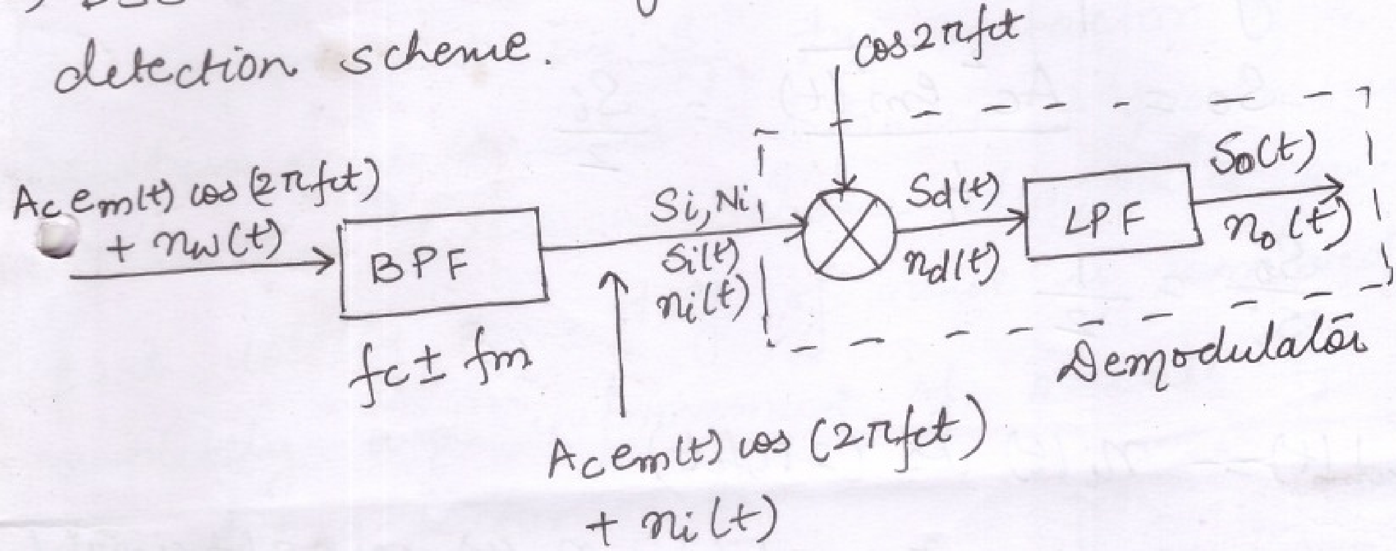
Unit 4 :- Noise in AM & FM

→ Noise is stationary, white & gaussian

Noise in AM systems

1) DSB-SC Scheme

→ DSB-SC R^x uses synchronous or coherent detection scheme.



$$s(t) = A_c\, e_m(t) \cos(2\pi f_c t)$$

$n_w(t) =$ white gaussian noise

$n_i(t) =$ Bandpass noise

$$\therefore S_i(t) = A_c\, e_m(t) \cos(2\pi f_c t) + n_i(t)$$

mean square value of signal power at i/p of the demodulator is,

$$S_i = \frac{A_c^2}{2}\, \overline{e_m^2(t)}$$

$$n_i(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$$

$$\therefore N_i = \overline{n_i^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)}$$

reduction of the signal power due to elimination of the hilbert transform of the msg. signal.

⇒ It seems that DSB-SC is superior to SSB-SC so far as the noise performance is concerned. But this is not true. as SSB-SC requires only 1/2 the BW required for DSB-SC. ∴ the i/p noise N_i in the case of DSB-SC is twice that of SSB-SC. Thus an improvement in SNR by a factor of 2 in the case of DSB-SC system is actually nullified by the larger i/p noise. ∴ for a given signal power at i/P, SNR at o/p is identical for DSB-SC + SSB-SC systems.

3) AM Scheme

→ Both sidebands + carrier are transmitted.

$$\therefore s(t) = A_c [1 + K_a\, m(t)] \cos(2\pi f_c t)$$

i) Envelope detection

→ consists simply of a non-linear device followed by a low-pass RC filter.

I/P to the demodulator of AM R^x is

$$S_i(t) = A_c [1 + K_a\, m(t)] \cos 2\pi f_c t + n_i(t)$$

$e_m(t)$ & $\hat{e}_m(t)$ occupy the same spectra range.

$$S_d(t) = S_c(t) \times \cos 2\pi f_c t$$

$$= \frac{A_c}{2}\left[e_m(t)\cos^2 2\pi f_c t \mp \hat{e}_m(t)\sin 2\pi f_c t \cos 2\pi f_c t\right]$$

$$= \frac{A_c}{4} e_m(t)\left[1+\cos 4\pi f_c t\right] \mp \frac{A_c}{4}\hat{e}_m(t)\left[\sin(4\pi f_c t)\right]$$

$$s_0(t) = \frac{A_c}{4} e_m(t)$$

$$S_0 = \frac{A_c^2}{16}\overline{e_m^2(t)}$$

$$S_0 = \frac{S_i}{4} \Rightarrow \frac{S_0}{S_i} = \frac{1}{4}$$

noise ratio will be same as that of DSB-SC,

$$\frac{N_0}{N_i} = \frac{1}{4}$$

$$\therefore \frac{\left(\frac{S_0}{N_0}\right)}{\left(\frac{S_i}{N_i}\right)} = 1.$$

⇒ Hence, SNR at O/P of SSB-SC is same as that of i/p & no improvement in SNR.

