JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

- Year & Sem – Ist Year , Ist Sem
- Subject – Engineering Physics
- Unit – Wave Optics
- Department- Applied Science (Physics)

VISION

To become a renowned institute of outcome based learning and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

MISSION

- Focus on valuation of learning outcomes and motivate students to inculcate research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academia and industry.
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders can emerge in a range of professions.

Syllabus & Course outcomes

- **Syllabus:-**

Wave Optics: Newton's Rings, Michelson's Interferometer, Fraunhoffer Diffraction from a Single Slit. Diffraction grating: Construction, theory and spectrum, Resolving power and Rayleigh criterion for limit of resolution, Resolving power of diffraction grating, X-Ray diffraction and Bragg's Law.

- **Course outcomes :-**

CO1:- Students will be able to explain the basic concepts, theoretical principles and practical applications of interference, diffraction phenomena and their related optical devices in visible range and X-ray diffraction by crystals (i.e., Bragg's law).

CONTENTS

Part :- 1

Lecture Plan

S. No	Topics	Lectures required	Lect. No.
1	Introduction	1	1
2	Newton's Rings:-, Theory, diagram and formation of circular rings.	1	2
3	Newton's Rings:- Mathematical derivation for wavelength of light.	1	3
4	Michelson's interferometer: Construction, working and application..	1	4

S. No	Topics	Lectures required	Lect. No.
5	Fraunhoffer diffraction, Single Slit:- formulation of resultant Intensity.	1	5
6	Diffraction Grating:- theory, construction and spectrum.	1	6
7	Resolving Power & Rayleigh criterion for limit of resolution.	1	7
8	Resolving power of diffraction grating	1	8
9	X-ray diffraction & Bragg’s law.	1	9

Introduction

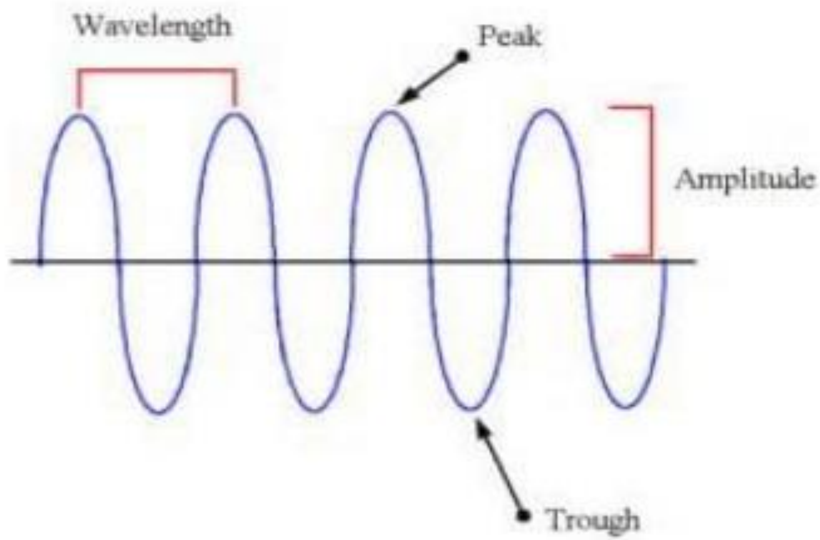
- Physics is the branch of science that deals with the nature and natural phenomena . eg. Formations of days and night , formation of seasons .....
- Types : Quantum Physics – Sir Isaac Newton

  Modern Physics -- Albert Einstein

  Optics is the study of light and its associated phenomenon like interference, diffraction and polarization etc.
- Light is an electromagnetic wave radiation(strong evidence of polarization).
- Study of light having two approaches:
- 1. Wave approach.
- 2. Particle approach (Photon concept of light)
- Using wave approach :Interference, Diffraction and polarization phenomena explained.
- Using particle approach :Photoelectric effect, Compton effect, Raman effect, LASERS etc. explained.

What is Light?

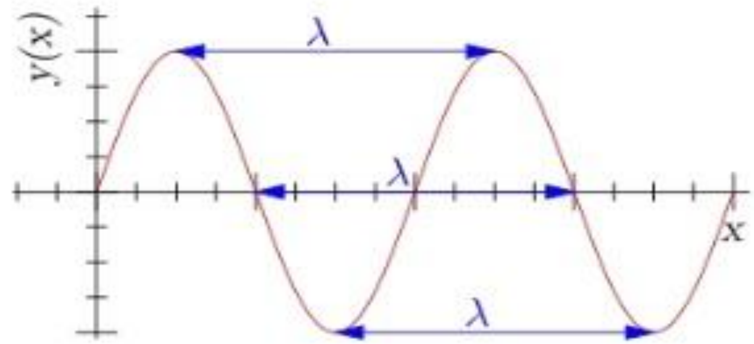
Light is an electromagnetic radiation refers to visible regions of electromagnetic spectrum corresponding to the wavelength range of 400nm to 760nm which has transverse vibrations.



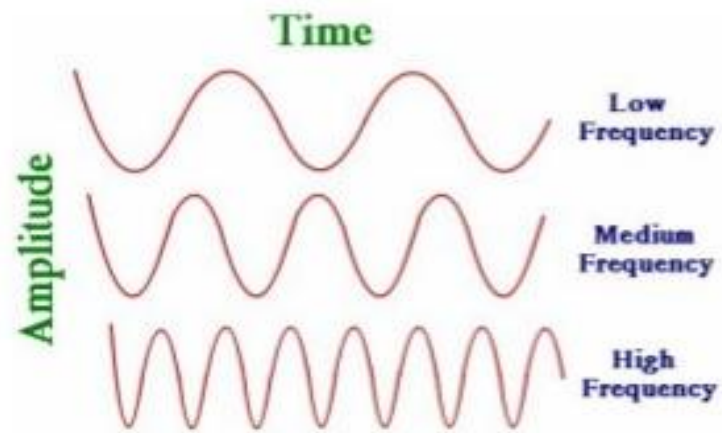
Wave

General Definitions

The Wavelength of a sin wave, λ, can be measured between any two points with the same phase, such as between crests, or troughs, as shown.



The frequency, f, of a wave is the number of waves passing a point in a certain time. We normally use a time of one second, so this gives frequency the unit hertz (Hz), since one hertz is equal to one wave per second.



Path Difference and Phase Difference

At time $t = 0$

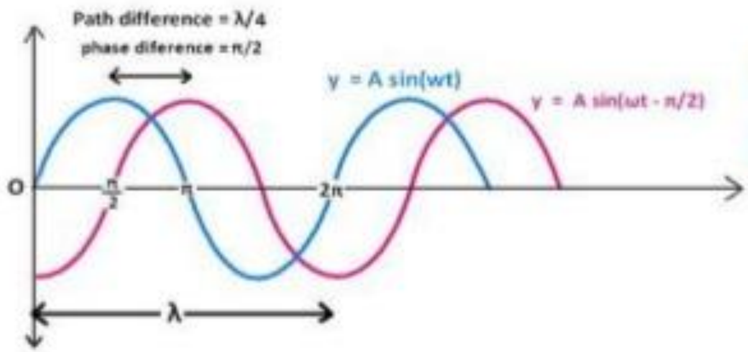
Blue wave displacement = 0

Red wave displacement = – A

At $\pi/2$

Blue wave has maximum displacement = + A

Red wave displacement = 0

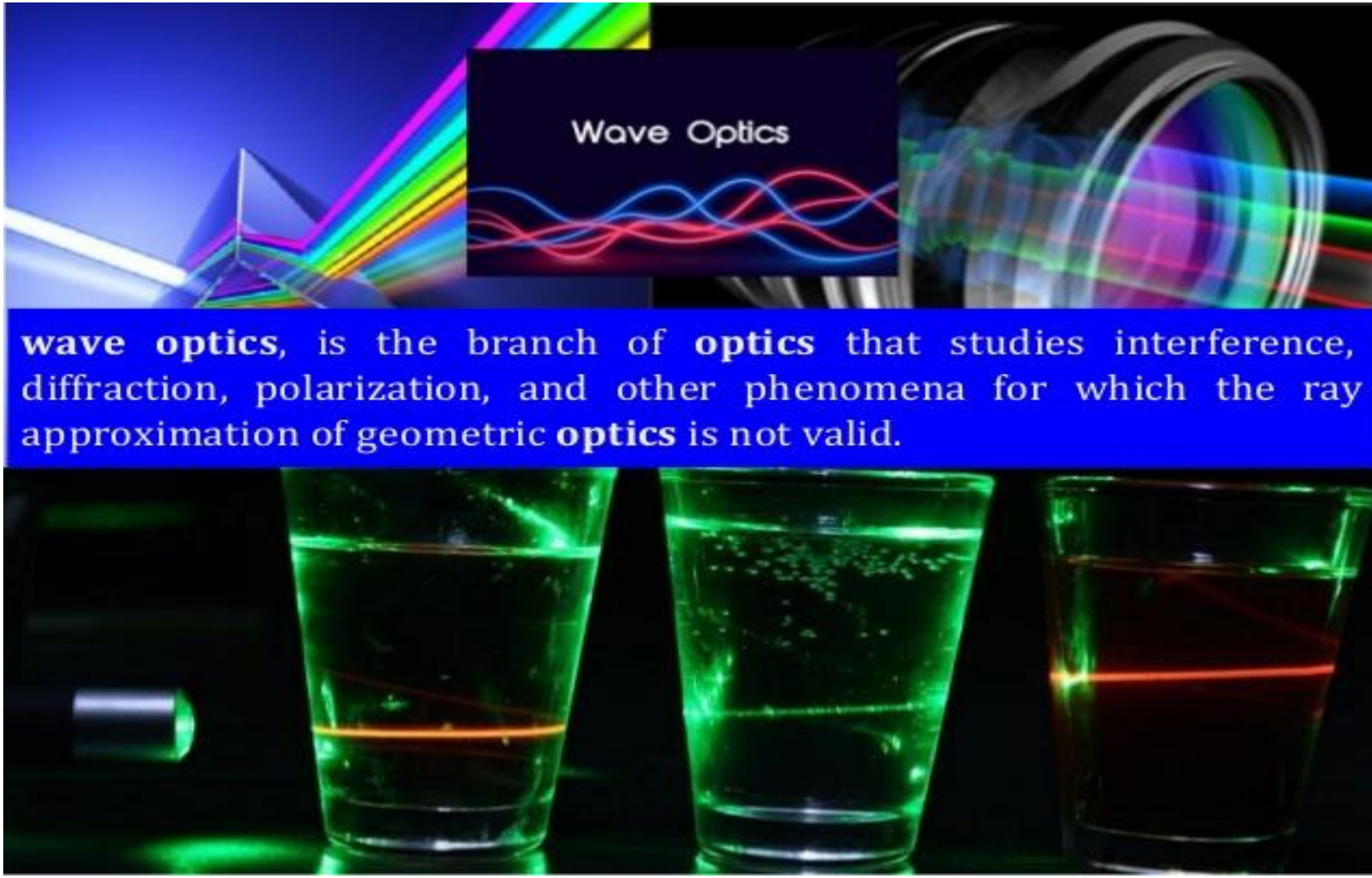


Blue wave is leading by a phase difference of $\pi/2$ and path difference of $\lambda/4$

One oscillation is completed in 2π radians which is equivalent to wavelength λ

(Path difference of one wavelength (λ) is equal to phase difference of 2π radians)

Wave optics


Wave Optics
wave optics, is the branch of optics that studies interference, diffraction, polarization, and other phenomena for which the ray approximation of geometric optics is not valid.

