# Electronics Measurement & Instrumentation
# 4EC3-06
# Unit -1

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# 4EC3-06: Electronics Measurement & Instrumentation

**Credit: 3** **Max. Marks: 150(IA:30, ETE:120)**

**3L+0T+0P** **End Term Exam: 3 Hours**

| SN | Contents | Hours |
|---|---|---|
| 1 | **Introduction:** Objective, scope and outcome of the course. | 1 |
| 2 | **THEORY OF ERRORS** - Accuracy & precision, Repeatability, Limits of errors, Systematic & random errors, Modeling of errors, Probable error & standard deviation, Gaussian error analysis, Combination of errors. | 8 |
| 3 | **ELECTRONIC INSTRUMENTS** - Electronic Voltmeter, Electronic Multimeters, Digital Voltmeter, and Component Measuring Instruments: Q meter, Vector Impedance meter, RF Power & Voltage Measurements, Introduction to shielding & grounding. | 8 |
| 4 | **OSCILLOSCOPES** – CRT Construction, Basic CRO circuits, CRO Probes, Techniques of Measurement of frequency, Phase Angle and Time Delay, Multibeam, multi trace, storage & sampling Oscilloscopes. | 7 |
| 5 | **SIGNAL GENERATION AND SIGNAL ANALYSIS** - Sine wave generators, Frequency synthesized signal generators, Sweep frequency generators. Signal Analysis - Measurement Technique, Wave Analyzers, and Frequency - selective wave analyser, Heterodyne wave analyser, Harmonic distortion analyser, and Spectrum analyser. | 8 |
| 6 | **TRANSDUCERS** - Classification, Selection Criteria, Characteristics, Construction, Working Principles and Application of following Transducers:- RTD, Thermocouples, Thermistors, LVDT, Strain Gauges, Bourdon Tubes, Seismic Accelerometers, Tachogenerators, Load Cell, Piezoelectric Transducers, Ultrasonic Flow Meters. | 8 |
|  | **Total** | **40** |

# COs

1. Describe the use of various electrical/electronic instruments, their block diagram, applications, and principles of operation, standards errors and units of measurements.
2. Develop basic skills in the design of electronic equipment.
3. Analyze different electrical/electronic parameters using state of equipment of measuring instruments which is require to all types of industries.
4. Identify electronics/ electrical instruments, understanding associated with the instruments.
5. Explain use of transducers in different types of field applications

# Text Books

1. Kalsi, H.S. “Electronic Instrumentation”, Tata McGraw-Hill Publishing Co. Ltd., 2017

2 A.K. Sawhney “A Course in Electronic and Electrical Measurement and Instrumentation”, Dhanpat Rai & Co. (P) Ltd., 2001

Text book

Reference book

For Digital voltmeters & transducers

Electronic Measurement by U.A. Bakshi, Technical pub.

# INTRODUCTION

- Instrumentation is a technology of measurement which serves sciences, engineering, medicine and etc.
- Measurement is the process of determining the amount, degree or capacity by comparison with the accepted standards of the system units being used.
- Instrument is a device for determining the value or magnitude of a quantity or variable.
- Electronic instrument is based on electrical or electronic principles for its measurement functions.

# Significance Of Measurement

Importance of Measurement is simply and eloquently expressed in the following statement of famous physicist Lord Kelvin: ”*I often say that when you can measure what you are speaking about and can express it in numbers, you know something about it; when you cannot express in it numbers your knowledge is of meager and unsatisfactory kind*”

**Methods of Measurement**

- DIRECT METHODS: In these methods, the unknown quantity (called the measurand ) is directly compared against a standard.

- INDIRECT METHOD: Measurements by direct methods are not always possible, feasible and practicable. In engineering applications measurement systems are used which require need of indirect method for measurement purposes.

# Evolution of Instruments

- MECHANICAL: These instruments are very reliable for static and stable conditions. But their disadvantage is that they are unable to respond rapidly to measurements of dynamic and transient conditions.

- ELECTRICAL: It is faster than mechanical, indicating the output are rapid than mechanical methods. But it depends on the mechanical movement of the meters. The response is 0.5 to 24 seconds.

- ELECTRONIC: It is more reliable than other system. It uses semiconductor devices and weak signal can also be detected.

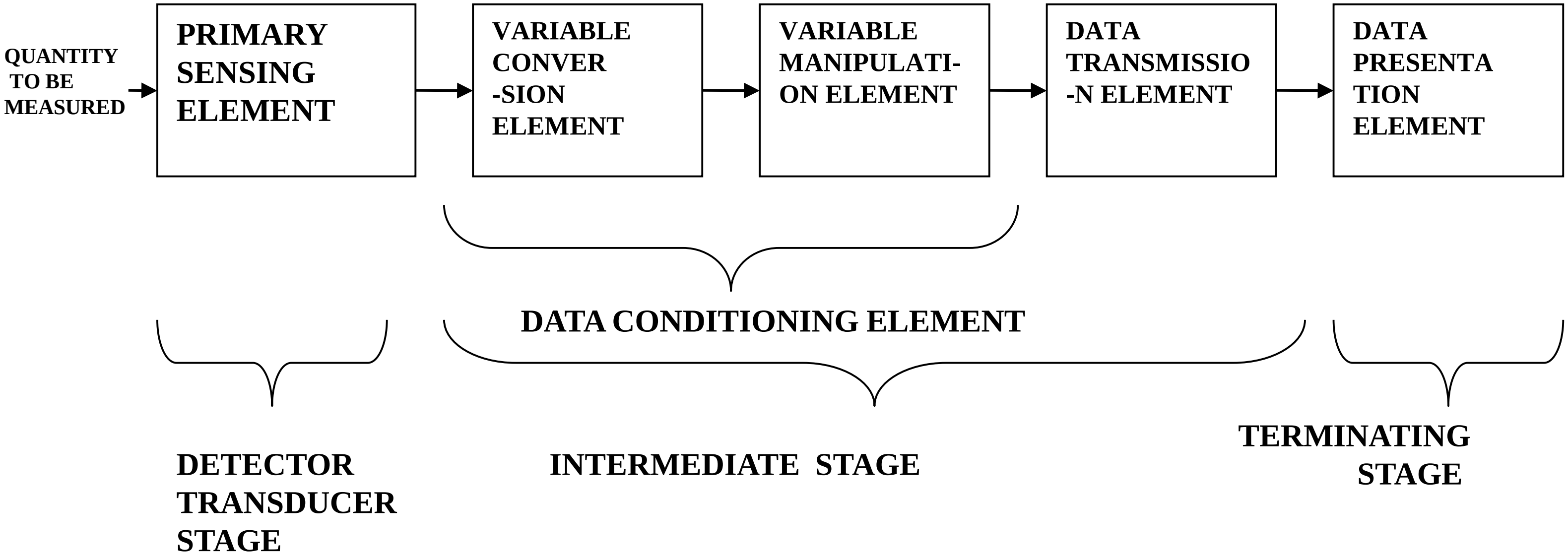
# A General Instrumentation System

- PRIMARY SENSING ELEMENT: The quantity under measurement makes its first contact with the primary sensing element of a measurement system.

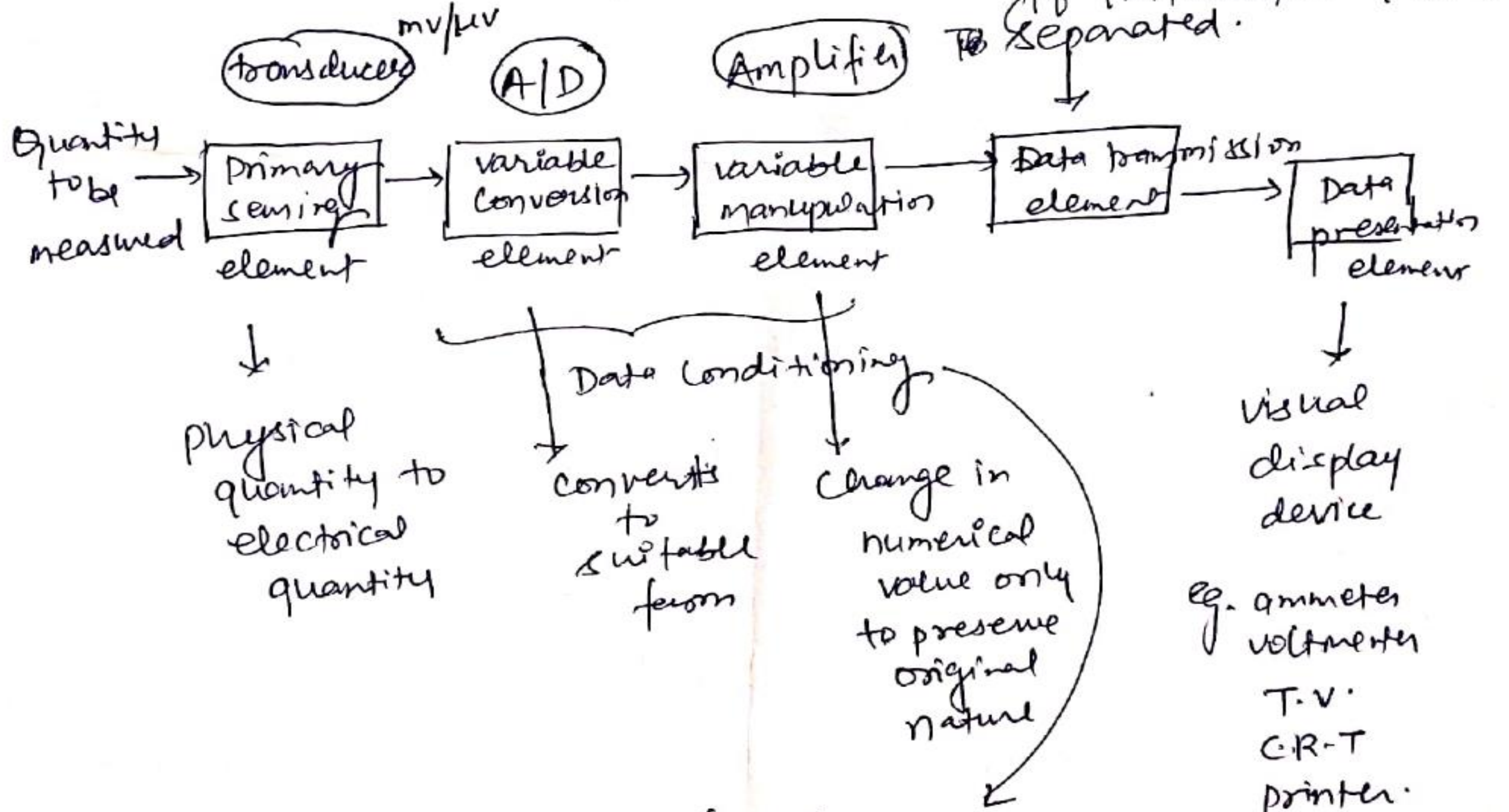
- VARIABLE CONVERSION ELEMENT: It converts the output of the primary sensing element into suitable form to preserve the information content of the original signal.

- DATA PRESENTATION ELEMENT: The information about the quantity under measurement has to be conveyed to the personnel handling the instrument or the system for monitoring, control or analysis purpose.

# Functional Elements of an Instrumentation System



# functional elements of an instrumentation system



- amplification
- attenuation
- integration
- filtering
- chopping
- clipping

# PERFORMANCE CHARACTERISTICS

- Performance Characteristics - Characteristics that shows the performance of an instrument.

Eg: accuracy, precision, resolution, sensitivity.

- Allow users to select the most suitable instrument for a specific measuring jobs.
- Two basic characteristics :
  1. Static – measuring a constant process condition.
  2. Dynamic - measuring a varying process condition.

# Static Characteristics

The set of criteria defined for the instruments, which are used to measure the quantities which are slowly varying with time is called 'static characteristics'.

**Accuracy** :- The accuracy of a measurement indicates the nearness value to the actual value of quantity. It can be expressed by different ways.
– the accuracy is expressed in terms of the percentage. Example… 0.3% of the actual value.

**Precision** :- It is the measure the degree to which successive measurement differ from each other. By this we can get fixed value of variable.

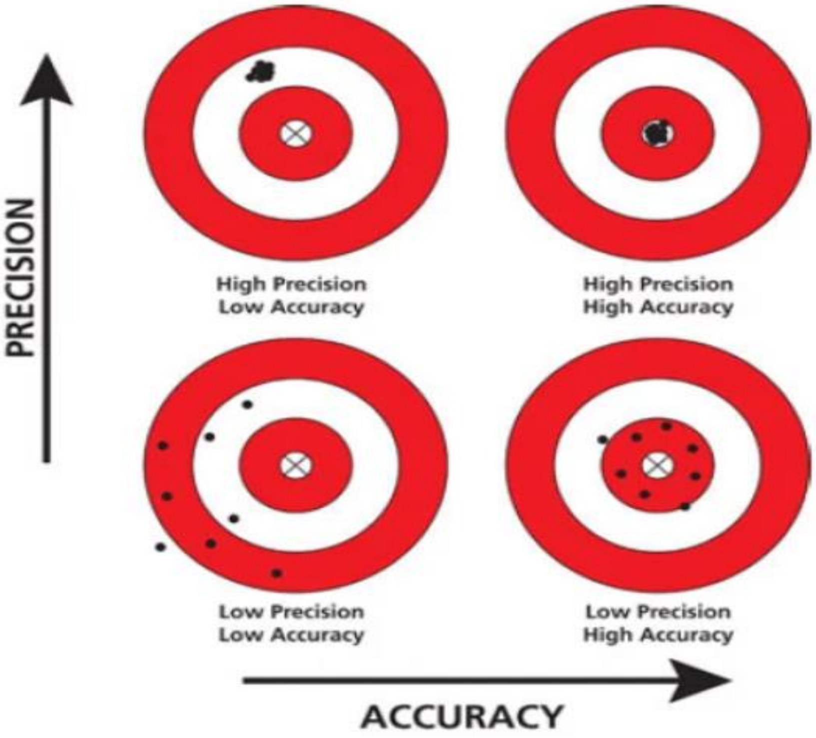
**Error** :- It is the algebraic difference between the actual value and measured value. It involved the comparison of unknown quantity with an standard quantity.

