ
$E_{x0} = E_{y0}$	1	45°
$E_{y0} > E_{x0}$	>1	$>45^\circ$
$E_{y0} < E_{x0}$	<1	<45

If $E_{y0} = 0$

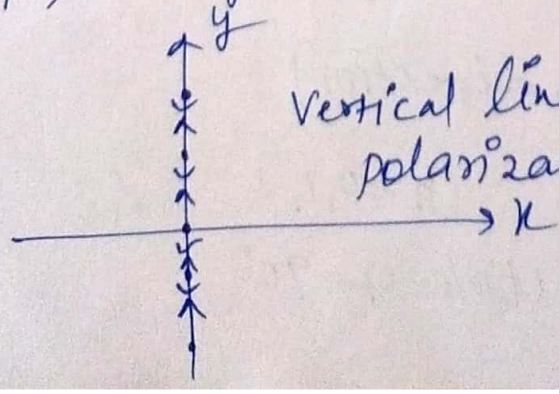
$$E(0,t) = E_{x0}\cos\omega t\ \vec{a}_x$$



Horizontal linear polarization

If $E_{x0} = 0$

$$E(0,t) = E_{y0}\cos\omega t\ \vec{a}_y$$



Vertical linear polarization.

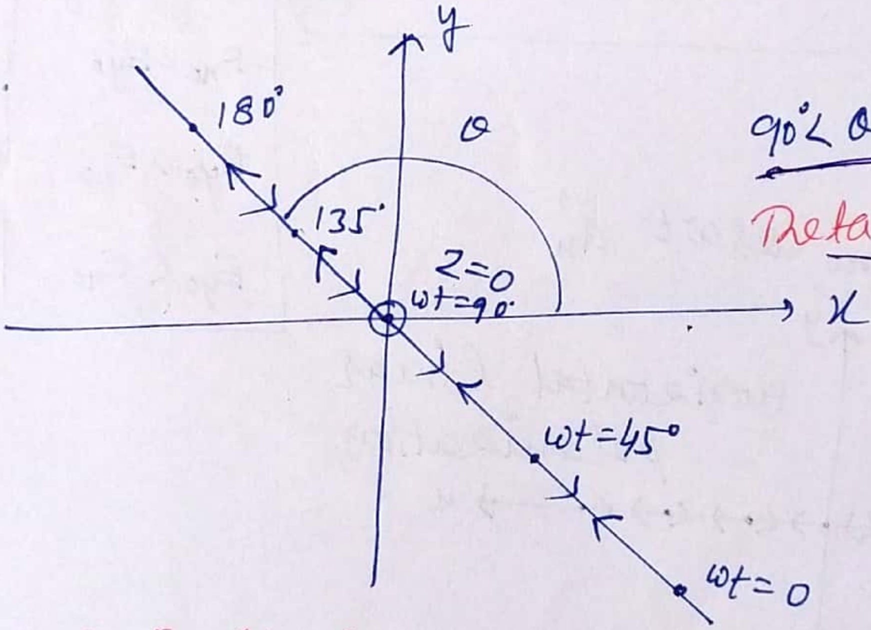
Case-2 $\delta = (2n+1)\pi \; (n = 0, 1, 2 \ldots)$

$\delta = \pi, 3\pi, 5\pi \ldots$

$e^{j\delta} = -1$

$E(Q,t) = E_{x0} \cos(\omega t)\vec{a}_x - E_{y0} \cos(\omega t)\vec{a}_y$

ωt	$E_{x0}\cos\omega t$	$E_{y0}\cos\omega t$
$0°$	E_{x0}	$-E_{y0}$
$45°$	$\frac{E_{x0}}{\sqrt{2}}$	$-\frac{E_{y0}}{\sqrt{2}}$
$90°$	0	0
$135°$	$-\frac{E_{x0}}{\sqrt{2}}$	$+\frac{E_{y0}}{\sqrt{2}}$



$90° < \theta < 180°$

Theta linear polarization

Circular polarization:

$E(z,t) = E_{x0}\cos(\omega t - \beta z)\vec{a}_x + e^{j\delta} E_{y0}\cos(\omega t - \beta z)\vec{a}_y$

$\vec{a}_k = \vec{a}_z$ [+z direction]

