# Unit 2:- AMPLITUDE MODULATION

⇒ MODULATION :- Systematic alteration of carrier according to characteristics of another waveform, the modulating signal or message signal.

→ Baseband :- band of frequencies representing the original signal as defined by the source of information.

modulation

- Linear modulation
- non-linear modulation
- Continuous wave
- Pulse modulation

⇒ Linear modulation is essentially direct frequency translation of the msg. spectrum.

## Amplitude modulation

→ process in which amplitude of the carrier wave is varied about a mean value, linearly with the baseband signal.

Carrier signal, $e_c(t) = A_c \cos 2\pi f_c t$

(phase $\angle = 0$) $+$ $A_c \cos 2\pi$ ... $e_m(t)\, K_a$

msg. signal, $e_m(t)$

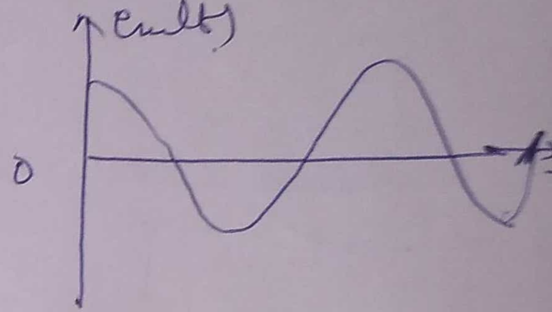
$\therefore$ modulated wave, $s(t) = \boxed{A_c [1 + K_a e_m(t)] \cos 2\pi f_c t}$

$K_a$ → constant (amplitude sensitivity)

$f_c \gg f_m$

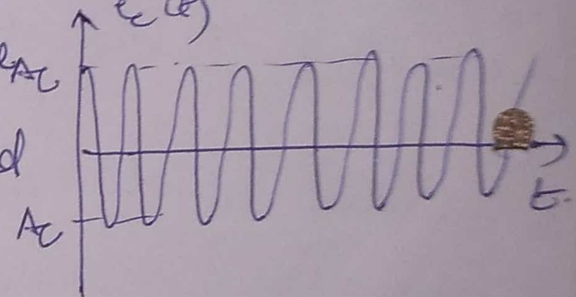
$\Rightarrow$ Envelope of the resultant modulated wave corresponds to the baseband signal when :-

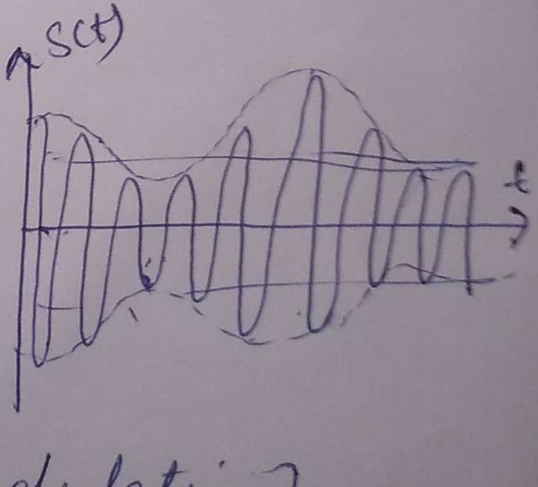
a) $|K_a e_m(t)| < 1$ for all $t$

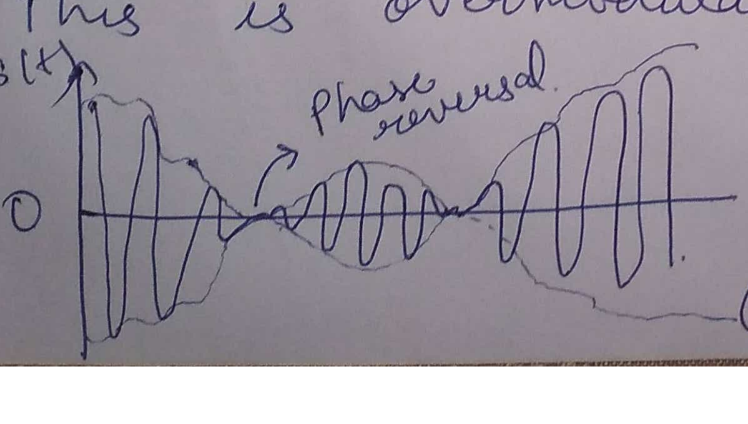



When $K_a$ is so large that $|K_a e_m(t)| > 1$, the envelope of the resultant modulated signal no longer resembles the baseband signal.

This is overmodulation




(Overmodulation)

→ phase reversal of the carrier every $f_c$ time $[1 + K_a e_m(t)]$ crosses zero.