There are different types of error. Gross error, systematic error, random error, schematic error.

# Difference between accuracy and precision

*Accuracy is the degree of closeness to true value. Precision is the degree to which an instrument or process will repeat the same value. In other words, accuracy is the degree of veracity while precision is the degree of reproducibility.*

- If a measurement is accurate, it means that it agrees closely with the accepted standard for that measurement. For example, if we estimate a project's size to x and the actual size of the finished project is equal to or very close to x, then it is accurate, but it might not be precise.
- A measurement that is precise means that it agrees with other measures of the same thing.



**Repeatability** :- It is defined as the variation of the scale readings. It is random in nature. Repeatability is measure of closeness with which a given input can be measure over and over again.

**Reproducibility** :- It is defined as the degree of closeness by which a given value can be repeatedly measured.

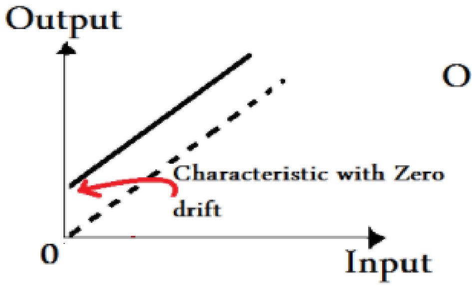
**Drift** :- Drift is defined as the gradual shift in the induction over a period of time where the input variable does not change.

Its divided in three parts, zero drift, span drift, zonal drift.

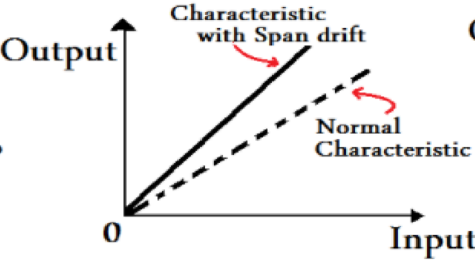
**Zero Drift:** The zero drift is defined as the deviation in the measured variable starts right from zero in the output with time.

**Span Drift:** If there is a proportionate change in its indication right along the upward scale the drift is termed span drift or sensitivity drift.

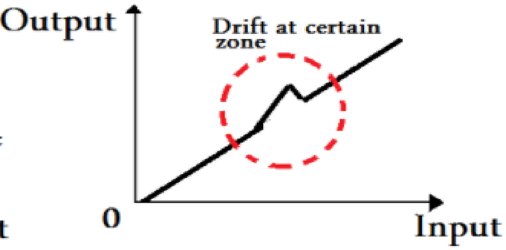
**Zonal Drift:** In case if the drift occurs only a certain portion of the span of an instrument. It is called zonal drift.



ZERO DRIFT



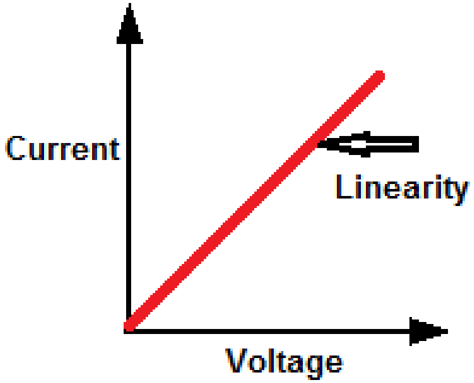
SPAN DRIFT



ZONAL DRIFT

**Sensitivity** :- it is the ratio of Change in output of an instruments to the change input.
The sensitivity of an instruments should be as high as possible.

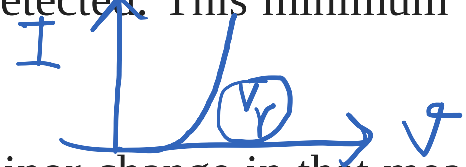
**Linearity** :- Accuracy and linearity is closely related to each other. it is defined as the ability of an instruments to reproduce its input linearly.



**Dead Zone** :- It is the largest change in input quantity for which there is no output.

**Dead time** :- It is the time before which instruments starts to responds after the input has been changed.

**Threshold** :- If the instrument input is increased very gradually from zero there will be some minimum value below which no output change can be detected. This minimum value defines the threshold of the instrument.



least count = 0.01 mm ✓

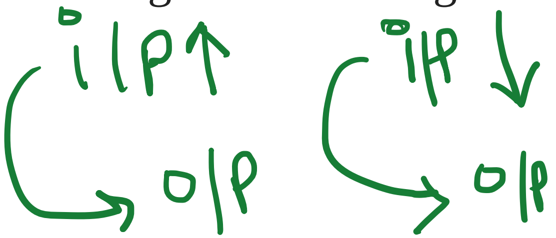
**Resolution :-** If we measure some value and if we do some minor change in that measured value, and if instruments responds, that smallest increment in the input value which can be detected by the instrument is known as resolution.

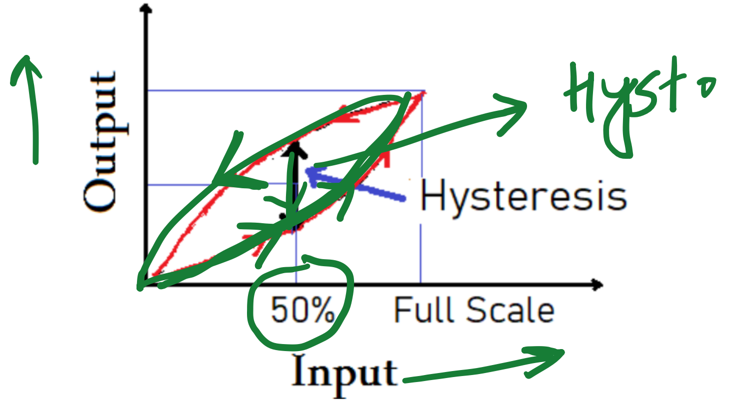
✓ **Stability** :- It is the ability of an instrument to retain its performance throughout is specified operating life.

R = 10 kΩ ± 10%

**Tolerance**:- The maximum allowable error in the measurement is specified in terms of some value which is called tolerance.

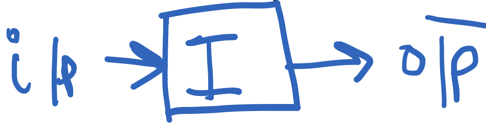
✓ **Hysteresis:** Hysteresis is a phenomenon that illustrates the different output effects when loading and unloading.

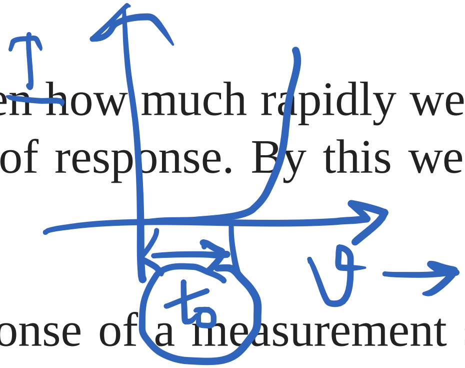




# Dynamic Characteristics of Measurement System

The set of criteria defined for the instruments, which changes rapidly with time, is called ‘dynamic characteristics’.





① **Speed of response :**– When we change the input value then how much rapidly we get the change in output value by instrument is known as speed of response. By this we get to know how fast or slow the system is.

② **Measuring lag:-** It is the retardation or delay in the response of a measurement system. The measuring lags are of two types:

1) **Retardation type:-** In this case the response of the measurement system begins immediately after the change in measured quantity has occurred.

2) **Time delay lag:-** In this case the response of the measurement system begins after a dead time when input is applied.

③ **Fidelity:** It is defined as the degree to which a measurement system indicates changes in the measured quantity without any dynamic error.

Hi-fi Wi-fi

④ **Dynamic error:-** It is the difference between the true value of the quantity changing with time & the value indicated by the measurement system if no static error is assumed. It is also called measurement error.

# TYPES OF STATIC ERROR

Types of static error

1) Gross error/human error
2) Systematic Error
3) Random Error

# TYPES OF STATIC ERROR

✓2K

20KΩ - 40 KΩ

1) **Gross Error**

- Cause by human mistakes in reading/using instruments
- May also occur due to incorrect adjustment of the instrument and the computational mistakes
- Cannot be treated mathematically
- Cannot eliminate but can minimize
- Eg: Improper use of an instrument.
- This error can be minimized by taking proper care in reading and recording from measurement parameter.
- In general, indicating instruments change ambient conditions to some extent when connected into a complete circuit.
- Therefore, several readings (at three readings) must be taken to minimize the effect of ambient condition changes.

× Single Reading

✓ avg of multiple Reading

# TYPES OF STATIC ERROR

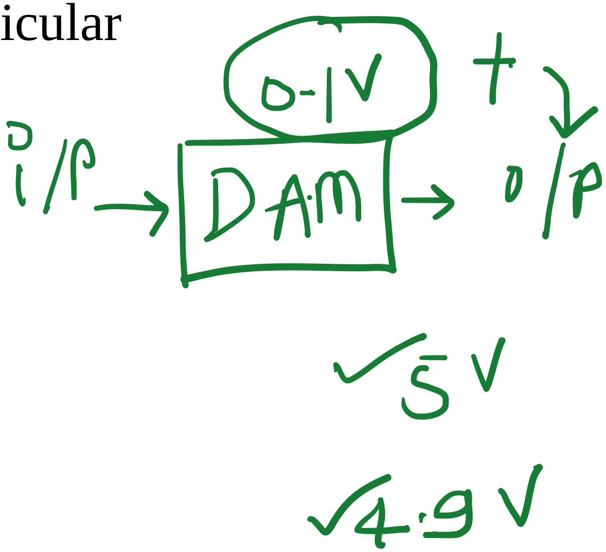
2) **Systematic Error**

- Due to shortcomings of the instrument (such as defective or worn parts, ageing or effects of the environment on the instrument)
- In general, systematic errors can be subdivided into
  - (i) Instrumental error
  - (ii) Environmental error
  - (iii) Observational error

# TYPES OF STATIC ERROR

(i) **Instrumental error**

- Inherent while measuring instrument because of their mechanical structure (eg: in a D'Arsonval meter, friction in the bearings of various moving component, irregular spring tension, stretching of spring, etc)
- Error can be avoid by:
  - (a) selecting a suitable instrument for the particular measurement application
  - (b) apply correction factor by determining instrumental error
  - (c) calibrate the instrument against standard



# TYPES OF STATIC ERROR

(ii) **Environmental error**

- due to external condition effecting the measurement including surrounding area condition such as change in temperature, humidity, barometer pressure, etc
- to avoid the error :-
  - (a) use air conditioner ✓
  - (b) sealing certain component in the instruments
  - (c) use magnetic shields

$$T = 2\pi\sqrt{\frac{L}{g}}$$

(iii) **Observational error**

- introduce by the observer ✓
- most common : ✓ parallax error and estimation error (while reading the scale)

Eg: an observer who tend to hold his head too far to the left while reading the position of the needle on the scale.

# TYPES OF STATIC ERROR

3) Random error

- due to unknown causes, occur when all systematic error has accounted
- accumulation of small effect, require at high degree of accuracy
- can be avoid by
  - (a) increasing number of reading
  - (b) use statistical means to obtain best approximation of true value

# Statistical Analysis of Error

## ① Arithmetic Mean

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}}$$

- The most probable value of measured variable is the arithmetic mean of the number of readings taken.
- It is given by ✓ $\boxed{\bar{x} = \dfrac{x_1 + x_2 + \ldots\ldots x_n}{n}} = \dfrac{\sum x}{n}$

  Where $\bar{x}$ = arithmetic mean
- x1,x2,.. x3= readings of samples
- n= number of readings

$$\left.\begin{array}{l} x_1 \\ x_2 \\ \vdots \\ x_n \end{array}\right\} \text{Observations}$$

$n$ = Total no. of obs.

# Deviation

②

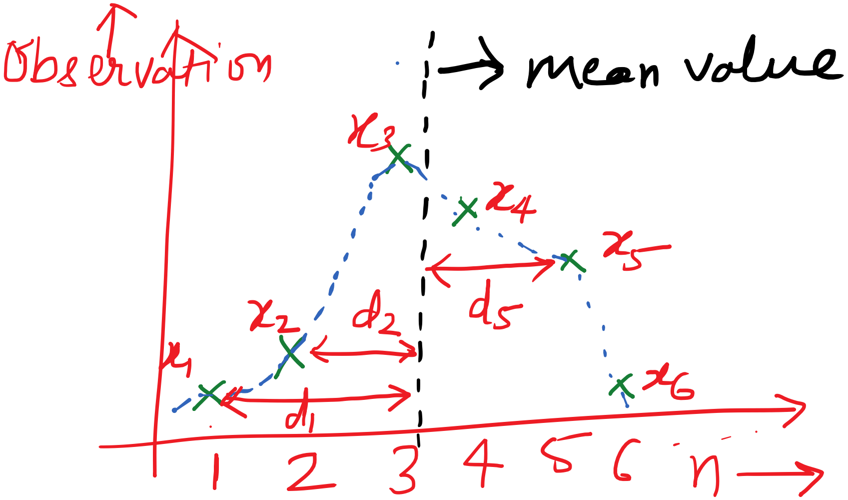
- Deviation is departure of the observed reading from the arithmetic mean of the group of readings.

$\bar{X}$ = mean value

$\bar{X} = \frac{\sum x}{n}$

$$d_1 = x_1 - \overline{X}$$
$$d_2 = x_2 - \overline{X}$$
$$d_3 = x_3 - \overline{X}$$

✓ $$d_n = x_n - \overline{X}$$



$$\boxed{d_1 + d_2 + d_3 + \ldots\ldots + d_n = 0}$$ ✓

$$= (x_1 - \overline{X}) + (x_2 - \overline{X}) + (x_3 - \overline{X}) + \ldots + (x_n - \overline{X})$$
$$= (x_1 + x_2 + x_3 + \ldots + x_n) - n\overline{X}$$
$$= n\overline{X} - n\overline{X} = 0$$

$\sum x = n\bar{X}$

# Standard Deviation

③

- The standard deviation of an infinite number of data is defined as the square root of the sum of the individual deviations squared divided by the number of readings.

