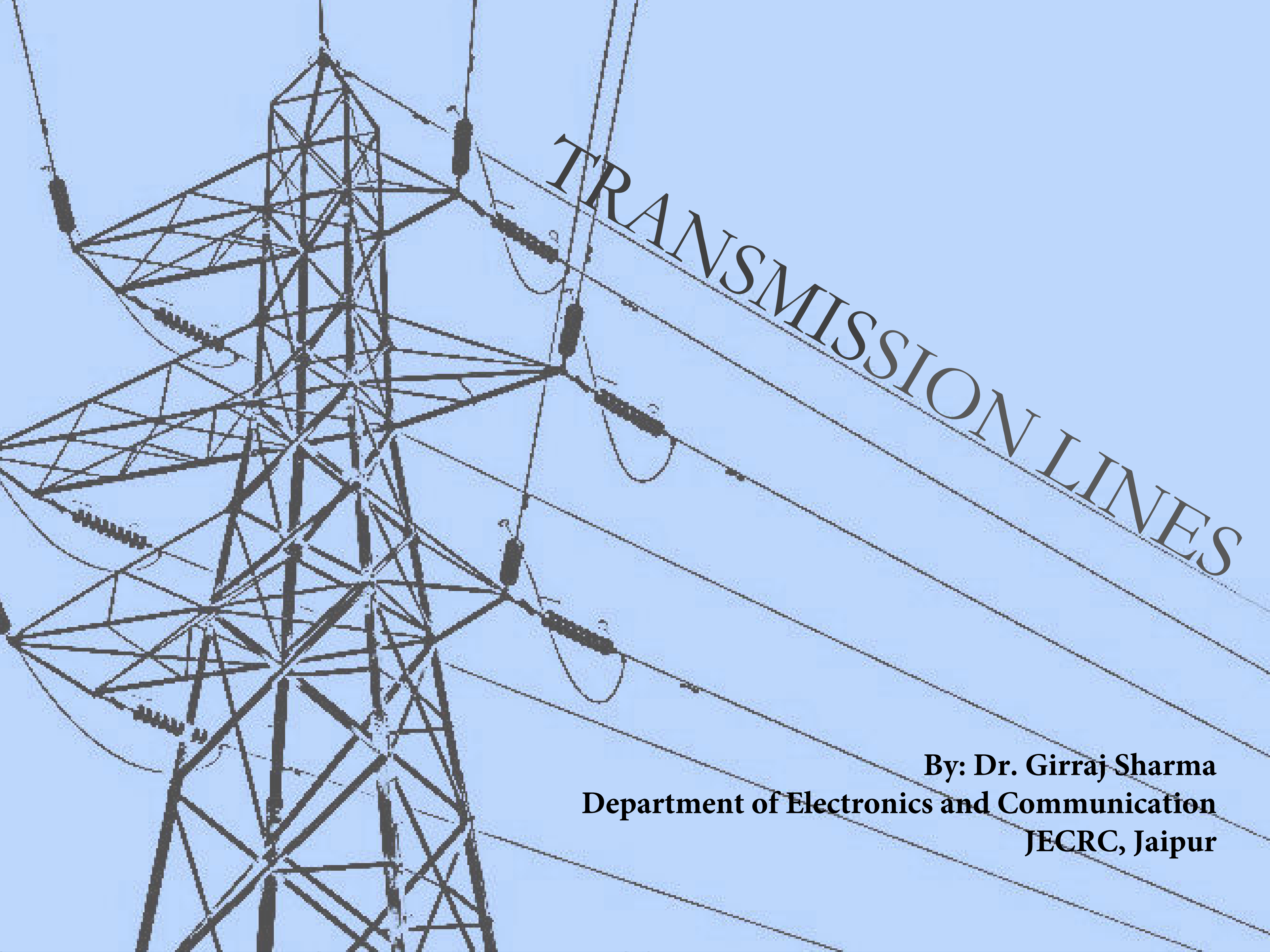

TRANSMISSION LINES

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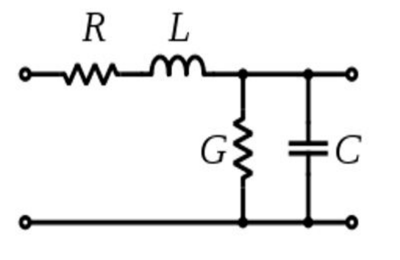
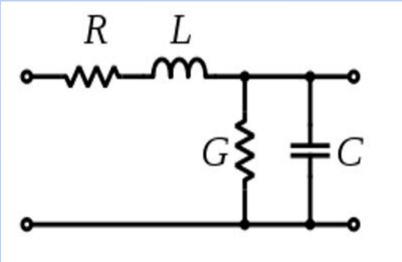
INTRODUCTION TO TRANSMISSION LINES



- Energy can be transmitted either by radiation of free electromagnetic waves as in the radio (or) it can be constant to move (or) carried in various conductor element known as transmission line.
- Thus transmission line is the conductive method of guiding electrical energy from one place to another place.
- The signal in transmission line flow in the form of voltage and current , these signal characteristics will be seen further.
- Transmission of information through transmission lines is guided bounded and point to point type of communication.
- Main goal of transmission is efficiency of power and less distortion in the signal.
- Examples of transmission lines are telephone lines , power transmission from generator to load , cage line ,etc.....

ANALYSIS OF TRANSMISSION LINE



- Transmission line is the two wires to which one end of the wires source is connected to the other end load is connected.
- Before going to analysis of transmission line, DO KCL AND KVL APPLICABLE FOR HIGH FREQUENCIES (OR) FOR LONG TRANSMISSION LINES?
- Consider a transmission line as shown in the figure 1.1

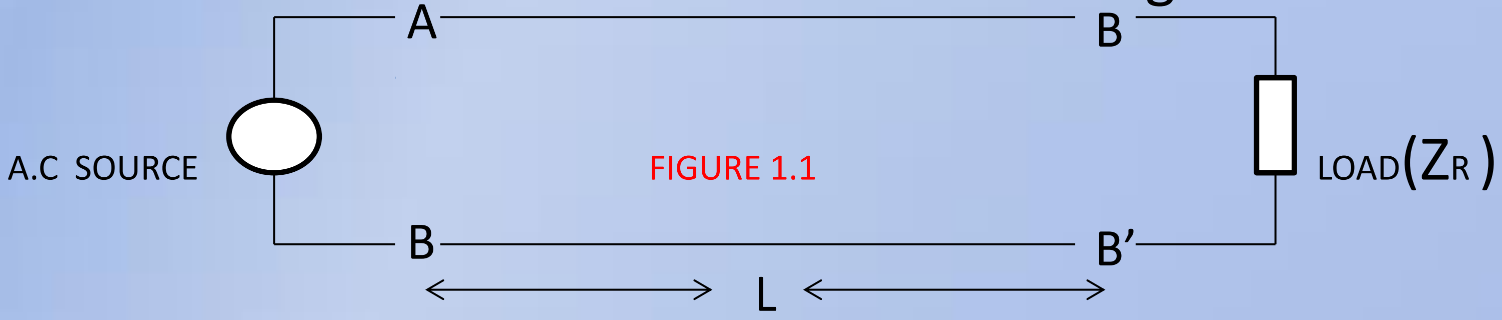
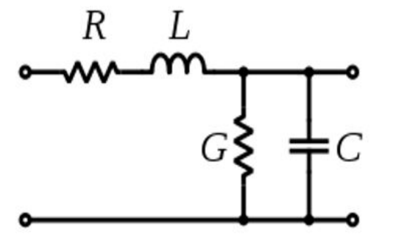


FIGURE 1.1

- Let the length of a transmission line be L, a sinusoidal signal of high frequency is applied at the source end and load impedance be Z_R.
- For low frequencies, the circuit analysis can be done by simply knowing the impedance values of components used.

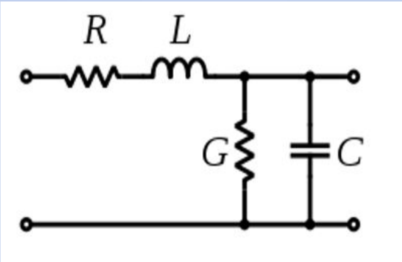


ANALYSIS OF TRANSMISSION LINE CONT…

- Consider sinusoidal signal is applied at AA' in the figure, At a particular instant let V_p be the voltage so in practical no signal travels with infinite speed so signal takes some time to travel from AA' to BB', So as the applied signal reaches the other end BB' voltage at AA' will be no longer the same as signal is varying.
- So due to this potential difference(P.D) is being developed in the transmission line.
- The delay signal to reach the other end of transmission line this time is called as transit time(t_r). Let the velocity of the signal be 'V' ,time period be 'T' and frequency be 'f'.

$t_r = L/V$ (if t_r is more then P.D will be more)

- If t_r is more its significance cannot be neglected.
- If t_r is far less when compared to time period(T) then the significance of transit time can be neglected as in normal cases.



ANALYSIS OF TRANSMISSION LINE CONT…

- So writing the equations,

  T >>>> t_r

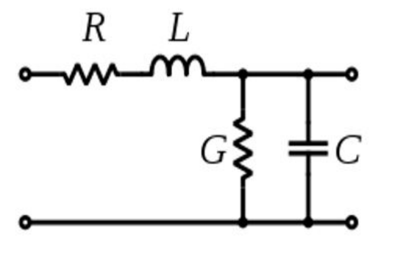
  1/f >>>> L/V

  V/f >>>> L

  wavelength >>>>L in this case transit time is neglected.

- But for practical cases, power will be transmitted to longer distances in this case transit time cannot be neglected and KVL,KCL cannot be applied for analysis.
- For analysis of transmission line, we use KVL ,KCL by considering the very small sections of transmission line so that in these sections transit time can be neglected.
- The equivalent diagram of small section of transmission line figure 1.1 is shown in figure 1.1(a) in the next slide.