⇒ elimination of the quadrature component of the noise is actually cancelled by the

$$s_d(t) = A_c\, e\, m(t) \cos(2\pi f_c t)\cos(2\pi f_c t)$$

$$= \frac{A_c\, e\, m(t)}{2}\left[1 + \cos 4\pi f_c t\right]$$

$$s_o(t) = \frac{A_c\, e\, m(t)}{2}$$

O/P signal power

$$S_o = \frac{A_c^2\, \overline{e\, m^2(t)}}{4} = \frac{S_i}{2}$$

$$\frac{S_o}{S_i} = \frac{1}{2}$$

$$n_d(t) = n_i(t)\cos(2\pi f_c t)$$

$$= n_c(t)\cos^2(2\pi f_c t) - n_s(t)\cos 2\pi f_c t \sin 2\pi f_c t$$

$$= \frac{n_c(t)}{2}\left[1 + \cos 4\pi f_c t\right] - \frac{n_s(t)}{2}\sin 4\pi f_c t$$

$$= \frac{1}{2}\Big[\underset{\substack{\downarrow \\ \text{in-phase} \\ \text{component}}}{n_c(t)} + \underbrace{n_c(t)\cos 4\pi f_c t - n_s(t)\sin 4\pi f_c t}_{\substack{\text{Spectra of } n_c(t) + n_s(t) \\ \text{shifted at } \pm f_c + \\ \text{filtered by LPF}}}\Big]$$

$$n_o(t) = \frac{1}{2}n_c(t)$$

$$N_o = \overline{n_o^2(t)}$$

$$= \frac{1}{4}\overline{n_c^2(t)} = \frac{1}{4}N_i$$

$$\Rightarrow \frac{N_o}{} = \frac{1}{}$$

carrier as a component of AM wave.

If single tone mod,

$$s(t) = A_c [1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$$

$$\overline{e_m^2(t)} = \frac{A_m^2}{2}$$

$$\frac{\left(\frac{S_o}{N_o}\right)}{\left(\frac{S_i}{N_i}\right)} = \frac{2 k_a^2 A_m^2}{2 + k_a^2 A_m^2} = \frac{2\mu^2}{2+\mu^2}$$

when $\mu = 1$, max. improvement in

SNR = 2/3.

b) <u>Large noise case</u>

$$n_i(t) \gg A_c [1 + k_a e_m(t)]$$

or

$n_c(t)$

or

$n_s(t)$

$$e(t) \cong [n_c^2(t) + n_s^2(t) + 2 n_c(t) A_c [1 + k_a e_m(t)]]^{1/2}$$

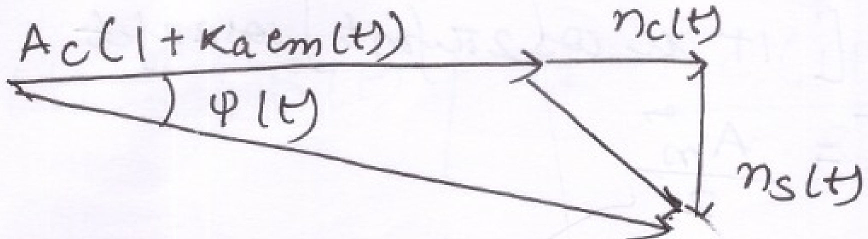
$$= [R^2(t) + 2 n_c(t) A_c (1 + k_a e_m(t))]^{1/2}$$

$$\cong R(t) \left[1 + \frac{2 A_c (1 + k_a e_m(t))}{R(t)} \cos \theta(t)\right]^{1/2}$$

$$\left\{ \begin{array}{l} n_c(t) = R(t) \cos \theta(t) \\ n_s(t) = R(t) \sin \theta(t) \end{array} \right. \Rightarrow \left. \frac{R(t) \cos \theta(t)}{R^2(t) \cos^2 \theta(t) + R^2(t) \sin^2 \theta(t)} \right\}$$

$$\frac{n_c(t)}{R^2(t)} = \frac{n_c(t)}{n_c^2(t) + n_s^2(t)}$$

$$\psi(t) \simeq 0$$



Useful signal at the o/p of demodulator is

$$S_0(t) = A_c K_a\, e_m(t)$$

$$n_0(t) = n_c(t)$$

$$S_0 = K_a^2 A_c^2\, \overline{e_m^2(t)}$$

$$N_0 = \overline{n_c^2(t)} = N_i$$

$$\frac{\left(\frac{S_0}{N_0}\right)}{\left(\frac{S_i}{N_i}\right)} = \frac{K_a^2 \cancel{A_c^2}\, \overline{e_m^2(t)}}{\frac{\cancel{A_c^2}}{2}\left(1 + K_a^2 \overline{e_m^2(t)}\right)}$$