Interference of Light

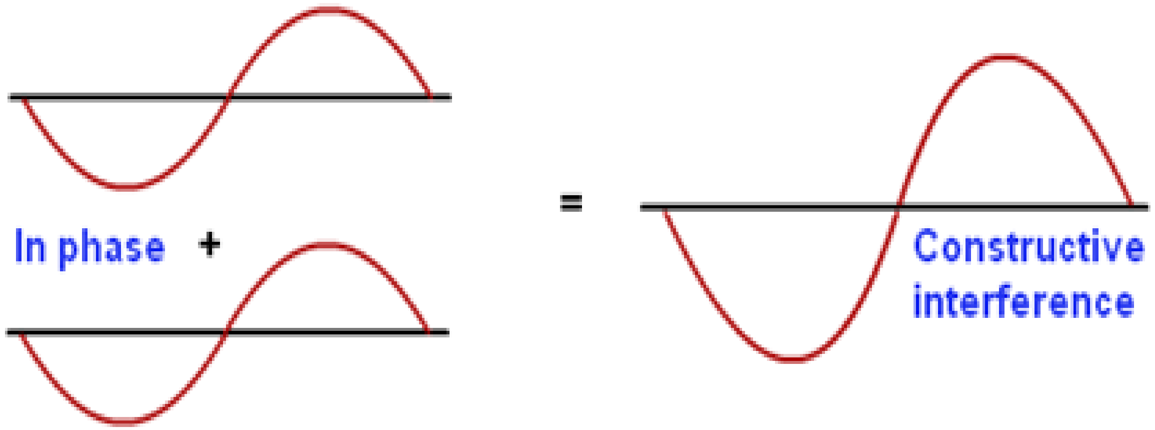
- When a single wave from a single source of light travels in a medium the intensity of light is distributed uniformly in space. But when the two or more waves of same frequency, same wavelength, nearly same amplitude and having a constant or zero phase difference between them , the intensity of light is not distributed uniformly in space . This non uniform distribution of light intensity due to superposition of two or more waves is called interference of light.

- At some points the intensity is found maximum and is called constructive interference and at some points the intensity is found minimum and it is called destructive interference.

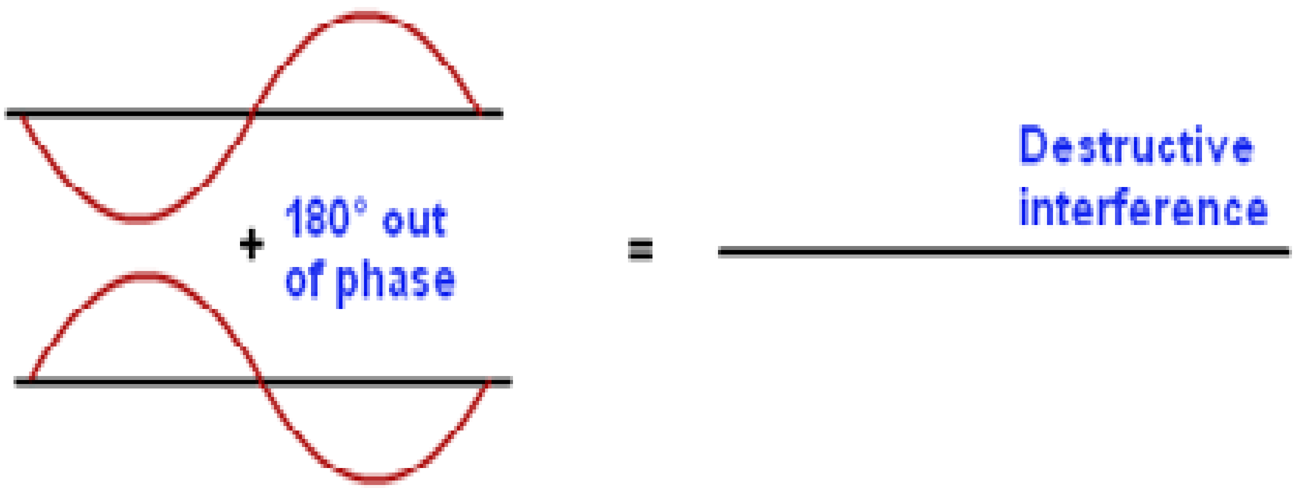
Types of Interference

Interference of light is of two type:-

(1) **Constructive Interference**:- when two waves superimpose in same phase and phase difference between them is zero or an integral multiple of 2π, the amplitude and intensity of the resultant light are maximum. This type of interference is called constructive interference.



(2) **Destructive Interference**:- when two waves superimpose in opposite phase and phase difference between them is 180^0 or odd multiple of π, the amplitude and intensity of the resultant light are minimum. This type of interference is called destructive interference.



Coherent Sources

Two source of light are said to be coherent if they emit light of same frequency, same amplitude and with constant phase difference between the light emitted.

The two coherent sources can be created by:-

(1) By the Division of Amplitude

(2) By the Division of wavefront

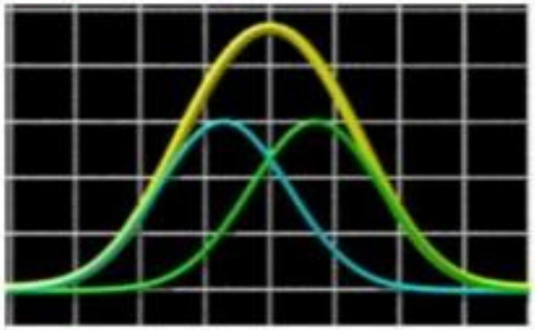
(1) **By the Division of Amplitude**:- In this method amplitude of incident waves is divided in two or more parts by partial reflection or refraction . These two wave of light beam act as a coherent sources. These two beams. When reunite, produce interference fringes. This method can be used in Newton's ring experiment and Michelson Interferometer experiment.

(2) **By the division of wavefront**:- In this method, the wavefront from a single monochromatic source is divided either into two parts. This can be achieved by the phenomenon of reflection, refraction or differection. These two part of incident wavefront can be treated as the wavefronts originating by two vertual coherent sources. In Young's double slit experiment and Fresnel's biprism experiments, we use this method to produce virtual coherent sources.

Principle of Superposition

"Whenever two or more waves superimpose in a medium, the total displacement at any point is equal to the vector sum of individual displacement of waves at that point"

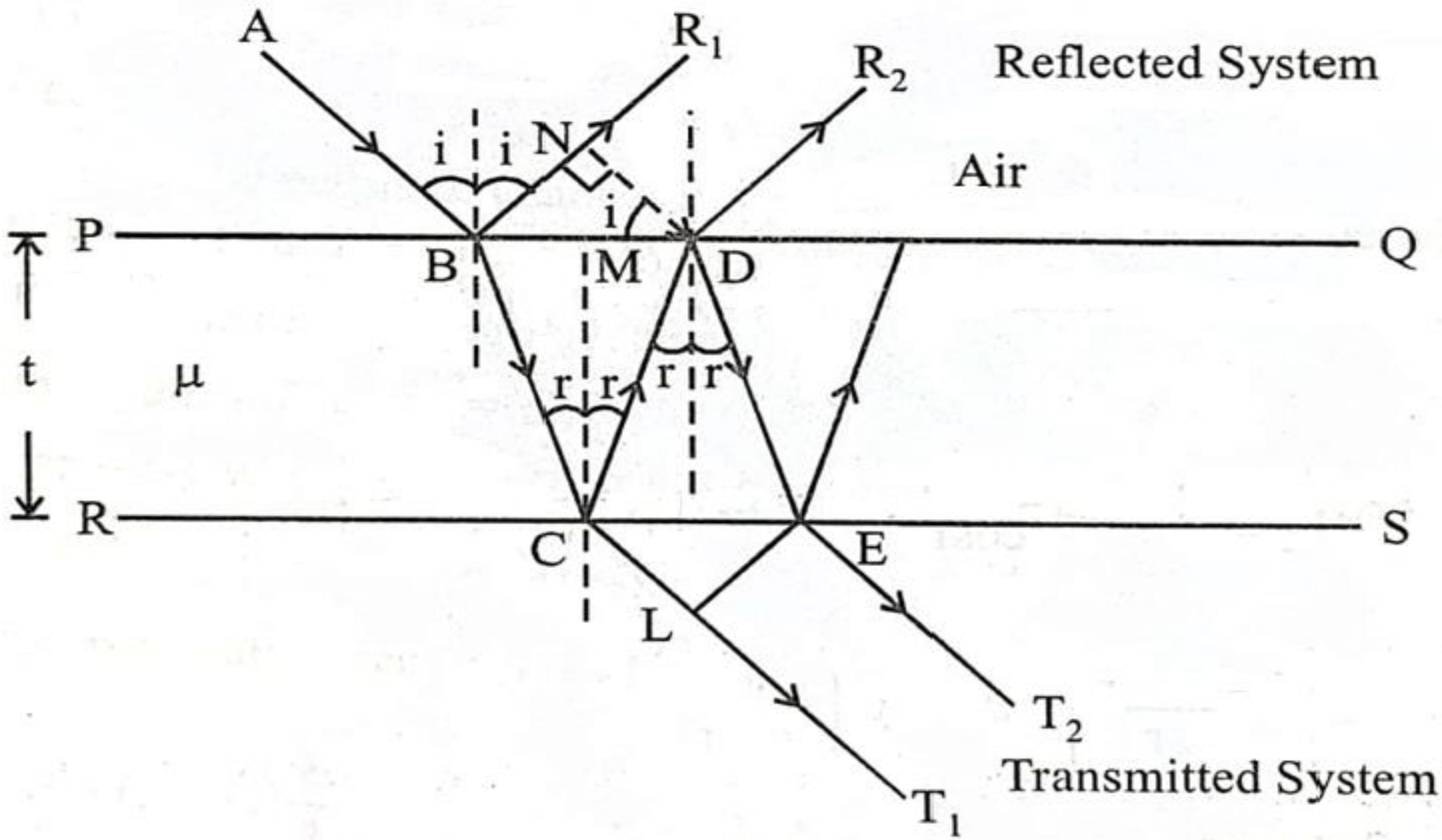
$y = y_1 + y_2 = a \sin(\omega t) + a \sin(\omega t + \phi)$



