# UNIT-5

**Numerical Integration:-** Sometimes it happens that f(x) is known and we are not able to calculate the value of the integral of f(x) by the usual method of integration.

e.g $\int_0^4 e^{-x^2} dx$. In such cases, the numerical methods for integration are required.

Numerical integration is a technique of finding the numerical value of a definite integral by means of the calculated values of the integrand.

$$I = \int_a^b f(x)\,dx \quad \text{----} \quad (1)$$

## Some Important approximate formulae:-

(1) **The Trapezodial Rule:-**

$$I = \int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right] \text{---}(ii)$$

(2) **Simpson's one third rule:-**

$$I = \int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n)+4[y_1+y_3+\cdots+y_{n-1}] + 2(y_2+y_4+\cdots+y_{n-2})\right] \cdots (3)$$

*** While applying the formulae (3), n should be even i.e, the given interval must be divided into even number of equal sub-intervals.

## Simpson's $\frac{3}{8}$ Rule :-

$$I = \int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\Big[(y_0+y_n)+3(y_1+y_2+y_4+y_5+\cdots+y_{n-1}) + 2(y_3+y_6+y_9+\cdots+y_{n-3})\Big]. \quad -(4)$$

*** While applying the above formulae (4), the number of sub-intervals should be taken as a multiple of 3.

** Examples based on Simpson's rule $\frac{1}{3}$ and $\frac{3}{8}$ rule:-

Q(1). Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using

(i) Simpson's $\frac{1}{3}$ Rule

(2) Simpson's $\frac{3}{8}$ Rule.

Soln:- Since we have to use Simpson's one third and three eight rule, we should divide the range (0, 6) in such a manner, that it is divisible by 2 and 3. therefore we divide the range (0, 6) into six equal parts each of the width $h = 1$, the values of $f(x) = \frac{1}{1+x^2}$ at the end points of the intervals are given below.

| $x$ | $y = f(x) = \frac{1}{1+x^2}$ |
| --- | --- |
| $x_0 = 0$ | $y_0 = \frac{1}{1+0} = 1.00$ |
| $x_1 = 1$ | $y_1 = \frac{1}{1+1} = 0.500$ |
| $x_2 = 2$ | $y_2 = \frac{1}{1+4} = 0.200$ |
| $x_3 = 3$ | $y_3 = \frac{1}{1+9} = 0.100$ |
| $x_4 = 4$ | $y_4 = \frac{1}{1+16} = 0.0588$ |
| $x_5 = 5$ | $y_5 \frac{1}{1+25} = 0.0384$ |
| $x_6 = 6$ | $y_6 = \frac{1}{1+36} = 0.02702$ |

(1) By simpson's $\frac{1}{3}$ Rule, we have

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \int_0^6 f(x)\,dx$$

$$= \frac{h}{3}\left[(y_0 + y_n) + 4[y_1 + y_3 + \cdots + y_{n-1}] + 2[y_2 + y_4 \cdots + y_{n-2})\right]$$

$$= \frac{1}{3}\left[(1 + 0.02702) + 4[0.500 + 0.100 + 0.0384) + 2[0.200 + 0.0588)\right]$$

$$= \frac{1}{3}\left[1.0270 + 4[0.6384) + 2[0.2588)\right]$$

$$= 1.3661$$ (approximate) (Ans)

(ii) By Simpson's $\frac{3}{8}$ Rule, we have:

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5\cdots+y_{n-1}) + 2(y_3+y_6+\cdots+y_{n-3})\right]$$

putting values from the given table, we have

$$= \frac{3\times1}{8}\left[(1.00+0.0270)+3(0.500+0.200+0.058+0.0384) + 2(0.100)\right]$$

$$= \frac{3}{8}\left[1.027+3(0.7972)+0.200\right]$$

$$= 1.3570 \quad \text{(Ans)}.$$

Q(2) Evaluate $\int_{-1.6}^{-1.0} e^{x}\,dx$ by Simpson's $\frac{1}{3}$ Rule with six intervals. [ RTU 2007 )]

Sol:-

| | $x$ | $y=f(x)=e^{x}$ |
|---|---|---|
| $x_0$ | $-1.6$ | 0.2019 |
| $x_1$ | $-1.5$ | 0.2231 |
| $x_2$ | $-1.4$ | 0.2466 |
| $x_3$ | $-1.3$ | 0.2725 |
| $x_4$ | $-1.2$ | 0.3012 |
| $x_5$ | $-1.1$ | 0.3329 |
| $x_6$ | $-1.0$ | 0.3679 |

Simpson's $\frac{1}{3}$ Rule we have.

$$\int_{x_0}^{x_0+6h} f(x)dx = \int_{-1.6}^{-1.0} e^{x} dx$$

$$= \frac{h}{3}\left[(y_0+y_n)+4[y_1+y_3+\cdots+y_{n-1})+2(y_2+y_4+\cdots+y_{n-2})\right]$$

$$= \frac{0.1}{3}\left[(y_0+y_6)+4[y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{0.1}{3}\left[(0.2019+0.3679)+4[0.2231+0.2725+0.3329) + 2[0.2466+0.3012)\right]$$

$$= \frac{1}{30}\left[0.5698+4[0.8285+2[0.5478)\right]$$

$$= 0.1660 \quad \text{(Ans)}$$

Ex(3) Evaluate $\int_0^{\pi/2} \sin x \, dx$ by Simpson's $\frac{1}{3}$ Rule by dividing the range into six equal parts.

Soln:- Given that $y = f(x) = \sin x$.

Now, we divide the interval into six equal parts with

$$h = \frac{\pi/2 - 0}{6} = \frac{\pi}{12}$$

The values of $y = f(x)$ are given below:-

| $x$ | $0$ | $\frac{\pi}{12}$ | $\frac{2\pi}{12}$ | $\frac{3\pi}{12}$ | $4\pi/12$ | $\frac{5\pi}{12}$ | $\frac{6\pi}{12} = \pi/2$ |
|---|---|---|---|---|---|---|---|
| $y = f(x) = \sin x$ | 0 | 0.258 | 0.500 | 0.707 | 0.866 | 0.965 | 1.00 |

By Simpson's $\frac{1}{3}$ Rule, we have

$$\therefore \int_0^{\pi/2} \sin x \, dx = \frac{h}{3}\left[ y_0 + y_6 + 4\,[y_1 + y_3 + y_5) + 2\,[y_2 + y_4] \right]$$

$$= \frac{\pi}{36}\Big[(0+1) + 4\,[0.258 + 0.707 + 0.965) + 2\,[0.500 + 0.866)\Big]$$

$$= 1.004 \quad \text{(Ans)}$$