$\delta = (2n+1)\pi/2 \quad (n = 0, 1, 2 \ldots)$

$\delta =$ odd multiple of $90°$

Case-I $\eta = 0$

$\delta = \frac{\pi}{2} = 90^\circ$ $\boxed{E_{x0} = E_{y0} = E_0}$

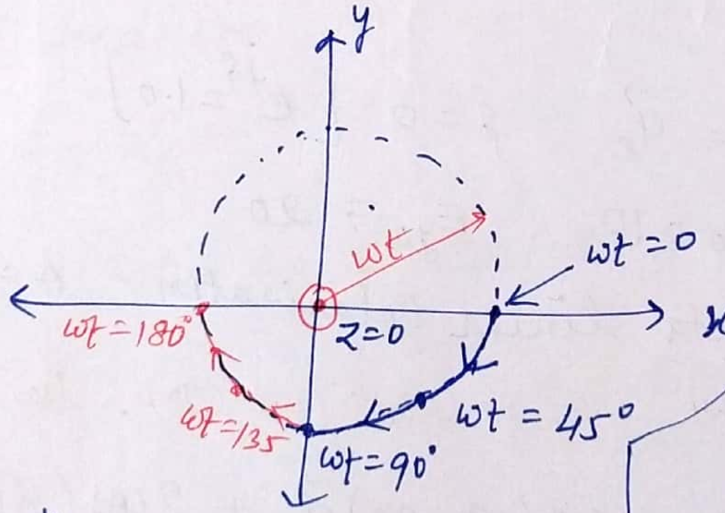
$$E(z,t) = E_0 \cos(\omega t - \beta z)\, a_x + E_y\, e^{j\pi/2} \cos(\omega t - \beta z)\, \vec{a_y}$$

$$E(z,t) = E_0 (a_x + j \vec{a_y}) \cos(\omega t - \beta z)$$

$$\boxed{E(z,t) = E_0 \cos(\omega t - \beta z)\, \vec{a_x} + E_0 \cos(\omega t - \beta z + 90^\circ)\, \vec{a_y}}$$

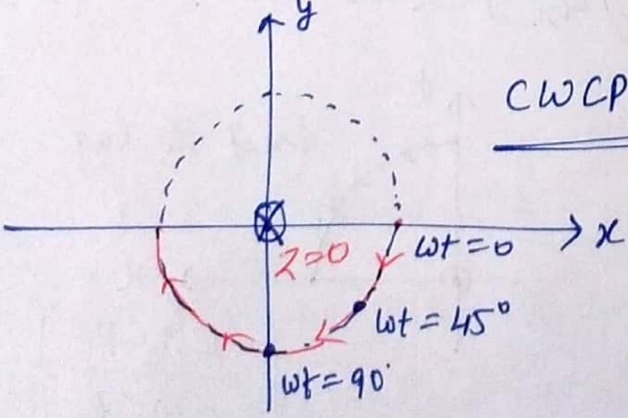
$$E(0,t) = E_0 \cos \omega t\, \vec{a_x} + E_0 \cos(\omega t + 90^\circ)\, \vec{a_y}$$

ωt	$E_{x0} \cos \omega t$	$E_{y0} \cos(\omega t + 90^\circ)$
0°	E_{x0}	0
45°	$\frac{E_{x0}}{\sqrt{2}}$	$-\frac{E_{y0}}{\sqrt{2}}$
⋮		