If Y_1, Y_2, Y_3…are different displacement vector due to the waves 1,2,3 …acting separately then according to the principle of superposition the resultant displacement is given by

$Y = Y_1 + Y_2 + Y_3 + \ldots\ldots$

Interference in thin film



- Let us consider a thin film of thickness t and refractive index μ.
- A ray of monochromatic light AB is incident with incident angle i on upper surface PQ at point B.
- At point B the ray is divided into two parts, one is partially reflected along BR_1 and the other partially refracted along BC.
- At point C again it is divided into two parts, one is transmitted along CT_1 and other is reflected along CD.
- Similarly reflection and refraction take place at D, E etc.
- The set of parallel rays BR_1 and DR_2 and transmitted rays CT_1 ET_2 are obtained which produce interference in reflected light and transmitted light respectively.

Draw DN perpendicular to reflected ray BR_1. Path difference between the waves from N and D is zero. So, the path difference between BR_1 and DR_2 is

$$\Delta = \text{path BCD in medium} - \text{path BN in air}$$

$$= \mu\,(BC + CD) - BN$$

$$= 2\mu(BC) - BN \qquad (\because BC = CD) \qquad \text{....... 1}$$

From fig. — $BC = CM \sec r = \dfrac{t}{\cos r}$ 2

and $BN = BD \sin i$

$$= 2\,BM \sin i \qquad \left(\because BM = \frac{1}{2}BD\right) \qquad \text{...... 3}$$

and $BM = CM \tan r = t \tan r$ 4

Putting value of BM from equation 4 into equation 3, we get

$$BN = 2t \tan r \sin i$$

$$= 2t \frac{\sin r}{\cos r}\left(\frac{\sin i}{\sin r}\right) \cdot \sin r \qquad \left[\mu - \frac{\sin i}{\sin r}\right]$$

$$BN = 2\mu t \cdot \frac{\sin^2 r}{\cos r} \qquad \text{....... } 5$$

Putting value of BN from equation 5 and BC from equation 2 into equation 1 we get

$$\Delta = 2\mu\left(\frac{t}{\cos r}\right) - 2\mu t \frac{\sin^2 r}{\cos r}; \quad \Delta = 2\mu t\left[\frac{1}{\cos r} - \frac{\sin^2 r}{\cos r}\right]$$

$$\Rightarrow \quad \Delta = 2\mu t\left[\frac{1-\sin^2 r}{\cos r}\right]; \quad \Delta = 2\mu t\left(\frac{\cos^2 r}{\cos r}\right)$$

$$\Delta = 2\mu t \cos r$$

- As ray BR_1 is reflected from the surface of an optically denser medium, a phase change of πoccurs.
- But, DR_2 is reflected at the surface of a rarer medium, so there is no phase change.

Hence the effective path difference is:-

$$\Delta = 2\mu t \cos r + \frac{\lambda}{2}$$

2. Interference in Transmitted Light

Similarly, the path difference between the transmitted waves CT_1 and ET_2 is can be computed as.

$$\Delta' = \text{path CDE in medium} - \text{path CL in air}$$

$$\Delta' = 2\mu t \cos r$$

In this case there will be no phase change due to reflection at C or at D because in either case the light is travelling from denser to rarer medium.

(i) Condition for Maxima (Bright fringe)

For maxima, path difference $= n\lambda$

$\Rightarrow$ $2\mu t \cos r = n\lambda$ where $n = 1, 2, 3, \ldots$

(ii) Condition for minima (Dark fringe)

For minima, path difference $= (2n+1)\dfrac{\lambda}{2}$

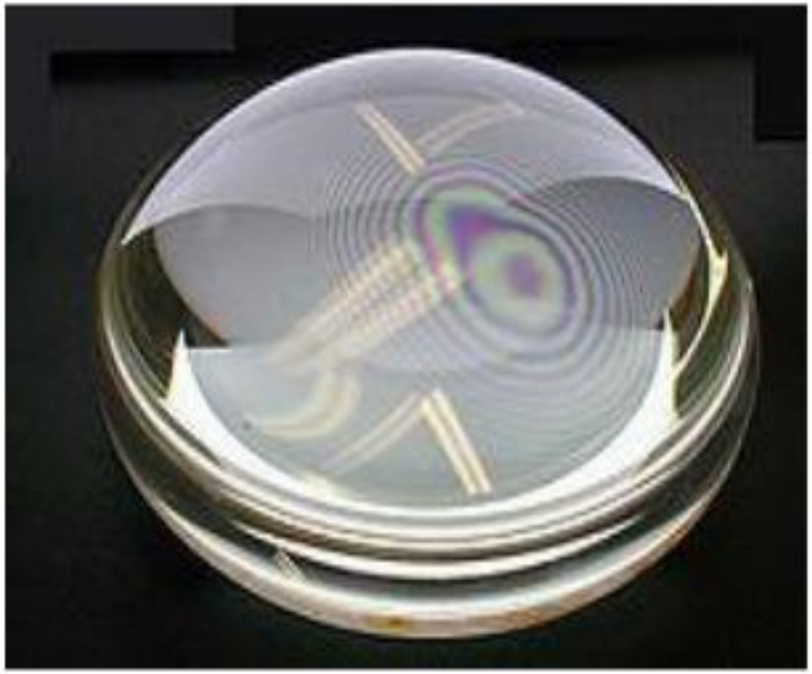
$\Rightarrow$ $2\mu t \cos r = (2n+1)\dfrac{\lambda}{2}$ where $n = 0, 1, 2, 3, \ldots$

Newton's Rings

- Optical device by which a series of alternate dark & bright circular rings are obtained though interference of light reflected from top & bottom layers of wedged shaped very thin film of air or some other transparent medium enclosed between glass plate and lens. This localized phenomenon is observed by travelling microscope.
- Also known as fringes of equal thickness (Fizeau fringes)
- Newton's Rings are very useful to check the plane ness of the glass surface in glass industries.