CIRUIT ANALYSIS OF TRANSMISSION LINE



- As there is potential difference across transmission line there will be resistance R(Ω/km) this resistance is known loop resistance which depends on the length of transmission line.
- Equivalent small section of a transmission line is as follows

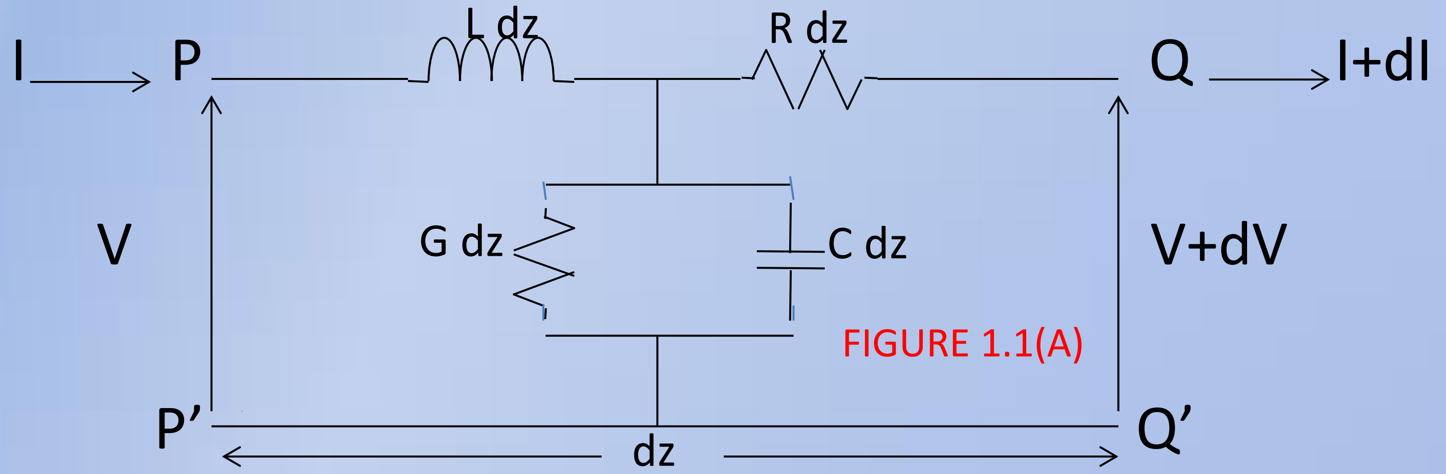
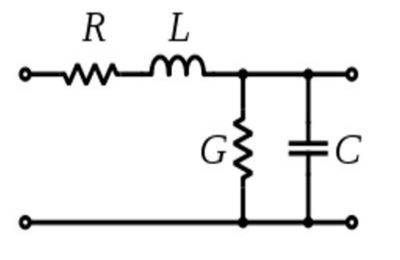


FIGURE 1.1(A)

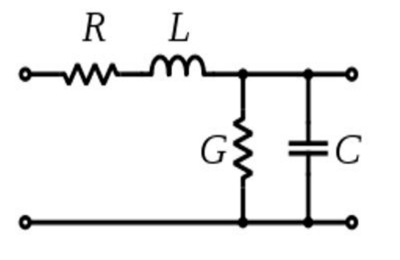
- When current is flowing in 2 parallel wire magnetic field will get induced around the wire so there will be inductance known as loop inductance(H/km) whose value depend upon the strength of current so it is in series with loop resistance.

CIRCUIT ANALYSIS OF TRANSMISSION LINE CONT…..



- As current flows, transmission line get charged so parallel lines are separated by a distance and the medium in between them act as dielectric so it will act as capacitance effect in shunt to transmission line. This capacitance is known as loop capacitance(F/km).
- As the capacitance is not ideal there will be leakage current so there will be conductance parallel to capacitance known as total shunt conductance(mho/km).
- So loop resistance , loop inductance, loop capacitance and total shunt conductance depends upon length of the transmission line.
- So equivalent impedance in series Z= (R + jwL) Ω/km.

  equivalent admittance in parallel Y = (G +jwC)mho/km.
- Consider a small section of line 'PQ' of length 'dz' in the direction of the power flow. At point 'P' let the voltage be 'V' and the current flowing be 'I' and at 'Q' voltage be 'V+dV' and current be 'I+dI'.



CIRCUIT ANALYSIS OF TRANSMISSION LINE CONT…..

- Current in the small section ‘PQ’ is ‘I’ but current coming out of the section is ‘I +dI’.
- Applying KVL in the section ‘PQ’ which have series impedance (R+jwL)dz ohm, we get

$$V - (V+dV) - ((R+jwL)dz)I = 0$$

$$-dV = (R+jwL)Idz$$

$$\boxed{(-dV/dz) = (R+jwL)I} \quad \text{--------- eq 1}$$

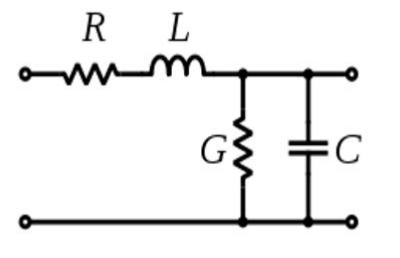
- Applying KVL in the section ‘PQ’ which have admittance (G+jwC)dz mho, we get

$$I - (I+dI) - ((G+jwC)dz)V = 0$$

$$-dI = (G+jwC)Vdz$$

$$\boxed{(-dI/dz) = (G+jwC)V} \quad \text{--------- eq 2}$$

- These are the two equations which lead us to get basic equations of transmission line.



CIRCUIT ANALYSIS OF TRANSMISSION LINE CONT…..

- Differentiating the eq 1 we get

$$(d^2V/dz^2)= -(R + jwL)(dI/dz) \text{ --------- eq 3}$$

- Substituting eq 2 in eq 3 we get

$$(d^2V/dz^2)= -(R + jwL) (-(G+jwC))V$$

$$(d^2V/dz^2) = (R + jwL) (G+jwC)V \text{ ---------eq 4}$$

- Similarly if we differentiate eq 2 and substitute eq 1 in that equation we get

$$(d^2I/dz^2) = (R + jwL) (G+jwC)I \text{ -----------eq 5}$$

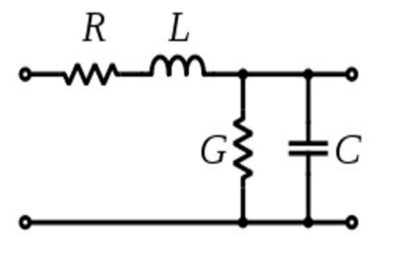
- If we generalise the equations 4 and 5 by considering

$$\gamma^2=(R + jwL) (G+jwC) \text{ ---------- eq 6}$$

- Gamma ‘γ’ is known as propagation constant and why it is known as propagation constant will be discussed later. So the eq 4 & 5 become

$$\boxed{(d^2I/dz^2) = \gamma^2 I \quad (d^2V/dz^2) = \gamma^2 V} \text{ ---------- eq 7}$$

SOLUTION FOR BASIC EQUATION 7



- Solution for Eq 7 is as follows

$$V(z) = V'e^{-\gamma z} + V''e^{+\gamma z}$$

- If one need to express the above equation in time domain or instantaneous value then we have to multiply with $e^{j\omega t}$.

$$V(z,t) = V'e^{-\gamma z}\, e^{j\omega t} + V''e^{+\gamma z}\, e^{j\omega t}$$

- 'γ' is expressed as α+jβ ,substituting 'γ' in above equation we get

$$V(z,t) = \underbrace{V'e^{-\alpha z}\, e^{j(\omega t-\beta z)}}_{\text{incident}} + \underbrace{V''e^{+\alpha z}\, e^{j(\omega t+\beta z)}}_{\text{reflected}}$$

- Considering the incident part

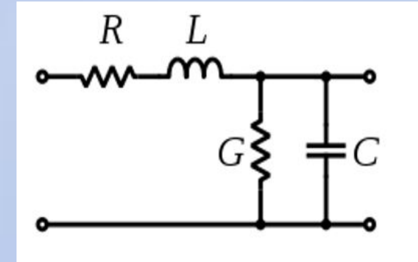
$$V'(e^{-\alpha z})\, e^{j(\omega t-\beta z)}$$

Amplitude of incident voltage depends on this part of expression.

Phase of incident voltage wave depends on this part of expression.

- So 'α' is called as attenuation factor because as z increases amplitude of signals goes on decreasing and 'β' is called as phase constant.
- Similarly for reflected part of voltage signal.