clockwise circular polarization

Left hand circular polarization



CWCP / RHCP

Thumb → direction of wave propogation

Rotation of fingures → polarization

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$n = 0$

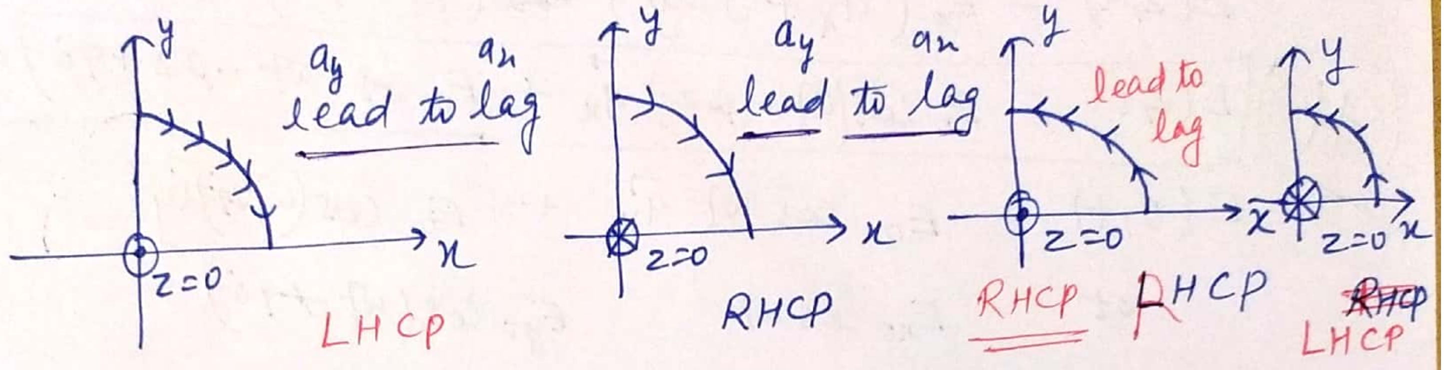
$\delta = 90°$

$\vec{a}_k = \vec{a}_z \qquad \vec{a}_k = -\vec{a}_z$

$n = 1$

$\delta = 270 / -90°$

$\vec{a}_k = \vec{a}_z \qquad \vec{a}_k = -\vec{a}_z$



Elliptical ~~circular~~ polarization:

$E_{x0} \neq E_{y0} \neq E_0$

Q ① $E(z,t) = 10\cos(\omega t - \beta z)\vec{a}_x + 20\cos(\omega t - \beta z)\vec{a}_y$

Here $\vec{a}_k = \vec{a}_z \qquad \delta = 0 \quad [e^{j\delta} = 1.0]$

$E_{x0} = 10 \qquad E_{y0} = 20$

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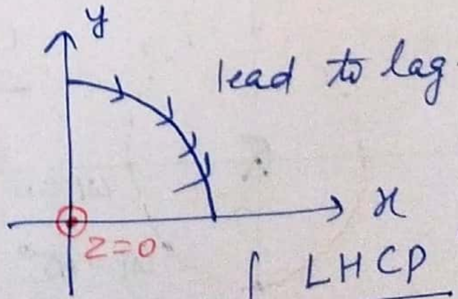
Theta linear polarization $\quad \theta = \tan^{-1}\left[\frac{E_{y0}}{E_{x0}}\right]$

② $E_x = 2\cos\omega t, \quad E_y = 2\cos(\omega t + 90°), \quad \vec{a}_k = \vec{a}_z$

$E(z,t) = 2\cos(\omega t - \beta z)\vec{a}_x + 2\cos(\omega t - \beta z + 90°)\vec{a}_y$

$\delta = 90°$

$E_{x_0} = E_{y_0} = 2.0$



$\left[\frac{\text{LHCP}}{\text{CWCP}}\right]$

9.6 POWER AND THE POYNTING VECTOR

As mentioned before, energy can be transported from one point (where a transmitter is located) to another point (with a receiver) by means of EM waves. The rate of such energy transportation can be obtained from Maxwell's equations:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \tag{9.48a}$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \tag{9.48b}$$

Dotting both sides of eq. (9.48b) with **E** gives

$$\mathbf{E} \cdot (\nabla \times \mathbf{H}) = \sigma E^2 + \mathbf{E} \cdot \varepsilon \frac{\partial \mathbf{E}}{\varepsilon t} \tag{9.49}$$

But for any vector fields **A** and **B** (see Appendix B.10)

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

Applying this vector identity to eq. (9.49) (letting **A** = **H** and **B** = **E**) gives

$$\begin{aligned} \mathbf{H} \cdot (\nabla \times \mathbf{E}) + \nabla \cdot (\mathbf{H} \times \mathbf{E}) &= \sigma E^2 + \mathbf{E} \cdot \varepsilon \frac{\partial \mathbf{E}}{\partial t} \\ &= \sigma E^2 + \frac{1}{2} \varepsilon \frac{\partial}{\partial t} E^2 \end{aligned} \tag{9.50}$$