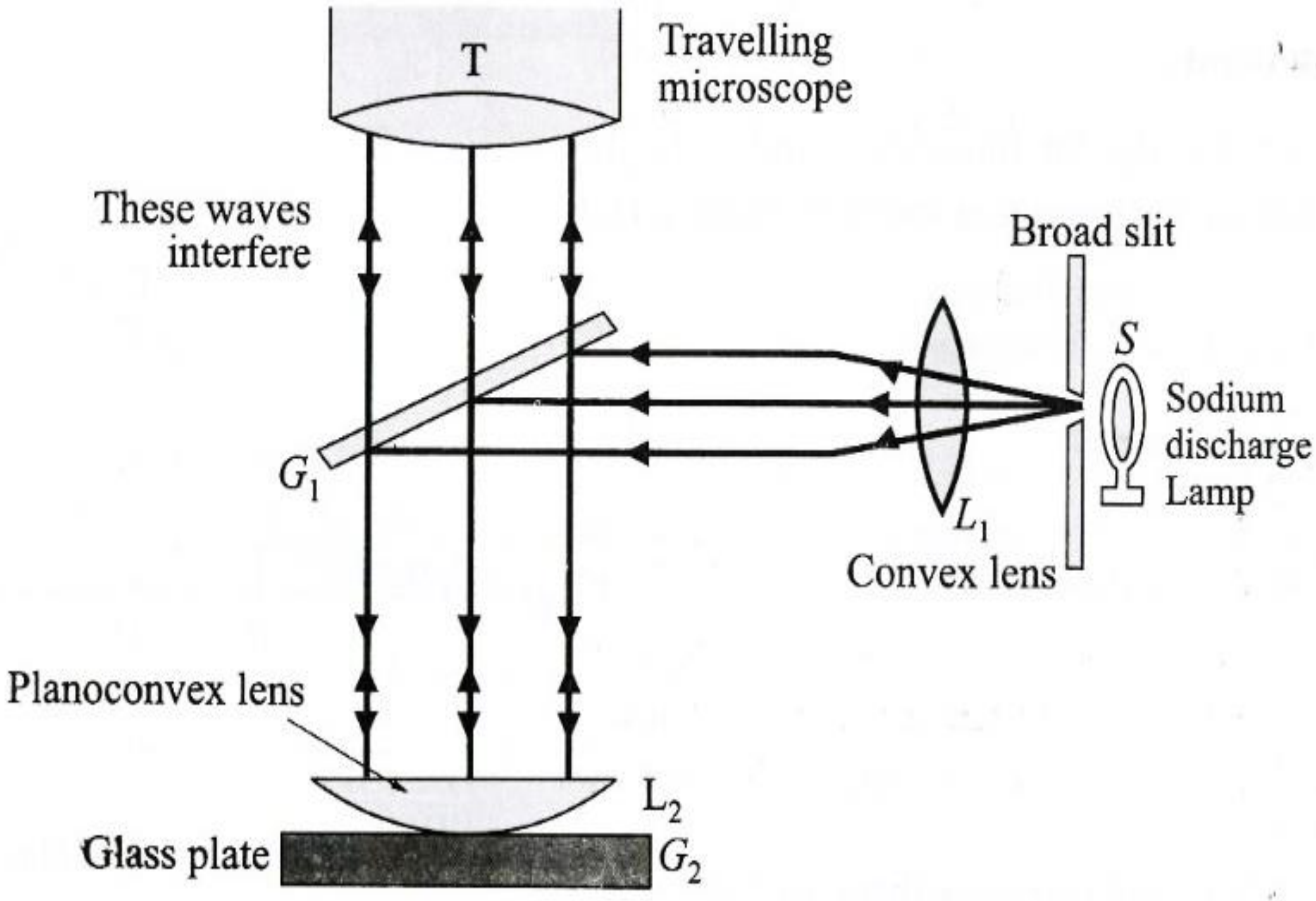
NEWTONS RING

Newton's rings is a phenomenon in which an interference pattern is created by the reflection of light between two surfaces—a spherical surface and an adjacent touching flat surface. It is named for Isaac Newton, who first studied the effect in 1717. When viewed with monochromatic light, Newton's rings appear as a series of concentric, alternating bright and dark rings centered at the point of contact between the two surfaces.



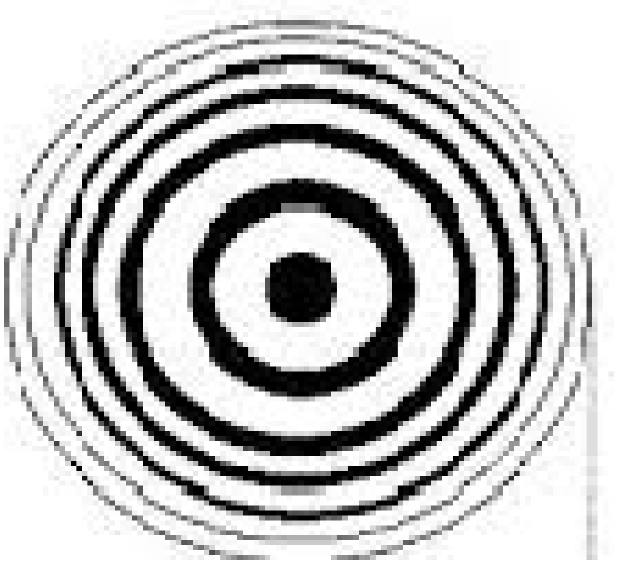
Newton's rings seen in two plano-convex lenses with their flat surfaces in contact. One surface is slightly convex, creating the rings. In white light, the rings are rainbow-colored, because the different wavelengths of each color interfere at different locations.

Experimental Setup of Newton's Rings



Newton’s Rings

Newton’s rings is a phenomenon in which an interference pattern is created by the Newton's rings reflection of light between two surfaces; a spherical surface and an adjacent touching flat surface. It is named after Isaac Newton, who investigate the effect in his 1704 treatise Optics.



Principle of Newton's Rings

- It works on the principle of by the way of division of Amplitude .
- In this the coming amplitude of light wave is divided into two or more parts by partial reflection or refraction and there by giving rise to two or more coherent beams .
- These beams superimpose to produce interference effects.

Working of Newton's Rings

- When a plano-convex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate , an air film is formed between the convex lens and a glass plate.

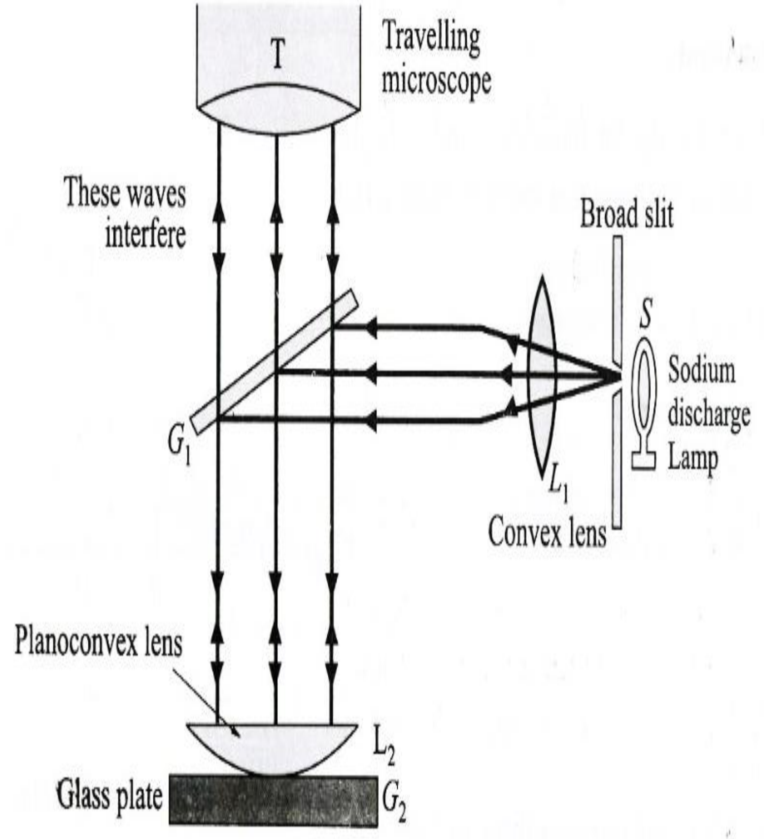
- The thickness of the air film at the point of contact is zero.

- When a sodium light is incident on such a system , light waves reflect from the top and bottom surfaces of the air film and when this air film is viewed in reflected light , alternate bright and dark rings are seen around the point of contact. These circular rings are called Newton's rings.

- These rings are circular as the locus of points of equal thickness of air film is a circle Newton's ring are the examples of interferences fringes of equal thickness. Since the thickness of air film remains constant along a circle with its centre at the point of contact , fringes are in the form of concentric circles.
- These rings were first discovered by Newton, so they are called Newton's rings.

Experimental Arrangement

1. A plano-convex lens P of large focal length is placed on a plane glass plate O.
2. S is a Monochromatic light Source.
3. L is a another lens, placed in front of source. It is converting the light rays into horizontal plane.
4. Glass plate G is inclined at 45^0 to the horizontal plane.
5. "M" is a travelling Microscope.

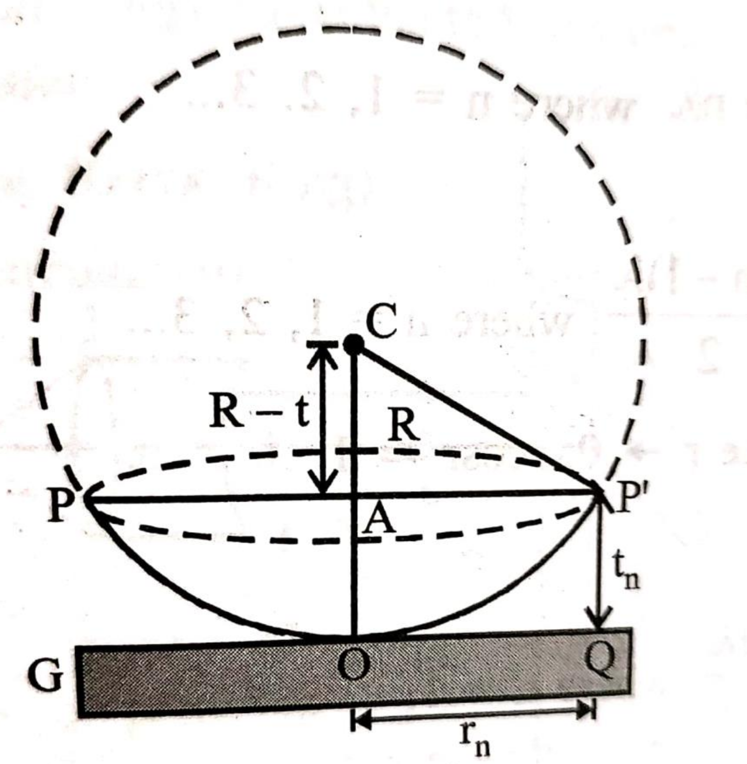


Determine the thickness of air film

1. Let R be the radius of curvature of the lens with its centre C suppose.

2. Let t_n be the thickness of air film at a point P' .

3. AP' is the radius of Newton's ring at t_n thickness of air film is r_n .



By Pythagoras theorem-

$$R^2 = (CA)^2 + r_n^2$$

but $CA = (R - t_n)$

$$R^2 = (R - t_n)^2 + r_n^2$$

$$R^2 = R^2 + t_n^2 - 2R t_n + r_n^2$$

$$t_n^2 - 2R t_n + r_n^2 = 0$$

$$r_n^2 = t_n^2 - 2R t_n$$

Since $t_n << R$ So $t_n^2 <<< 2R t_n$

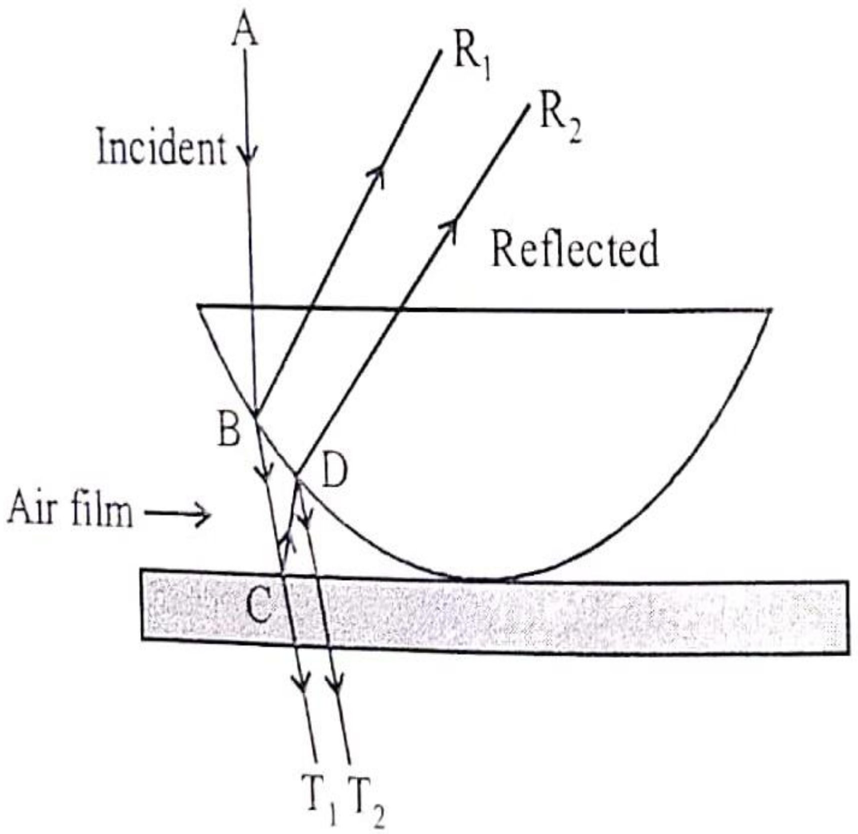
$$\mathbf{t_n = r_n^2 / 2R}$$

Which gives the thickness of air film corresponding to the Newton's ring of radius r_n

