(a)

Ex :- The values of a function $y = f(x)$ at some points are given in the following table

x :	0	1	2	3	4	5	6
$y = f(x)$:	176	183	197	205	215	223	231

Find the value of the function $y = f(x)$ at $x = 0.2$

Sol :- Since the value $x = 0.2$ at which, we have to make the interpolation lies in the begining of the given data, therefore, we use the Newton's forward interpolation formula:

$$y_n(x) = y_n(x_0 + uh) = y_0 + u\Delta y_0 + \frac{u(u-1)}{\lfloor 2} \Delta^2 y_0$$
$$+ \frac{u(u-1)(u-2)}{\lfloor 3} \Delta^3 y_0 + \cdots \quad - (1)$$

where $u = \dfrac{x - x_0}{h}$.

Here, we take $x_0 = 0$, and $h = 1 - 0 = 1$, $x = 0.2$.

so $u = \dfrac{x - x_0}{h} = \dfrac{0.2 - 0}{1} = 0.2 \quad - - (2)$

<u>forward difference table</u>

x	$y = f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$	$\Delta^6 f(x)$
$x_0 = 0$	$y_0 = 176$						
		Δy_0 7					
$x_1 = 1$	$y_1 = 183$		$\Delta^2 y_0$ 7				
		14		$\Delta^3 y_0$ -13			
$x_2 = 2$	$y_2 = 197$		-6		$\Delta^4 y_0$ 21		
		8		8		$\Delta^5 y_0$ -33	
$x_3 = 3$	$y_3 = 205$		2		12		$\Delta^6 y_0$ 51
		10		-4		18	
$x_4 = 4$	$y_4 = 215$		-2		6		
		8		2			
$x_5 = 5$	$y_5 = 223$		0				
		8					
$x_6 = 6$	$y_6 = 231$						

(b)

Substituting the values of $y_0, \Delta y_0, \Delta^2 y_0, \Delta^3 y_0, \ldots$ etc from difference table in to equation (1), we get

$$y(0.2) = 176 + (0.2)(7) + \frac{(0.2)(0.2-1)}{\lfloor 2} \times (7)$$

$$+ \frac{(0.2)(0.2-1)(0.2-2)}{\lfloor 3} (-13)$$

$$+ \frac{(0.2)(0.2-1)(0.2-2)(0.2-3)}{\lfloor 4} (21)$$

$$+ \frac{(0.2)(0.2-1)(0.2-2)(0.2-3)(0.2-4)}{\lfloor 5} (-33)$$

$$+ \frac{(0.2)(0.2-1)(0.2-2)(0.2-3)[(0.2)-4](0.2-5)}{\lfloor 6} \times (51)$$

$$= 176 + 1.4 + (-0.56) + (-0.624) + (-0.7056) + (-0.8426) + (-1.0418)$$

$$= 173.6259 \quad \text{Ans}$$

EX:- From the data given below, find the number of students whose weight is between 60 and 70

Weight	0-40	40-60	60-80	80-100	100-120
Number of students	250	120	100	70	50

Sol:- Firstly, convert the given frequency distribution in to cummulative distribution.

Weight: (say x:)	40	60	80	100	120
No of students Less Than the above weight (say y:)	250	250+120 =370	370+100 =470	470+70 =540	540+50 =590

(c)

The number of students whose weight lie between 60 and 70 $= y_{70} - y_{60}$

= no of students having the weight up to 70 − no. of students having the weight up to 60

ie y_{60} is known and is given as 370.

To find y_{70} we use the Newton-gregory forward Interpolation formula.

$$y(70) = y_0 + u\Delta y_0 + \frac{u(u-1)}{\lfloor 2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 y_0 + \cdots \quad (1)$$

Forward difference table for given data.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0 = 40$	$y_0 = 250$				
		120			
			−20		
$x_1 = 60$	$y_1 = 370$			−10	
		100			20
			−30		
$x_2 = 80$	$y_2 = 470$			10	
		−70			
			−20		
$x_3 = 100$	$y_3 = 540$				
		50			
$x_4 = 120$	$y_4 = 590$				

here $x = 70$, let $x_0 = 40$

$$\therefore \quad u = \frac{x - x_0}{h} = \frac{70-40}{20} = 1.5$$

$$y(x=70) = 250 + 1.5 \times 120 + \frac{(1.5)\times(1.5-1)}{\lfloor 2}(-20) + \frac{(1.5)\times(1.5-1)(1.5-2)}{\lfloor 3}(-10) + \frac{(1.5)(1.5-1)(1.5-2)(1.5-3)}{\lfloor 4}(20)$$

$\therefore \; y(x=70) = y_{70} = 423.593 = 424$ (approx)

Number of students having the weight between 60 and 70 are

$$y_{70} - y_{60} = 424 - 370 = 54 \quad \text{Ans}$$

Ex:- Using Newton's backward interpolation, formula find the interpolating polynomial that approximates the function given by following table and hence find $f(2.5)$

x:	0	1	2	3
$f(x)$:	1	3	7	13

Sol:- Backward diff. table is

x	$y=f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$
0	1			
		2		
			2	
1	3			0
		4		
			2	
2	7			
		6		
3	13			

here $x_0 = 3$, $y_n = 13$, $\nabla y_n = 6$, $\nabla^2 y_n = 2$, $\nabla^3 y_n = 0$

For $x = x_0 + uh$, $h = 1$

$$u = \frac{x - x_0}{h} = \frac{x-3}{1} = x - 3.$$

(e)

The Newton's backward interpolation formula is

$$y(x) = y_n + u \nabla y_n + \frac{u(u+1)}{\lfloor 2} \nabla^2 y_n + \cdots \quad (1)$$

$$= 13 + (x-13)(6) + \frac{2}{\lfloor 2}(x-3)(x-3+1)$$

$$= x^2 + x + 1 \quad - - - - (2)$$

Eq^n (2) is required interpolating polynomial

put $x = 2.5$ in eq (2), we get

$$y = f(2.5) = (2.5)^2 + 2.5 + 1 = 9.75 \text{ Ans}$$