Dotting both sides of eq. (9.48a) with **H**, we write

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}) = \mathbf{H} \cdot \left(-\mu \frac{\partial \mathbf{H}}{\partial t} \right) = -\frac{\mu}{2} \frac{\partial}{\partial t} (\mathbf{H} \cdot \mathbf{H}) \tag{9.51}$$

and thus eq. (9.50) becomes

$$-\frac{\mu}{2} \frac{\partial H^2}{\partial t} - \nabla \cdot (\mathbf{E} \times \mathbf{H}) = \sigma E^2 + \frac{1}{2} \varepsilon \frac{\partial E^2}{\partial t}$$

Rearranging terms and taking the volume integral of both sides,

$$\int_v \nabla \cdot (\mathbf{E} \times \mathbf{H})\, dv = -\frac{\partial}{\partial t} \int_v \left[\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2 \right] dv - \int_v \sigma E^2\, dv \tag{9.52}$$

Applying the divergence theorem to the left-hand side gives

$$\oint_S (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \int_v \left[\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2 \right] dv - \int_v \sigma E^2\, dv \tag{9.53}$$

$$\underset{\text{leaving the volume}}{\overset{\downarrow}{\text{total power}}} = \underset{\text{and magnetic fields}}{\overset{\downarrow}{\text{rate of decrease in energy stored in electric}}} - \overset{\downarrow}{\text{ohmic power dissipated}} \tag{9.54}$$

Equation (9.53) is referred to as *Poynting's theorem*.[2] The various terms in the equation are identified using energy-conservation arguments for EM fields. The first term on the right-hand side of eq. (9.53) is interpreted as the rate of decrease in energy stored in the electric and magnetic fields. The second term is the power dissipated because the medium is conducting ($\sigma \neq 0$). The quantity $\mathbf{E} \times \mathbf{H}$ on the left-hand side of eq. (9.53) is known as the *Poynting vector* $\mathcal{P}$, measured in watts per square meter (W/m²); that is,

$$\boxed{\mathcal{P} = \mathbf{E} \times \mathbf{H}} \tag{9.55}$$

It represents the instantaneous power density vector associated with the EM field at a given point. The integration of the Poynting vector over any closed surface gives the net power flowing out of that surface.

Poynting's theorem states that the net power flowing out of a given volume v is equal to the time rate of decrease in the energy stored within v minus the ohmic losses.

The theorem is illustrated in Figure 9.6.

It should be noted that $\mathcal{P}$ is normal to both $\mathbf{E}$ and $\mathbf{H}$ and is therefore along the direction of wave propagation $\mathbf{a}_k$ for uniform plane waves. Thus

$$\mathbf{a}_k = \mathbf{a}_E \times \mathbf{a}_H \tag{9.39}$$

The fact that $\mathcal{P}$ points along $\mathbf{a}_k$ causes $\mathcal{P}$ to be regarded as a "pointing" vector.

Again, if we assume that

$$\mathbf{E}(z, t) = E_o e^{-\alpha z} \cos(\omega t - \beta z)\mathbf{a}_x$$

then

$$\mathbf{H}(z, t) = \frac{E_o}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\mathbf{a}_y$$

and

$$\begin{aligned} \mathcal{P}(z, t) &= \frac{E_o^2}{|\eta|} e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_\eta)\mathbf{a}_z \\ &= \frac{E_o^2}{2|\eta|} e^{-2\alpha z} [\cos \theta_\eta + \cos(2\omega t - 2\beta z - \theta_\eta)]\mathbf{a}_z \end{aligned} \tag{9.56}$$

[2] After J. H. Poynting, "On the Transfer of Energy in the Electromagnetic Field," *Philosophical Transactions*, vol. 174, 1883, p. 343.

① Gauss law: Total flux passing through closed surface is equal to charge enclosed by that closed surface.

$$\psi = \oint_S \vec{D}\cdot d\vec{s} = Q_{enc} \quad \text{——— ①}$$

Let ρ_v be the volume charge density

$$Q = \int_v \rho_v \, dv \quad \text{——— ②}$$

$$\boxed{\oint \vec{D}\cdot d\vec{s} = \int_v \rho_v \, dv}$$

1st equation of Maxwell's in Integration theorem.