⇒ absolute max. value of $K_a e_m(t)$ multiplied by 100 is often referred to as % modⁿ.

b) $f_c >> W$ (highest freq. component)

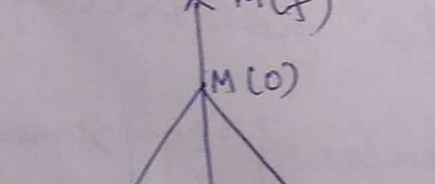
## Spectrum of AM Signal.

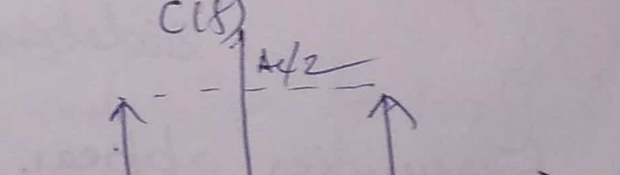
$$s(t) = A_c[1 + K_a e_m(t)] \cos 2\pi f_c t$$

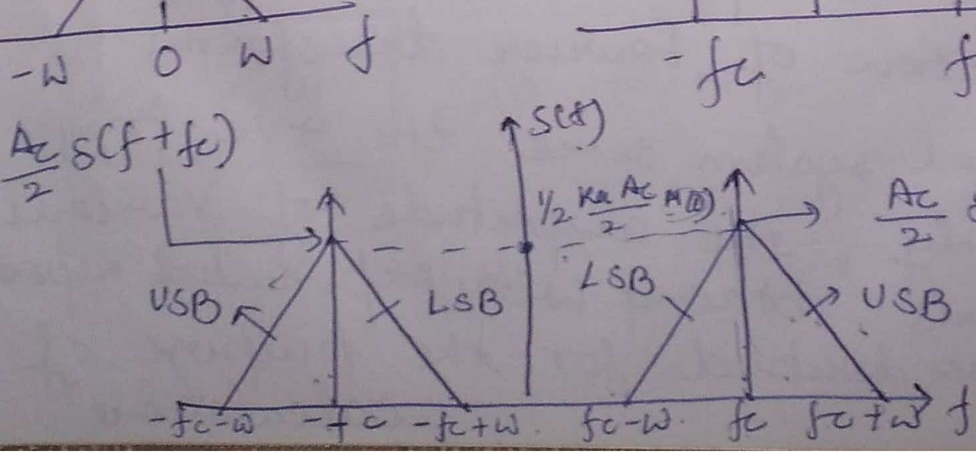
$$= A_c \cos 2\pi f_c t + A_c K_a e_m(t) \cos 2\pi f_c t$$

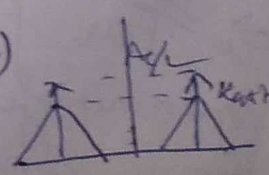
$\cos 2\pi f_c t = \frac{1}{2}[\delta(f-f_c) + \delta(f+f_c)]$

$$= \frac{A_c}{2}[\delta(f-f_c) + \delta(f+f_c)] + \frac{K_a A_c}{2}[M(f-f_c) + M(f+f_c)]$$



→ Two impulse functions occuring at fc and -fc and the original spectrum M(f) shifted in the frequency domain by fc and -fc.

→ Spectrum of AM signal consists of two delta functions weighted by a factor $\frac{A_c}{2}$ occuring $\pm f_c$ together with version of baseband spectrum translated by $\pm f_c$ and scaled in amplitude by $\frac{k_a A_c}{2}$.

⇒ The band of frequency which is lying above fc is called the upper sideband (USB) whereas the symmetrical portion below fc is called the lower sideband (LSB).

$f_c > W$ (ensures that the two sidebands do not overlap)

⇒ -ve frequencies appear because of exponential representation of Fourier transforms.

⇒ For practical system since the +ve freq. are meaningful only, the amplitude of various freq. components obtained with exponential representations can just be doubled for the purpose of calculations.

→ For +ve freq, highest freq. comp. $f_c + W$
lowest " " $f_c - W$.

∴ Bandwidth, $B_T = f_c + W - (f_c - W)$

$= \underline{\underline{2W}}$.

## Single - tone sinusoidal modulation

→ $e_m(t) = A_m \cos(2\pi f_m t)$

$S(t) = A_c [1 + K_a e_m(t)] \cos 2\pi f_c t$

$= A_c [1 + \boxed{K_a A_m} \cos 2\pi f_m t] \cos 2\pi f_c t$

$= A_c [1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$

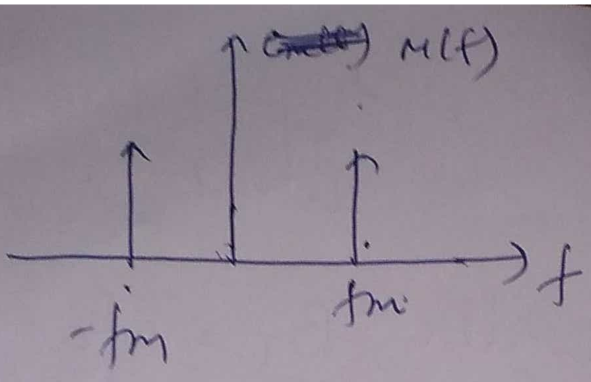
where $\boxed{\mu = K_a A_m}$ {max. deviation from the unmodulated carrier wave.}