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + \dots d_n^2}{n}}$$

$$d_1 = x_1 - \bar{x} \quad \dots\dots \quad d_n = x_n - \bar{x}$$

$$d_2 = x_2 - \bar{x}$$

$$S.D = \sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_4^2}{n}} = \sqrt{\frac{\sum d^2}{n}} (> 20 \underline{\underline{observation}})$$

$$S.D = s = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_4^2}{n-1}} = \sqrt{\frac{\sum d^2}{n-1}} (< 20\, observation)$$

# Variance

④

$$\boxed{Variance = (S.D)^2} = \underline{\sigma^2} = \frac{\sum d^2}{n}$$

$(> 20\ observation)$

$$Variance = (S.D)^2 = s^2 = \frac{\sum d^2}{n-1}$$

$(< 20\ observation)$

# Probable Error

⑤

Gaussian error analysis

- Probable error of one reading($r_1$)=$\underline{0.6745}$s
- Probable error of mean ($r_m$)

$$\boxed{r_m = \frac{r_1}{\sqrt{\underline{n}-1}}}$$

Standard dev.

Total no. of observatio

# Problem

Question: The following 10 observation were recorded when measuring a voltage:

41.7,42.0,41.8,42.0,42.1,

41.9,42.0,41.9,42.5,41.8 volts.

1. ✓ Mean
2. ✓ Standard Deviation
3. ✓ Probable Error
4. ✓ $\boxed{\text{Range.}}$

Step①

$$\bar{x} = \frac{x_1 + x_2 \cdots x_{10}}{10}$$

$$= \frac{41.7 + 42.0 + \cdots \cdots 41.8}{10}$$

$$= 41.97 \text{ volt}$$

$n = 10$

step②

$$d_1 = x_1 - \bar{x} = 41.7 - 41.97 =$$

$$d_2 = x_2 - \bar{x} = -$$

$$\vdots$$

$$d_{10} = x_{10} - \bar{x}$$

Step③

$$S = \sqrt{\frac{d_1^2 + d_2^2 \cdots d_{10}^2}{9}}$$

$$= 44.24$$

Step④

$$\checkmark\ r_1 = 0.4745 \times S =$$

Step⑤

$$r_m = \frac{1}{\sqrt{n-1}}$$

Step⑥

✓ Range = 42.5 − 41.7 = 0.8 Volt

# Answer

- Mean=41.97 volt
- S.D=0.22 volt
- Probable error=0.15 volt
- Range=0.8 volt.

# Absolute Error/Limiting Error

Denoted by $\partial A$ / $\varepsilon_0$

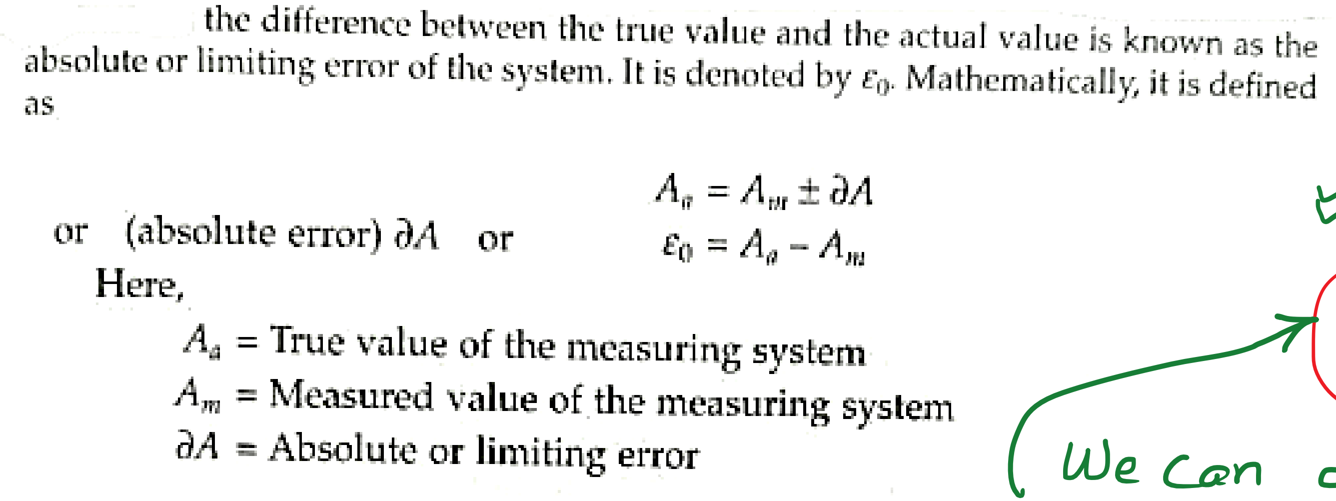
$$\boxed{\partial A = A_a - A_m}$$

True

measured

$$\boxed{A_a = A_m \pm \partial A}$$

We can always represent a parameter in this form if we know limiting error



the difference between the true value and the actual value is known as the absolute or limiting error of the system. It is denoted by $\varepsilon_0$. Mathematically, it is defined as

$$A_a = A_m \pm \partial A$$

or (absolute error) $\partial A$ or $\varepsilon_0 = A_a - A_m$

Here,

$A_a$ = True value of the measuring system

$A_m$ = Measured value of the measuring system

$\partial A$ = Absolute or limiting error

# Relative or Fractional Error

$\boxed{\epsilon_r}$

denoted by ↗

The relative or fractional error is defined as the ratio of the absolute or limiting error ($\partial A$) to the measured or actual value ($A_m$) of the system. It is denoted by $\varepsilon_r$. Mathematically, it is defined as

Absolute error → $\partial A$

Measured value → $Am$

Correction factor

$$\boxed{\delta C = -\delta A}$$

$$\text{Relative error, } \varepsilon_r = \frac{\partial A}{A_m} = \frac{\varepsilon_0}{A_m}$$

$$\boxed{\epsilon_r = \frac{\partial A}{Am}}$$

or

$$\varepsilon_0 = \varepsilon_r \cdot A_m$$

Absolute error

Putting this value in the expression of $A_a$,

$$A_a = A_m \pm \varepsilon_r \cdot A_m$$

or

$$A_a = A_m(1 \pm \varepsilon_r)$$

$\therefore$ Percentage relative error ($\%\ \varepsilon_r$) = $\varepsilon_r \times 100$

$$\boxed{\%\ \epsilon_r = \epsilon_r \times 100}$$

$$\partial A = \epsilon_r Am$$

$$Aa = Am \pm \partial A$$

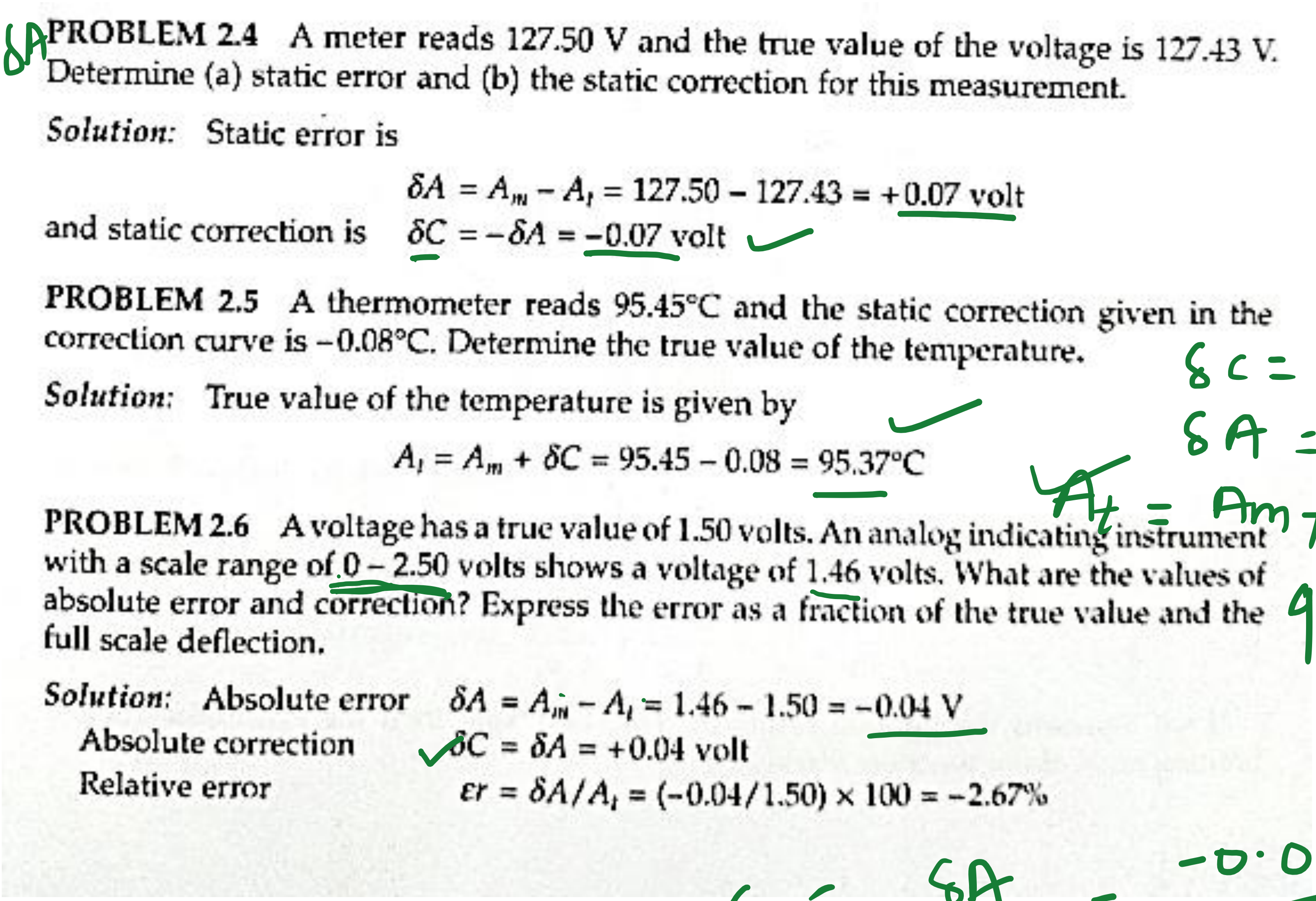
We can always represent a parameter in terms of relative error like →

$$\boxed{Aa = Am \pm \epsilon_r Am}$$

$$\boxed{Aa = Am(1 + \epsilon_r)}$$

$\delta A = A_m - A_t$   $A_t = 127.43\ V$,   $A_m = 127.50\ V$

$\delta c = -\delta A$



**PROBLEM 2.4** A meter reads 127.50 V and the true value of the voltage is 127.43 V. Determine (a) static error and (b) the static correction for this measurement.

*Solution:* Static error is

$$\delta A = A_m - A_t = 127.50 - 127.43 = +0.07 \text{ volt}$$

and static correction is $\delta C = -\delta A = -0.07$ volt ✓

**PROBLEM 2.5** A thermometer reads 95.45°C and the static correction given in the correction curve is −0.08°C. Determine the true value of the temperature.

*Solution:* True value of the temperature is given by ✓

$$A_t = A_m + \delta C = 95.45 - 0.08 = 95.37°C$$

**PROBLEM 2.6** A voltage has a true value of 1.50 volts. An analog indicating instrument with a scale range of 0 – 2.50 volts shows a voltage of 1.46 volts. What are the values of absolute error and correction? Express the error as a fraction of the true value and the full scale deflection.

*Solution:* Absolute error $\delta A = A_m - A_t = 1.46 - 1.50 = -0.04$ V

Absolute correction ✓ $\delta C = \delta A = +0.04$ volt

Relative error $\varepsilon r = \delta A / A_t = (-0.04/1.50) \times 100 = -2.67\%$

$\delta c = -0.08$

$\delta A = -\delta c$

$A_t = A_m + \delta c$

$95.45 - 0.08$

$\varepsilon r \times 100 = \frac{\delta A}{A_t} = \frac{-0.04}{1.50} \times 100$

$= -2.67$

Ex. A resistance R of 600 Ω is known to have possible absolute error as ± 60 Ω. Express the value of resistor in relative error.

R= 600 ± 60 Ω

Relative error= ± 60/600 = ± 0.1 = ± 10 %

Thus R = 600 ± 10% Ω

Percentages are usually employed to express errors in resistances and electrical quantities. The terms *Accuracy* & *Tolerance* are also used. A resistor with ± 10% error is said to be accurate to ± 10 % or having tolerance of ± 10%.