Using divergence theorem

$$\oint \vec{D}\cdot d\vec{s} = \int_v (\nabla\cdot\vec{D})\, dv$$

$$\int_v \nabla\cdot\vec{D}\, dv = \int_v \rho_v \, dv$$

$$\boxed{\nabla\cdot\vec{D} = \rho_v}$$

Maxwell's 1st equation in differential form (point form)

For surface free region $\rho_v = 0$

$$\boxed{\nabla\cdot\vec{D} = 0}$$

$$\boxed{\nabla\cdot\varepsilon\vec{E} = 0}$$

$\bar{D}$ = electric flux density or electric displacement vector

(2) Faraday's law (Time varying field)

$$V_{emf} = -\frac{\partial \lambda}{\partial t} = -\frac{\partial}{\partial t}(N\psi)$$

$\psi \rightarrow$ magnetic flux

If: $N = 1$

$$V_{emf} = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot d\vec{s} \quad \text{--- (1)}$$

$V = \oint_l \vec{E}\cdot dl \rightarrow$ calculation of potential difference

$$\boxed{\oint_l \vec{E}\cdot \vec{dl} = \int_s -\frac{\partial B}{\partial t}\cdot ds}$$

Maxwell's 2nd equation (Integral form)

stokes theorem: $\oint_l \vec{A}\cdot\vec{dl} = \int(\nabla\times A)\cdot ds$

$$\int_s (\nabla\times\vec{E})\cdot d\vec{s} = \int_s -\frac{\partial B}{\partial t}\cdot ds$$

$$\boxed{\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

Maxwell's 2nd equation. (point form or differential form)

$\vec{B}$ = magnetic flux density

$\vec{B} = \mu\vec{H}$

$$\boxed{\nabla\times\vec{E} = -j\omega\mu\vec{H}}$$

$$\frac{\partial}{\partial t} = j\omega$$

(#) Ampere's law:- closed loop integration of tangential component of magnetic field is equal to current enclosed by that closed loop.

$$\oint\vec{H}\cdot d\vec{l} = I_{enc} \quad \text{--- (1)}$$

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Let $\vec{J}_c$ be conduction current density $\left(\frac{\text{Current}}{\text{Area}}\right)$

$$I = \int_s \vec{J}_c \cdot d\vec{s} \quad \text{—— (2)}$$

$$\boxed{\oint H \cdot \vec{dl} = \int_s \vec{J}_c \cdot d\vec{s}}$$ Maxwell's 3rd eq^n (In integral form)

Stokes theorem : $\oint \vec{A} \cdot \vec{dl} = \int_s (\nabla \times \vec{A}) \cdot d\vec{s}$

$$\int_s (\nabla \times \vec{H}) \cdot ds = \int_s \vec{J}_c \cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{H} = \vec{J}_c}$$ Maxwell's 3rd eq^n (In Differential form)

→ valid only for static field.

For time varying field $\nabla \times \vec{H} = \vec{J}_c + \vec{J}_D$

$\vec{J}_c$ - conduction current density $= \sigma E$

$\vec{J}_D$ = displacement current density

$$= \frac{\partial \vec{D}}{\partial t} = \frac{\partial (\varepsilon \vec{E})}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$= j\omega \varepsilon \vec{E}$$

$$\boxed{\nabla \times \vec{H} = (\sigma + j\omega\varepsilon)\vec{E}}$$

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④ Isolated magnetic monopole does not exist

$$\boxed{\oint \vec{B} \cdot d\vec{s} = 0}$$ maxwell's 4th equation (Integral eq^n)

Apply divergence theorem

$$\int_V (\nabla \cdot \vec{B})\, dV = 0$$

$$\boxed{\nabla \cdot \vec{B} = 0}$$ Maxwell's 4th eq^n (differential or point form)

$$\nabla \cdot (\mu \vec{H}) = 0$$

$$\boxed{\nabla \cdot \vec{H} = 0}$$ when medium is homogeneous.

Derivation of uniform plane wave equation:

Maxwell's equations

① $\nabla \cdot \vec{E} = 0$

② $\nabla \cdot \vec{H} = 0$

③ $\nabla \times \vec{E} = -j\omega\mu \vec{H}$

④ $\nabla \times \vec{H} = (\sigma + j\omega\epsilon)\vec{E}$

Take curl of eq ③

$$\nabla \times \nabla \times \vec{E} = -j\omega\mu (\nabla \times \vec{H})$$

vector identity. $\nabla^2 \vec{A} = \nabla(\nabla\cdot\vec{A}) - \nabla\times\nabla\times\vec{A}$

$$\cancel{\nabla(\nabla\cdot\vec{E})}^{\,0} - \nabla^2 E = -j\omega\mu(\nabla\times\vec{H})$$