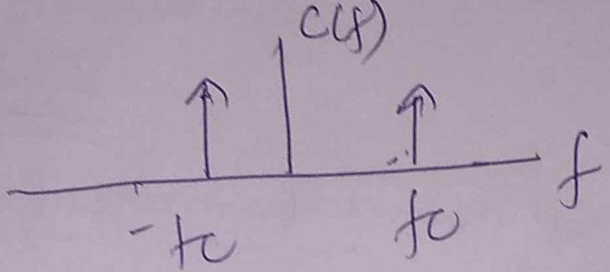
↓

dimensionless

[Modulation index or factor]

$\boxed{\mu < 1}$ to avoid overmodulation.





$A_{max}$ & $A_{min}$ - max. & min. values of the envelope of the modulated wave.

$$A_{max} = A_c [1 + \mu]$$

$$A_{min} = A_c [1 - \mu]$$

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{A_{max} - A_c}{A_c} = \frac{A_c - A_{min}}{A_c}$$

$$s(t) = A_c \cos 2\pi f_c t + \mu A_c \cos 2\pi f_c t \cos 2\pi f_m t$$

$$s(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2}\left[\cos(2\pi(f_c+f_m)t) + \cos(2\pi(f_c-f_m)t)\right]$$

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{\mu A_c}{4}\left[\delta(f-f_c-f_m) + \delta(f+f_c+f_m)\right] + \frac{\mu A_c}{4}\left[\delta(f-f_c+f_m) + \delta(f+f_c-f_m)\right]$$

Delta functions at $\pm f_c$, $f_c \pm f_m$, $-f_c \pm f_m$.

Q. The signals $e_1(t) = 2\cos(2\pi f_1 t) + \cos(2\pi \cdot 2 f_1 t)$ and $e_2(t) = \cos(2\pi f_2 t) + 2\cos(2\pi \cdot 2 f_2 t)$ are multiplied. Plot the amplitude-frequency charac. of the resultant signal, assuming $f_2 = 2f_1$.

Sol $$S(f) = \frac{1}{4}\delta(f) + \frac{1}{2}\left[\delta(f-f_1) + \delta(f+f_1)\right] + \frac{1}{2}\left[\delta(f-2f_1) + \delta(f+2f_1)\right] + \text{---} \text{---} + \frac{1}{2}\left[\delta(f-6f_1) + \delta(f+6f_1)\right]$$

# Multi Tone sinusoidal modulation

- 2 tones of freq. $f_1$ and $f_2$.

AM wave,

$$s(t) = A_c\left[1 + K_a A_{m_1} \cos(2\pi f_1 t) + K_a A_{m_2} \cos(2\pi f_2 t)\right] \cos 2\pi f_c t$$

$$= A_c\left[1 + \mu_1 \cos(2\pi f_1 t) + \mu_2 \cos(2\pi f_2 t)\right] \cos 2\pi f_c t$$

where $\mu_1 = K_a A_{m_1}$ & $\mu_2 = K_a A_{m_2}$

$$s(t) = A_c \cos(2\pi f_c t) + \frac{\mu_1 A_c}{2}\left[\cos(2\pi(f_c+f_1)t) + \cos(2\pi(f_c-f_1)t)\right] + \frac{\mu_2 A_c}{2}\left[\cos(2\pi(f_c+f_2)t + \cos(2\pi(f_c-f_2)t)\right]$$

→ Delta functions occuring at $\pm f_c$, $f_c \pm f_1$, $f_c \pm f_2$ and $-f_c \pm f_2$.

→ Similar effect can be produced by the modulating the carrier individually by the two modulating waves separately and adding the results.

∴ superposition holds in case of AM.

∴ AM is a linear mod$^n$.

content of sidebands and carrier in AM

$$s(t) = A_c \cos 2\pi f_c t + K_a A_c e_m(t) \cos 2\pi f_c t.$$

$$P_c = \frac{A_c^2}{2}$$ [mean square value of $A_c \cos 2\pi f_c t$]

$$P_S = \frac{K_a^2 A_c^2}{2} \overline{e_m^2(t)}$$

$$P_t = P_c + P_S = \frac{A_c^2}{2} + \frac{K_a^2 A_c^2 \overline{e_m^2(t)}}{2}$$

∵ the information is contained in the sidebands only, the carrier power has no contribution in the transmission of information.