$R = 600 \pm 60\,\Omega$

$R = 600 \pm \boxed{x}\%$ → Relative error

$\boxed{R = 600 \pm 10\%}$

$E_r = \dfrac{\delta A}{A_t}$

$= \dfrac{60}{600} = 0.1 \times 1$

$= 10\%$

# Combination of two quantities with Limiting Error

Suppose we have two parameters $a_1$ with absolute error $\partial a_1$ & $a_2$ with absolute error $\partial a_2$

Case I: When we add / subtract the parameters :-

Let A is resultant parameter with absoulte error $\partial A$

so $A = a_1 + a_2$

True value of $a_1 = a_1 \pm \partial a_1$

True value of $a_2 = a_2 \pm \partial a_2$

Hence we can write

$$A \pm \partial A = a_1 \pm \partial a_1 + a_2 \pm \partial a_2$$

$$\Rightarrow A \pm \partial A = A \pm (\partial a_1 + \partial a_2)$$

$$\Rightarrow \boxed{\partial A = \pm(\partial a_1 + \partial a_2)}$$

* In both addition & subtraction the resultant limiting error always added

# Combination of two quantities with Limiting Error

Suppose we have two parameters $a_1$ with absolute error $\partial a_1$ & $a_2$ with absolute error $\partial a_2$

**Case I**: When we multiply/divide the parameters :-

Let A is resultant parameter with absoulte error $\partial A$

we can write $A = a_1 \cdot a_2$

True value of $a_1 = a_1 \pm \partial a_1$

True value of $a_2 = a_2 \pm \partial a_2$

so $A \pm \partial A = (a_1 \pm \partial a_1)(a_2 \pm \partial a_2)$

$$A \pm \partial A = a_1 a_2 \pm a_1 \partial a_2 \pm a_2 \partial a_1 \pm \cancel{\partial a_1 \partial a_2}$$

neglected

divide both sides with A

$$\frac{A \pm \partial A}{A} = \frac{a_1 a_2 \pm a_1 \partial a_2 \pm a_2 \partial a_1}{a_1 a_2}$$

$$\Rightarrow 1 \pm \frac{\partial A}{A} = 1 \pm \frac{\partial a_2}{a_2} \pm \frac{\partial a_1}{a_1}$$

$$\Rightarrow \boxed{\frac{\partial A}{A} = \pm\left(\frac{\partial a_1}{a_1} + \frac{\partial a_2}{a_2}\right)}$$

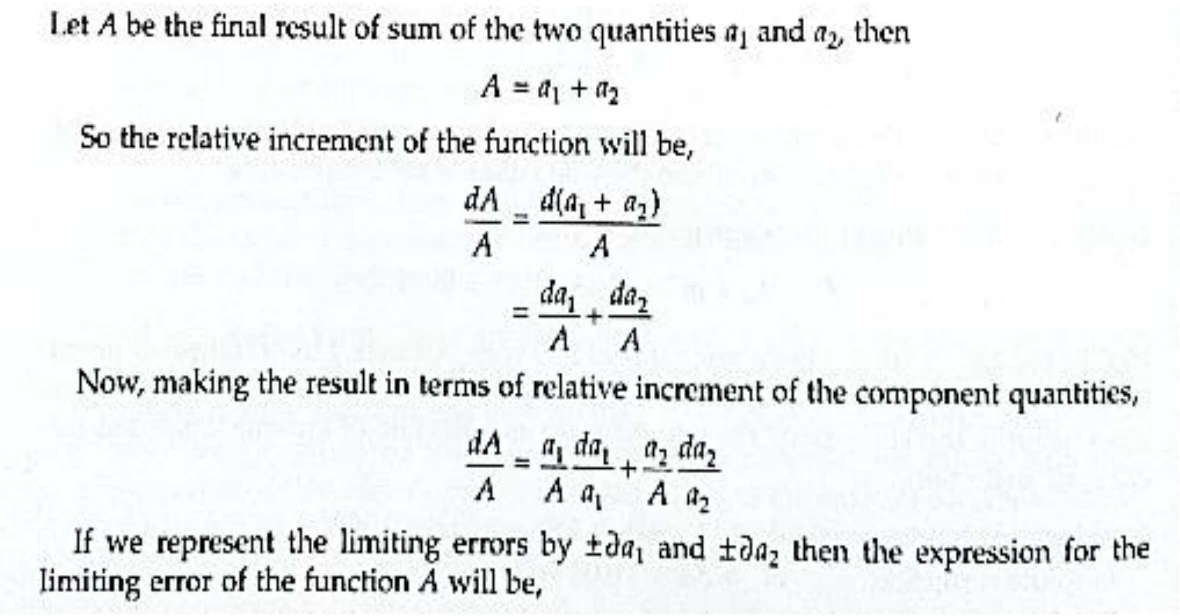
$\frac{\partial A}{A}$ → Relative error , $\partial A$ → limiting/Absolute error

Note :- ⇒ In case of addition & subtraction the absoute/limiting error is added in the resultant

⇒ In case of multiplication & division the relative error is added in the resultant

# Combination of two quantities with Limiting Error

## Sum of two quantities



Let $A$ be the final result of sum of the two quantities $a_1$ and $a_2$, then

$$A = a_1 + a_2$$

So the relative increment of the function will be,

$$\frac{dA}{A} = \frac{d(a_1 + a_2)}{A}$$

$$= \frac{da_1}{A} + \frac{da_2}{A}$$

Now, making the result in terms of relative increment of the component quantities,

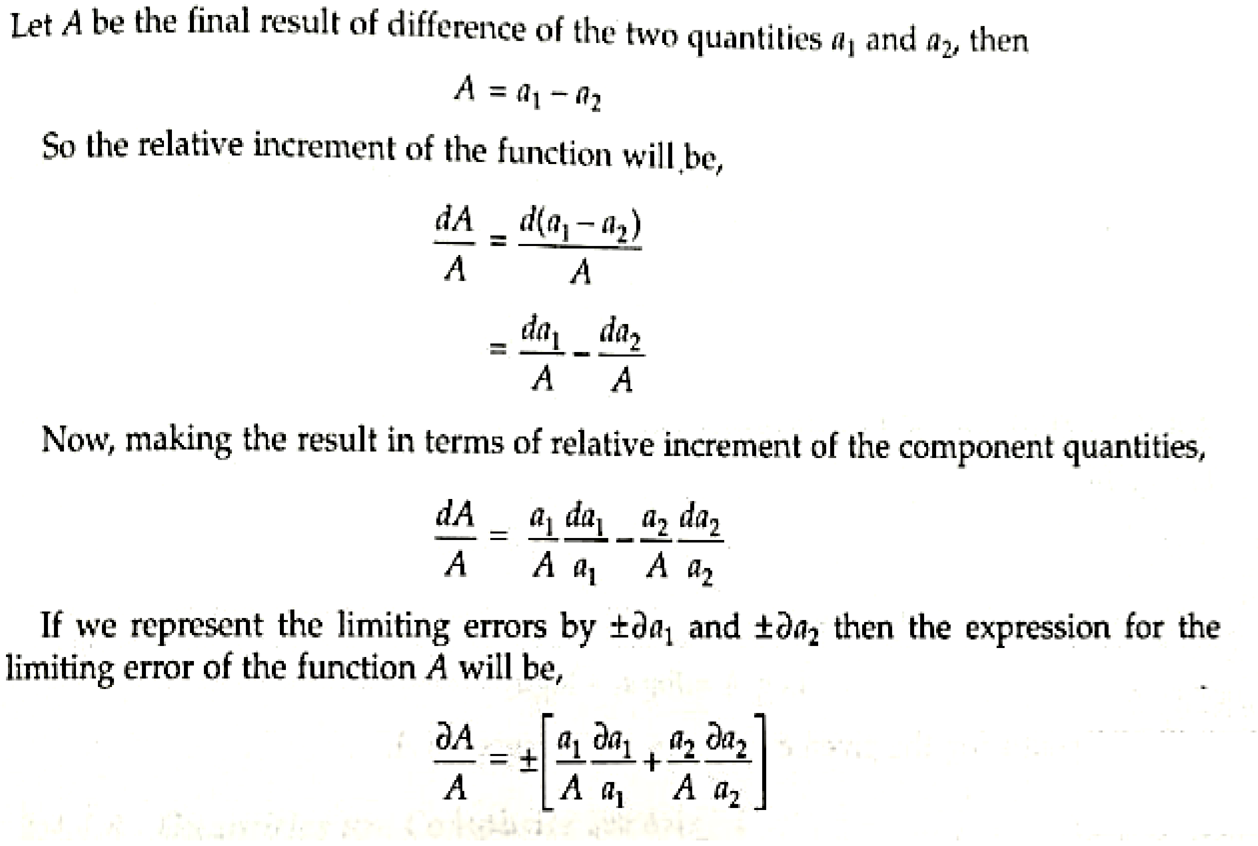
$$\frac{dA}{A} = \frac{a_1}{A}\frac{da_1}{a_1} + \frac{a_2}{A}\frac{da_2}{a_2}$$

If we represent the limiting errors by $\pm\partial a_1$ and $\pm\partial a_2$ then the expression for the limiting error of the function $A$ will be,

$$\frac{\partial A}{A} = \pm\left[\frac{a_1}{A}\frac{\partial a_1}{a_1} + \frac{a_2}{A}\frac{\partial a_2}{a_2}\right]$$

✓

# Difference of two quantities



Let $A$ be the final result of difference of the two quantities $a_1$ and $a_2$, then

$$A = a_1 - a_2$$

So the relative increment of the function will be,

$$\frac{dA}{A} = \frac{d(a_1 - a_2)}{A}$$

$$= \frac{da_1}{A} - \frac{da_2}{A}$$

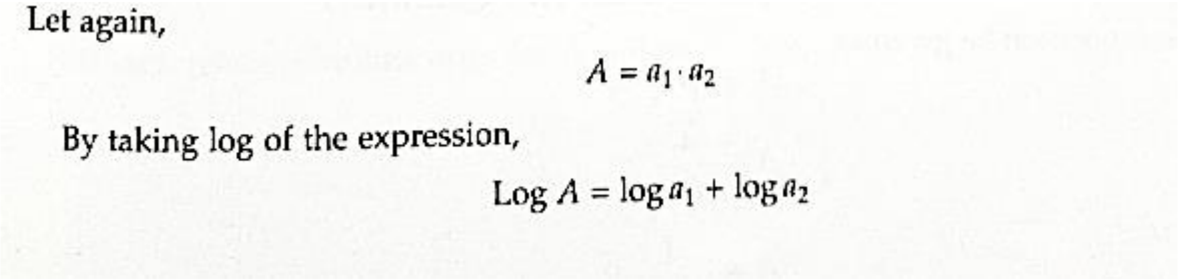
Now, making the result in terms of relative increment of the component quantities,

$$\frac{dA}{A} = \frac{a_1}{A}\frac{da_1}{a_1} - \frac{a_2}{A}\frac{da_2}{a_2}$$

If we represent the limiting errors by $\pm\partial a_1$ and $\pm\partial a_2$ then the expression for the limiting error of the function $A$ will be,

$$\frac{\partial A}{A} = \pm\left[\frac{a_1}{A}\frac{\partial a_1}{a_1} + \frac{a_2}{A}\frac{\partial a_2}{a_2}\right]$$

# Product of two quantities

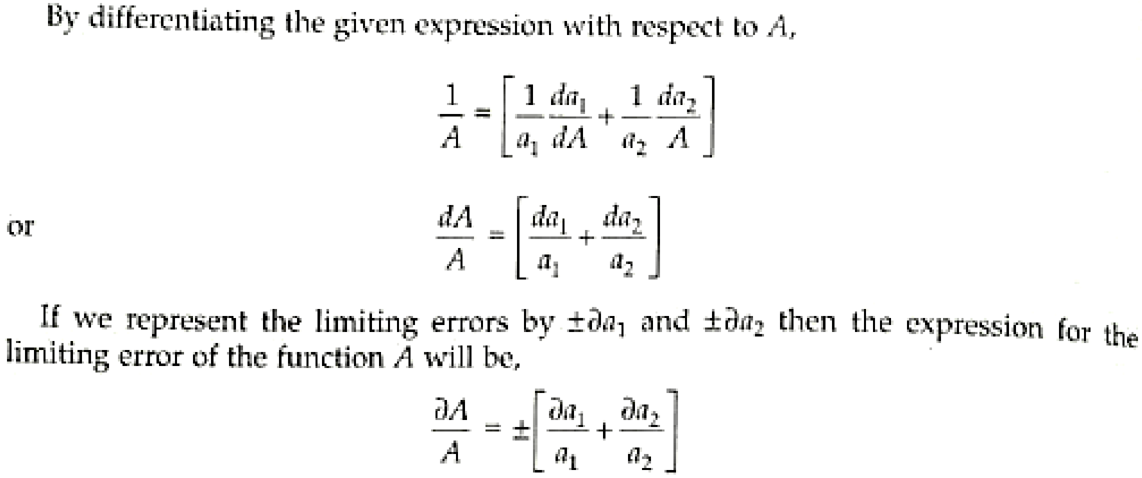


Let again,

$$A = a_1 \cdot a_2$$

By taking log of the expression,

$$\text{Log } A = \log a_1 + \log a_2$$



By differentiating the given expression with respect to $A$,

$$\frac{1}{A} = \left[\frac{1}{a_1}\frac{da_1}{dA} + \frac{1}{a_2}\frac{da_2}{A}\right]$$

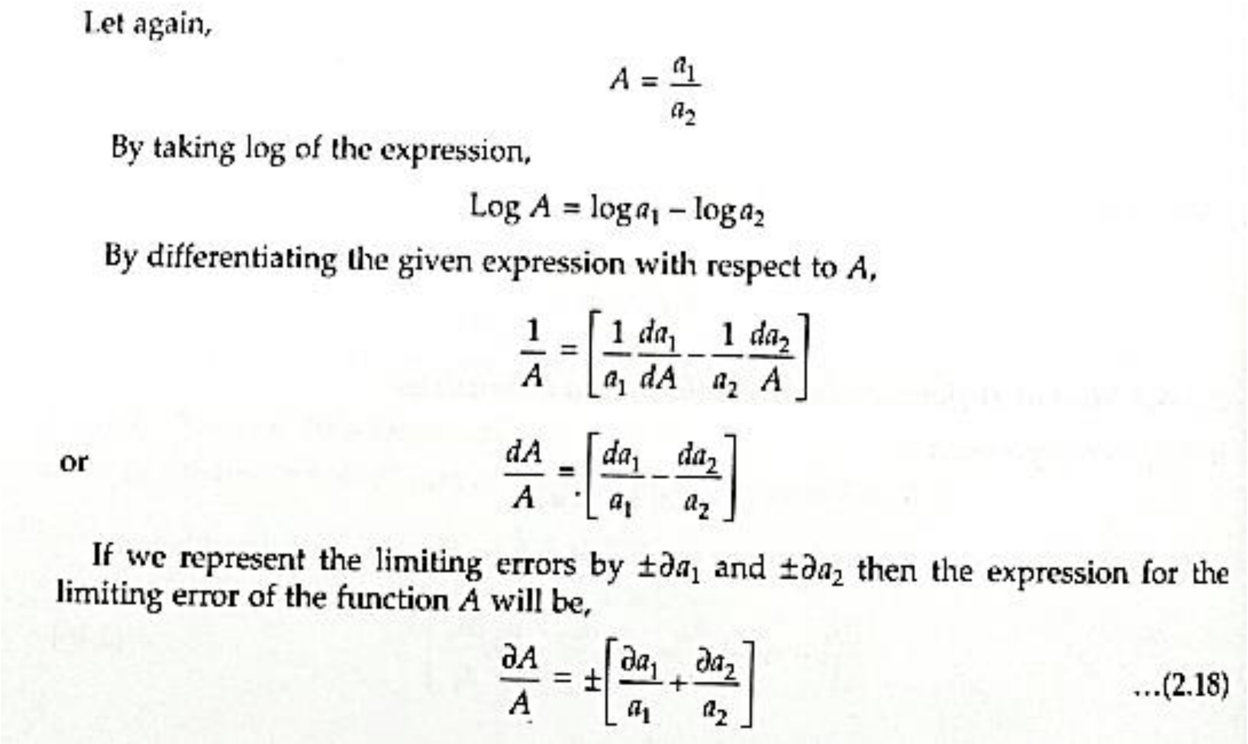
or

$$\frac{dA}{A} = \left[\frac{da_1}{a_1} + \frac{da_2}{a_2}\right]$$

If we represent the limiting errors by $\pm\partial a_1$ and $\pm\partial a_2$ then the expression for the limiting error of the function $A$ will be,

$$\frac{\partial A}{A} = \pm\left[\frac{\partial a_1}{a_1} + \frac{\partial a_2}{a_2}\right]$$