$$= \frac{2 K_a^2\, \overline{e_m^2(t)}}{1 + K_a^2\, \overline{e_m^2(t)}}$$

$\Rightarrow$ SNR at O/P of the demodulator to its i/p using envelope detection is always less than unity. $\therefore$ noise performance of an AM R^x is always inferior to that of a DSB-SC or SSB-SC R^x. This is due to the wastage of T^x power which results from transmitting the

$$S_i = \frac{A_c^2}{2}\left[1 + K_a^2\,\overline{e_m^2(t)}\right]$$

$$N_i = \overline{n_i^2(t)}$$

$$S_i(t) = A_c[1 + K_a e_m(t)]\cos 2\pi f_c t + n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t$$

$$= e(t)\cos(2\pi f_c t + \psi(t)]$$

$$e(t) = \left[\{A_c(1 + K_a e_m(t)) + n_c(t)\}^2 + n_s^2(t)\right]^{1/2}$$

$$\psi(t) = \tan^{-1}\left\{\frac{n_s(t)}{A_c(1 + K_a e_m(t)) + n_c(t)}\right\}$$

<u>2 cases</u>

i) <u>Small noise case</u> :-

$A_c[1 + K_a e_m(t)] >> n_i(t)$ or $n_c(t)$ or $n_s(t)$

$$e(t) \simeq \left[\{A_c(1 + K_a e_m(t))\}^2 + 2A_c(1 + K_a e_m(t))\,n_c(t)\right]^{1/2}$$

$$= A_c(1 + K_a e_m(t))\left[1 + \frac{2n_c(t)}{A_c[1 + K_a e_m(t)]}\right]^{1/2}$$

$$= A_c[1 + K_a e_m(t)]\left[1 + \frac{n_c(t)}{A_c[1 + K_a e_m(t)]}\right]$$

$$= A_c[1 + K_a e_m(t)] + n_c(t)$$

where $R(t) = [n_c^2(t) + n_s^2(t)]^{1/2}$

$\theta(t) = \tan^{-1}\left[\frac{n_s(t)}{n_c(t)}\right]$

$$e(t) \simeq R(t)\left[1 + \frac{A_c(1+k_a e_m(t))\cos\theta(t)}{R(t)}\right]$$

$$\simeq R(t) + A_c(1+k_a e_m(t))\cos\theta(t)$$

$$= R(t) + A_c\cos\theta(t) + A_c k_a e_m(t)\cos\theta(t)$$

⇒ When noise is larger for the CNR is low, detector O/P has no components proportional to $e_m(t)$. Only term which contains the msg. signal $e_m(t)$ is multiplied by $\cos\theta(t)$ where $\theta(t)$ is the phase of BP noise $n_i(t)$. Hence it is impossible to separate message signal from $A_c k_a e_m(t)\cos\theta(t)$.

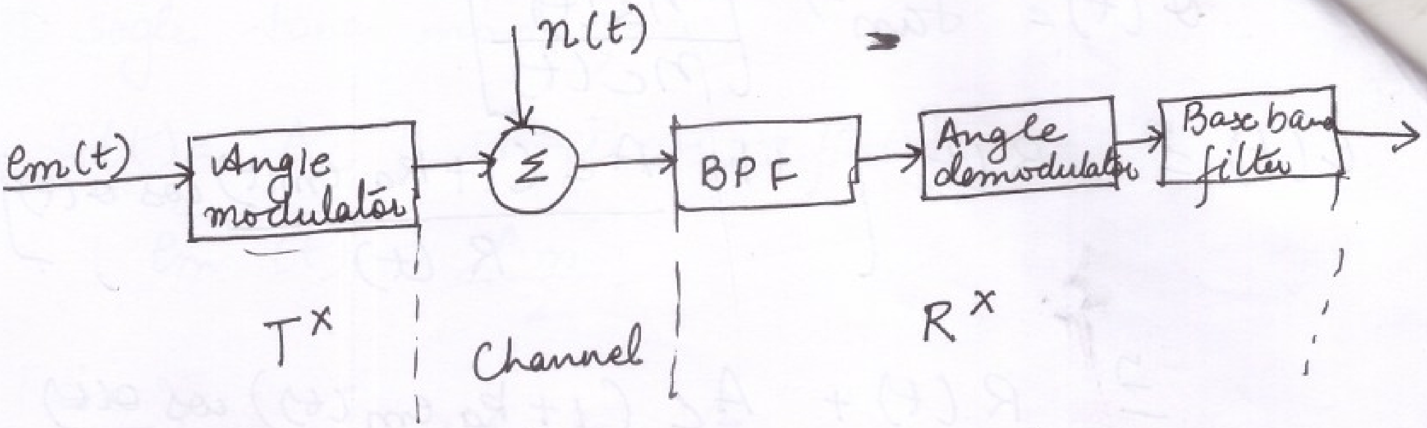
⇒ loss of msg. in an envelope detector which operates at a larger noise level is referred to as threshold effect in AM.