WAVEFORMS OF INCIDENT AND REFLECTED VOLTAGE SIGNALS



- why $V'(e^{-\alpha z})\, e^{j(\omega t-\beta z)}$ is called as incident part and $V''e^{+\alpha z}\, e^{j(\omega t+\beta z)}$ is called as reflected part ?
- consider $\alpha=0$ then equation of incident and reflected part becomes as and consider the real part then

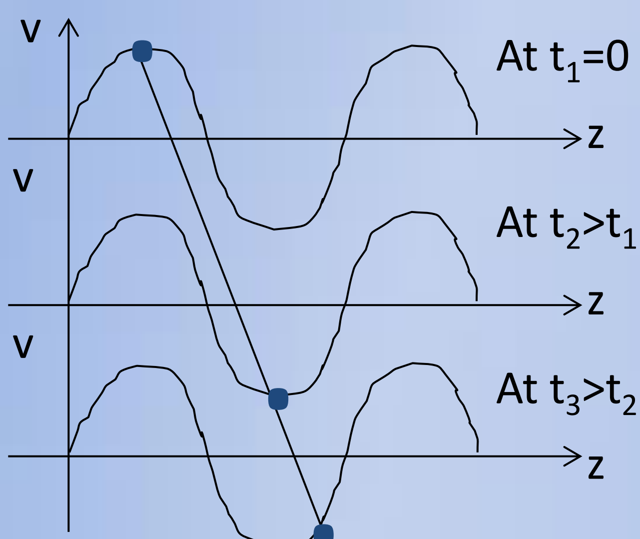
Incident part

$V'e^{j(\omega t-\beta z)}$

$Re\{\, V'e^{j(\omega t-\beta z)} \,\}$

$V'\cos(\omega t-\beta z)$

Wave is propagating in +z direction as shown



So it is called as incident part

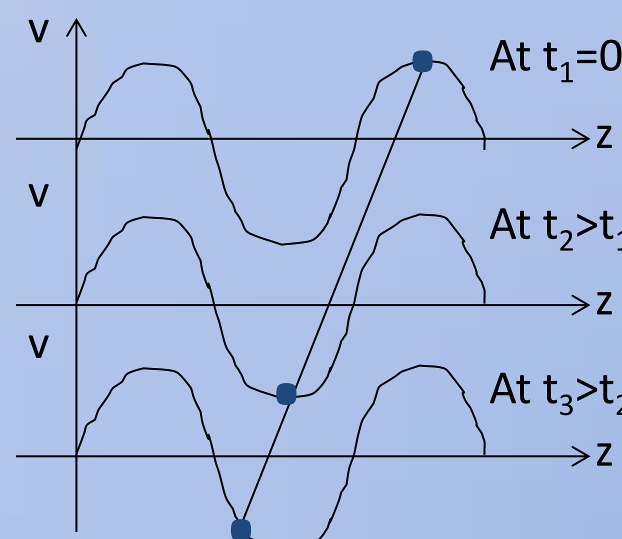
Reflected part

$V''e^{j(\omega t+\beta z)}$

$Re\{\, V''e^{j(\omega t+\beta z)} \,\}$

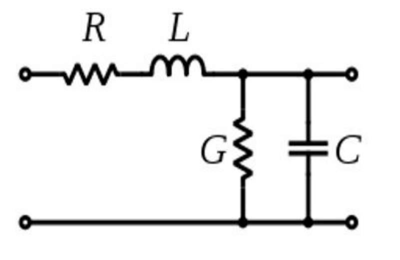
$V'\cos(\omega t-\beta z)$

Wave is propagating in -z direction as shown



So it is called as reflected part

CIRCUIT ANALYSIS OF TRANSMISSION LINE CONT…..



- Solution of equations 7 can be expressed in the exponential (or) hyperbolic functions as follows

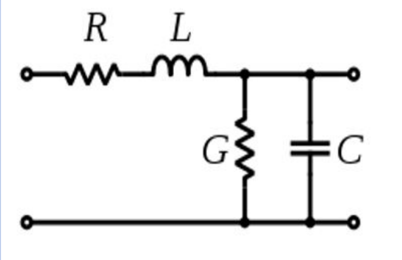
$$V(z) = V'e^{-\gamma z} + V''e^{+\gamma z} \text{ --------- eq 8}$$

$$I(z) = \underset{\text{(incident)}}{I'e^{-\gamma z}} + \underset{\text{(reflected)}}{I''e^{+\gamma z}} \text{ --------- eq 9}$$

- From the above equation it is clear that voltage and current is different at every point.
- V',I' are the voltage ,current signals travelling in +z direction known as incident signal . V'',I'' are the voltage ,current signals travelling in –ve z direction known as reflected signal.
- Now we will get introduced to new term characteristic impedance

CHARACTERSITIC IMPEDANCE : It is defined as the ratio of voltage and current of a single wave travelling in a positive direction and it is called as characteristic. It is denoted by Zo.

DERIVATION OF CHARACTERISTIC IMPEDANCE



- Differentiating the equation 8 we get

$$(dV/dz) = -\gamma V'e^{-\gamma z} + \gamma V''e^{+\gamma z}$$

substituting $(dv/dz) = -(R + jwL)I$ from eq 1 in above equation.

$$-(R + jwL)I = -\gamma V'e^{-\gamma z} + \gamma V''e^{+\gamma z}$$

substituting $I = I'e^{-\gamma z} + I''e^{+\gamma z}$ from eq 9 in above equation we get

$$-(R + jwL)(I'e^{-\gamma z} + I''e^{+\gamma z}) = -\gamma V'e^{-\gamma z} + \gamma V''e^{+\gamma z} \text{----- eq 10}$$

- Equalising the positive direction terms we get

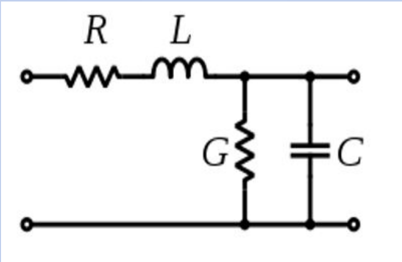
$-(R + jwL)(I'e^{-\gamma z}) = -\gamma V'e^{-\gamma z}$ cancelling '$e^{-\gamma z}$' term we get

$$\frac{V'}{I'} = \frac{(R + jwL)}{\gamma} \qquad (\text{where } \gamma=((R + jwL)(G+jwC))^{1/2})$$

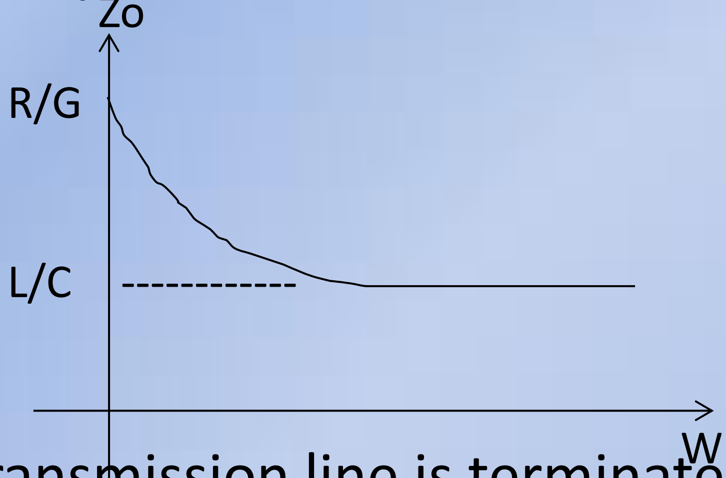
$$Zo = \frac{(R + jwL)^{1/2}}{(G+jwC)^{1/2}} \text{ --------- eq 11} \quad (\text{Where } Zo=-(v''/I''))$$

- Value of Zo doesn't depend upon the length of transmission line but on R,L,G and C.

CHARACTERSTIC IMPEDANCE AND REFLECTION CO-EFFICIENT



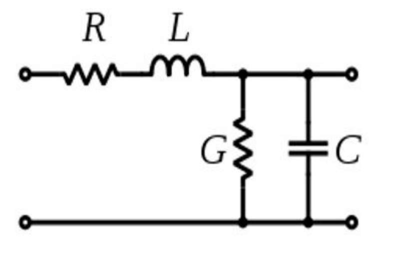
- Graphical representation of Zo is shown below



- Thus if, transmission line is terminated with a load(Z_R) as figure 1.1. The value of Z_R is the ratio of voltage and current at that load.

$$Z_R = \frac{V}{I} = \frac{(V' + V'')}{(I + I'')} = \frac{Zo\,(I' - I'')}{(I + I'')} \text{ ------------- eq 12}$$

- **REFLECTION CO-EFFIECIENT** : According to the maximum power transfer theorem if load impedance and source impedance doesn’t match each other then there will be reflection takes place so the parameter which signifies the amount of reflection is reflection co-efficient and it is denoted by ‘γ_R’. It is the ratio of reflected voltage and incident voltage.



REFLECTION CO-EFFIECIENT

- Derivation of reflection coefficient is as follows

$$\frac{V'}{I'} = Zo \qquad \frac{V''}{I''} = -Zo$$

$$\frac{V'}{I'} = -\frac{V''}{I''}$$

$$Z_R = Zo\,\frac{I' - I''}{I' + I''}$$

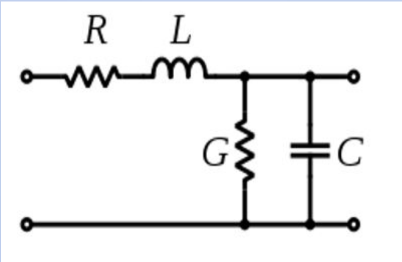
$$Z_R(I' + I'') = Z_O(I' - I'')$$

$$(Z_R + Zo)I'' = (Z_O - Z_R)I'$$

$$\boxed{\frac{I''}{I'} = \frac{(Zo - Z_R)}{(Z_R + Zo)} \qquad \frac{V''}{V'} = \frac{(Z_R - Zo)}{(Z_R + Zo)}} \text{ ---------- eq 13}$$

- If $\gamma_R = 1$ then $Z_R = 0$
- If $\gamma_R = 0$ then $Z_R = Zo$
- If $\gamma_R = -1$ then $Z_R = \infty$