$$\cancel{-}\nabla^2\vec{E} = \cancel{-}j\omega\mu(\sigma + j\omega\epsilon)\vec{E}$$

put eq 4

$$\nabla^2 E = j\omega\mu(\sigma + j\omega\epsilon)E$$

$$\boxed{\nabla^2\vec{E} - \gamma^2\vec{E} = 0}$$

Plane wave equation for electric field

$(m^{-1})\ \gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

Propogation constant

Take curl both sides of equation 4 and substitute

$$\boxed{\nabla^2\vec{H} - \gamma^2\vec{H} = 0}$$

Plane wave eq^n for magnetic field.

$$\gamma = \alpha + j\beta$$

α = attenuation constant $\left(\frac{\text{Nepers}}{m} \text{ or } \frac{dB}{m}\right)$

$$\boxed{1\ Np = 8.686\ dB}$$

$\frac{2\pi}{\lambda} = \beta$ = phase constant $\left(\frac{rad}{m}\right)$

= phase shift per unit distance

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Solution of plane wave equation

If wave is travelling in z-direction

$$\nabla^2 \vec{E} - \gamma^2 \vec{E} = 0$$

$$(D^2 - \gamma^2)\vec{E} = 0$$

Roots of this auxiliary differential equation

$$D = \pm \gamma$$

If the roots are distant and real then

$$E(z) = E_0^+ e^{-\gamma z} + E_0^- e^{\gamma z}$$

$$E(z,t) = \left[\underbrace{E_0^+ e^{-\gamma z}}_{\text{Positive z direction}} + \underbrace{E_0^- e^{\gamma z}}_{\text{negative z direction}}\right] e^{j\omega t}$$

If wave is travelling in +z direction

$$E(z,t) = E_0^+ e^{-\gamma z} e^{j\omega t}$$

If wave is travelling in −z direction

$$E(z,t) = E_0^- e^{\gamma z} e^{j\omega t}$$

$$E(z,t) = E_0^+ e^{-(\alpha + j\beta) z} e^{j\omega t}$$

$$= \underbrace{E_0^+ e^{-\alpha z}}_{\text{Amplitude}} \, e^{-j\beta z} e^{j\omega t}$$

$$\hookrightarrow e^{j(\omega t - \beta z)}$$

$$E(z,t) = \underbrace{E_0^+ e^{-\alpha z}}_{\text{Amplitude}} \; \underbrace{e^{j(\omega t - \beta z)}}_{\text{phase}}$$

$$E(z,t) = Re\left[E_0^+ e^{-\alpha z} e^{j(\omega t - \beta z)} \right]$$

$$\boxed{\vec{E}(z,t) = E_0 \, e^{-\alpha z} \cos(\omega t - \beta z) \, \hat{a}_E}$$

Vector Representation

$$\vec{H}(z,t) = H_0 \, e^{-\alpha z} \cos(\omega t - \beta z) \, \hat{a}_H$$

In uniform plane wave, electric field, magnetic field and direction of propogation are perpendicular to each other.

$$\boxed{\hat{a}_k = \hat{a}_E \times \hat{a}_H}$$

$$\boxed{\hat{a}_H = \hat{a}_k \times \hat{a}_E}$$ Right Shift

$$\boxed{\hat{a}_E = \hat{a}_H \times \hat{a}_k}$$ Left Shift

wave propogation through different medium:

① Lossy medium or general medium

$$\boxed{\sigma \neq 0, \quad \mu = \mu_0 \mu_r, \quad \epsilon = \epsilon_0 \epsilon_r \quad \sigma << \omega\epsilon}$$

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$|\gamma| = \alpha^2 + \beta^2 = \sqrt{\omega^2\mu^2\sigma^2 + \omega^4\mu^2\epsilon^2}$$

$$\alpha^2+\beta^2 = \sqrt{\omega^4\mu^2\varepsilon^2\left(1+\frac{\omega^2\mu^2\sigma^2}{\omega^4\mu^2\varepsilon^2}\right)}$$

$$\alpha^2+\beta^2 = \omega^2\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} \quad \text{—— (1)}$$

$$\alpha^2-\beta^2+2j\alpha\beta = j\omega\mu\sigma - \omega^2\mu\epsilon$$

$$\alpha^2-\beta^2 = -\omega^2\mu\epsilon \quad \text{—— (2)}$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]}$$