⇒ Transmission efficiency, $\eta$ :- % of total power carried by the sidebands.

$$\eta = \frac{P_S}{P_t} \times 100\% = \frac{\frac{K_a^2 A_c^2 \overline{e_m^2(t)}}{2}}{\frac{A_c^2}{2} + \frac{K_a^2 A_c^2 \overline{e_m^2(t)}}{2}} \times 100\%$$

$$= \frac{K_a^2 \overline{e_m^2(t)}}{1 + K_a^2 \overline{e_m^2(t)}} \times 100\%$$

$$e_m(t) = A_m \cos(2\pi f_m t)$$

$$\overline{e_m^2(t)} = \frac{A_m^2}{2}.$$

$$\eta = \frac{P_s}{P_t} \times 100\%$$

$$= \frac{K_a^2 A_m^2}{2 + K_a^2 A_m^2} \times 100\%$$

$$= \frac{\mu^2}{2 + \mu^2} \times 100\%$$

If $\mu = 1$,

$$\eta_{max} = \frac{1}{3} \times 100\% = 33.33\%$$

⇒* For the highest modulation index ($\mu = 1$), the efficiency of transmission is approx. 33%. the total power is carried by About 67% of the carrier and as such, represents waste. For $\mu$ less than unity, the eff. is less than 33%.

Q A 400 W carrier is modulated to a depth of 75%. Calculate the total power in the modulated wave.

Sol

$$\frac{P_s}{P_t} = \frac{\mu^2}{2 + \mu^2}$$

$$P_t = P_c + P_s$$

$$= P_c\left(1 + \frac{\mu^2}{2}\right) = 400\left[1 + \frac{(0.75)^2}{2}\right] = 512.5 W$$

A broadcast radio transmitter radiates 5 KW power when modulation % is 60%. How much is the carrier power?

Sol $P_c = \dfrac{P_t}{\left(1+\frac{\mu^2}{2}\right)} = \dfrac{5\times 10^3}{1+\frac{(0.6)^2}{2}} = 4.235\text{kW}$

Current calculation

$$\frac{P_t}{P_c} = \frac{I_t^2 R}{I_c^2 R} = \left(1+\frac{\mu^2}{2}\right)$$

or $I_t = I_c \left(1+\frac{\mu^2}{2}\right)^{1/2}$

Q The antenna current of an AM $T^x$ is 8 A when only carrier is sent, but it increases to 8.96 A when the carrier is ~~not~~ modulated by a single tone sinusoid. Find the % mod$^n$. Find the antenna current when the depth of mod$^n$ changes to 0.8.

Sol $\mu = \left[2\left\{\left(\frac{I_t}{I_c}\right)^2 - 1\right\}\right]^{1/2}$

$= \left[2\left\{\left(\frac{8.96}{8}\right)^2 - 1\right\}\right]^{1/2} = 0.7133$

$$I_t = I_c \left(1 + \frac{\mu^2}{2}\right)^{1/2}$$

$$= 8\left(1 + \frac{(0.8)^2}{2}\right)^{1/2}.$$

Effective mod$^n$ index.

$e_{m_1}(t) = A_{m_1} \cos(2\pi f_1 t)$

$e_m(t) = A_{m_2} \cos(2\pi f_2 t)$ - - - - -

$e_c(t) = A_c \cos(2\pi f_c t)$

$s(t) = A_c \left[1 + K_a A_{m_1} \cos(2\pi f_1 t) + K_a A_{m_2} \cos(2\pi f_2 t) + \cdots\right] \cos(2\pi f_c t)$

$= A_c\left[1 + \mu_1 \cos 2\pi f_1 t + \mu_2 \cos 2\pi f_2 t + \cdots\right] \cos 2\pi f_c t$

$$P_t = \frac{A_c^2}{2} + \frac{\mu_1^2 A_c^2}{4} + \frac{\mu_2^2 A_c^2}{4} + \cdots$$

$$= \frac{A_c^2}{2}\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2} + \cdots\right]$$

$$= P_c\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2} + \cdots\right]$$

$$P_t = \frac{A_c^2}{2} + \frac{\mu_t^2 A_c^2}{4} = \frac{A_c^2}{2}\left(1 + \frac{\mu_t^2}{2}\right) = P_c\left(1 + \frac{\mu_t^2}{2}\right)$$

$$\mu_t^2 = [\mu_1^2 + \mu_2^2 + \mu_3^2 + \cdots]$$

$$\boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2 + \cdots}}$$

Q The antenna current of an AM broadcast $T_x$, modulated to a depth of 40% by an audio sine wave is 11 A. It increases to 12 A as a result of sinusoidal modn by another audio sine wave. What is the modulation index due to second wave?