DERIVATION OF VOLTAGE AND CURRENT EXPRESSIONS



- Circuit diagram for transmission line is as follows

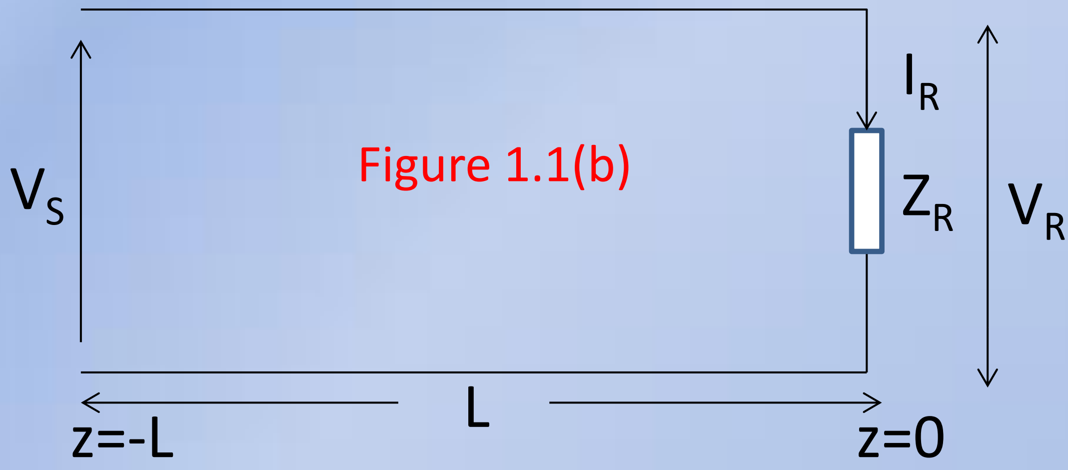


Figure 1.1(b)

- Solutions for the eq 7 which we have derived earlier can also be represented in the hyperbolic functions as follows

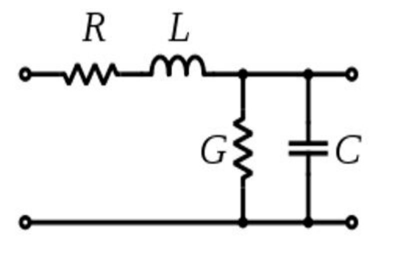
$$\boxed{(d^2I/dz^2) = \gamma^2 I \quad (d^2V/dz^2) = \gamma^2 V}$$ ---------- eq 7

$$V = A_1 \cosh(\gamma z) + B_1 \sinh(\gamma z)$$ -------------- eq 14

$$I = A_2 \cosh(\gamma z) + B_2 \sinh(\gamma z)$$ ---------------eq 15

- Thus the constants A_1, A_2, B_1 and B_2 can be determined by the applying boundary conditions.

DERIVATION OF VOLTAGE AND CURRENT EXPRESSIONS



<u>BOUNDARY CONDITIONS:</u>

- At z=0, $V=V_R$, $I=I_R$,$Z_{IMPEDEANCE}=Z_R$
  At z=-L, $V=V_S$, $I=I_S$,$Z_{IMPEDEANCE}=Z_{IN}$
- Substituting boundary condition 1 i.e., z=0 in eq 13 & 14 we get

  $A_1 = V_R$, $A_2 = I_R$

  $A_1 = V_R$, $A_2 = I_R$ ---------- eq 15
- So the main equations will become as

  $V = V_R \cosh(\gamma z) + B_1 \sinh(\gamma z)$ ------------ eq 16

  $I = I_R \cosh(\gamma z) + B_2 \sinh(\gamma z)$ ------------ eq 17
- On differentiating the equation 16

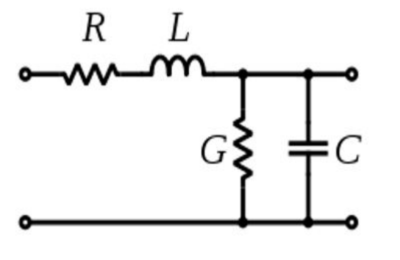
  $(dV/dz) = V_R \gamma \sinh(\gamma z) + B_1 \gamma \cosh(\gamma z)$, from the eq 1 we will get

  $-(R+jwL)I = V_R \gamma \sinh(\gamma z) + B_1 \gamma \cosh(\gamma z)$, from the eq 17 we will get

  $-(R+jwL)(I_R \cosh(\gamma z) + B_2 \sinh(\gamma z)) = V_R \gamma \sinh(\gamma z) + B_1 \gamma \cosh(\gamma z)$

  equalizing the cosh and sinh terms on both sides we will get

DERIVATION OF VOLTAGE AND CURRENT EXPRESSIONS



- $B1 = \dfrac{-(R+j\,\omega L)I_R}{\gamma} = -\dfrac{I_R(R+j\,\omega L)^{1/2}}{(G+j\omega C)^{1/2}} = -ZoI_R$
- $B2 = \dfrac{-(G+j\omega C)V_R}{\gamma} = -\dfrac{V_R(G+j\omega C)^{1/2}}{(R+j\omega L)^{1/2}} = (-V_R/Zo)$
- Substituting the constants in eq 14 and eq 15 we ge

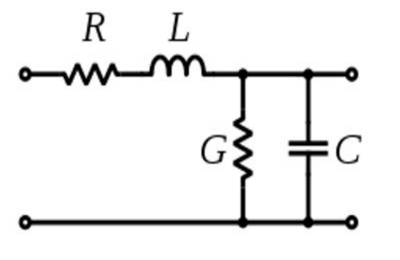
$$V = V_R \cosh(\gamma z) - ZoI_R \sinh(\gamma z) \text{ ------------ eq 17}$$
$$I = I_R \cosh(\gamma z) - (V_R/Zo)\sinh(\gamma z) \text{------------ eq 18}$$

- At $z=z_1$ current and voltages equations are,

$$V = V_R \cosh(\gamma z_1) - ZoI_R \sinh(\gamma z_1)$$
$$I = I_R \cosh(\gamma_1) - (V_R/Zo)\sinh(\gamma z_1)$$

- **Input impedance** : The ratio of voltage and current at z=-L in the transmission line is called as input impedance.
- In order to get calculate the input impedance substitute z= - L from the circuit diagram of transmission line figure 1.1(b).

INPUT IMPEDANCE(Zin)



- So substituting z= - L we get

$$V_S = V_R \cosh(\gamma L) + Zo I_R \sinh(\gamma L)$$
$$I_S = I_R \cosh(\gamma L) + (V_R/Zo)\sinh(\gamma L)$$

- Ratio of voltage and current at z=-L

$$Zin = \frac{V_R \cosh(\gamma L) + Zo I_R \sinh(\gamma L)}{I_R \cosh(\gamma L) + (V_R/Zo)\sinh(\gamma L)}$$

- After rearranging the terms and I_R common from numerator and denominator we get the final equation of Zin as

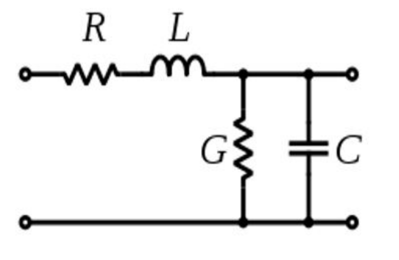
$$Zin = Zo \frac{V_R/I_R \cosh(\gamma L) + Zo \sinh(\gamma L)}{Zo \cosh(\gamma L) + (V_R/I_R)\sinh(\gamma L)}$$

$$Zin = \frac{Zo(Z_R \cosh(\gamma L) + Zo \sinh(\gamma L))}{Zo \cosh(\gamma L) + Z_R \sinh(\gamma L)} \text{ ------------ eq 19}$$

- If load impedance i.e., $Z_R = 0$ then Zin = Zsc

$$Zsc = Zo(\tanh(\gamma L)) \text{--------- eq 20}$$

INPUT IMPEDANCE(Zin) CONT……



- If load impedance is open i.e., $Z_R=\infty$ then Zin =Zoc

  Zoc = Zo(coth(γL)) --------- eq 21

- Multiplying the eq 20 & 21 we get

  $\boxed{Zoc*Zsc = Zo^2}$

- So with the quantities Zoc and Zsc known we can calculate Zo easily.

LOW LOSS AND ULTRA HIGH FREQUENCY TRANSMISSION LINE

- Power loss is due to resistive and conductance part of transmission line so if these get neglected then transmission line is low loss and this occurs in ultra high frequency ranges only.so

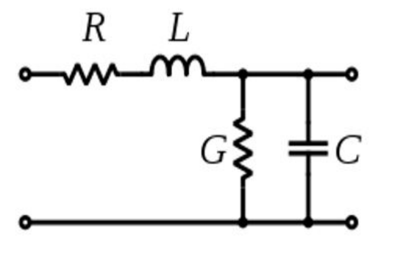
  $R \ll \omega L$, $G \ll \omega C$

- At very high frequencies the practical transmission lines are low loss line so substituting the above conditions in Zo expression we get,

$$Zo= \frac{(R+j\omega L)^{1/2}}{(G+j\omega C)^{1/2}}$$

$\boxed{Zo=(L/C)^{1/2}}$ --------- for low loss transmission line.

LOW LOSS AND ULTRA HIGH FREQUENCY TRANSMISSION LINE CONT…..



- Substituting conditions in γ we get,

  $\gamma = ((R+j\omega L)(G+j\omega C))^{1/2}$

  $\gamma = (j^2\omega^2 LC)^{1/2}$

  $\gamma = J\omega(LC)^{1/2}$

  as $\gamma = \alpha + j\beta$ where α – attenuation factor,β-phase velocity

- Comparing the equations we get

  $\alpha = 0,\ \beta = \omega(LC)^{1/2}$

- For all practical cases 'α' cannot be zero so undertaking the approximations we get 'γ' as

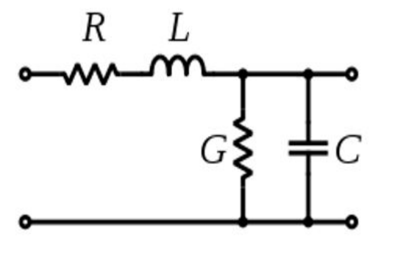
  $\gamma = ((R+j\omega L)(G+j\omega C))^{1/2}$

  $= \sqrt{(j\omega L(1+(R/j\omega L))j\omega C(1+(G/j\,\omega C))}$

  $= j\omega\sqrt{LC}\,\sqrt{((1+(R/j\omega L))(1+(G/j\,\omega C))}$ (Since $(R/j\omega L) \ll 1$ and $(G/j\,\omega C) \ll 1$)

  $= j\omega\sqrt{LC}\ ((1+(R/2j\omega L))(1+(G/2j\,\omega C))$

LOW LOSS AND ULTRA HIGH FREQUENCY TRANSMISSION LINE CONT…..