where $\frac{\sigma}{\omega\epsilon}$ = loss tangent = $\tan\theta$

$$\vec{E}(z,t) = E_0 e^{-\alpha z}\cos(\underbrace{\omega t-\beta z}_{\text{constant}})\hat{a}_E$$

$$\omega t - \beta z = \text{constant}$$

$$\omega - \beta\frac{dz}{dt} = 0$$

$$\omega = \beta\frac{dz}{dt}$$

$$\boxed{\text{Phase velocity} = \frac{\omega}{\beta}}$$

Intrinsic Impedance :

$$\eta = \frac{E_0}{H_0} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

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$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\epsilon\left(1+\frac{\sigma}{j\omega\epsilon}\right)}}$$ Complex quantity

$$= \frac{\sqrt{\mu/\epsilon}}{\sqrt{1-j\frac{\sigma}{\omega\epsilon}}}$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}$$

$$\theta_\eta = \tan^{-1}\left(\frac{0}{\sqrt{\mu/\epsilon}}\right) - \frac{1}{2}\tan^{-1}\left(-\frac{\sigma}{\omega\epsilon}\right)$$

$$\boxed{\theta_\eta = \frac{1}{2}\tan^{-1}\frac{\sigma}{\omega\epsilon}}$$

(2) Lossless medium

$$\sigma = 0, \quad \mu = \mu_0\mu_r, \quad \epsilon = \epsilon_0\epsilon_r$$

$$\alpha = 0 \qquad \beta = \omega\sqrt{\mu\epsilon}$$

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

$$\boxed{120\pi\sqrt{\frac{\mu_r}{\epsilon_r}} = |\eta| = \sqrt{\frac{\mu}{\epsilon}}} \qquad \theta_\eta = 0$$

Note

(1) Thus E and H are in time phase with each other.

(2) For non-magnetic medium ($\mu_r = 1$). Practically ~~at~~ all lossless medium are non-magnetic nature.

(3) wave propogation through free space

$$\sigma = 0 \quad \mu = \mu_r \quad \epsilon = \varepsilon_0$$

$$\alpha = 0$$

$$\beta = \frac{\omega}{c}$$

$$V_p = c$$

$$\eta = 120\pi = 377\Omega$$

$\therefore c =$ Velocity of light $= 3 \times 10^8$ m/sec

(4) wave propogation through Good conductor

$$\sigma \simeq \infty, \quad \varepsilon = \varepsilon_0, \quad \mu = \mu_0 \mu_r, \quad \sigma >> \omega\epsilon$$

$$\alpha = \sqrt{\frac{\mu\omega\sigma}{2}} = \sqrt{\frac{2\pi f \mu\sigma}{2}}$$

$$\alpha = \sqrt{\pi f \mu\sigma} = \beta$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma\left(1 + j\frac{\omega\epsilon}{\sigma}\right)}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\, e^{j\pi/4}$$

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\pi f \mu\sigma}} \quad \therefore \boxed{V_p \propto \frac{1}{\sqrt{\sigma}}}$$

Note (1) For perfect conductor $(\sigma = \infty)$

$$\boxed{\eta = 0}$$

② $\eta = \sqrt{\frac{\omega\mu}{\sigma}}\left[\cos\pi/4 + j\sin\frac{\pi}{4}\right]$

$= \sqrt{\frac{2\pi f\mu}{\sigma}}\,\frac{1}{\sqrt{2}}\,(1+j)$

$= \sqrt{\frac{\pi f\mu}{\sigma}}\,(1+j)$ Inductive impedance

Skin depth or depth of penetration :-

$$\vec{E}(z,t) = E_0\, e^{-\alpha z}\cos(\omega t - \beta z)\,\hat{a}_x$$

As the $\vec{E}$ or $\vec{H}$ wave travels in a conducting medium, its amplitude is attenuated by the factor $e^{-\alpha z}$. The distance (δ) through which the wave amplitude ~~its~~ decreases to a factor e^{-1} (about 37% of the original value) is called skin depth or Penetration depth of the medium

$$E_0\, e^{-\alpha\delta} = \frac{E_0}{e}$$

$$\alpha\delta = 1$$

$$\boxed{\delta = \frac{1}{\alpha}}$$