Sol

$$I_c = \frac{I_t}{\sqrt{1 + \frac{\mu_t^2}{2}}} = \frac{11}{\sqrt{1 + 0.08}} = 10.58A$$

$$I_c = \frac{I_t}{\sqrt{1 + \frac{\mu_t^2}{2}}}$$

$$\Rightarrow \mu_t = \sqrt{\left[2\left\{\left(\frac{I_t}{I_c}\right)^2 - 1\right\}\right]} = \sqrt{\left[2\left\{\left(\frac{12}{10.58}\right)^2 - 1\right\}\right]}$$

$$= 0.757$$

$$\mu_t^2 = \mu_1^2 + \mu_2^2 \Rightarrow \mu_2 = \sqrt{\mu_t^2 - \mu_1^2}$$

$$= \sqrt{(0.757)^2 - (0.4)^2}$$

$$= \underline{0.643}$$

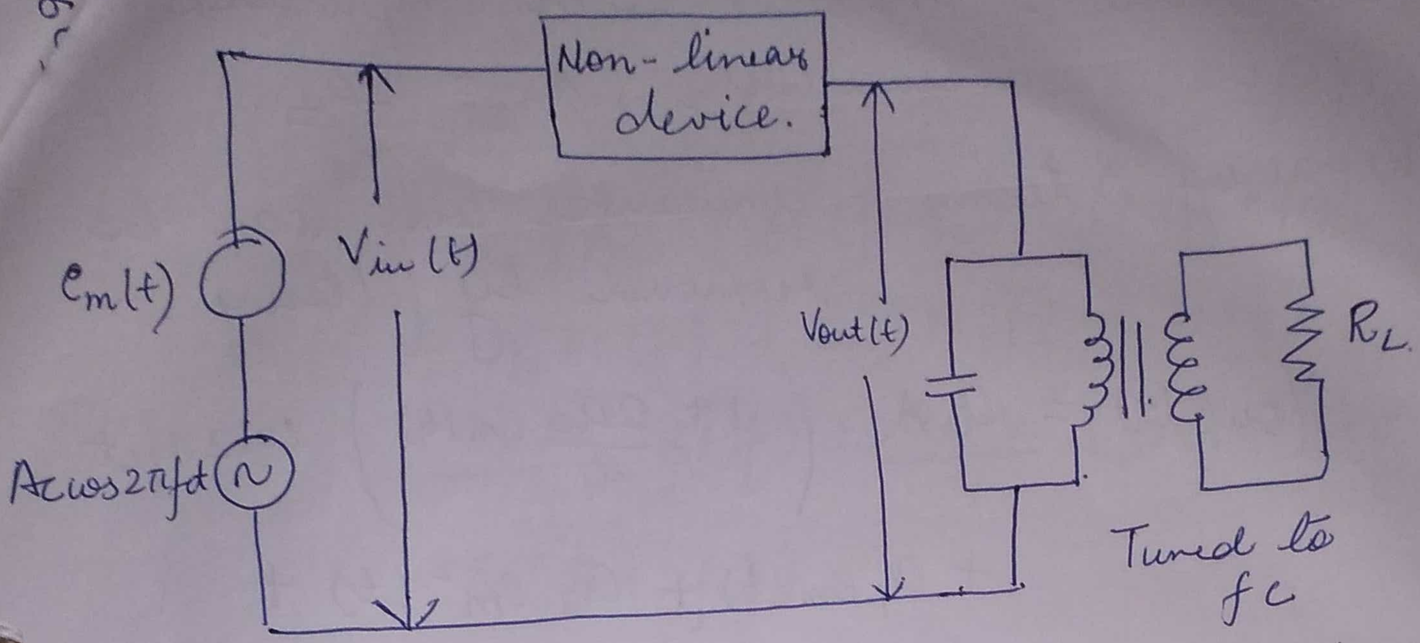
Q/ A 300W carrier is simultaneously modu
by two audio waves with modn % of 50 & 6
What is the total sideband power
radiated ?

Sol
$$P_t = \frac{A_c^2}{2}\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right]$$
$$= P_c\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right]$$
$$\frac{P_t}{P_c} = \left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right]$$
$$\frac{P_S}{P_c} = \left[\frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right]$$
$$P_S = 300\left[\frac{(0.50)^2}{2} + \frac{(0.60)^2}{2}\right] = 96.5W$$

## Generation of AM wave

### Square-law modulator

→ Requires a means to add up the carrier and modulating waves, a non-linear element and a BPF for extracting the desired modulated wave.




→ The non-linear element used in square-law modulator may be a diode or a transistor.

According to square law,

$$V_{out}(t) = a_1 V_{in}(t) + a_2 V_{in}^2(t)$$

Where $a_1$ and $a_2$ are constants.

$$V_{in}(t) = A_c \cos(2\pi f_c t) + e_m(t)$$

$$V_{out}(t) = a_1 [A_c \cos 2\pi f_c t + e_m(t)] + a_2 [A_c \cos 2\pi f_c t + e_m(t)]^2$$