- On expanding the above equation we get

$$\gamma = \left(1 + \frac{R}{2j\omega L} + \frac{G}{2j\omega C} - \frac{RG}{4j\omega^2 LC}\right) j\omega\sqrt{LC}$$

- As ω^2 is so large for low loss transmission line $1/\omega^2=0$

$$\gamma = \left(1 - j\frac{R}{2\omega L} - j\frac{G}{2\omega C}\right) j\omega\sqrt{LC}$$

So for practical cases

$$\beta = \omega\sqrt{LC}$$

$$\alpha = \frac{1}{2}\left(R\left(\frac{L}{C}\right)^{1/2} + G\left(\frac{C}{L}\right)^{1/2}\right) = \frac{1}{2}\left(R\,Zo + \frac{G}{Zo}\right)$$

- Equations of voltage and current for low loss transmission line is as follows

$$V_s = V_R \cosh(j\beta L) + Zo I_R \sinh(j\beta L)$$

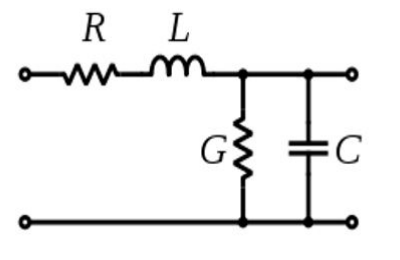
$$I_s = I_R \cosh(j\beta L) + (V_R/Zo)\sinh(j\beta L)$$

Since $\alpha=0, \gamma=j\beta L$

- Applying some mathematics we get,

$$Sinh(jx) = \frac{e^{jx} - e^{-jx}}{2} = \frac{(\cos x + j\sin x) - (\cos x - j\sin x)}{2} = j\sin x$$

INPUT IMPEDANCE OF LOW LOSS TRANSMISSION LINE



- Similarly coshjx = cosx
- Substituting these terms in voltage and current equat--ions we get $V_s = V_R \cos(\beta L) + jZoI_R \sin(\beta L)$

$$I_s = I_R \cos(\beta L) + j(V_R/Zo)\sin(\beta L)$$

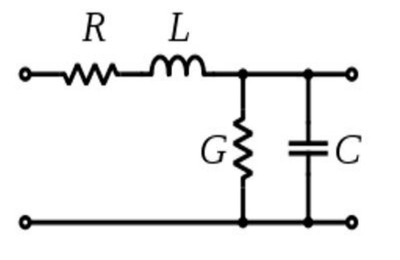
- Input impedance is Z_S is given by ratio of V_S and I_S

$$Z_S = Zo\,\frac{V_R/I_R \cos(\beta\, L) + jZo \sin(\beta\, L)}{Zo \cos(\gamma L) + j(V_R/I_R)\sin(\beta L)}$$

$$Z_S = \frac{Z_R \cos(\beta\, L) + jZo \sin(\beta\, L)}{Zo \cos(\beta\, L) + j\, Z_R \sin(\beta L)}\, Zo$$

- This is the input impedance at z=-L in terms of load impedance and characteristic impedance i.e ., Z_R , Zo respectively.
- Now replacing the terms z=0 and sending it to x=L from terminating end L=x we get ie., L=-x we get voltage and current expressions at a distance ‘x’ from the load end.

STUDY OF VOLTAGE AND CURRENT EXPRESSIONS



- The equations of voltage and current is as follows

$$V_s = V_R \cos(\beta x) - jZoI_R \sin(\beta x)$$
$$I_s = I_R \cos(\beta x) - j(V_R/Zo)\sin(\beta x)$$

- Taking V_R and I_R common from voltage and current equations respectively we get

$$V_x = V_R[\cos(\beta x) – j(Zo/Z_R) \sin(\beta x)]$$
$$I_x = I_R [\cos(\beta x) - j(Z_R/Zo)\sin(\beta x)]$$

- Magnitude of voltage and current at some arbitrary point in the transmission line is as follows

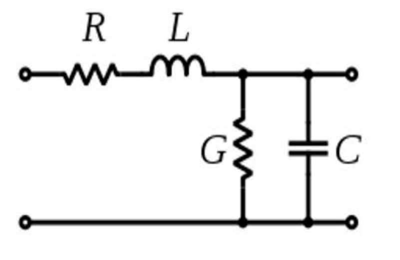
$$|V_x| = V_R\sqrt{[\cos^2(\beta x) – (Zo/Z_R)^2 \sin^2(\beta x)]}$$
$$|I_x| = I_R\sqrt{[\cos^2 (\beta x) - (Z_R/Zo)^2\sin^2(\beta x)]}$$

NEGATIVE SIGN??

- For a loss less transmission line $Zo=(L/C)^{1/2}$ which is purely resistive the terminating load should be also resistive let Zo=Ro and Z_R=R

$$|V_x| = V_R\sqrt{[\cos^2(\beta x) – (Ro/R)^2 \sin^2(\beta x)]}$$
$$|I_x| = I_R\sqrt{[\cos^2 (\beta x) - (R/Ro)^2\sin^2(\beta x)]}$$

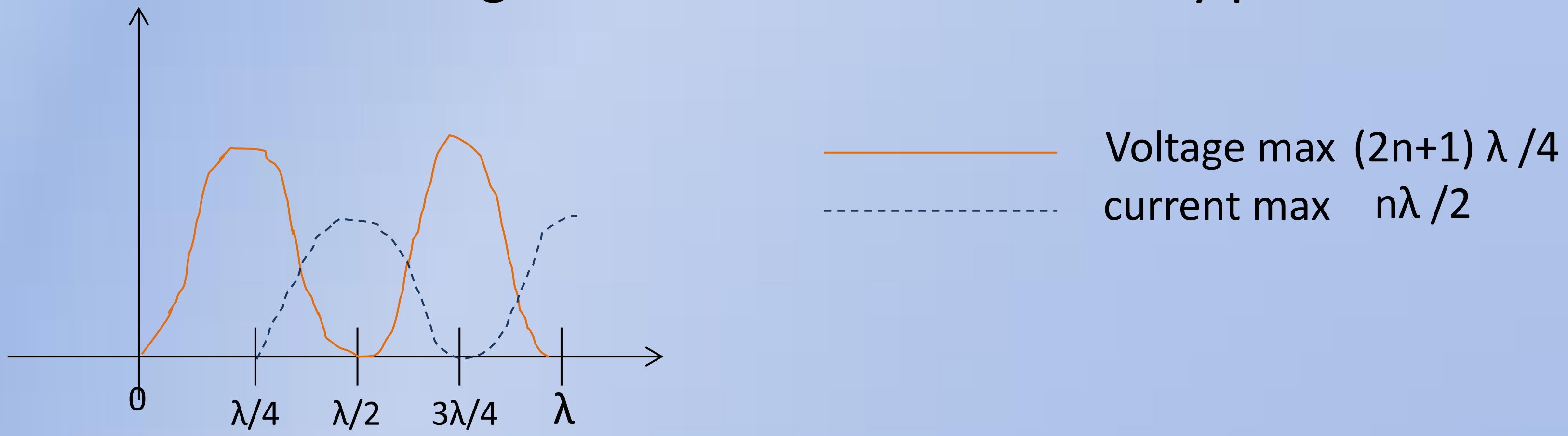
STUDY OF VOLTAGE AND CURRENT EXPRESSIONS



- When R << Ro

  $|V_X|$ will be maximum when cos(βx) is minimum or sin(βx) is maximum. Similarly $|I_X|$ will be maximum when cos(βx) is maximum or sin(βx) is minimum.

- Waveforms of voltage and current at an arbitrary point is as follows

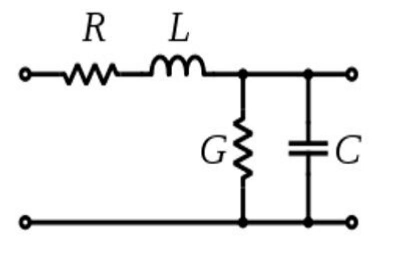


- For voltage max =>βx = (2n-1)π/2 For current max =>βx = nπ

$$\frac{2\pi x}{\lambda} = (2n-1)\pi/2 \qquad \frac{2\pi x}{\lambda} = n\pi$$

$$x = (2n+1)\lambda/4 \qquad x = n\lambda/2$$

‘α’ AND ‘β’ EXPRESSIONS



- From the expression γ

$$\gamma = ((R+j\omega L)(G+j\omega C))^{1/2} \quad \text{since } \gamma = \alpha + j\beta$$

$$(\alpha + j\beta)^2 = (R+j\omega L)(G+j\omega C)$$

$$\alpha^2 - \beta^2 + (2\alpha\beta)j = (RG - \omega^2 LC) + j\,(R\omega C + G\omega L)$$

- Equalizing real and imaginary terms on both sides we get

$$\alpha^2 - \beta^2 = RG - \omega^2 LC \text{ -------- eq 22}$$

$$2\alpha\beta = R\omega C + G\omega L$$

$$\alpha^2 + \beta^2 = ((R^2+\omega^2L^2)(G^2+\omega^2C^2))^{1/2} \text{ --------- eq 23}$$

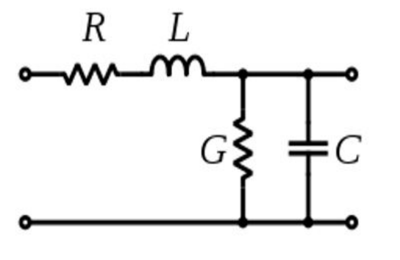
- Adding eq 22 & eq 23 we get

$$\alpha = \pm\sqrt{\frac{1}{2}[RG - \omega^2 LC] + \sqrt{((R^2+\omega^2L^2)(G^2+\omega^2C^2))}} \text{ ------------ eq 24}$$