Newton's Ring in reflected Light

In reflected system, the path difference between the reflected rays is given by-

$$\Delta = 2 \mu t \cos r + \lambda/2$$



Condition for Maxima (Bright Fringes)

For n^{th} order bright fringes (Maxima)

$$\Delta = 2\ \mu t_n \cos r + \lambda/2 = n\lambda \qquad \text{where } n = 1,2,3\ldots$$

$$2\ \mu t_n \cos r = (2n - 1)\,\lambda / 2 \qquad \text{where } n = 1,2,3\ldots$$

for normal incidence $r \quad 0$; $\cos r \approx 1$; for air $\mu = 1$

$$2\, t_n = (2n - 1)\,\lambda / 2$$

Bright ring of any particular order (n) will occur at a particular thickness (t_n), which remains constant along a circle with its centre at the point by contact, so the rings are circular in shape.

Substitute $t_n = \frac{r_n^2}{2R}$ in equation

$$2\mu \frac{r_n^2}{2R} = \frac{(2n-1)\lambda}{2} \quad \text{where } n = 1,\ 2,\ 3\ \ldots$$

$$r_n = \left[\frac{(2n-1)\lambda R}{2\mu}\right]^{\frac{1}{2}} \quad \text{where } n = 1,\ 2,\ 3\ \ldots$$

r_n^2 is the radius of n^{th} bright fringe

$$\text{For air } \mu = 1 \quad r_n = \left[\frac{(2n-1)\lambda R}{2}\right]^{\frac{1}{2}} \quad \text{where } n = 1,\ 2,\ 3\ \ldots$$

Diameter of n^{th} bright fringe

$$D_n = 2r_n = [2(2n-1)\lambda R]^{1/2} \quad \text{where } n = 1,\ 2,\ 3\ \ldots$$

$$\Rightarrow \boxed{D_n \alpha \sqrt{2n-1}}$$

When $n = 1, \quad D_1 = \sqrt{1}$

$n = 2, \quad D_2 = \sqrt{3}$

$n = 3, \quad D_3 = \sqrt{5}$ …. So on

Therefore

$$D_1 : D_2 : D_3 = \sqrt{1} : \sqrt{3} : \sqrt{5} \ \ldots.$$

So the diameter of bright rings in reflected system, are proportional to square root of odd natural numbers.

Condition for Minima (Dark Fringes)

For n^{th} order dark fringes (Minima)

$$\Delta = 2\mu t_n \cos r + \lambda/2 = (2n-1)\lambda/2 \quad \text{where } n = 1,2,3\ldots$$

$$2\mu t_n \cos r = n\lambda \quad \text{where } n = 1,2,3\ldots$$

for normal incidence $r = 0$; $\cos r \approx 1$; for air $\mu = 1$

$$2t_n = (2n-1)\lambda/2$$

Dark ring of any particular order (n) will occur at a particular thickness (t_n), which remains constant along a circle with its centre at the point by contact, so the rings are circular in shape.

Substitute $t_n = \frac{r_n^2}{2R}$ in equation

$$2\mu \frac{r_n'^2}{2R} = n\lambda \quad \text{where } n = 0, 1, 2, 3, \ldots$$

$$r_n' = \left[\frac{n\lambda R}{\mu}\right]^{1/2} \quad \text{where } n = 0, 1, 2, 3, \ldots$$

For air $\mu = 1$

$$r'_n = [n\lambda R]^{1/2} \quad \text{where } n = 0, 1, 2, 3, \ldots$$

At point of contact between lens and glass plate $t_n = 0$

and $t_n = \frac{r_n'^2}{2R}$

so $r_n'^2 = 0$ (point fringe)

this is satisfied for $r'_n = [n\lambda R]^{1/2}$ for $n = 0$

At point of contact fringes is dark and point fringe and is center of concentric ring. Diameter of n^{th} dark fringe is

$$D'_n \ \alpha \ 2r'_n = [4n\lambda R]^{1/2}$$

$$D'_n \ \alpha \ \sqrt{n}$$

When $n = 1$, $D_1 = \sqrt{1}$

$n = 2$, $D_2 = \sqrt{2}$

$n = 3$, $D_3 = \sqrt{3}$ …. So on

Therefore

$$D_1 : D_2 : D_3 = \sqrt{1} : \sqrt{2} : \sqrt{3} \; ....$$

So the diameter of dark rings in reflected system, are proportional to square root of natural numbers.

Newton's Ring in Transmitted Light

When a light ray AB falls normally on the glass plate at C it is partly transmitted along CT_1 and partly reflected along CD. Similarly reflection and refraction occurs at D. Thus we get a set of parallel transmitted ray T_1 and T_2 . The path difference between these two rays for normal incidence is nearly-

$$\Delta = 2\mu t \quad (\text{for air } \mu = 1)$$

$$\Delta = 2t$$

where t is thickness of air film at a point C

1. Condition for Maxima (Bright Ring)

$\Delta = 2t = n\lambda$ where n = 0, 1, 2, 3, . . .

From equation (1.74), we have

$$t_n = \frac{r_n^2}{2R}$$

So the radius of n^{th} bright ring is

$$r_n = \sqrt{2Rt_n}$$

$$\Rightarrow \quad r_n = \sqrt{nR\lambda}$$

Thus, the diameter of the n^{th} bright ring is

$$D_n = 2r_n = 2\sqrt{nR\lambda} = \sqrt{4nR\lambda}$$

$$\Rightarrow \quad \boxed{D_n \propto \sqrt{n}}$$

2. Condition for Minima (Dark Rings)

$$\Delta = 2t = (2n-1)\frac{\lambda}{2} \quad \text{where } n = 1, 2, 3.......$$

Again, we have $t_n = \frac{r_n^2}{2R}$ From equation

So, the radius of ^{nth} dark ring is

$$r'_n = \sqrt{2t_nR}$$

$$r'_n = \sqrt{(2n-1)\frac{\lambda}{2}\cdot R} \quad \text{where } n = 1, 2, 3.....$$

and the diameter of n^{th} dark ring is

$$D'_n = 2r_n = 2\sqrt{\frac{(2n-1)\lambda R}{2}}$$

$$\Rightarrow \quad D'_n = \sqrt{2(2n-1)\lambda R}$$

The condition of maxima and minima in the reflected light are just reverse to those in transmitted light.

Applications of Newton's Ring

Determination of wavelength of Sodium Light:

The diameter of n^{th} order ring is given by-

$$D_n = [4n\lambda R]^{1/2}$$

$$D_n^2 = 4n\lambda R \quad \ldots\ldots\ldots\ldots\ldots 1$$

Similarly diameter of $(n+p)^{th}$ order ring is given by-

$$D_{n+p}^2 = 4(n+p)\lambda R \quad \ldots\ldots\ldots\ldots\ldots 2$$

By subtraction of equation 1 from equation 2, we get-

$$D_{n+p}^2 - D_n^2 = 4(n+p)\lambda R - 4n\lambda R$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4\,p\,R}$$

Different Cases of Newton's rings :-

1 **If lower plane glass plate $\Rightarrow$ plane mirror**

– Uniform illumination (No contrast between dark and bright ring)

2 **If Na discharge lamp $\Rightarrow$ Hg discharge lamp**

– Poor contract, Coloured fringes observed in place of yellow colour Tringes.

3 **If Air film $\Rightarrow$ Liquid film**

– Shrinking (or contraction) in diameter of ring takes place.

4 **If dust particles are present between lense & plate.**

– Central fringe becomes bright.

5 **If glass plate put on plano convex lense.**

– No fringe formation – No air film.

6 If plane glass plate is rough in place of smooth plate.

– Distorted shaped rings formation (not perfectly circular rings, not see equidistant rings.)

7 If extended light source ⇒ Narrow light source.

– Full range N' rings formation ⇒ short range formation.

8 If distance between lense & plate becomes large.

– Thick air film, fringes disappearance occur.

9 If plano convex lense of smaller radius of curvature.

– Reduce the radius of Newton rings.

Numerical Problems

1. In Newton's ring experiment the diameter of n^{th} and $(n+14)^{th}$ ring are 4.2 mm and 7 mm. If the radius of curvature of lens is 1 m then find the wavelength of light used. (Ans. 5600Å)

2. Newton's rings are observed in reflected light of wavelength 5.9×10^{-5} cm. The diameter of the 10^{th} dark ring is 0.50 cm. Find the radius of curvature of lens & thickness of air film at the ring.