# Division of two quantities



Let again,

$$A = \frac{a_1}{a_2}$$

By taking log of the expression,

$$\text{Log } A = \log a_1 - \log a_2$$

By differentiating the given expression with respect to $A$,

$$\frac{1}{A} = \left[\frac{1}{a_1}\frac{da_1}{dA} - \frac{1}{a_2}\frac{da_2}{A}\right]$$

or

$$\frac{dA}{A} = \left[\frac{da_1}{a_1} - \frac{da_2}{a_2}\right]$$

If we represent the limiting errors by $\pm\partial a_1$ and $\pm\partial a_2$ then the expression for the limiting error of the function $A$ will be,

$$\frac{\partial A}{A} = \pm\left[\frac{\partial a_1}{a_1} + \frac{\partial a_2}{a_2}\right] \quad \text{...(2.18)}$$

# Quantities are Power of a Factor

Let the expression be,

$$A = a_1^x$$

Taking log of the expression

$$\text{Log } A = x \log a_1$$

Now differentiating the expression with respect to $A$,

$$\frac{1}{A} = x \cdot \frac{1}{a_1} \frac{da_1}{dA}$$

or

$$\frac{dA}{A} = x \cdot \frac{da_1}{a_1}$$

Hence, relative limiting error for $A$ will be

$$\frac{\partial A}{A} = \pm x \cdot \frac{\partial a_1}{a_1}$$

# Quantities are composite Factor

Let the expression be,

$$A = a_1^x \cdot a_2^x$$

Taking log of the expression

$$\text{Log } A = x \log a_1 + y \log a_2$$

Now differentiating the expression with respect to $A$,

$$\frac{1}{A} = x \frac{1}{a_1} \frac{da_1}{dA} + y \frac{1}{a_2} \frac{da_2}{dA}$$

or,

$$\frac{dA}{A} = x \frac{da_1}{a_1} + y \frac{da_2}{a_2}$$

Hence, relative limiting error for $A$ will be

$$\frac{\partial A}{A} = \pm \left[ x \frac{\partial a_1}{a_1} + y \frac{\partial a_2}{a_2} \right]$$

Ex:

$$A = a^2 = a \cdot a$$

$$A \rightarrow \partial A$$

$$a \rightarrow \partial a$$

$$A \pm \partial A = (a \pm \partial a)(a \pm \partial a)$$

$$A \pm \partial A = a^2 \pm 2\overset{a}{\wedge}\partial a \pm \cancel{(\partial a)^2}$$

divide both side 'A'

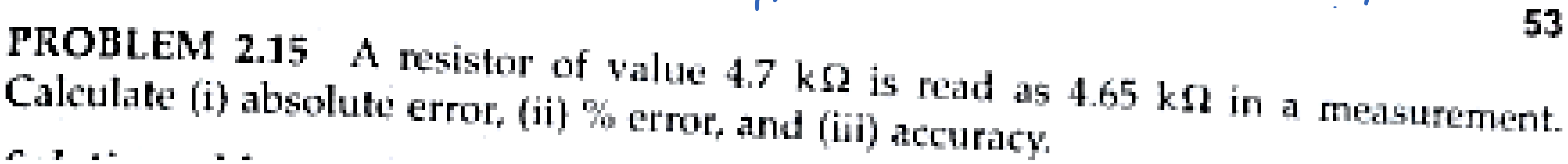
$$\frac{A \pm \partial A}{A} = \frac{a^2 \pm 2 \cdot \partial a \times a}{a^2}$$

$$1 \pm \frac{\partial A}{A} = 1 \pm \frac{2\partial a}{a}$$

$$\Rightarrow \boxed{\frac{\partial A}{A} = \pm \frac{2\partial a}{a}}$$

# Numerical Problems and Solution

$$\% \text{ error} = \frac{\text{Abs. error}}{\text{True value}} \times 100 = \frac{R_m - R_t}{R_t} \times 100$$



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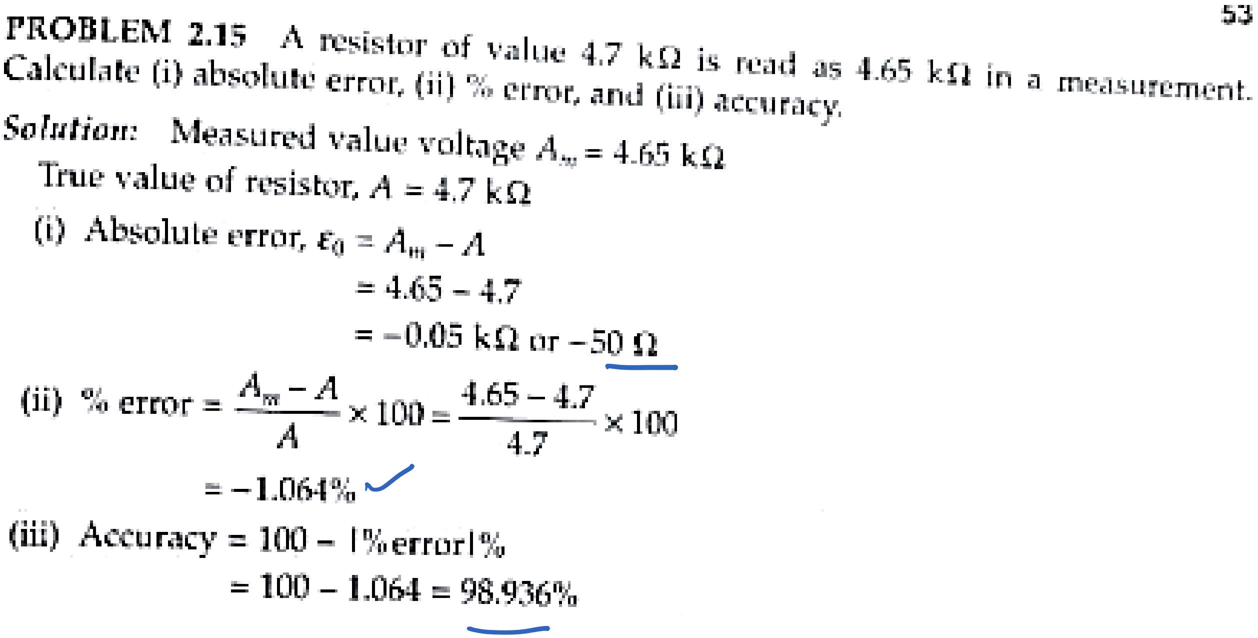
**PROBLEM 2.15** A resistor of value 4.7 kΩ is read as 4.65 kΩ in a measurement. Calculate (i) absolute error, (ii) % error, and (iii) accuracy.

Sol$^n$ $R_t = 4.7 \, k\Omega$, $R_m = 4.65 \, K\Omega$

(i) $\partial R = R_m - R_t = 4.65 - 4.7 = -0.05 \, K\Omega$

(ii) Rel. error. $\epsilon_r = \frac{\partial R}{R_t} = \frac{-0.05}{4.7 \, K\Omega} \cancel{\times 100\%}$

(iii) Accuracy $= 100 - |\%\text{ error}| =$ ____



**PROBLEM 2.15** A resistor of value 4.7 kΩ is read as 4.65 kΩ in a measurement. Calculate (i) absolute error, (ii) % error, and (iii) accuracy.

*Solution:* Measured value voltage $A_m = 4.65$ kΩ

True value of resistor, $A = 4.7$ kΩ

(i) Absolute error, $\varepsilon_0 = A_m - A$

$$= 4.65 - 4.7$$

$$= -0.05 \text{ k}\Omega \text{ or } -50\ \Omega$$

(ii) $$\%\text{ error} = \frac{A_m - A}{A} \times 100 = \frac{4.65 - 4.7}{4.7} \times 100$$

$$= -1.064\%$$

(iii) $$\text{Accuracy} = 100 - |\%\text{error}|\%$$

$$= 100 - 1.064 = 98.936\%$$

**PROBLEM 2.16** A moving coil voltmeter has a uniform scale with 100 divisions and gives full-scale reading of 200 V. The instrument can read up to $\frac{1}{5}$th of a scale division with a fair degree of certainty. Determine the resolution of the instrument in volts.

↳ least count

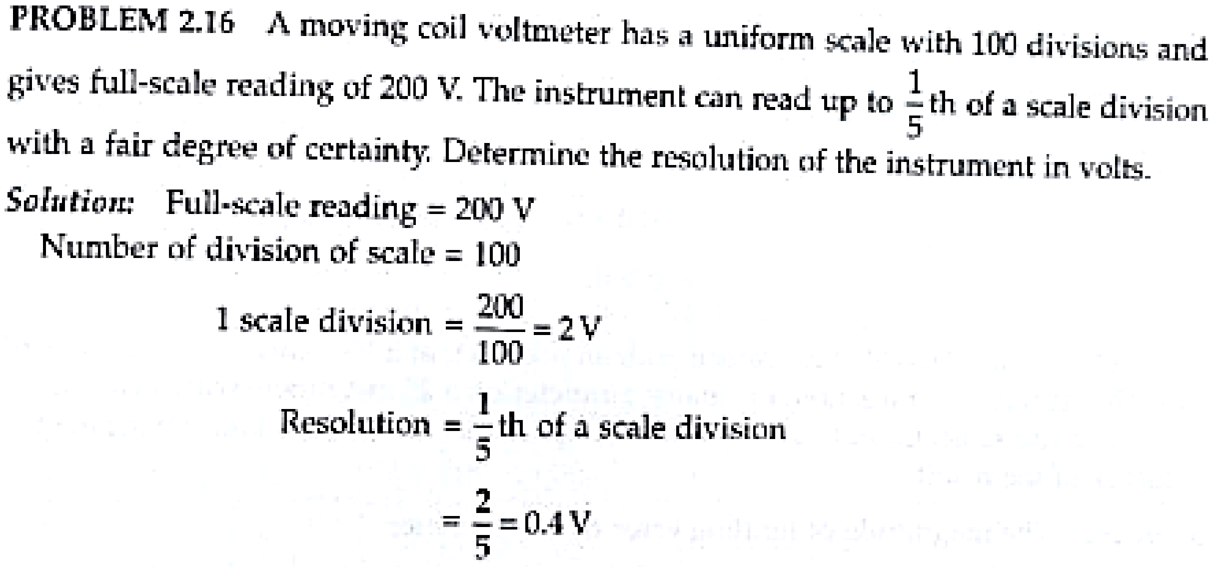
Soln

No. of div. = 100

full scale Reading = 200 V

$$1 \text{ scale Reading} = \frac{200}{100} = 2 \text{ V} \checkmark$$

$$\text{Res.} = \frac{1}{5} \times 1 \text{ scale Reading} = \frac{1}{5} \times 2 = 0.4 \text{ V} \checkmark$$



**PROBLEM 2.16** A moving coil voltmeter has a uniform scale with 100 divisions and gives full-scale reading of 200 V. The instrument can read up to $\frac{1}{5}$th of a scale division with a fair degree of certainty. Determine the resolution of the instrument in volts.

***Solution:*** Full-scale reading = 200 V

Number of division of scale = 100

$$1 \text{ scale division} = \frac{200}{100} = 2\text{ V}$$

$$\text{Resolution} = \frac{1}{5}\text{th of a scale division}$$

$$= \frac{2}{5} = 0.4\text{ V}$$

**PROBLEM 2.17** Three registers have the following ratings:

$R_1 = 200\ \Omega \pm 5\%$, $R_2 = 100\ \Omega \pm 5\%$, $R_3 = 50\ \Omega \pm 5\%$

Determine the magnitude of resultant resistance in ohms, if the above resistances are connected in (a) series and (b) parallel.