- Similarly subtracting eq 22 and eq 23 we get

$$\beta = \pm\sqrt{\frac{1}{2}[RG - \omega^2 LC] + \sqrt{((R^2+\omega^2L^2)(G^2+\omega^2C^2))}} \text{ ------------- eq 25}$$

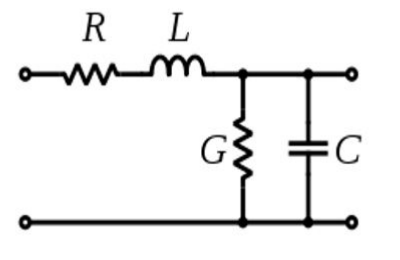
DISTORTION FREE TRANSMISSION LINE



- As said before ‘α’ as attenuation factor if α don’t vary with frequency then the transmission line is almost said to be distortion free transmission line.
- At a load side the signal is amplified by using some amplifier circuit.
- It is because, transmission line is used for different frequencies so if attenuation factor depends upon frequency, we have to make a amplifier circuit which amplifies depending upon frequency which is difficult and also sometimes transmission line is used for transmitting group of frequencies so if α depends upon frequency, each signal of different frequency is attenuated by different amounts which leads to distortion.
- So an attempt is made by varying L such that attenuation factor α is minimum such that ($d\alpha/dL$) =0.
- So differentiating the eq 24 we get

$\boxed{L = CR/G}$ ---------- distortion less condition

DISTORTION FREE TRANSMISSION LINE CONT….



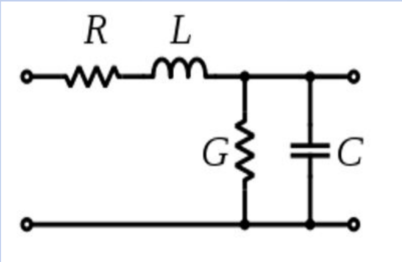
- Substituting the above condition in eq 24 & 25 we get

  $\alpha = \sqrt{RG} \qquad \beta = \omega C\left(\frac{R}{G}\right)^{1/2}$

- Under these conditions attenuation distortion is completely eliminated because there is no frequency component in expression.
- Underground cables have usually large value of R because of small diameters of conductors are used. It has also value of large shunt capacitance because of small spacing between the conductors . Similarly leakage conductance is also negligible. Under this condition addition of suitable value of inductance to the cable conductors can be used to achieve distortion less condition such arrangement is termed as loading .

STANDING WAVE RATIO

- When the reflection takes place along the line then the incident and reflected wave get combine to give standing wave. The ratio of maximum to minimum voltage or current is called standing wave ratio.



STANDING WAVE RATIO CONT…..

- Example of a standing wave is Mexican wave which is usually done to encourage cricketers by audience in stadium.

  voltage standing wave ratio(VSWR) = $\left|\dfrac{V_{MAX}}{V_{MIN}}\right|$

  current standing wave ratio(CSWR) = $\left|\dfrac{I_{MAX}}{I_{MIN}}\right|$

- If only RMS value is considered

$$|V_{MAX}| = |V_i| + |V_r|$$

$$|V_{MIN}| = |V_i| - |V_r|$$

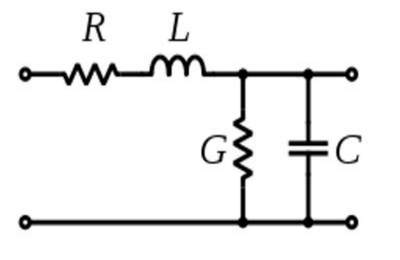
$$VSWR = \frac{|V_i| + |V_r|}{|V_i| - |V_r|} = \frac{1 + \dfrac{|V_r|}{|V_l|}}{1 - \dfrac{|V_r|}{|V_l|}} = \frac{1 + \gamma}{1 - \gamma}$$

- Similarly

$$CSWR = \frac{|I_i| + |I_r|}{|I_i| - |I_r|} = \frac{1 + \dfrac{|I_r|}{|I_l|}}{1 + \dfrac{|I_r|}{|I_l|}} = \frac{1 + \gamma}{1 - \gamma}$$

$$0 \leq \gamma \leq 1$$

$$1 \leq VSWR < \infty$$



STANDING WAVE RATIO CONT…..

- When there is a reflection then they sometimes add up or they might subtract depends upon phase differ--ence between incident and reflected wave.

$$I_{MIN}=\frac{|V_i|-|V_r|}{Zo} \qquad I_{MAX}=\frac{|V_i|+|V_r|}{Zo}$$

- Maximum impedance occurs at voltage maximum and current minimum.

$$Z_{MAX}=\frac{|V_i|+|V_r|}{I_{MIN}} = \frac{|V_i|+|V_r|}{|V_i|-|V_r|}Zo = (VSWR)Zo$$

$$Z_{MIN}=\frac{|V_i|-|V_r|}{I_{MAX}} = \frac{|V_i|-|V_r|}{|V_i|+|V_r|}Zo = Zo/(VSWR)$$

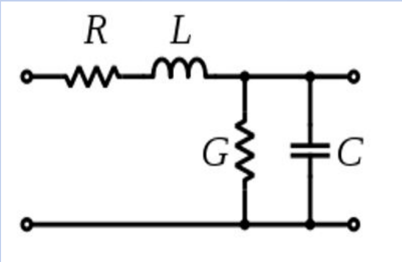
INPUT IMPEDANCE IN TERMS OF REFLECTION CO-EFFICIENT

$$Z_{IN}=Zo\frac{(Z_R\cosh(\gamma L)+Zo\sinh(\gamma L))}{Zo\cosh(\gamma L)+Z_R\sinh(\gamma L)}$$

$$Z_{IN}=Zo\frac{(Z_R(e^{\gamma L}+e^{-\gamma L})+Zo(e^{\gamma L}-e^{-\gamma L}))}{(Zo(e^{\gamma L}+e^{-\gamma L})+Z_R(e^{\gamma L}-e^{-\gamma L}))} = Zo\frac{((e^{\gamma L}(Z_R+Zo)+e^{-\gamma L}(Z_R-Zo))}{(e^{\gamma L}(Z_R+Zo)-e^{-\gamma L}(Z_R-Zo))}$$

$$Z_{IN}=Zo\frac{(e^{\gamma L}+e^{-\gamma L}\gamma_R)}{(e^{\gamma L}-e^{-\gamma L}\gamma_R)}$$ Where γ_R –Reflection coefficient

MORE ABOUT REFLECTION COEFFICIENT



- From the earlier equations it is known to us that along the line, voltage at any point V_X is always sum of incid- -ent and reflected voltages V_I and V_R and these voltages goes to maximum at respective points depending on whether V_I and V_R is in phase or in phase opposition.
- Since we are dealing with lossless line (i.e., α =0)

  $\gamma = j\beta$

  $V_X = Ae^{j\beta L} + Be^{-j\beta L}$ --------- Eq 26

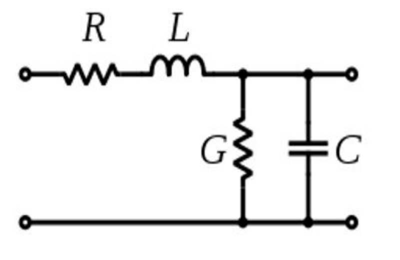
  Where 'L' is distance measured from the receiver end.
- If 'k' is reflection co-efficient expressed in magnitude and direction both in polar form

  $k = |k|\angle\phi = |k|e^{j\phi}$ $\qquad K = \frac{V_R}{I_R} = \frac{Be^{-j\beta L`}}{Ae^{-j\beta L}} = B/A$

  $B/A = |k|e^{j\phi}$

  $B = A\,|k|e^{j\phi}$ substitute it in Eq 26 we get

  $V_X = Ae^{j\beta L} + A\,|k|e^{j\phi}\,e^{-j\beta L}$



DEPENDANCE OF VOLTAGE ON 'K'

$$V_X = Ae^{j\beta L}[1 + |k|e^{j\phi}\, e^{-2j\beta L}]$$

$$V_X = Ae^{j\beta L}[1 + |k|e^{-j[2j\beta L - \phi]}]$$

- Taking only the modulus , we can get

$$|V_X| = |A|[1 + |k|e^{-j[2j\beta L - \phi]}]$$

- Now voltage has maximum value when two components are in phase . i.e., at $L=L_{max}$

$$2\beta L_{max} - \phi = 2n\Pi \text{ ------Eq a} \quad n=0,1,2,3,.....$$

$$|V_{min}|=|A|[1+|K|] \quad \text{i.e., } L=L_{max}$$

- Similarly the voltage has minimum magnitude when components are out of phase

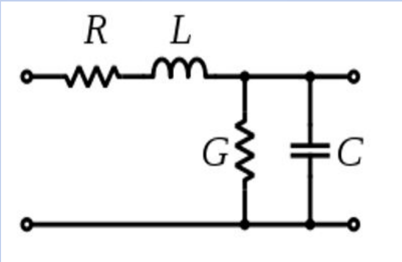
$$2\beta L_{min} - \phi = (2n+1)\Pi \text{ --------Eq b} \quad n=0,1,2,3,.....$$

$$|V_{min}|=|A|[1-|K|]$$

- From Eq a

$$2\beta L_{max} - 2n\Pi = \phi$$

CALCULATION OF ϕ



$$\phi = 2\left(\frac{2\Pi}{\lambda}\right)L_{max} - 2\Pi \qquad \text{when } n=1$$

$$\phi = 2\left(\frac{2\Pi}{\lambda}\right)L_{max} \qquad \text{when } n=0$$

$$\phi = 2\Pi(2L_{max}/\lambda) \text{ radian}$$

$$\phi = 360^{o}\ \frac{(2L_{max})}{\lambda} \text{ degrees}$$

- From equation b

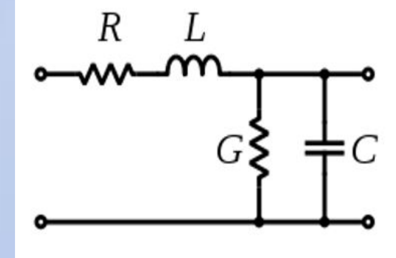
$$2\beta L_{min} - \phi = \Pi \qquad \text{when } n=0$$

$$\phi = 2\,\beta L_{min} - \Pi$$

$$\phi = 2\left(\frac{2\Pi}{\lambda}\right)L_{min} - \Pi \quad \text{radian}$$

$$\phi = 360^{o}\left[\frac{(2L_{min})}{\lambda} - \frac{1}{2}\right] \text{ degrees}$$