ii) Coherent Detector

⇒ Same as that for small noise in Envelope detector.

$$\frac{\left(\frac{S_o}{N_o}\right)}{\left(\frac{S_i}{N_i}\right)} = \frac{2k_a^2\overline{e_m^2(t)}}{1+k_a^2\overline{e_m^2(t)}}$$

Noise in $\angle$ modulated systems.



$$S(t) = A_c \cos(2\pi f_c t + \phi(t))$$

$$\phi(t) = k_p e_m(t) \quad \text{for PM}$$

$$\phi(t) = 2\pi K_f \int_0^t e_m(t)\,dt \quad \text{for FM.}$$

Channel noise $n_i(t) \rightarrow$ BP noise

$$n_i(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$$

$$= R(t) \cos[2\pi f_c t + \theta(t)]$$

where $R(t) = \sqrt{n_c^2(t) + n_s^2(t)}$ $\quad n_c t = R(t)$

$$\theta(t) = \tan^{-1}\left(\frac{n_s(t)}{n_c(t)}\right)$$

$$n_c(t) = R(t) \cos\theta(t)$$

$$n_s(t) = R(t) \sin\theta(t)$$

Noise in FM.

$$S(t) = A_c \cos\left(2\pi f_c t + 2\pi K_f \int_0^t e_m(t)\,dt\right)$$

$$+ R(t) \cos(2\pi f_c t + \theta(t))$$

$\simeq K_f \, e_m(t) + n_d(t)$

where $n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} \left[R(t) \sin[\theta(t) - \phi(t)] \right]$

$\phi(t)$ which is the signal term can be neglected from the above noise term.

$\therefore n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} \left[R(t) \sin \theta(t) \right]$

We know that $n_s(t) = R(t) \sin \theta(t)$

$\therefore n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} n_s(t)$

$\therefore s_d(t) = K_f \, e_m(t) + \frac{1}{2\pi A_c} \frac{d}{dt} n_s(t)$

when passed through LPF

$S_o(t) = K_f \, e_m(t)$

$\Rightarrow S_o = K_f^2 \, \overline{e_m^2(t)}$ —— (1)

→ $n_d(t)$ can be obtained by passing $n_s(t)$ through a linear filter whose transfer function is

$H(f) = \frac{j2\pi f}{2\pi A_c} = \frac{jf}{A_c}$

→ Let $S_{ns}(f)$ & $S_{nd}(f)$ denote the psd of the quadrature comp. of BP noise & noise at the o/p of the demodulator.

$$\boxed{\frac{\left(\frac{S_o}{N_o}\right)}{\left(\frac{S_i}{N_i}\right)} = \frac{6 K_f^2\, \overline{e_m^2(t)}\, B_T}{W^3}}$$

⇒ carrier freq. deviation in FM is directly proportional to K_f. Hence, the BW of FM is proportional to K_f.

∴ SNR at the FM demodulator o/p is propo-rtional to the square of the BW of transmission.

Threshold in FM

→ It can be seen that the SNR in FM cannot be improved indefinitely by increasing the BW. As the BW increases, the i/p noise also increases and ultimately a point is reached where the carrier power becomes of the order of the i/p noise power. ∴ under this condition threshold occurs.

→ threshold effect in FM is much more pronounced than in AM.

FM signal at the demodulator i/p can be expressed as

$$S_i(t) = A_c \cos\left(2\pi f_c t + \phi(t)\right) + n_i(t)$$

phasis + Pre-emphasis

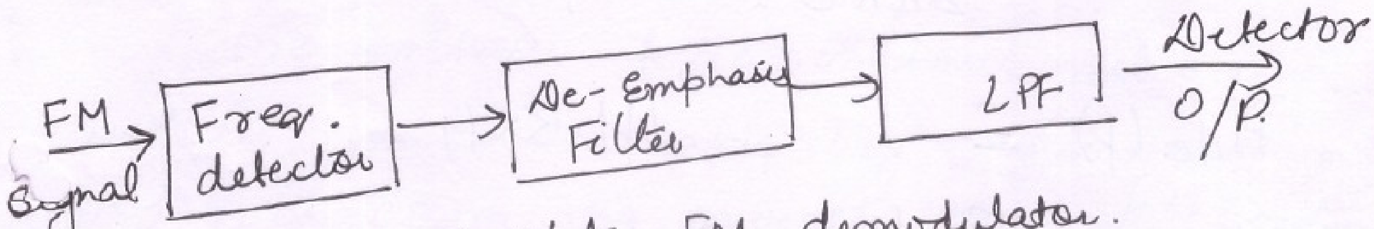
To reduce the effect of noise and interference, a method for improving system performance i.e.

de-emphasis

→ Let the demod^r is followed by a LPF having an amp. ratio that begins ~~to~~ decrease gradually below W, the msg. BW.