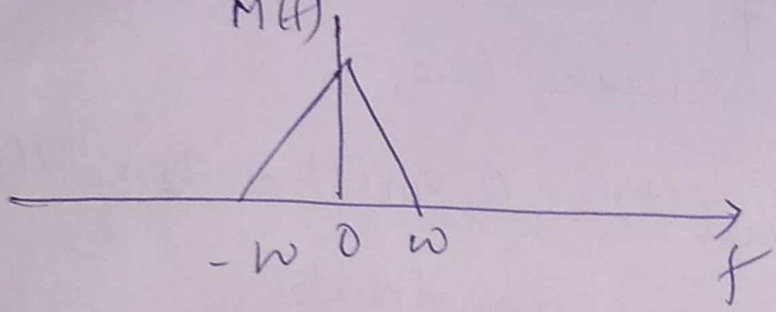
$$= a_1 A_c \cos 2\pi f_c t + a_1 e_m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 e_m^2(t) + 2a_2 A_c \cos 2\pi f_c t \, e_m(t)$$

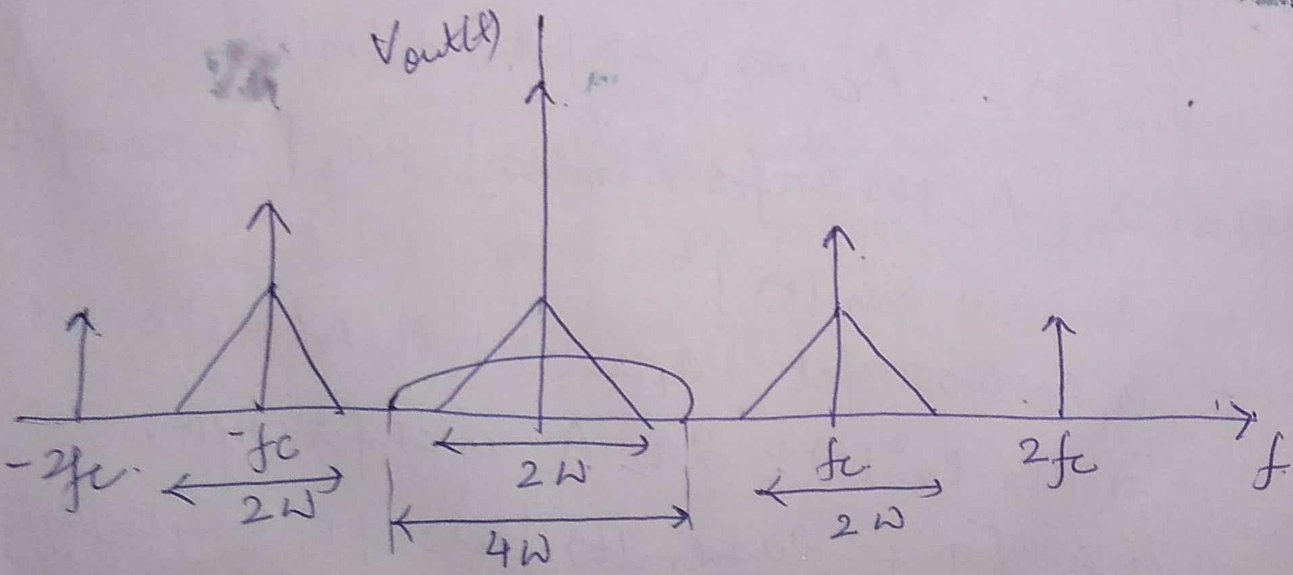
$$= a_1 A_c \left[1 + \frac{2a_2 e_m(t)}{a_1}\right] \cos 2\pi f_c t + a_1 e_m(t) + a_2 e_m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t$$

First term :- AM wave with amplitude sensitivity , $K_a = \frac{2a_2}{a_1}$

Remaining 3 terms :- unwanted and can be removed by filtering .

$$V_{out}(t) = a_1 A_c \left(1 + \frac{2a_2}{a_1} e_m(t)\right) \cos 2\pi f_c t$$
$$+ a_1 e_m(t) + a_2 e_m^2(t) +$$
$$\frac{a_2 A_c^2}{2}\left(1 + \cos(2\pi . 2f_c t)\right)$$



Taking FT,

$$V_{out}(f) = \frac{a_1 A_c}{2}\left(\delta(f-f_c) + \delta(f+f_c)\right) + a_2 A_c \left(M(f-f_c) + M(f+f_c)\right) + a_1 M(f) + a_2 M(f) * M(f) + \frac{a_2 A_c^2}{4}\delta(f) + \frac{a_2 A_c^2}{4}\left(\delta(f-2f_c) + \delta(f+2f_c)\right)$$

→ Unwanted freq. components can be removed by using suitable BPF.