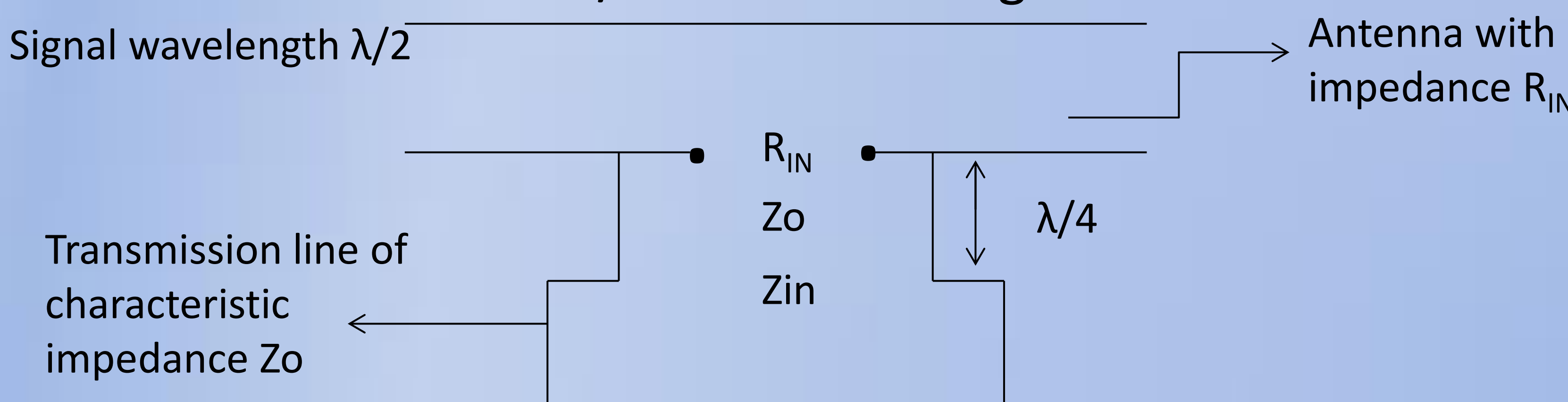
IMPEDANCE MATCHING



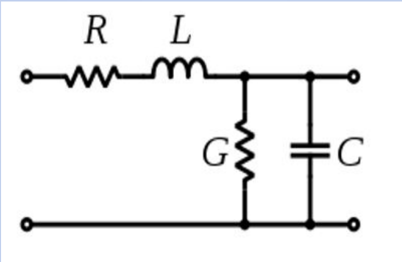
- When load impedance equals to the characteristic impedance then there will be no reflection of signal. This impedance matching can be achieved by various methods.
  - Quarter wave transformer.
  - Stub matching.
    - Single stub matching.
    - Double stub matching.

QUARTER WAVE TRANSFORMER

- Consider an antenna, the signal that is to be transmitted is given by a transmission line. Let the signal wavelength be ‘λ/2’,then length of transmission line is λ/4 as shown in figure



QUARTER WAVE TRANSFORMER CONT….



- Antenna has an impedance R_{IN} which is a load for the transmission line of characteristic impedance Zo.
- A quarter wave transformer like low frequency transformers changes the impedance of the load to another value so that matching is possible.
- This method of matching impedance uses a section of transmission line Zo of length λ/4 long. So for a loss less line input impedance at a distance λ/4 from the load is

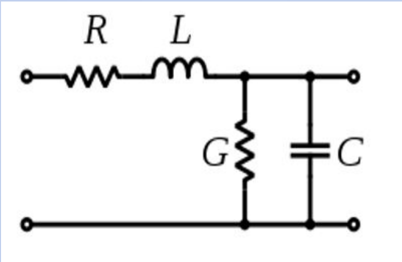
$$Zin = \frac{R_{IN} + j\,Zo \tan(\beta(\lambda/4))}{Zo + j\,R_{IN}\tan(\beta(\lambda/4))}\,Zo \quad \left(\because \beta * (\lambda/4) = \frac{2\Pi}{\lambda} * \frac{\lambda}{4} = \frac{\Pi}{2}\right)$$

$$Zin = \frac{R_{IN} + j\,Zo \tan(\Pi/2)}{Zo + j\,R_{IN}\tan(\Pi/2)}\,Zo$$

$$Zin = \frac{Zo^2}{R_{IN}}$$

$$\boxed{Zo^2 = Zin * R_{IN}} \quad \text{----------- Eq 27}$$

QUARTER WAVE TRANSFORMER CONT….

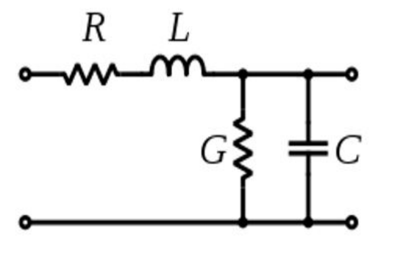


- Thus it is clear from the Eq 27, the product of input impedance Zin and R_{IN} equal to the square of charact- -eristic impedance of the line. From equation, we can say that quarter wavelength line transforms a load impedance R_{IN} i.e., smaller than Zo into a value Zin i.e., larger than Zo and vice versa.
- In this way impedance is matched by choosing values of R_{IN} , Zo and Zin which satisfy the Eq 27 there by information or signal can be transmitted without any loss to antenna(application).

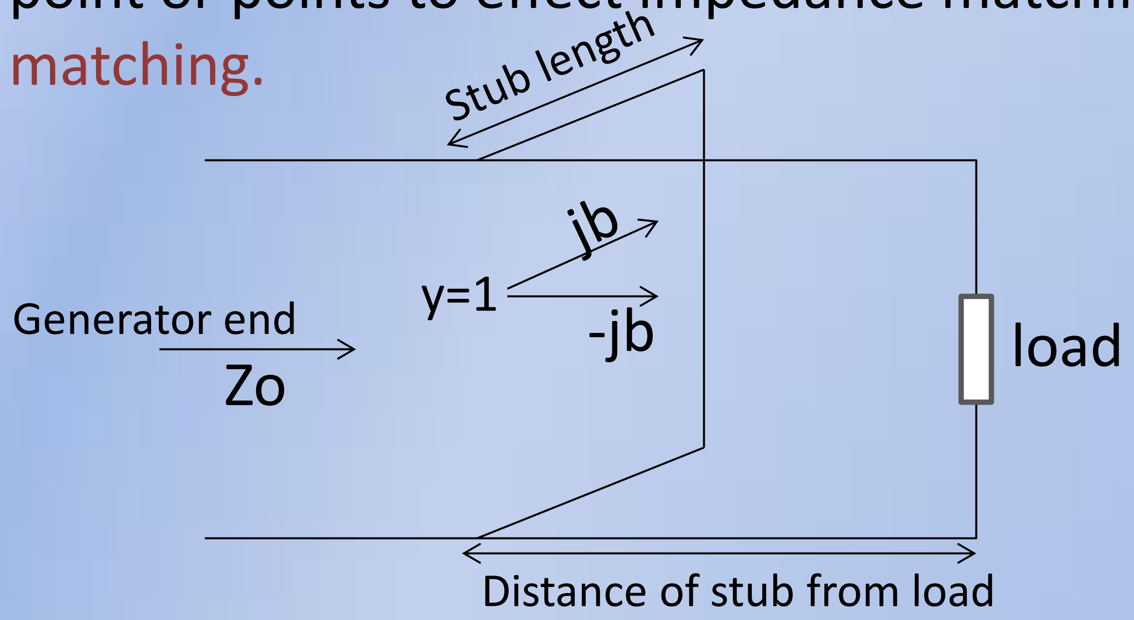
DISADVANTAGES:

- Quarter wave transformer is sensitive to change in frequency for a new wavelength the section will no longer the same λ/4 line.