→ This will de-emphasize the high freq. portion of the msg. band & ∴ reduces the more serious interference.

→ However, a sharp cut-off filter is still required to remove any residual components above W.



Complete FM demodulator.

→ De emphasis filtering attenuates the high freq. components of the msg. itself.

→ To compensate for de-emphasis distortion by predistortion or pre-emphasizing the msg. signal at the transmitter before modulation.

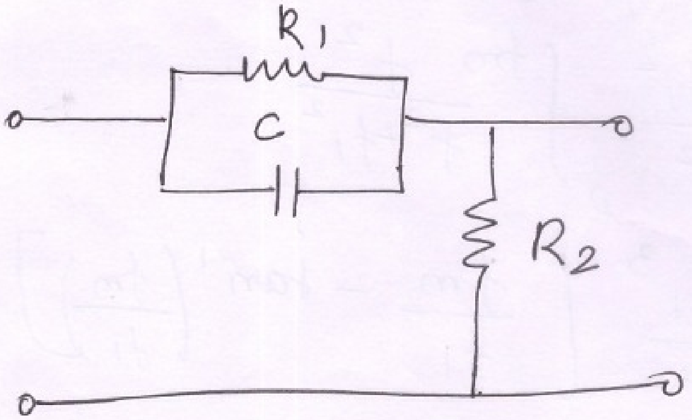
Fold improvement through De-emphasis

At T^x, the high freq. components of the msg. signal $e_m(t)$ are boosted using a ckt. arrangement called pre-emphasis which gives $e_m'(t)$ which is used to frequency modulate the carrier.

→ The desired signal is finally obtained by passing the demod. o/p through a filter which restores the high freq. components to the original level (de-emphasis).

→ This ~~get~~ yields the original msg. signal $e_m(t)$ & the noise whose psd is reduced.

→ ∴ pre-emphasis & de-emphasis not only reduces the undesirable high noise but also cuts down the overall noise at the o/p & improves the o/p SNR. ∴ threshold of FM is improved.

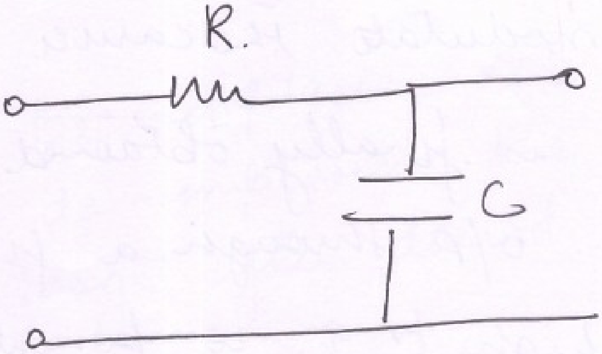


pre-emphasis filter

Break-point freq.

$$f_1 = \frac{1}{2\pi R_1 C}$$

$$f_2 = \frac{1}{2\pi R_2 C}$$



De-emphasis ckt.

$$H(f) = \frac{1}{1 + j(f/f_1)}$$

$$\Rightarrow |H(f)|^2 = \frac{f_1^2}{f^2 + f_1^2}$$

If $n_o'(t)$ represents the final o/p, then $S_{no}'(f)$

$$S_{no}'(f) = \frac{f^2}{A_c^2}\eta\, |H(f)|^2$$

$$= \frac{f_1^2 f^2 \eta}{A_c^2 (f^2 + f_1^2)}$$

$$N_o' = \frac{2\eta f_1^2}{A_c^2} \int_0^{f_m} \frac{f^2}{f^2 + f_1^2}$$

$$= \frac{2\eta f_1^3}{A_c^2}\left[\frac{f_m}{f_1} - \tan^{-1}\left(\frac{f_m}{f_1}\right)\right]$$

2) An AM R^x, operating with a sinusoi[dal] modulating wave and 80% modulation has o/p SNR of 30 dB. What is the corresponding carrier-to-noise ratio?

Solution

For an AM system with modulation index m, the o/p SNR is given by

$$\left(\frac{S}{N}\right)_o = \frac{A_R^2 \, m^2 \, \overline{x^2(t)}}{\overline{n_i^2(t)}}$$

$$(SNR)_o = 30\,dB = 10^3$$

$$m = 0.8$$

$$\overline{x^2(t)} = \frac{1}{2} \quad \text{(single-tone)}$$

$$\Rightarrow \frac{A_R^2 \times 0.64 \times 1/2}{\overline{n_i^2(t)}} = 1000$$

$$\Rightarrow \frac{A_R^2}{\overline{n_i^2(t)}} = \frac{1000}{0.32}$$

$$CNR = \frac{A_R^2/2}{\overline{n^2(t)}} \Rightarrow CNR = \frac{1000}{0.64} = 1562.5$$