   (Ans. R=106 cm. & t= 3×10^{-4} cm)

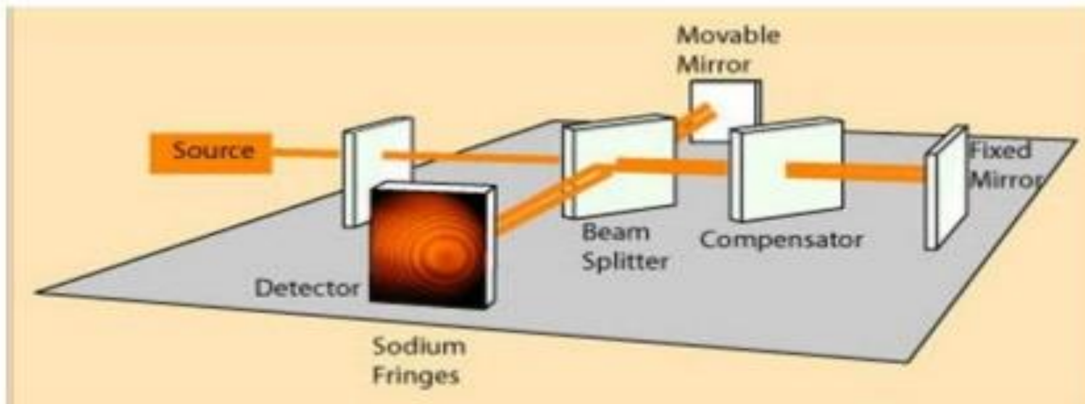
1. In Newton's ring experiment the diameter of 15^{th} ring and 5^{th} ring are .590 cm and 0336 cm. If the radius of curvature of lens is 100 cm then find the wavelength of light used.

   (Ans. 5880Å)

MICHELSON'S INTERFERROMETER

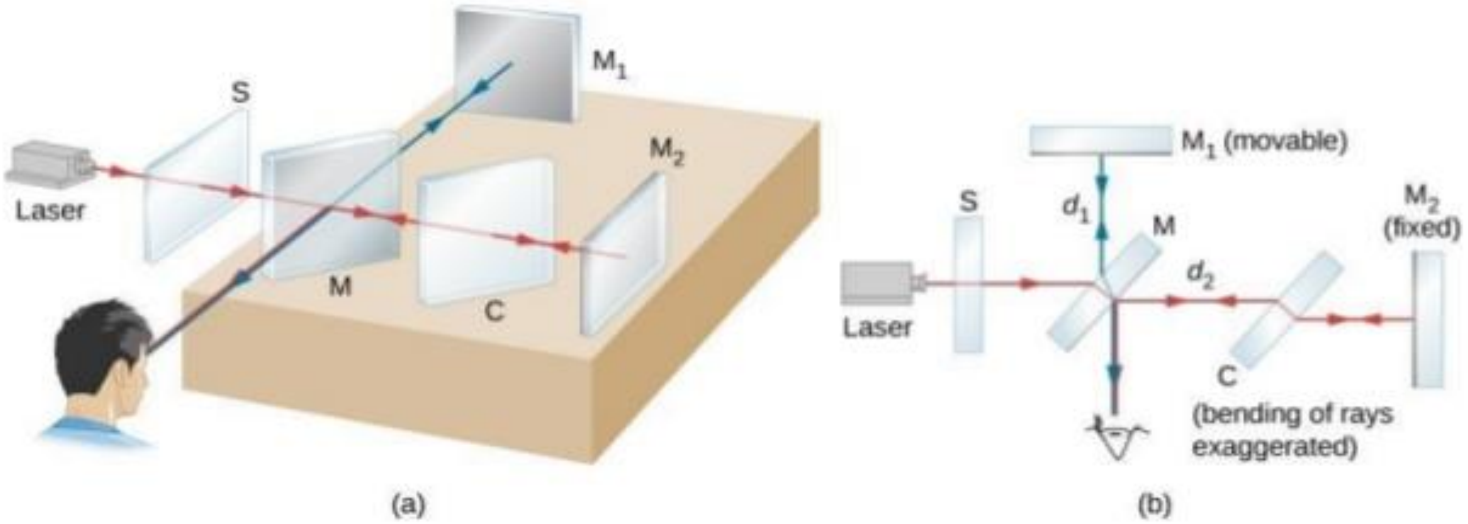
Albert Abraham Michelson
(1852-1931)

The Michelson interferometer is a common configuration for optical interferometer and was invented by Albert Abraham Michelson in 1887. Using a beam splitter, a light source is split into two arms.

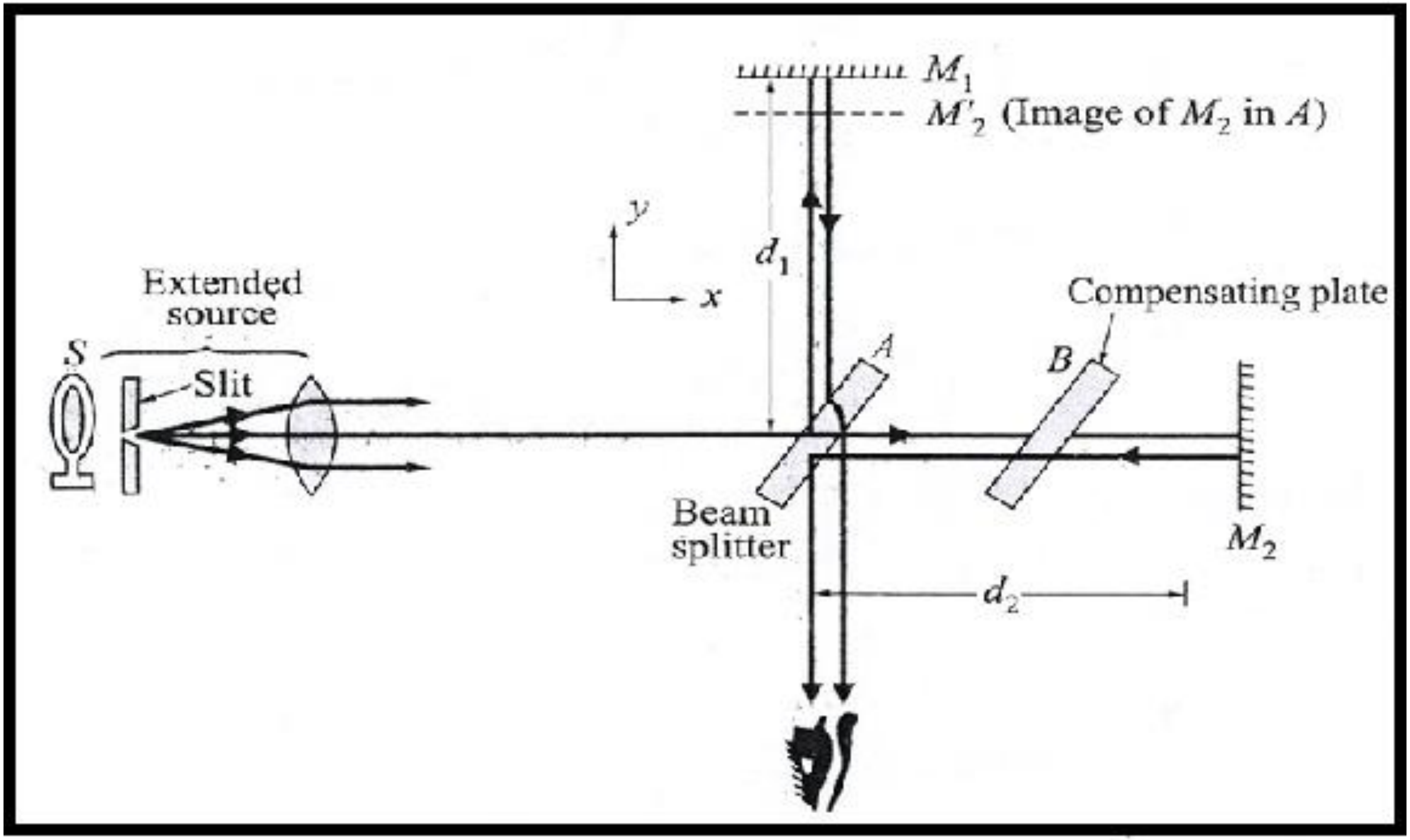


Experimental set up

Principle:- The MI works on the principle of division of amplitude. When the incident beam of light falls on a beam splitter which divided light wave in two part in different directions. These two light beams after traveling different optical paths, are superimposed to each other and due to superposition interferences fringes formed.



Construction and Working



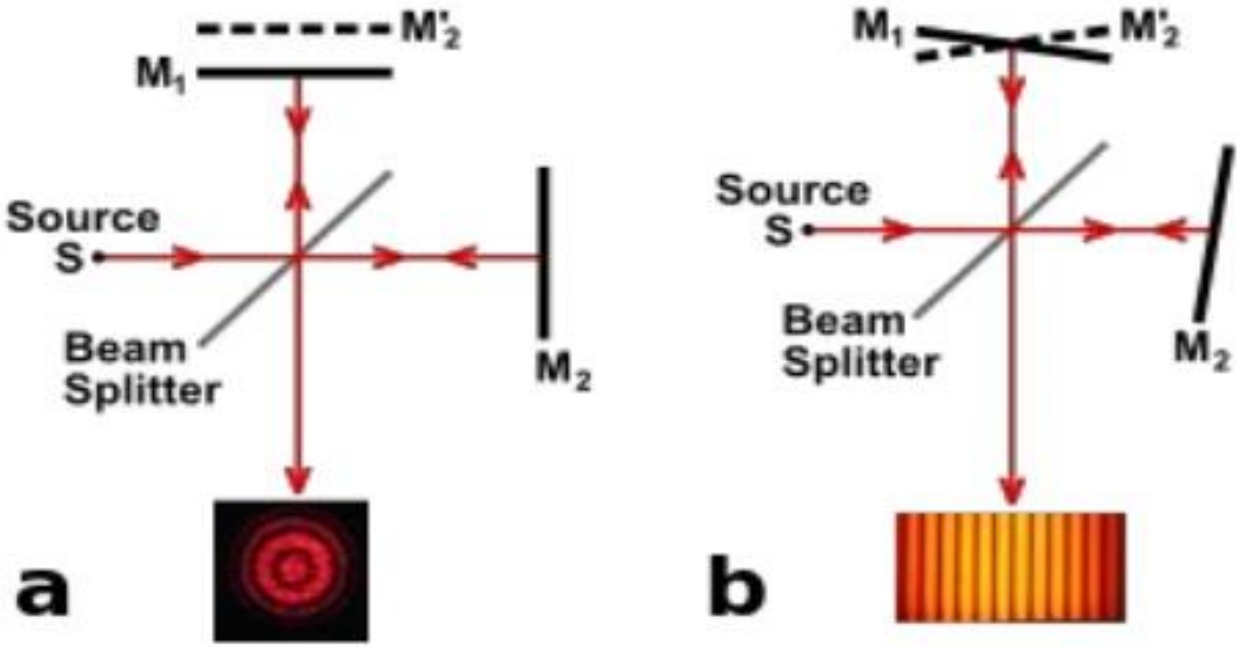
Construction:- It consists of two highly polished plane mirror M_1 and M_2, with two optically plane glass plate G_1 and G_2 which are of same material and same thickness. The mirror M_1 and M_2 are adjusted in such a way that they are mutually perpendicular to each other. The plate G_1 and G_2 are exactly parallel to each other and placed at 45° to mirror M_1 and M_2. Plate G_1 is half silvered from its back while G_2 is plane and act as compensating plate. Plate G_1 is known as beam-splitter plate.