INTRODUCTION TO STUB MATCHING

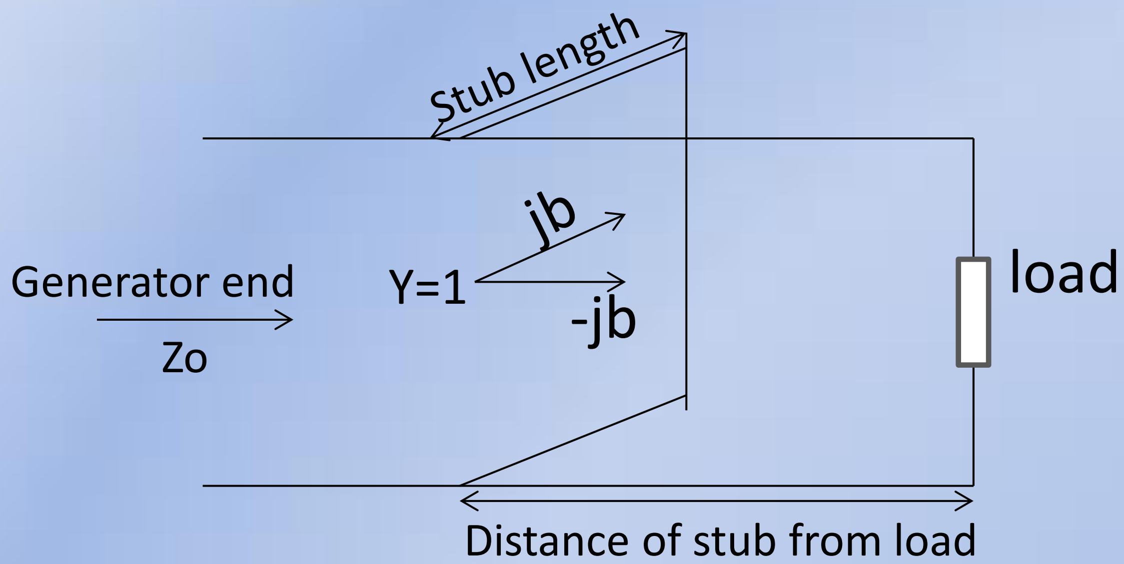


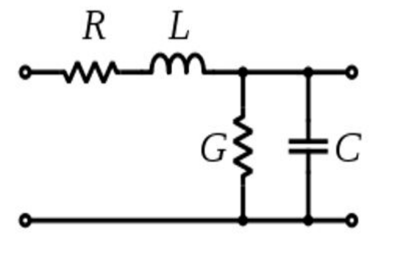
- We have seen that a section of transmission line can be used as matching section by inserting them betwe--en source and load. It is also possible to connect sections of open or short circuited line called stub in shunt with the main line at some point or points to effect impedance matching. This is called as stub matching.



- Real part will be matched by position(should move stub from load).
- Imaginary part will be matched by the length of the stub.

SINGLE STUB MATCHING





- Input impedance at any point of transmission line is given by

$$Zin= \frac{Zo(Z_R + Zo \tanh(\gamma L))}{Zo + Z_R \tanh(\gamma L)}$$

- Now converting impedance to admittance by

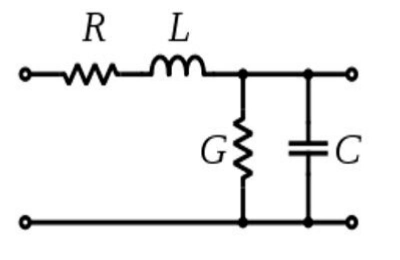
$Y_o = 1/Z_o$ [Characteristic admittance]

$Y_R = 1/Z_R$ [Load admittance]

$Y_{in} = 1/Z_{in}$ [input admittance]

$$Y_s = Y_o \frac{Y_R + Yo \tanh(\gamma L))}{Yo + Y_R \tanh(\gamma L)}$$

SINGLE STUB MATCHING cont....



- For a lossless line

$$\gamma = \alpha + j\beta \quad \text{since } \alpha=0$$

$$\gamma = j\beta$$

$$Y_S = Y_o \frac{Y_R + Yo \tanh(j\beta L))}{Yo + Y_R \tanh(j\beta L)}$$

- Using normalised admittance i.e.,

$$Y_s = \frac{Y_S}{Y_o} \qquad Y_r = \frac{Y_R}{Y_o}$$

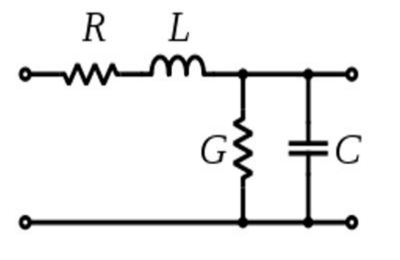
$$\frac{Y_S}{Y_o} = \frac{Y_R/Yo + j\tan(j\beta L))}{1 + j(Y_R/ Yo) \tan(j\beta L)}$$

$$Y_s = \frac{Y_r + j \tan(\beta L)}{1 + jY_r \tan(\beta L)}$$

- On rationalising the above equation with 1-jtan(βL), we get normalised source impedance as $Y_s = G_s + jB_s$.

$$G_s = \frac{Y_r(1+\tan^2\beta L)}{1+Y_r^2 \tan^2\beta L} \qquad B_s = \frac{\tan\beta L(1-Y_r^2)}{1+Y_r^2\tan^2(\beta L)}$$

SINGLE STUB MATCHING cont….



- But for no reflection, $Y_s = G_s + j\,B_s = 1 + j\,0$

$$\frac{Y_r(1+\tan^2\beta L_s)}{1+Y_r^2\tan^2\beta L_s} = 1$$

L_s-distance of stub from load

- As it is normalised, real part should be zero.

$$1+Y_r^2\tan^2\beta L_s = Y_r(1+\tan^2\beta L_s)$$

$$Y_r\tan^2\beta L_s(Y_r-1)=Y_r-1$$

$$\tan^2\beta L_s = \frac{1}{Y_r} = \frac{Y_o}{Y_R}$$

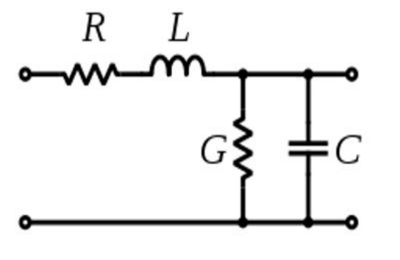
$$\beta L_s=\tan^{-1}\sqrt{Y_o/Y_R}$$

$$L_s= \frac{1}{\beta}\tan^{-1}\sqrt{Y_o/Y_R}$$

$$L_s= \frac{1}{\beta}\tan^{-1}\sqrt{Z_R/Z_O} \quad \text{---------- Eq a}$$

- **DISADVANTAGES :**
- It is difficult to locate the position where imaginary part is exactly zero by using single stub.
- L_s depends on wavelength of the signal.

SINGLE STUB MATCHING cont….



- If L_s is very small, it is difficult to locate the stub.

$$\beta = \frac{2\Pi}{\lambda}$$

$$L_s = \frac{\lambda}{2\Pi} \tan^{-1}\sqrt{Z_R/Z_O}$$

$$L_s \,\alpha\, \lambda$$

$$L_s \,\alpha\, \frac{1}{f}$$

- For high frequency variation, it is difficult.
- Eq a gives the location of stub from the load end. Now the susscept-ance at the point of attachment of stub is given by

$$B_r = \frac{B_s}{Y_0} = \frac{\mathrm{Tan}(\beta L_S)(1 - Y_r^{\,2})}{1 + Y_r^{\,2}\tan^2(\beta L_S)} \bullet \frac{1}{Y_0}$$

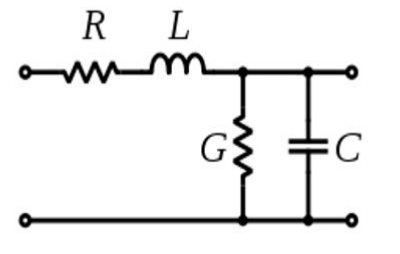
- Substitute

$$\tan(\beta L_S) = \left(\frac{Y_0}{Y_R}\right)^{1/2} \;\&\; Y_r = \frac{Y_R}{Y_0}$$ in above equation we get B_r as follows

$$B_r = \frac{B_s}{Y_0} = \left(\frac{Y_0}{Y_R}\right)^{1/2}(Y_0 - Y_R)$$

Hence this is the susceptance which should be added at the point of attachment of stub

SINGLE STUB MATCHING cont….



- This addition of susceptance can be obtained by either short-circuited stub or open circuited stub. The desired length which provide susceptance B_s is readily obtained with the help of fundamental equation.

$$V_R = V_s\cos(\beta L) – Jz_0\, I_s\sin(\beta L).$$

- For a lossless short circuited stub $V_R= 0$ because it is shorted at one end. Let the impedance be Z_t so

$$Z_t = \frac{V_s}{I_s} = j\, Z_0 \tan(\beta L_t)$$

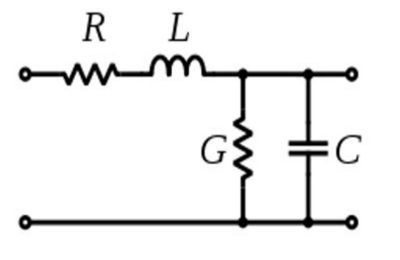
- Short circuited admittance $Y_t =1/Z_t$. So

$$Y_t = \frac{1}{j\, Z_0 \tan(\beta L_t)} = \frac{-j}{Z_0 \tan(\beta L_t)}$$

$$Y_t = G_t + j\, B_t = - j\, Y_0 \cot(\beta L_t)$$

- Equating real and imaginary part we get $G_t = 0$, $B_t = -jY_0\cot(\beta L_t)$.
- Now at the point of attachment, line susceptance and stub suscept-ance must be equal to zero.

SINGLE STUB MATCHING cont….



- So $B_t + B_s = 0$

$$Y_0 \cot(\beta L_t) + \left(\frac{Y_0}{Y_R}\right)^{1/2} (Y_0 - Y_R) = 0$$

$$\cot(\beta L_t) = \frac{(Y_0 - Y_R)}{Y_0} \left(\frac{Y_0}{Y_R}\right)^{1/2}$$

- Substituting $Y_R = (Z_R/Z_0)$ and $\beta = (\lambda/2\Pi)$ in above equation we get

$$L_t = \frac{\lambda}{2\Pi} \mathrm{Tan}^{-1} \left(\sqrt{\frac{Z_R Z_0}{(Z_R - Z_0)}}\right)$$

- **DISADVANTAGES :**
- The range of terminating impedances which can be transferred is limited.
- It is used for fixed frequency only (limited bandwidth).