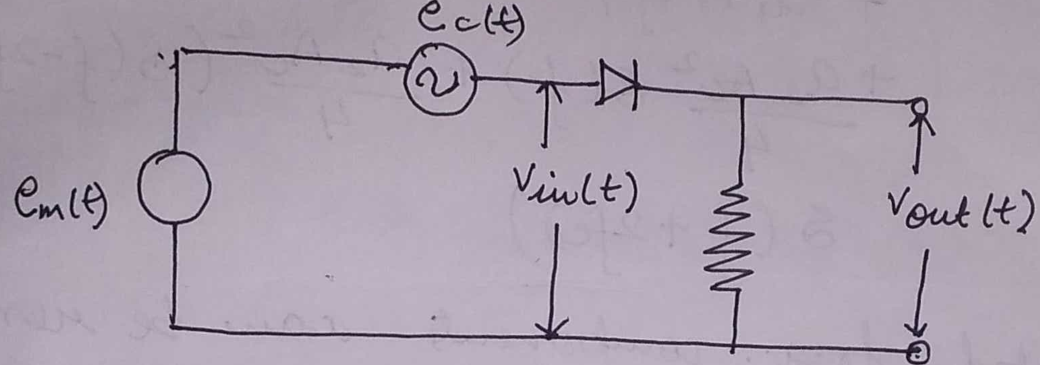
Since $M(f) * M(f)$ is bandlimited in $2W$, there ~~is~~ can be overlapping of ~~spore~~ spectra when $f < 3W$. ∴ in order to avoid spectral overlapping the carrier freq., $f_c$ must be greater than three times the BW of low freq. signal.

$$f_c > 3W.$$

∴ The desired amp. modulated wave can be obtained by passing the resultant signal through a BPF of $B_T = 2W$ having the centre freq. $f_c$.

# Switching Modulator

- Eff. high-level modulators are arranged so that undesired mod$^n$ products never fully develop & need not be filtered out.




→ $e_c(t)$ applied to diode has been assumed to be of large amp. so as to swing right across the charac. curve of the diode.

→ diode offers zero res. in F/w direc$^n$ ($e_c(t) > 0$)
∞ " " Reverse direc$^n$ ($e_c(t) < 0$)

$$V_{in}(t) = e_c(t) + e_m(t)$$
$$= A_c \cos 2\pi f_c t + e_m(t)$$

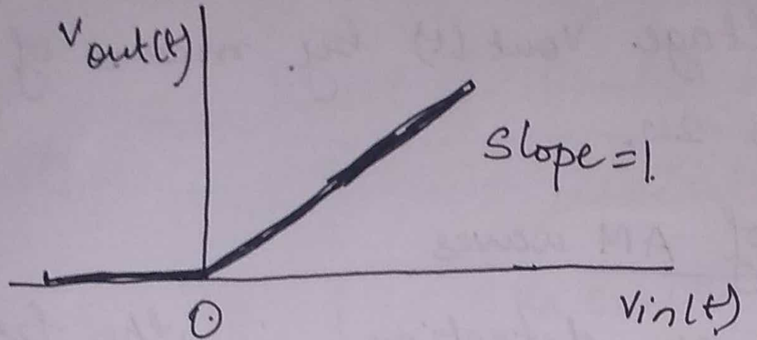
where $|e_m(t)| << A_c$.

Resulting load voltage $V_{out}(t)$ is

$$V_{out}(t) \cong V_{in}(t) , \quad e_c(t) > 0$$
$$\cong 0 , \quad e_c(t) < 0$$

ad voltage $V_{out}(t)$ varies periodically between the values $V_{in}(t)$ and zero at a rate equal to $f_c$.

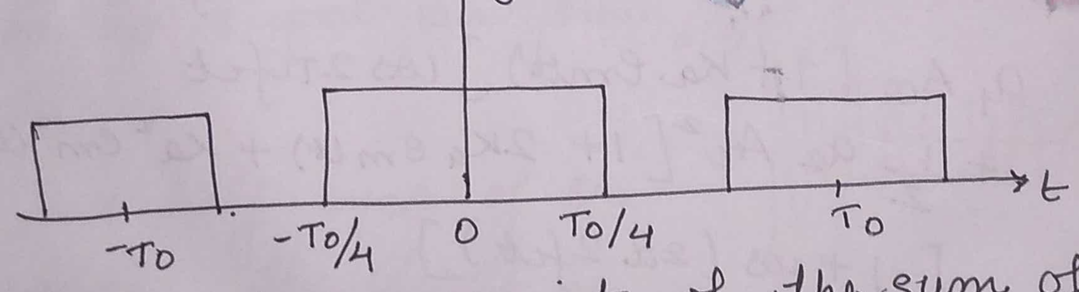
$$V_{out}(t) \simeq [A_c \cos 2\pi f_c t + e_m(t)]\, g_p(t)$$




$g_p(t)$ is a periodic pulse train of duty cycle equal to one-half and period $T_0 = \frac{1}{f_c}$.

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c t(2n-1))$$

$$= \frac{1}{2} + \frac{2}{\pi}\left[\cos 2\pi f_c t - \frac{\cos 2\pi f_c t}{3} + \cdots\right]$$




→ O/P voltage consists of the sum of two components

$$\frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} e_m(t)\right] \cos 2\pi f_c t$$

which is the desired AM wave with $K_a = \frac{4}{\pi A_c}$.