$R_1 = 200\Omega \pm \underline{5\%}$ → Abs. error

$A = A_m \pm \boxed{\Delta A}$ → Ab err

I Method

$R_1 = 200\Omega$, $R_2 = 100\Omega$

$R_3 = 50\Omega$

$R_{eq} = \underline{R_1 + R_2 + R_3} = 200 + 100 + 50$

$= 350\ \Omega \pm (?)$

$\Delta R_{eq} = \Delta R_1 + \Delta R_2 + \Delta R_3$

$= \frac{200 \times 5}{100} + \frac{100 \times 5}{100} + \frac{50 \times 5}{100} = 10 + 5 + 2.5 = \underline{17.5\Omega}$

$R = 350\Omega \pm 17.5\Omega = 350\ \Omega \pm (?)\ \%$

## II Method

for series combination → $R_s = R_1 + R_2 + R_3$

Relative error ⇒ $\frac{\delta R}{R_s} = \frac{R_1}{R_s}\frac{\partial R_1}{R_1} + \frac{R_2}{R_s}\frac{\partial R_2}{R_2} + \frac{R_3}{R_s}\frac{\partial R_3}{R_3}$

***Solution :*** (a) for series combination $R = R_1 + R_2 + R_3 = 200 + 100 + 50 = 350\Omega$

$$\boxed{\frac{\delta R}{R} = \frac{R_1}{R}\frac{\delta R_1}{R_1} + \frac{R_2}{R}\frac{\delta R_2}{R_2} + \frac{R_3}{R}\frac{\delta R_3}{R_3}}$$

Please correct

$$= \frac{200}{350}\times \underset{0.05}{\cancel{0.1}} + \frac{100}{350}\times 0.05 + \frac{50}{350}\times 0.05 = 0.079 = 7.9\%$$

Correct

$$\boxed{R = 350\Omega \pm 7.9\%} \rightarrow \text{Correct}$$

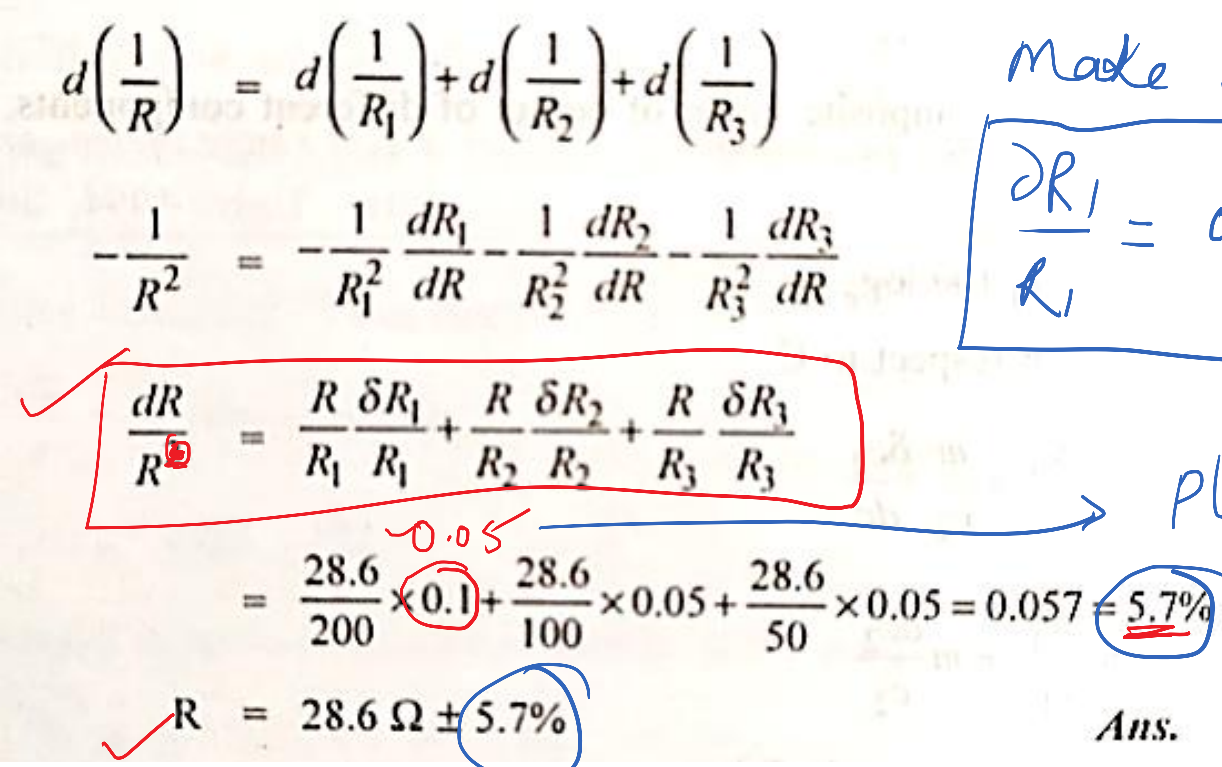
For parallel combination

↳ $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ ✓

Relative error

$$\frac{\delta R_p}{R_p} = \frac{R_p}{R_1}\frac{\delta R_1}{R_1} + \frac{R_p}{R_2}\frac{\partial R_2}{R_2} + \frac{R_p}{R_3}\frac{\partial R_3}{R_3}$$

# Parallel Combination :



$$d\left(\frac{1}{R}\right) = d\left(\frac{1}{R_1}\right)+d\left(\frac{1}{R_2}\right)+d\left(\frac{1}{R_3}\right)$$

$$-\frac{1}{R^2} = -\frac{1}{R_1^2}\frac{dR_1}{dR}-\frac{1}{R_2^2}\frac{dR_2}{dR}-\frac{1}{R_3^2}\frac{dR_3}{dR}$$

$$\boxed{\frac{dR}{R} = \frac{R}{R_1}\frac{\delta R_1}{R_1}+\frac{R}{R_2}\frac{\delta R_2}{R_2}+\frac{R}{R_3}\frac{\delta R_3}{R_3}}$$

$$= \frac{28.6}{200}\times\overset{0.05}{0.1}+\frac{28.6}{100}\times 0.05+\frac{28.6}{50}\times 0.05 = 0.057 = 5.7\%$$

$$R = 28.6\,\Omega \pm 5.7\%$$

*Ans.*

Make Correction

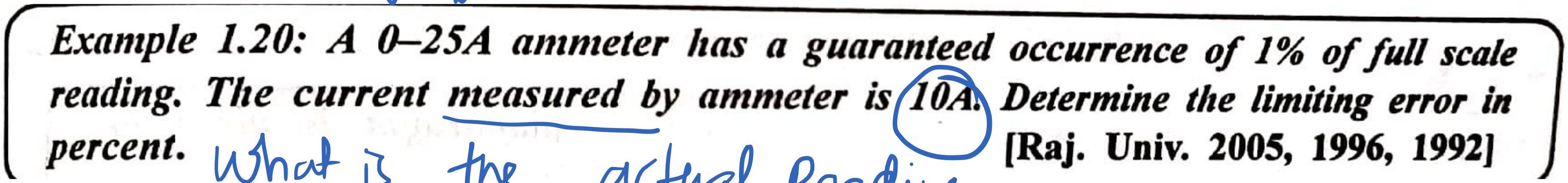
$$\frac{\partial R_1}{R_1} = 0.05$$

Please correct

Please Correct

$$\epsilon_r = \frac{\text{Abs. error}}{\text{true value / measured value}}$$

full scalle Reading = 25A



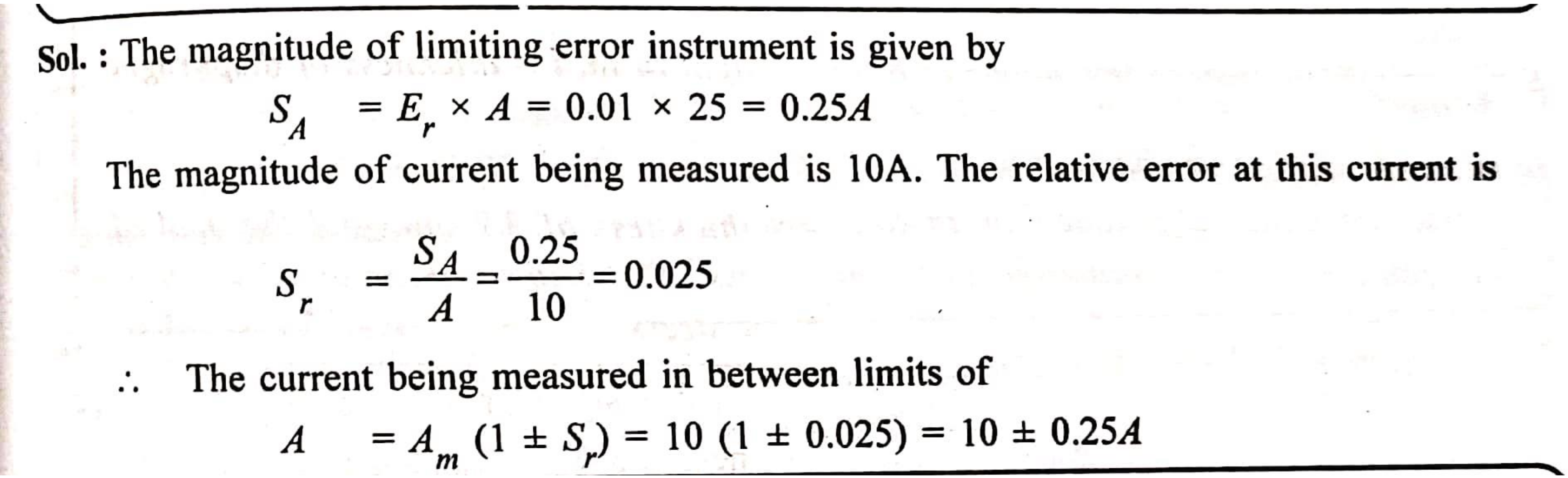
**Example 1.20:** *A 0–25A ammeter has a guaranteed occurrence of 1% of full scale reading. The current measured by ammeter is 10A. Determine the limiting error in percent.* **[Raj. Univ. 2005, 1996, 1992]**

What is the actual Reading.

Sol$^n$. Abs. error $= \frac{25 \times 1}{100} = 0.25A$

$$\epsilon_r = \text{Rel. error} = \frac{\text{Abs. error}}{\text{measured value}} = \frac{0.25}{10}$$

$$\boxed{A_t = A_m(1 \pm \epsilon_r)}$$



**Sol. :** The magnitude of limiting error instrument is given by

$$S_A = E_r \times A = 0.01 \times 25 = 0.25A$$

The magnitude of current being measured is 10A. The relative error at this current is

$$S_r = \frac{S_A}{A} = \frac{0.25}{10} = 0.025$$

∴ The current being measured in between limits of

$$A = A_m (1 \pm S_r) = 10 (1 \pm 0.025) = 10 \pm 0.25A$$

**Example 1.15:** *A 4 – dial decade box has*
*decade a of 10 × 1000 Ω + 0.1%;*
*decade b of 10 × 100 Ω + 0.1%;*
*decade c of 10 × 10 Ω + 0.5%;*
*decade d of 10 × 1 Ω + 1.0%;*
*It is set at 4639Ω. Find the percentage limiting error and range of resistance value.*
**[Raj. Univ. 1994, 2002, 2004]**

% Rel. error

$R \pm (?)$

Soln

$R_m = 4639\,\Omega = \boxed{4000} + \boxed{600} + \boxed{30} + \boxed{9}$

a: % error
b: % error
c: % error
d: % error

Total error = 4 + 0.6 + 0.15 + 0.09

= ______ % error

$\% \text{ Rel. error} = \frac{\text{Abs. error}}{R_m} \times 100$

$\frac{4000 \times 0.1}{100}$

$= \frac{\_\_\_}{4639} \times 100 = 4$

$\frac{600 \times 0.1}{100} = 0.6$

$\frac{30 \times 0.5}{100} = \_\_\_$

$\frac{9 \times 1}{100} = 0.09$ 09

$R = R_m(1 \pm \% \text{ Rel. error})$

**Sol.:** Decade $a$ is set at 4000 Ω and therefore,

$$\text{error} = \pm 4000 \times \frac{0.1}{100} = \pm 4\ \Omega$$

Decade $b$ is set at 600 Ω and therefore,

$$\text{error} = \pm 600 \times \frac{0.1}{100} = \pm 0.6\ \Omega$$

$$\text{error in decade } c = \pm 30 \times \frac{0.5}{100} = \pm 0.15\ \Omega \text{ and}$$

$$\text{error in decade } d = \pm 9 \times \frac{1}{100} = \pm 0.09\ \Omega$$

$$\text{Hence total error} = \pm (4 + 0.6 + 0.15 + 0.09) = \boxed{4.84\Omega}$$

$$\text{Relative limiting error } E_r = \pm \frac{4.84}{4639} = \pm 0.00104$$

$$\text{Percentage limiting error } \% = \pm (0.00104 \times 100) = \pm 0.104\%$$

$$\text{Limiting values of resistance } A_m = 4639\ (1 \pm 0.00104) = \boxed{4639 \pm 5\Omega}$$

# Gaussian error Analysis :-

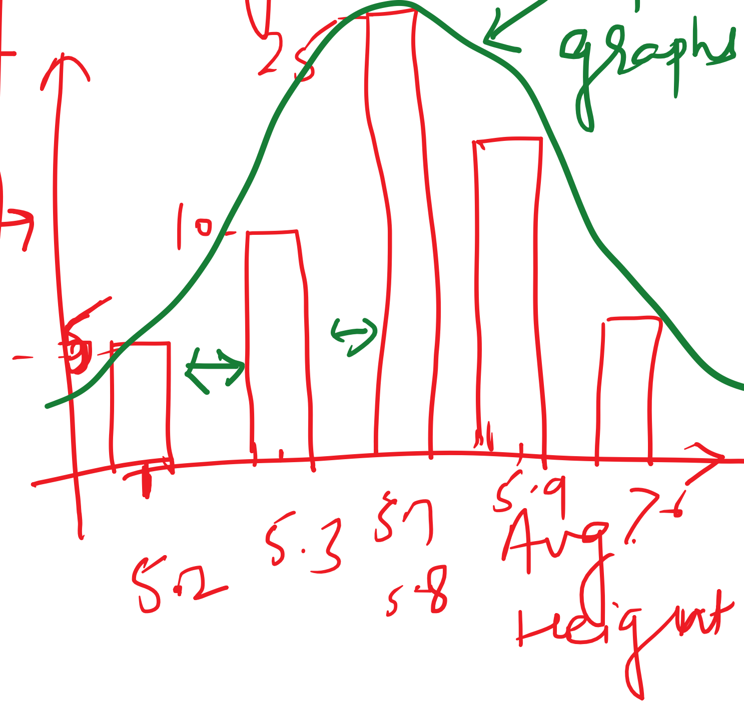
## Gaussian Process :-

what if the no. of student ↑

Ex. Avg. height of students in a class $\boxed{60}$

| Avg. height | No. of students |
|---|---|
| 5.2 | 5 |
| 5.3 | 10 |
| 5.7 \| 5.8 | 25 |
| 5.9 | 15 |
| 7.6 | 5 |