The mirror M_2 with screw on its back can slightly titled about vertical and horizontal direction to make it exactly perpendicular to mirror M_1. The mirror M_1 can be moved forward or backward with the help of micrometer screw and this movement can be measured very accurately.

Working: Light from a broad source is made paralied by using a convex lens L. Light from lens L is made to fall on glass plate G_1 which is half silver polished from its back. This plate divides the incident beam into two light rays by the partial reflection and partial transmission, known as Beam splitter plate. The reflected ray travels towards mirror M_1 and transmitted ray towards mirror M_2. These rays after reflection from their respective mirrors meet again at 'O' and superpose to each other to produce interference fringes. This firings pattern is observed by using telescope.

Functioning of Compensating Plate: In absence of plate G_2 the reflected ray passes the plate G_1 twice, whereas the transmitted ray does not passes even once. Therefore, the optical paths of the two rays are not equal. To equalize this path the plate G_2 which is exactly same as the plate G_1 is introduced in path of the ray proceeding towards mirror M_2 that is why this plate is called compensating plate because it compensate the additional path difference.

Formation of fringes in MI



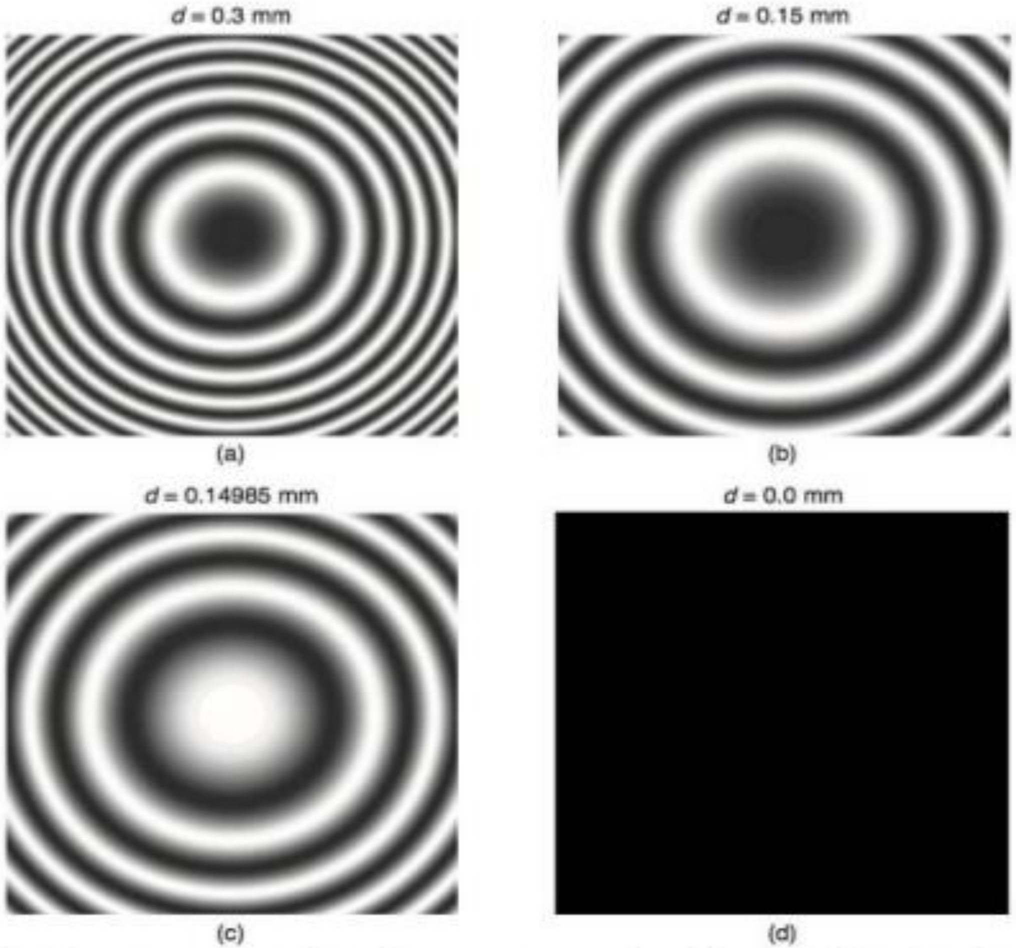


Fig. 15.36 Computer-generated interference pattern produced by a Michelson interferometer.

Formation of Circular fringes

The shape of fringes in MI depends on inclination of mirror M_1 and M_2. Circular fringes are produced with monochromatic light, if the mirror M_1 and M_2 are perfectly perpendicular to each other. The virtual image of mirror M_2 and the mirror M_1 must be parallel. Therefore it is assumed that an imaginary air film is formed in between mirror M_1 and virtual image mirror M'_2. Therefore, the interference pattern will be obtained due to imaginary air film enclosed between M_1 andM'_2.

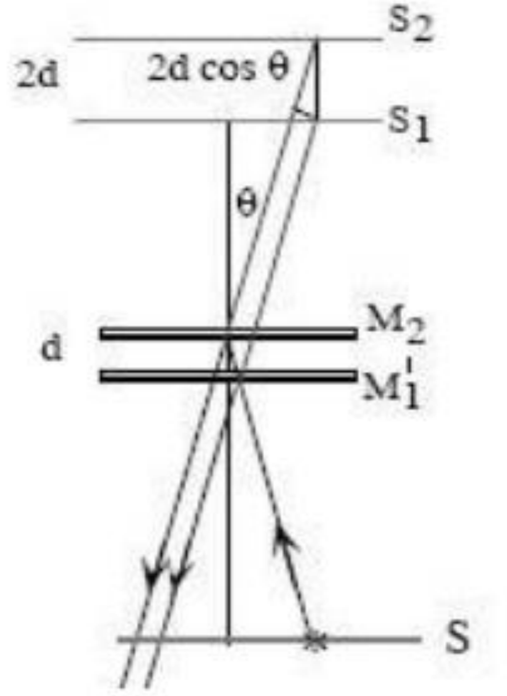
From Fig. if the distance M_1 and M_2 and M'_2 is'd', the distance between S'_1 and S'_2 will be 2D.

If the light ray coming from two virtual sources making an angle θ with the normal then the path difference between the two beams from S_1 and S_2 will becomes

$$\Delta = 2d\cos\theta$$

As one of the ray is reflecting from denser medium mirror M1, a path change of λ/2 occurs in it. Hence the effective path difference between them will be

$$\Delta = 2d\cos\theta \pm \frac{\lambda}{2}$$



If this path difference is equal to an integral number of wavelength λ, the condition for constructive interference is satisfied. Thus the bright fringe will formed.

$$2d\cos\theta \pm \frac{\lambda}{2} = n\lambda \qquad 2d\cos\theta = \left(n \pm \frac{1}{2}\right)\lambda$$

here

$$n = 1,2,3......$$

If this path difference is equal to an integral number of wavelength (2n±1)λ/2, the condition for destructive interference is satisfied. Thus the dark fringe will formed.

$$2d\cos\theta \pm \frac{\lambda}{2} = \left(n \pm \frac{1}{2}\right)\lambda \qquad 2d\cos\theta = n\lambda$$

here

$$n = 1,2,3......$$

Formation of straight line and curved fringes

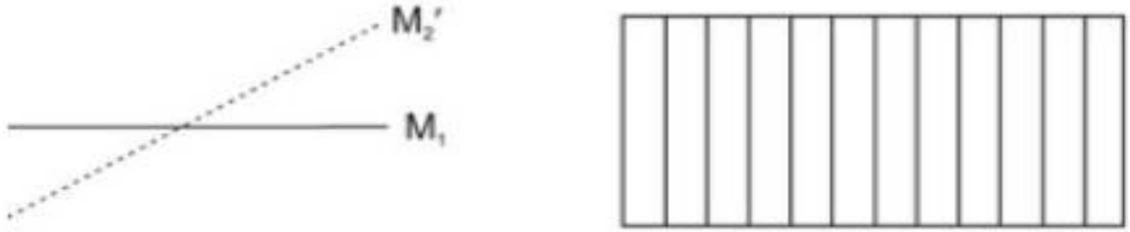


Fig. 4: Formation of straight line fringes

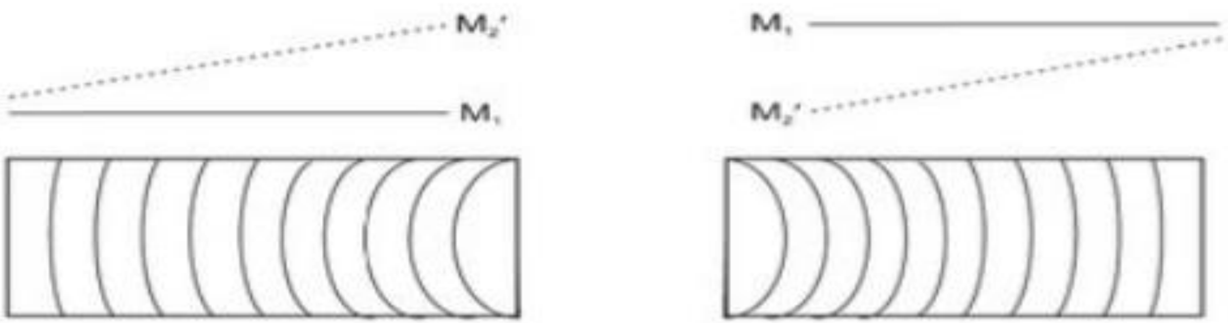


Fig. 3: Formation of curved fringes

Radius of Fringes

The Condition for maxima and minima in MI is given by

$2d\cos\theta = \left(n - \frac{1}{2}\right)\lambda$ For maxima $\qquad$ $2d\cos\theta = n\lambda$ For minima

