JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – I Year & II Sem
Subject –Engineering Mathematics-II
Unit – I
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# VISION AND MISSION OF INSTITUTE

## VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

## MISSION OF INSTITUTE

- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the area of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

# CONTENTS (TO BE COVERED)

Eigen values and Eigen vectors

# Characteristic Equation of a Matrix

Let $A$ be a square matrix of order $n$ and $I$ be the unit matrix of order $n$. Then, for any scalar quantity $\lambda$, the matrix $[A-\lambda I]$ is known as characteristic matrix of $A$.

The equation $|A-\lambda I|=0$ is known as characteristic eq$^n$ of $A$.

$$|A-\lambda I| = \begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{n1} & a_{n2} & & a_{nn}-\lambda \end{vmatrix} = 0.$$

# Eigen Values and Vectors

The roots of the characteristic eqn of a square matrix A are known as eigen values.

For every eigen values there exists a eigen vector

Q.1 Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Sol$^n$ The characteristic eq$^n$ of the given matrix A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda(\lambda^2 - 18\lambda + 45) = 0$$

$$\lambda(\lambda-3)(\lambda-15) = 0$$

$\lambda_1 = 0, \lambda_2 = 3, \lambda_3 = 15$ are the eigen values

(i) For $\lambda_1 = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 2 & -4 & 3 \\ -6 & 7 & -4 \\ 8 & -6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + 3R_1$ , $R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 2 & -4 & 3 \\ 0 & -5 & 5 \\ 0 & 10 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 2 & -4 & 3 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 - 4x_2 + 3x_3 = 0$$

$$-5x_2 + 5x_3 = 0$$

Let $x_2 = x_3 = k \Rightarrow x_1 = \frac{k}{2}$

Hence for $\lambda_1 = 0$ the eigen vector is $\left(\frac{k}{2}, k, k\right)$

for $k = 1$ $\left(\frac{1}{2}, 1, 1\right)$

(ii) For $\lambda_2 = 3$

$$[A - 3I]X = 0$$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to \frac{1}{2} R_3$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3 \quad , \quad R_3 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & -2 & 0 \\ 5 & -6 & 2 \\ -6 & 4 & -4 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 5R_1 \quad , \quad R_3 \rightarrow R_3 + 6R_1$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 4 & 2 \\ 0 & -8 & -4 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left. \begin{array}{l} x_1 - 2x_2 = 0 \\ 4x_2 + 2x_3 = 0 \end{array} \right] \quad x_2 = \frac{-x_3}{2} = k$$

$$X_2 = (2k_1, k_1, -2k_1)$$

For $\lambda_2 = 3$ at $k_1 = 1$ $\quad X_2 = (2, 1, -2)$

(iii) $\lambda_3 = 15$

$$[A - 15I]X = 0$$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} -1 & 2 & 6 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - 6R_1, \quad R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} -1 & 2 & 6 \\ 0 & -20 & -40 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 + 2x_2 + 6x_3 = 0$

$-20x_2 - 40x_3 = 0$

$x_2 = -2x_3 = k_2 \quad , x_1 = -k_2$

$x_3 = \left(-k_2, k_2, \frac{-k_2}{2}\right)$ For $\lambda_3 = 15$ $\quad x_3 = \left(-1, 1, \frac{-1}{2}\right)$

Q. 2 Find the eigen values and eigen vectors of

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Sol$^n$ Let $\lambda$ be eigen value of $A$.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (6-\lambda)[(3-\lambda)^2-1]+2[(-2)(3-\lambda)+2]+2[2-2(3-\lambda)]=0$$

$$\lambda^3-12\lambda^2+36\lambda-32=0$$

$$\lambda=2,2,8$$

Case I for $\lambda=2$

Put $\lambda=2$ in given characteristic eq$^n$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & +1 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$4x_1-2x_2+2x_3=0$$

$$-2x_1+x_2-x_3=0$$

$$2x_1 \cancel{+} - x_2+x_3=0$$

This is a set of one equations with three unknowns.

Hence it has an infinite no. of solutions

$$2x_1 - x_2 + x_3 = 0$$

Choosing $x_2 = 0$ , $x_1 = 1$, $x_3 = -2$

$x_3 = 0$ $x_1 = 1$ , $x_2 = 2$

So eigen vector corresponding to

$\lambda = 2$ $\qquad \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$$k_1 \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + k_2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{matrix} k_1 + k_2 \\ 2k_2 \\ -2k_1 \end{matrix}$$

(ii) For $\lambda = 8$ we get

$$\begin{bmatrix} -2 & 2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + 2x_2 + 2x_3 = 0$$
$$-2x_1 - 5x_2 - x_3 = 0$$
$$2x_1 - x_2 - 5x_3 = 0$$

$$\frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{1} = k_3$$

$$X_2 = \begin{bmatrix} 2k_3 \\ -k_3 \\ k_3 \end{bmatrix}$$

For $k_3 = 1$ $\quad X_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

# Refrences


1.Advanced Engineering Mathematics by Prof.ERWIN KREYSZIG (Ch.10,page no.557-580)
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3.Advanced Engineering Mathematics by B.V RAMANA (Ch.20,pageno.20.1.20.5)
4.NPTEL Lectures available on

http://www.infocobuild.com/education/audio-video-courses/mathematics/TransformTechniquesForEngineers-IIT-Madras/lecture-47.html

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# Thank you!