$$= \underline{31.9\,dB}$$

$$\{A_R = KA_c\}$$

$$\left(\frac{S}{N}\right)_0 = 3\left(\frac{k_f^2 S_x}{W^2}\right)\gamma \qquad \left\{\begin{array}{l} \Delta\omega = |K_f x(t)| \\ \quad = K_f \\ \& \ \frac{\Delta\omega}{\omega} = m_f \end{array}\right.$$

$$= \frac{3}{2}\left(\frac{\Delta\omega}{\omega}\right)^2 \gamma$$

$$\boxed{\left(\frac{S}{N}\right)_0 = \frac{3}{2} m_f^2 \gamma}$$

5) Compute the B_T & the S_T of DSB, SSB & AM systems for transmitting an audio signal which has a BW of ~~10H~~ 10 KHz with an O/P SNR of 40dB. Assume that the channel introduces a 40 dB power loss and channel noise is AWGN with PSD $\frac{\eta}{2} = 10^{-9}$ W/Hz. Assume $\mu^2 S_x = 0.5$ for AM.

Sol

$$B_T = \begin{cases} 20\text{ KHz} & \text{for DSB and AM} \\ 10\text{ KHz} & \text{for SSB} \end{cases}$$

$$\left(\frac{S}{N}\right)_0 = \frac{S_i}{\eta f_m} = 10^4 \ (= 40\text{dB})$$

$$S_i = \eta f_m (10^4) = 2(10^{-9})(10^4)(10^4) = 0.2\text{W}$$

Because the channel power loss is 40 dB, the required transmitted power S_T is

$$S_T = 0.2(10^4) = 2000\text{ W} = 2\text{KW}$$

For an AM system using envelope detection with $m^2 S_x = 0.5$, we have $\left(\frac{S}{N}\right)_0 = \frac{1}{3}\left(\frac{S_i}{\eta f_m}\right)$

Hence, the required transmitted power is 3 times than that for DSB or SSB system, $\underline{S_T = 6\text{ KW}}$ Ans

SNR at O/P of demod to that at the i/p (2)
of DSB-SC is

$$\frac{\left(\frac{S_o}{N_o}\right)}{\left(\frac{S_i}{N_i}\right)} = 2$$

∴ improvement in $\frac{S}{N}$ by a factor of 2.

Bandpass noise contains inphase + quadrature comp
∴ synchronous detection scheme eliminates one part totally. ∴ Half of noise is removed completely and improvement of overall $\frac{S}{N}$ ratio by a factor of 2.

2) <u>SSB-SC</u>

⇒ R^x is same as that of DSB-SC except the BW of the BPF of SSB-SC should be one half of DSB-SC.

$$S(t) = \frac{A_c}{2}\left[e_m(t)\cos 2\pi f_c t \mp \hat{e}_m(t)\sin 2\pi f_c t\right]$$

$$S_i = \frac{A_c^2}{4}\left[\frac{\overline{e_m^2(t)}}{2} + \frac{\overline{\hat{e}_m^2(t)}}{2}\right]$$

$$= \frac{A_c^2}{4}\,\overline{e_m^2(t)}$$

avg. power of $e_m(t)$ is same as that of its hilbert transform.

FM broadcast system with max. deviation of 75 KHz & BW, B = 15 KHz. Assume PSD $S_n = 1/2$, find the O/P SNR.

Sol :-
$$(SNR)_0 = \frac{3 K_f^2 \overline{m^2(t)} A_c^2}{2\eta W^3}$$

$\{ \Delta f = K_f A_m$, if $A_m = 1$ $\}$

$\Delta f = K_f$

$$= \frac{3 \times (75 \times 10^3)^2 \times \frac{1}{2}}{2 \times \frac{1}{2} \times 15 \times 10^3}$$

$$= \frac{3 \times 75 \times \cancel{75}^{5} \times 10^3}{2 \times \cancel{15}}$$

$$= 562.5 \times 10^3 \text{ Ans}$$

4) Prove that in an FM system the O/P SNR assuming sinusoidal modulation is given by

$$\left(\frac{S}{N}\right)_0 = \frac{3}{2} m^2 r$$

where m is the modulation index for FM.

Sol For sinusoidal modulation,

$X(t) = \cos \omega_m t$

$$\overline{x^2(t)} = \frac{1}{2}$$

Noise in AM & FM - Numericals

(Tutorial Sheet)

1) Find the figure of merit of an AM system when the depth of modulation is
i) 100% ii) 50% iii) 30%

Sol

$$FOM = \frac{m^2 \overline{x^2}}{1 + m^2 \overline{x^2}}$$

i) $m = 1$ i.e. 100% mod^n

$$FOM = \frac{\overline{x^2}}{1 + \overline{x^2}}$$

If single tone $\overline{x^2} = \frac{1}{2}$

$$\therefore \quad FOM = \frac{1}{3}$$

ii) $m = 0.5 \Rightarrow 50\%$

$$FOM = 0.111$$

iii) $m = 0.3 \Rightarrow 30\%$

$$FOM = 0.04306$$

PCS (19/10/15)