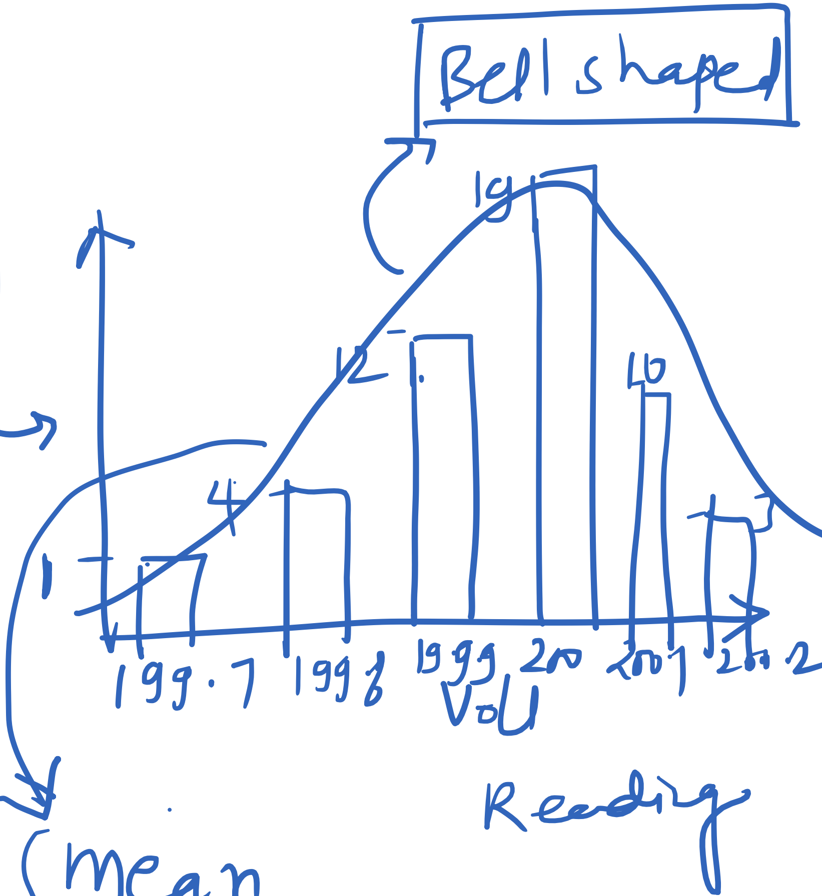
Histogram



Bell shaped graphs

# Ex: Voltage Measurement

| Voltage Reading(V) | No. of Reading |
|---|---|
| 199.7 | 1 |
| 199.8 | 4 |
| 199.9 | 12 |
| 200.0 | 19 |
| 200.1 | 10 |
| 20 .2 | 3 |

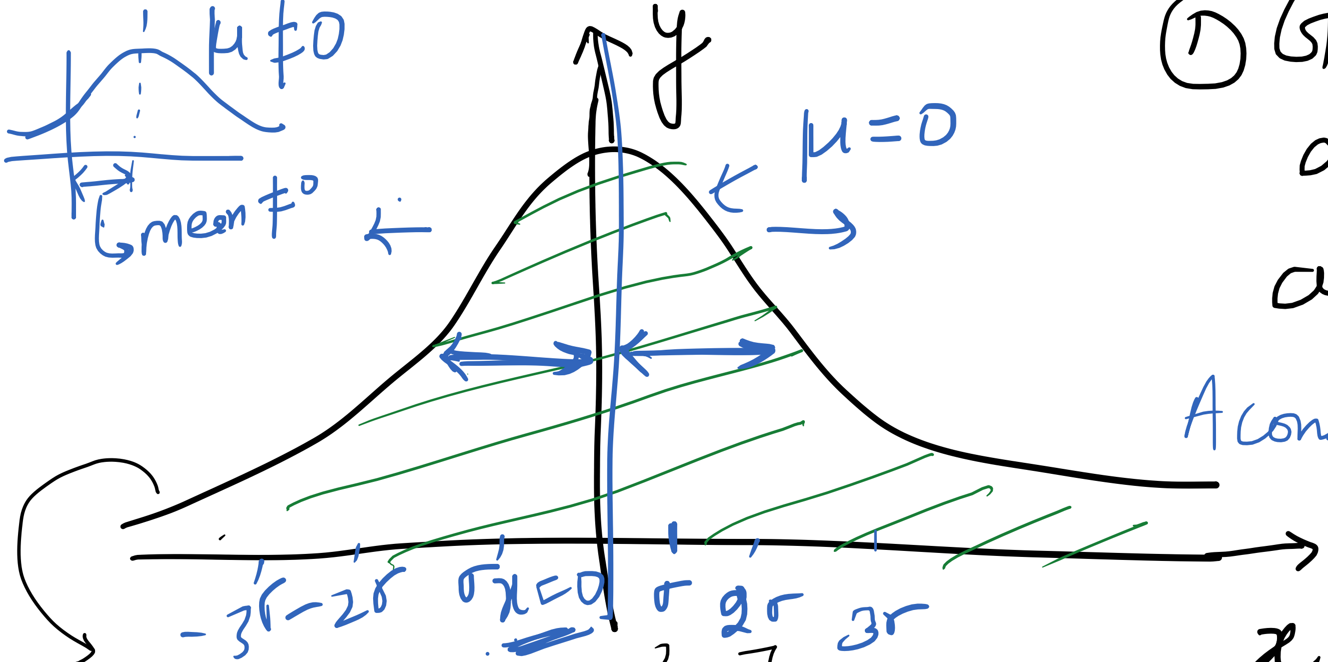


mean

std dev.

var.

# Properties of Gaussian Process:-



① Gaussian process is always symm- about y axis

A constant value h → Precision index

$$\boxed{h = \frac{1}{\sqrt{2}\sigma}}$$

② Area under the curve (Gaussian) is unity

X $y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$ ] = for mean value = 0

σ = std. dev.

$$\boxed{y = \frac{h}{\sqrt{\pi}} e^{-h^2x^2}}$$

↳ this type of distribution is most frequently met in practice.

Normal occurance of deviations from avg. value of infinite no. of measurements is gaussian & mathematically expressed by.

$$\boxed{y = \frac{h}{\sqrt{\pi}} \exp(-h^2x^2)}$$

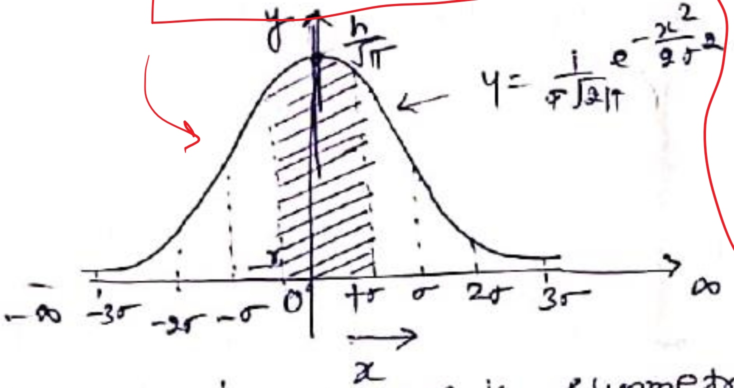
x = mag. of dev. from mean

y = no. of readings at any dev. x
prob. of occurance of dev. x

h = const. (precision index)

Convenient eqn

$$\boxed{y = \frac{1}{\sigma\sqrt{2\pi}} \exp(-x^2/2\sigma^2)}$$



properties ! (i) curve is symmetrical abt. mean value.
(ii) Area under the curve is unity.

$$\text{Area} = \int_{-\infty}^{\infty} y\,dx = \int_{-\infty}^{\infty} \frac{h}{\sqrt{\pi}} e^{-(h^2x^2)} dx$$

$$= \textcircled{1}$$

**Precision index;** if $x=0$ $y = \frac{h}{\sqrt{\pi}}$

→ max of y depends on h; larger the value of h sharper the curve

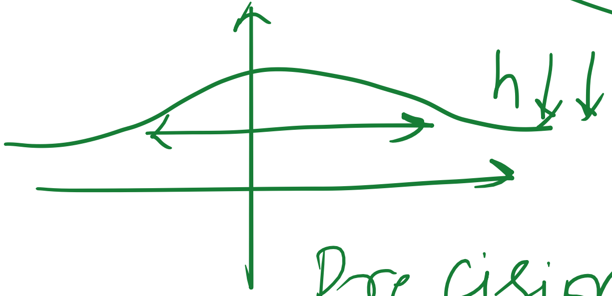
→ Sharp curve indicates that deviations are more closely grouped together. it also indicates greater precision

→ A large value of h → high precision.

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} \quad \text{put } x = 0$$

$$y = \frac{h}{\sqrt{\pi}} \qquad h\uparrow\uparrow \Rightarrow y\uparrow\uparrow$$

Precision ↑

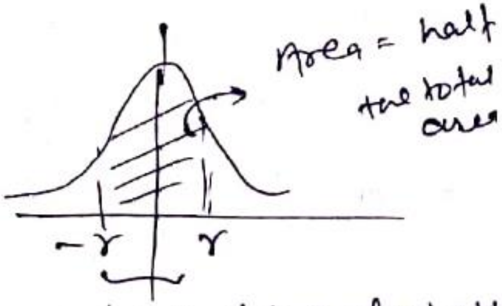


Precision ↓

## Probable error :-

→ most probable value of a gaussian distribution = arithmatic mean.

→ most probable value is connected with sharpness of distribution curve.

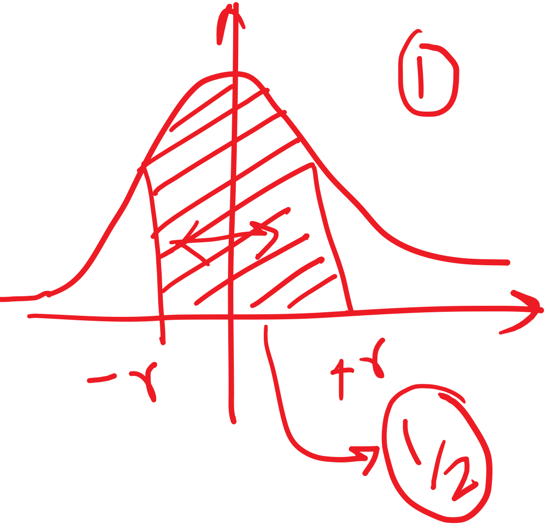


{ Chances are even that the one reading will have an error no greater than ± r
r. is called P.E.

↳ half the deviation lie b/w ± r

r can be found $\frac{h}{\sqrt{\pi}} \int_{-r}^{+r} e^{-h^2x^2} = 1/2$

$$\Rightarrow \boxed{r = \frac{0.4769}{h}}$$

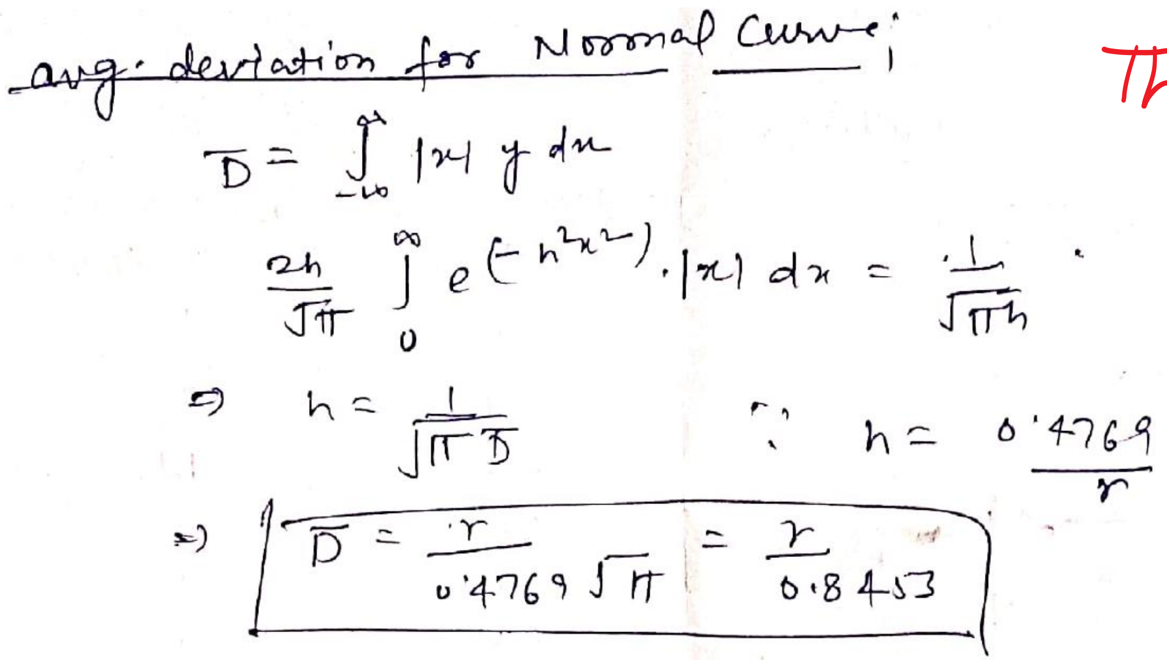


$$\int_{-r}^{r} y\,dx = \frac{1}{2}$$

$$= \int_{-r}^{r} \frac{h}{\sqrt{\pi}} e^{-x^2h^2} = 1/2 \Rightarrow \boxed{r = \frac{0.4769}{h}}$$

P.E.