It is clear that on moving away from center the value of angle θ increases and the value of cos θ decreases hence the order of fringe also decrease so n maximum at center, The condition for nth dark ring at center is

$2d = n\lambda$ ……………Eq 1

On moving m number of rings away from the center, the order of m^{th} ring will be (n-m). If m^{th} ring make an angle θ_m with the axis of telescope then from equation

$2d\cos\theta_m = (n - m)\lambda$ ……………Eq 2

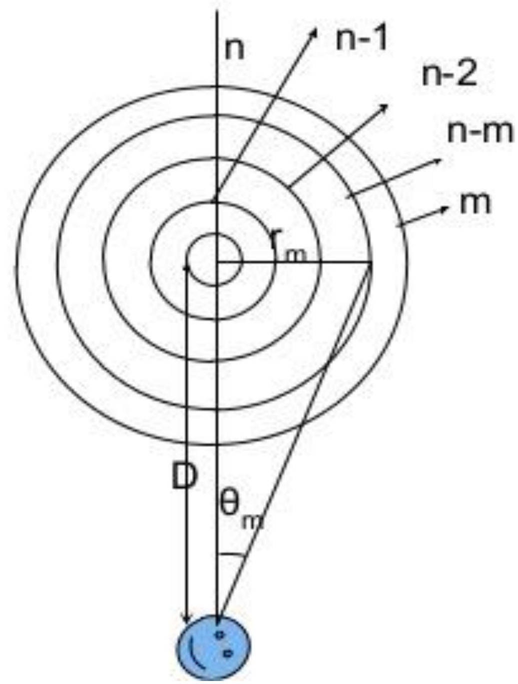
On Subtracting eq 1 and 2

$$2d(1 - \cos\theta_m) = m\lambda$$

$(1 - \cos\theta_m) = \frac{m\lambda}{2d}$ $\quad$ $,\cos\theta_m = 1 - \frac{m\lambda}{2d}$ …Eq 3

Here

$\cos\theta_m = \frac{D}{\sqrt{r_m^2 - D^2}}$ …Eq 4



By eq 3 and 4

$$\frac{D}{\sqrt{r_m^2 - D^2}} = 1 - \frac{n\lambda}{2d} \qquad \text{……………Eq 5}$$

$$\frac{D}{1 - \frac{m\lambda}{2d}} = \sqrt{r_m^2 - D^2} \qquad \text{……………Eq 6}$$

$$\left(\frac{D}{1 - \frac{m\lambda}{2d}} \right)^2 = r_m^2 - D^2 \qquad \text{……………Eq 7}$$

$$r_m^2 = D^2 \left[\left(1 - \frac{m\lambda}{2d} \right)^{-2} - 1 \right] \qquad \text{……………Eq 8}$$

$$r_m^2 = D^2 \left[1 - \frac{2m\lambda}{2d} + \ldots (-1) \right] = D^2 \frac{m\lambda}{d} \qquad \text{……………Eq 9}$$

$$r_m = D\sqrt{\frac{m\lambda}{d}} \qquad \text{……………Eq 10}$$

This equation gives the radius of m^th^ ring

Applications of MI

(1) Measurement of the wavelength of monochromatic light : The mirror M_1 and M_2 adjusted such that circular fringes are formed. For this purpose mirror M1 and M2 are made exactly perpendicular to each other.

Now set the telescope at the center of fringe and move the mirror M1 in any direction, number of fringes shifted in field of view of telescope is counted.

Let on moving mirror M1 through x distance number of fringes shifted is N So the path difference

$$2x = N\lambda \qquad\qquad 2(x_2 - x_1) = N\lambda$$

By using both equations we will calculate wavelength corresponding to distance and number of fringes shifted through telescope.

(2) Determination of the difference in between two nearby wavelengths :- Suppose a source has two nearby wavelengths λ1 and λ2. Each wavelength gives rise its own fringe pattern in MI. By adjusting the position of the mirror M1, aposition will be found where fringes from both wavelength will coincide and form highly contrast fringes.

So the condition is given by

$$\delta = n_1\lambda_1 = n_2\lambda_2 \qquad \text{...............Eq 1}$$

When a mirror M_1 has been moved through a certain distance, the bright fringe due to wavelength λ_1 coincide with dark fringe due to wavelength λ_2 and no fringe will be seen. On further movies mirror M_1 the bright fringes again distinct, this is the position where n_1+m order coincide with n_2+m+1.

So the condition given by

$$\delta + 2x = (n_1 + m)\lambda_1 = (n_2 + m + 1)\lambda_2 \qquad \text{...............Eq 2}$$

Subtracting eq 2 by eq 1

$$2x = m\lambda_1 = (m+1)\lambda_2 \qquad \text{...............Eq 3}$$

$$m(\lambda_1 - \lambda_2) = \lambda_2, m = \frac{\lambda_2}{(\lambda_1 - \lambda_2)} \qquad \text{...............Eq 4}$$

So by eq 4 and 3 $$2x = \frac{\lambda_2}{\lambda_1 - \lambda_2}\lambda_1 = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} = \frac{\lambda_1\lambda_2}{\Delta\lambda}$$

$$\Delta\lambda = \frac{\lambda_1\lambda_2}{2x} = \frac{\lambda^2}{2x}\, where, \lambda = \sqrt{\lambda_1\lambda_2}$$

Problems & Solution

Q.1. In MI 200 fringes cross the field of view when the movable mirror is displaced through 0.05896mm. Calculate the wavelength of the monochromatic light used.

Solution:- Given

N=200

x= 0.05896mm = 0.05896 X 10^{-3} m

So the wavelength $$\lambda = \frac{2x}{N} = \frac{2\times 0.05896\times 10^{-3}\,m}{200} = 5896\times 10^{-10}\,m = 5896\text{Å}$$

Q.2. The initial and final readings of MI screw are 10.7347 mm and 10.7057mm respectively, when 100 fringes pass trough the field of view. Calculate the wavelength of light used.

Solution:- Given

N=100

$x=x_2-x_1$= 10.7347-107057=0.029mm=0.029 X 10^{-3}m

So the wavelength $$\lambda = \frac{2x}{N} = \frac{2\times 0.029\times 10^{-3}\,m}{100} = 5800\times 10^{-10}\,m = 5800\text{Å}$$

Problems & Solution

Q.3. MI is set to form circular fringes with light of wavelength 5000Å. By Changing the path length of movable mirror slowly, 50 fringes cross the center of view How much path length has been changed?

Solution:- Given

N=50

$\lambda = 5000 \times 10^{-10} m$

So the path length

$$d = \frac{n\lambda}{2} = \frac{50 \times 5000 \times 10^{-10} m}{2} = 12.5 \times 10^{-6} m$$

Q.4. In a Michelson Interferometer, when 200 fringes are shifted, the final reading of the screw was found to be 5.3675mm. If the wavelength of light was 5.92 X 10^{-7} m, What was the critical reading of the screw?

Solution:- Given

N=200

$x = x_2 - x_1 = 5.3675 \times 10^{-3} m$ - ?

and wavelength $\lambda = 5.92 \times 10^{-2} m$

So the wavelength

$$\lambda = \frac{2x}{N}, x = \frac{N\lambda}{2} = \frac{200 \times 5.92 \times 10^{-7} m}{2} = 5.92 \times 10^{-5} m$$

Now initial reading of screw $d_1 = d_2 \pm x = 5.3675 \times 10^{-3} m + 0.0592 \times 10^{-3} m = 5.4267 \times 10^{-3} m$

Numericals

- Light containing two wavelengths λ_1 & λ_2 falls normally on a plano convex lens of radius of curvature R resting on a glass plate. If the n^{th} dark ring due to λ_1 coincides with the $(n+1)^{th}$ dark ring due to λ_2. Find the radius of n^{th} dark ring due to $\lambda_{1.}$

- Michelson interferometer experiment is performed with a source which has two wavelengths 4882Å and 4886Å. By what distance does the mirror have to be moved between two positions of disappearance of fringes? (Ans. .00596mm)

- In Newton's ring experiment the diameter of n^{th} and $(n+1)^{th}$ ring are 4.2 mm and 5 mm. If the radius of curvature of lens is 3 m then find the wavelength of light used. (Ans. 6133Å)

Lecture contents with a blend of NPTEL contents and other plateforms

- https://www.youtube.com/watch?v=jtsqsdkjr7g by Prof. G.D. Verma, IIT Roorkee.
- https://nptel.ac.in/courses/115/105/115105120/ by Prof. A. K. Das, IIT Kharagpur.
- https://www.youtube.com/watch?v=UFSniycjqyY by Prof. G. S. Raghuvanshi, JIET Jodhpur.
- https://www.youtube.com/watch?v=F8Cn6jAMa-A by Prof. G. S. Raghuvanshi , JIET Jodhpur.
- https://www.youtube.com/watch?v=n65gZGwiZtk by Prof. M. K. Srivastava, IIT Roorkee.

References and Bibliography

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- Fundamental of Optics by Jetkins and White, Tata McGraw Hill, New Delhi
- Engineering Physics by Prof. Y. C. Bhatt, Ashirwad Publications
- Optics by Subhramanium and Brij lal, S. Chand Publications.

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Thank You