1, 2, 4, 5, 9, 10, 11, 13, 14, 15, 18, 21, 22,
23, 27, 28, 29, 31, 32, 33, 34, 37, 38, 40,
41, 42, 43, 44, 45, 46, 48, 49, 50, 52,
58, 59, 62, 65, 66, 67, 69, 70, 73, 74, 72,
76, Harshita, Pallavi, 61, 60

43223.15
8677.34
223231.00

275) 131.49

nent

.10

$$f = \frac{N_0}{N_0'}$$

previously $N_0 = \dfrac{2\eta W^3}{3A_C^2}$

$$f = \frac{1}{3}\left[\frac{\left(\frac{f_m}{f_1}\right)^3}{\frac{f_m}{f_1} - \tan^{-1}\left(\frac{f_m}{f_1}\right)}\right]$$

→ decreasing the first break freq. f_1 improves the performance.

$$\frac{314 \text{ (old)}}{1003 \text{ (new)}}$$

- Shalvi sharma
Swati soni

The pre-emphasis & de-emphasis fi[lters]
should be related by

$$H_{pe}(f) = \frac{1}{H_{de}(f)} \quad \text{for } |f| < W$$

→ we pre-emphasize the msg. before mod. so we de-emphasize the interference relative to msg. after demod.

→ De-emphasis filter used in FM demodulators is

$$H_{de}(f) = \frac{1}{1 + j\left(\frac{f}{f_1}\right)}$$

where $f_1 = \frac{1}{2\pi RC}$

$$H_{de}(f) \simeq 1 \qquad |f| \ll f_1$$

$$\simeq \frac{f_1}{jf} \qquad |f| \gg f_1$$

$$H_{pe}(f) = \left[1 + j\left(\frac{f}{f_1}\right)\right]$$

$$H_{pe}(f) = 1, \quad |f| \ll f_1$$

$$= j\left(\frac{f}{f_1}\right), \quad |f| \gg f_1$$

$$\therefore \beta(t) \simeq \frac{A_c}{R(t)} \sin\left(\phi(t) - \theta(t)\right)$$

O/P of FM demodulator is

$$S_d(t) = \frac{d}{dt}\left[2\pi f_c t + \theta(t) + \beta(t)\right]$$

$$= 2\pi f_c + \underbrace{\frac{d\theta(t)}{dt}}_{\text{entirely due to noise.}} + \underbrace{\frac{d\beta(t)}{dt}}_{\text{message.}}$$

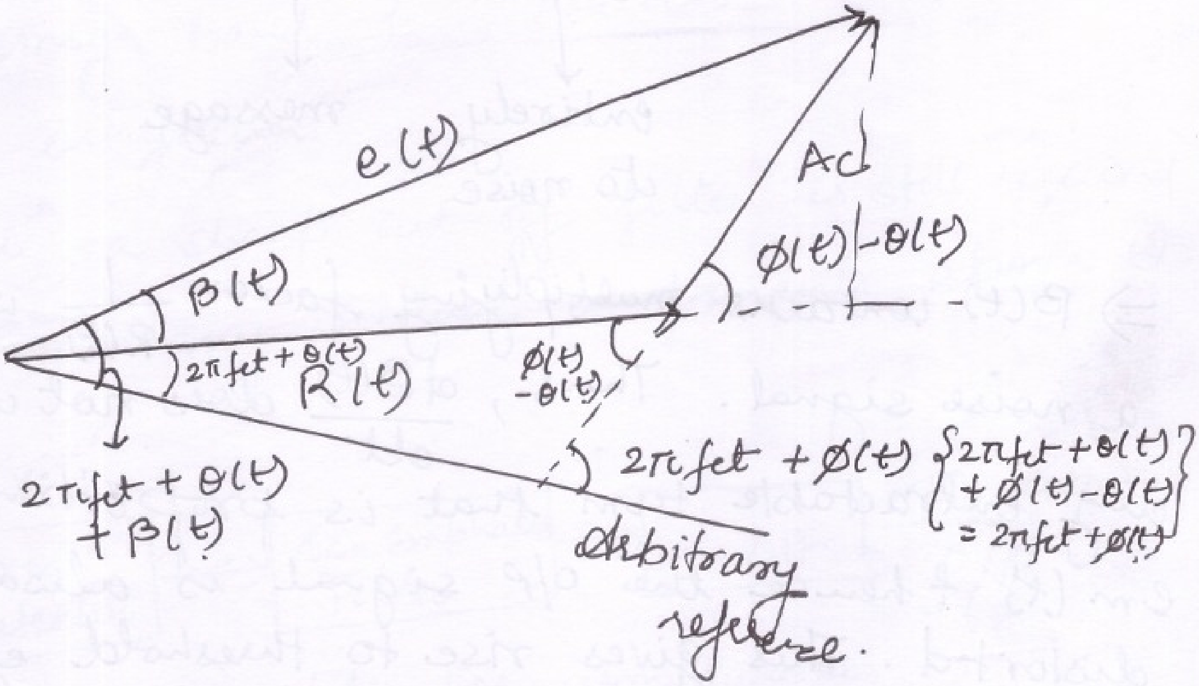
$\Rightarrow$ $\beta(t)$ contains multiplying factor $\frac{1}{R(t)}$ which is a noise signal. Thus, $\frac{d\beta(t)}{dt}$ does not contain any extractable term that is proportional to $e\, m(t)$ & hence the O/P signal is always distorted. This gives rise to threshold effect.