- An unwanted component, the spectrum of which contains delta functions at $0, \pm 2f_c, \pm 4f_c$ & so on and which occupies freq. intervals of width $2W$ centered at $0, \pm 3f_c, \pm f_c$ & so on. The unwanted terms can be removed from the load voltage $V_{out}(t)$ by means of BPF with $f_c$ & BW $2W$.

## Demodulation of AM waves

→ Demodulation or detection, is the process by which the msg. is recovered from the modulated wave at the $R^x$.

### Square-law demodulator

→ Square-law device.

$$V_{out}(t) = a_1 V_{in}(t) + a_2 V_{in}^2(t)$$

$$V_{in}(t) = A_c [1 + K_a e_m(t)] \cos 2\pi f_c t$$

$$V_{out}(t) = a_1 A_c [1 + K_a e_m(t)] \cos 2\pi f_c t + \frac{1}{2} a_2 A_c^2 [1 + 2K_a e_m(t) + K_a^2 e_m^2(t)] [1 + \cos(2\pi 2f_c t)]$$

→ Desired signal $a_2 A_c^2 K_a e_m(t)$ is due to the square term $a_2 V_{in}^2(t)$ & hence the name square law demodulator.

Using LPF, this component can be extracted.

→ $\frac{1}{2} a_2 A_c^2 k_a^2 e_m^2(t)$ → plurality of similar freq. components.

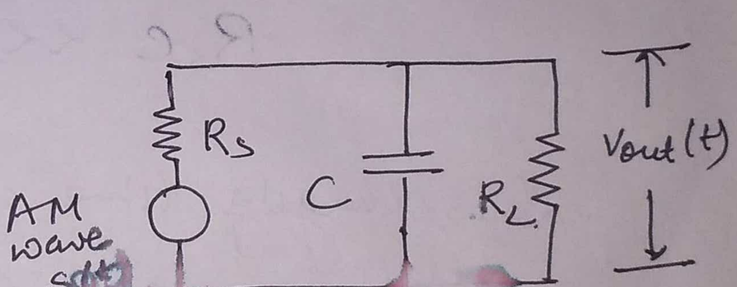
$$e_m^2(t) \xleftrightarrow{FT} M(f) * M(f)$$

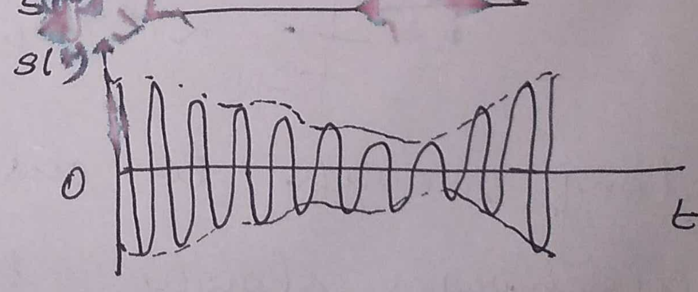
$$\frac{2}{k_a e_m(t)} = \frac{\text{Ratio of wanted signal}}{\text{distortion}}$$

To ↑ , $k_a e_m(t) < 1$

∴ % mod$^n$ should be very small.

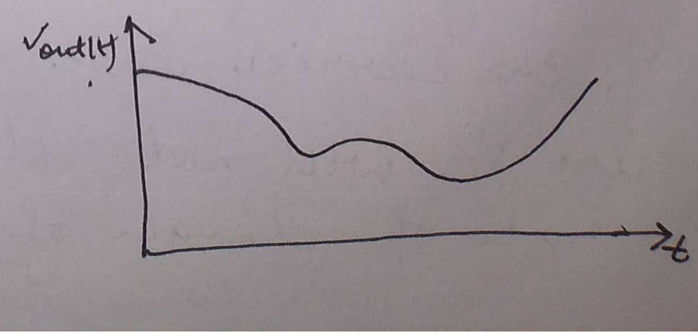
## Envelope detector




→ the envelope detector produces an O/P signal that follows the envelope of the i/p signal waveform exactly.

→ On +ve half-cycle of the i/p signal, the diode is F/W-biased and the capacitor C charges up rapidly to the peak of the i/p signal.



- When the i/p signal falls below this value, the diode becomes reverse biased & C discharges through $R_L$.

Discharge process continues until the next +ve half cycle.

When i/p signal > ~~vo~~ $V_C$, diode conducts again & the process is repeated.

Charging time constant $R_sC$ must be short compared with the carrier time period.

$$R_sC << \frac{1}{f_c}.$$

Discharging time constant

$$\frac{1}{f_c} << R_LC << \frac{1}{W}$$

long enough to ensure that the capacitor discharges slowly through $R_L$ b/w +ve peaks of the carrier wave, but not so long that the $V_C$ will not discharge at the max. rate of change of modulating wave

Capacitor voltage — detector O/P which is same as envelope wave.