h ↑↑



avg. deviation for Normal Curve;

$$\bar{D} = \int_{-\infty}^{\infty} |x|\, y\, dx$$

$$\frac{2h}{\sqrt{\pi}} \int_{0}^{\infty} e^{(-h^2x^2)} \cdot |x|\, dx = \frac{1}{\sqrt{\pi} h}$$

$$\Rightarrow h = \frac{1}{\sqrt{\pi}\,\bar{D}} \qquad \because h = \frac{0.4769}{r}$$

$$\Rightarrow \boxed{\bar{D} = \frac{r}{0.4769\sqrt{\pi}} = \frac{r}{0.8453}}$$

The avg. value of a function f(x) can be determine as

$$\int_{a}^{b} f(x)\, dx$$

r = probable error

h = Precesion index

$\bar{D}$ = Avg. deviation

so . dev. –

$$\bar{D} = \int_{-\infty}^{\infty} |x|\, y\, dx$$

here $y = \frac{h}{\sqrt{\pi}} e^{-x^2h^2}$

deviation $d_1 = x_1 - \bar{x}$

↳ mean value

↳ first observation

Standard deviation:

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

$$\sigma = \frac{2h}{\sqrt{\pi}} \int_0^\infty e^{-h^2x^2} \cdot x^2 dx$$

$$\sigma = \frac{1}{\sqrt{2}h} = \frac{r}{\sqrt{2} \times 0.4769} = \frac{r}{0.6745}$$

$$\boxed{\begin{aligned} P.E\ r &= 0.8453\ \bar{D} \\ &= 0.6745\sigma \end{aligned}}$$

Generally we defined S.D. as

$$= \sqrt{\frac{d_1^2 + d_2^2 + \cdots d_n^2}{n}}$$

$$= \sqrt{\frac{\Sigma d^2}{n}}$$

mean square value

In general we can determine mean square value as

$$\int_a^b x^2 y \cdot dx$$

here

$$y = \frac{h}{\sqrt{\pi}} e^{-\,{}^{2}x^{2}}$$

# Probable Error of a finite No. of Readings

$r = 0{\cdot}8453\ \bar{\sigma}$

$= 0{\cdot}6745\ \sigma$

$r = 0{\cdot}6745 \sqrt{\frac{\Sigma d^2}{n}}$

$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + \cdots d_n^2}{n}}$

$\sigma = \sqrt{\frac{\Sigma d_1^2}{n}}$

prob. error of one reading

$r_1 = 0{\cdot}6745\sigma$

Prob. error of mean

→ $r_m = 0{\cdot}6745 \frac{\sigma}{\sqrt{n}}$

Stand. dev. of mean $\sigma_m = \frac{\sigma}{\sqrt{n}}$

St'd. dev. of std. dev. $\sigma_\sigma = \frac{\sigma}{\sqrt{2n}} = \frac{\sigma_m}{\sqrt{2}}$

***Example 1.18: The following 10 observations were recorded when measuring a voltage: 41.7, 42.0, 41.8, 42.0, 42.1, 41.9, 42.0, 41.9, 42.5 and 41.8. Find (i) the mean (ii) the standard deviation (iii) the probable error of one reading (iv) the probable error of mean and (v) range.***

**Sol.:** For the sake of ease in calculations, the observations are tabulated and manipulated as under.

| $x$ | $d$ | $d^2$ |
|---|---|---|
| 41.7 | −0.27 | 0.0729 |
| 42.0 | +0.03 | 0.0009 |
| 41.8 | −0.17 | 0.0289 |
| 42.0 | +0.03 | 0.0009 |
| 42.1 | +0.13 | 0.0169 |
| 41.9 | −0.07 | 0.0049 |
| 42.0 | +0.03 | 0.0049 |
| 41.9 | −0.07 | 0.0049 |
| 42.5 | +0.53 | 0.2809 |
| 41.8 | −0.17 | 0.0289 |
| □x = 419.7 | | □$d^2$ = 0.441 |

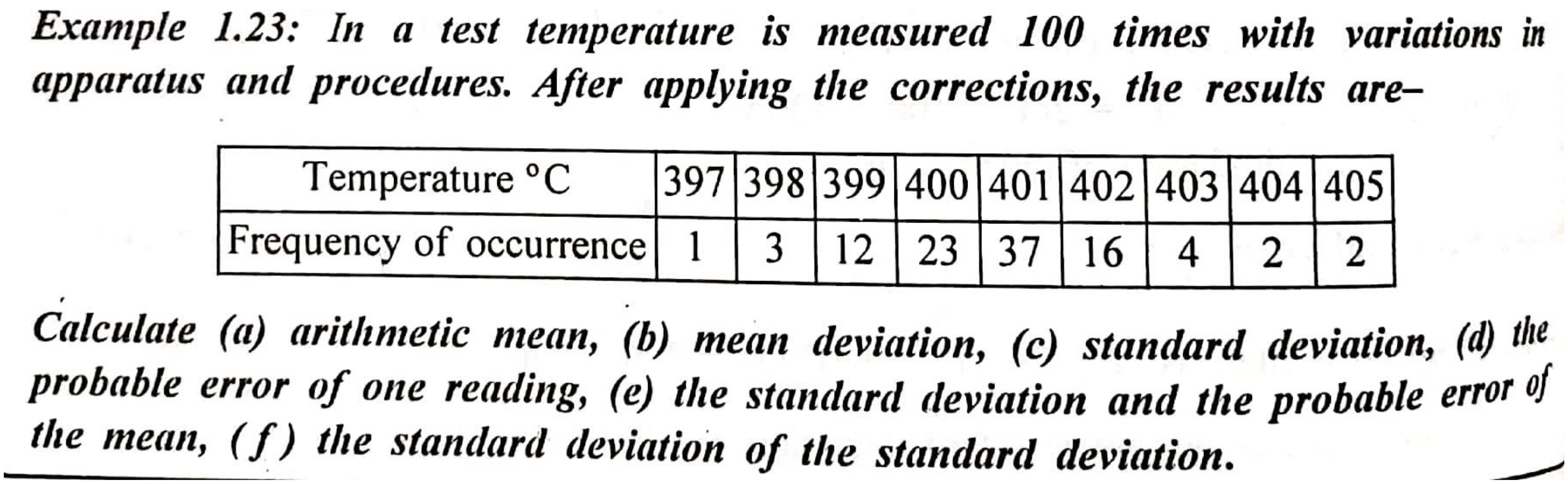
*(i)* Mean length $\bar{X} = \dfrac{419.7}{10} = 41.97$ volt

*(ii)* The value of standard deviation is $\sigma = \sqrt{\dfrac{d^2}{n}} = \sqrt{\dfrac{0.441}{10}} = 0.21$ volt

If the data is considered to be a set of infinite readings. However, the number of observations is only 10 and therefore, the standard deviation is

$$\sigma = \sqrt{\frac{d^2}{n-1}} = \sqrt{\frac{0.441}{(10-1)}} = 0.22 \text{ volt}$$

*(iii)* Probable error of one reading $r_1$ = 0.6745; = 0.15 volt



***Example 1.23:** In a test temperature is measured 100 times with variations in apparatus and procedures. After applying the corrections, the results are–*

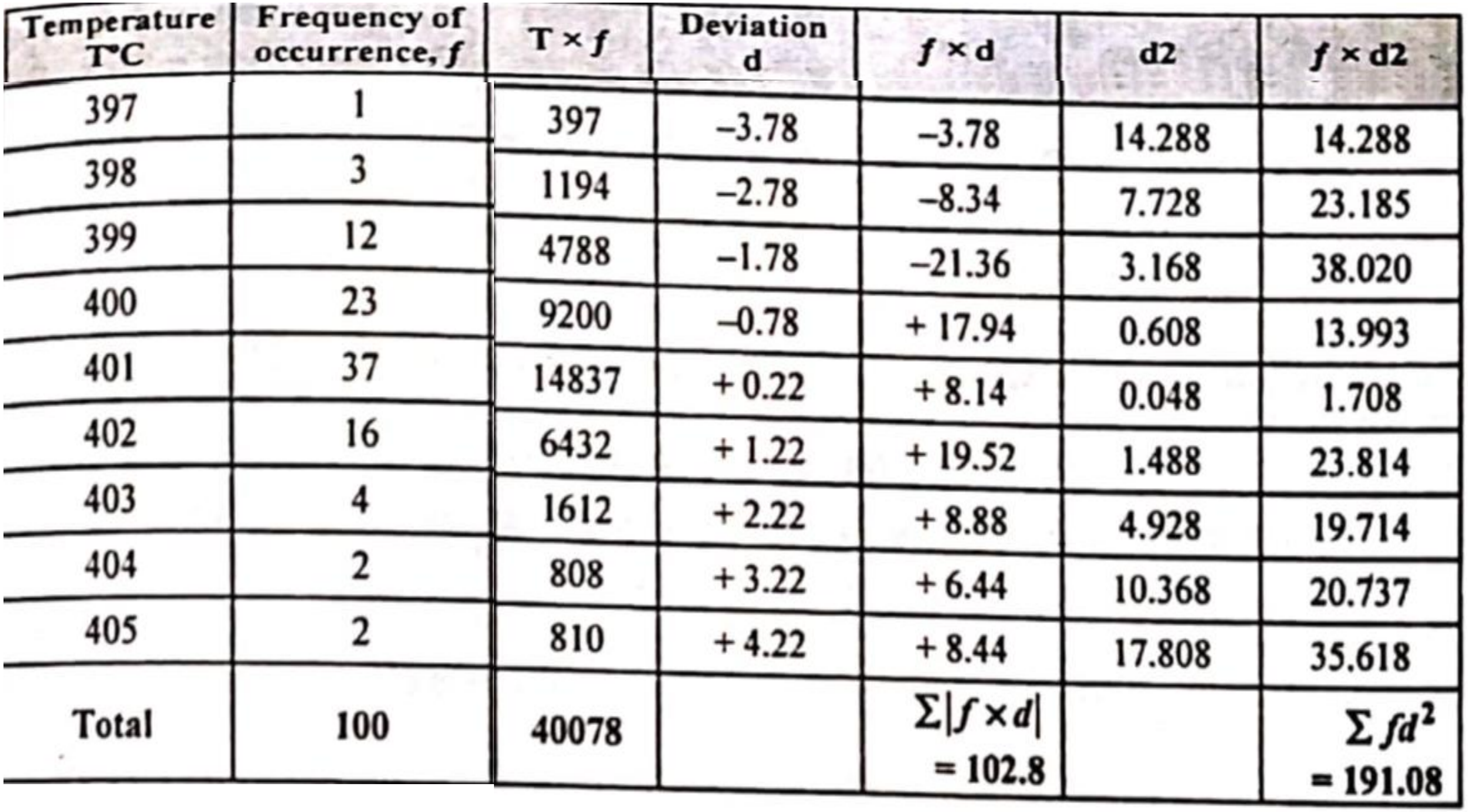
| Temperature °C | 397 | 398 | 399 | 400 | 401 | 402 | 403 | 404 | 405 |
|---|---|---|---|---|---|---|---|---|---|
| Frequency of occurrence | 1 | 3 | 12 | 23 | 37 | 16 | 4 | 2 | 2 |

***Calculate (a) arithmetic mean, (b) mean deviation, (c) standard deviation, (d) the probable error of one reading, (e) the standard deviation and the probable error of the mean, (f) the standard deviation of the standard deviation.***

square

| Temperature T°C | Frequency of occurrence, *f* | T × *f* | Deviation d | *f* × d | d2 | *f* × d2 |
|---|---|---|---|---|---|---|
| 397 | 1 | 397×1 | $d_1 = 397 - \bar{x}$ | | | |
| 398 | 3 | 398×3 | $d_2 = 398 - \bar{x}$ | | | |
| 399 | 12 | 399×12 | | | | |
| 400 | 23 | | | | | |
| 401 | 37 | | | | | |
| 402 | 16 | | | | | |
| 403 | 4 | | | | | |
| 404 | 2 | | | | | |
| 405 | 2 | | | | | |
| Total | 100 | $\Sigma Tf$ | | | | |

$$\bar{x} = \frac{\Sigma Tf}{\Sigma f} =$$



| Temperature T°C | Frequency of occurrence, *f* | T × *f* | Deviation d | *f* × d | d2 | *f* × d2 |
|---|---|---|---|---|---|---|
| 397 | 1 | 397 | –3.78 | –3.78 | 14.288 | 14.288 |
| 398 | 3 | 1194 | –2.78 | –8.34 | 7.728 | 23.185 |
| 399 | 12 | 4788 | –1.78 | –21.36 | 3.168 | 38.020 |
| 400 | 23 | 9200 | –0.78 | + 17.94 | 0.608 | 13.993 |
| 401 | 37 | 14837 | + 0.22 | + 8.14 | 0.048 | 1.708 |
| 402 | 16 | 6432 | + 1.22 | + 19.52 | 1.488 | 23.814 |
| 403 | 4 | 1612 | + 2.22 | + 8.88 | 4.928 | 19.714 |
| 404 | 2 | 808 | + 3.22 | + 6.44 | 10.368 | 20.737 |
| 405 | 2 | 810 | + 4.22 | + 8.44 | 17.808 | 35.618 |
| **Total** | **100** | **40078** | | $\Sigma\lvert f \times d\rvert$ **= 102.8** | | $\Sigma fd^2$ **= 191.08** |

(a) Mean temperature $= \frac{400.78}{100} = 400.780\ °\mathrm{C}$

(b) Mean deviation $\bar{D} = \frac{102.8}{100} = 1.028\ °\mathrm{C}$

(c) Standard deviation $\sigma = \sqrt{\frac{191.08}{100}} = 1.380\ °\mathrm{C}$

(d) Probable error of one reading $r_1 = 0.6745\ s = 0.6745 \times 1.38 = 0.93\ °\mathrm{C}$

(e) Probable error of the mean $r_m = \frac{0.93}{\sqrt{100}} = 0.093\ °\mathrm{C}$

Standard deviation of the mean $r_m = \frac{0.93}{\sqrt{100}} = 0.138\ °\mathrm{C}$

(f) Standard deviation of the standard deviation

$$\sigma_s = \frac{\sigma_m}{\sqrt{2}} = \frac{0.138}{\sqrt{2}} = 0.0796\ °\mathrm{C}$$