where $\phi(t) = 2\pi K_f \int_0^t e_m(t)\,dt$

& $n_i(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t$

$= R(t)\cos(2\pi f_c t + \theta(t))$

$S_i(t) = A_c\cos(2\pi f_c t + \phi(t)) + R(t)\cos(2\pi f_c t + \theta(t))$



$S_i(t) = e(t)\cos[2\pi f_c t + \theta(t) + \beta(t)]$

$$\beta(t) = \tan^{-1}\left[\frac{A_c \sin(\phi(t) - \theta(t))}{R(t) + A_c\cos(\phi(t) - \theta(t))}\right]$$

For large noise case, $R(t) \gg A_c$

$$\therefore \beta(t) = \tan^{-1}\left[\frac{A_c \sin(\phi(t) - \theta(t))}{R(t)}\right]$$

then

$$S_{nd}(f) = |H(f)|^2 S_{ns}(f)$$

$$= \frac{f^2}{A_c^2} S_{ns}(f)$$

$$S_{nd}(f) = \frac{f^2 \eta}{A_c^2} \quad , \quad |f| \le \frac{B_T}{2}$$

$\eta \rightarrow$ PSD of $n_s(t)$

$$= 0 \quad , \text{ otherwise}$$

$$S_{no}(f) = \frac{f^2 \eta}{A_c^2}$$

$$N_o = \frac{\eta}{A_c^2} \int_{-W}^{W} f^2 \, df = \frac{2\eta W^3}{3 A_c^2} \quad — ②$$

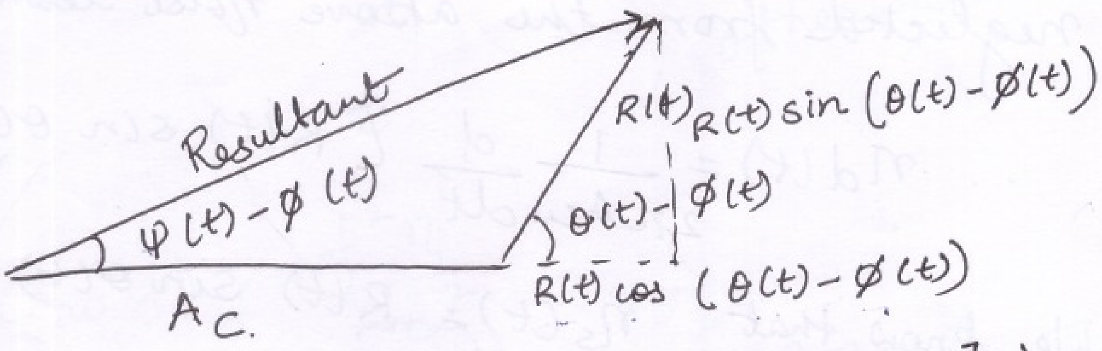
$$S_i = \frac{A_c^2}{2} \quad — ③$$

$$N_i = 2\eta B_T \quad — ④$$

$$\cancel{\frac{\left(\frac{S_o}{N_o}\right)}{\left(\frac{S_i}{N_i}\right)}} = \cancel{\frac{\frac{K_f^2 \overline{e_m^2(t)}}{\frac{2\eta W^3}{3 A_c^2}}}{\frac{\frac{A_c^2}{2}}{2\eta B_T}}} = \cancel{K_f^2 \overline{e_m^2(t)}\, 2\eta B_T}$$

$$\frac{S_o}{N_o} = \frac{3 K_f^2 \overline{e_m^2(t)} \, A_c^2}{2\eta W^3}$$

where $R(t)$ & $\theta(t)$ are the amplitude & phase of Bandpass noise.



$$\psi(t)-\phi(t) = \tan^{-1}\left(\frac{R(t)\sin[\theta(t)-\phi(t)]}{A_c+R(t)\cos[\theta(t)-\phi(t)]}\right)$$

where $\phi(t) = 2\pi K_f \int_0^t e_m(t)\,dt$.

→ We assume that the amplitude of the unmodulated carrier is very large so that the carrier-to-noise ratio measured at the discriminator i/p is large compared to unity.

$$\therefore \psi(t) \simeq \phi(t) + \frac{R(t)}{A_c}\sin[\theta(t)-\phi(t)]$$

$$\simeq 2\pi K_f \int_0^t e_m(t)\,dt + \frac{R(t)}{A_c}\sin[\theta(t)-\phi(t)]$$

O/P of the discriminator can be written as

$$S_d(t) = \frac{1}{2\pi}\,\frac{d\psi(t)}{dt}$$