JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem. – B. Tech I year, Sem.-I

Subject –Engineering Mathematics-1

Unit – 5 (Vector **)**

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Designation - Associate Professor

Department - Mathematics

VISSION OF INSTITUTE

To become a renowned centre of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

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❖**Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.**

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❖**Offer opportunities for interaction between academia and industry.**

❖**Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.**

Engineering Mathematics-1: Course Outcomes

Students will be able to:

On completion of this course students will be expected to:

CO1. Understand fundamental concepts of improper integrals, beta and gamma functions and their properties. Evaluation of Multiple Integrals in finding the areas, volume enclosed by several curves after its tracing and its application in proving certain theorems.

CO2. Interpret the concept of a series as the sum of a sequence, and use the sequence of partial sums to determine convergence of a series. Understand derivatives of power, trigonometric, exponential, hyperbolic, logarithmic series.

CO3. Recognize odd, even and periodic function and express them in Fourier series using Euler's formulae.

CO4. Understand the concept of limits, continuity and differentiability of functions of several variables. Analytical definition of partial derivative. Maxima and minima of functions of several variables Define gradient, divergence and curl of scalar and vector functions.

Line Integral:

- let c be a curve defined by
- $x = f(t), y = f(t), z = f(t)$ for t lying in a certain interval . let F be a vector point function defined at all points of the curve c.
- Unit vector t at any point P on the curve is
- $t = \frac{dr}{ds}$
- The component of F along the tangential direction is F.$\frac{dr}{ds}$
- The integral of the tangential components of F integratd along the curve c is called line integral of F and is written as
- $I = \int \left(F.\frac{dr}{ds}\right) ds = \int_c F.dr$
- If $F = f_1 i + f_2 j + f_3 k$ and $dr = dxi + dyj + dzk$
- Then $\int_c F.dr = \int_c (f_1 dx + f_2 dy + f_3 dz).$

Line integral

- If F represent the velocity of the fluid particle and c be a closed curve the integral $\int_c \; F.dr$ is called circulation of F around the curve c.
- The integral $\int_A^B F.dr$ represent the work done by force F, acting on a particle in moving it from A to B along the arc AB.
- If $F = \nabla\emptyset$ for some scalar function$\emptyset$ then $\int_A^B F.dr = \emptyset(B) - \emptyset(A)$ so in this case the work done depends on the values of $\emptyset$ at the curve ends points A and B , not on the path joining A and B.

Surface Integral:

Let F be a vector point function and S be the given surface.

Surface integral of a vector point function F over the surface S is defined as the integral of the the components of F along the normal to the surface.

Components of F along the normal=

$= F.\grave{n}$ where $\grave{n}$ is the unit normal vector to an element ds and $\grave{n} = \frac{\nabla\emptyset}{|\nabla\emptyset|}$ $and\ \ ds = \frac{dxdy}{(\grave{n}.\grave{k})}$

So Surface integral of F over S= $\iint F.\grave{n}\, ds$

If F represent velocity of the liquid the surface integral represent flux.

Q: find the value of the surface integral $\iint_S F.\grave{n}\, ds, where\ F = yzi + zxj + xyk\ and$ S is the part of the surface of sphere $x^2 + y^2 + z^2 = 1$ which lies in first octant.

- Solution:Vector normal to the surface
- $$\nabla\emptyset = 2xi + 2yj + 2zk$$
- Then $\acute{n} =$ a unit vector at any point of S
- $$= (xi + yj + zk)$$
- Nor $\iint_S F.\acute{n}ds = \iint_R F.\acute{n}\frac{dxdy}{|n.k|}$ where R is the projection of the surface S on xy plane . the region R is bounded by x axis , y axis and the circle $x^2 + y^2$=1,and z=0.
- Now $F.\acute{n} = 3xyz$ and $n.k$=z
- So $\iint_S F.\grave{n}\, ds = \iint_R 3xydxdy = 3/8$ by changing to polar coordinate.
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Q: Evaluate the $\iint_S \ F.\acute{n}ds, \ whrer \ F = 18zi + 12j + 3yk$ and S is the surface of the plane $2x + 3y + 6z = 12$ in the first octant.

- Solution: Vector normal to the surface
- $\nabla\emptyset = 2i + 3j + 6k$
- Then $\acute{n} =$ a unit vector at any point of S
- $= \frac{2i+3j+6k}{\sqrt{4+9+36}} = \frac{1}{7}(2i + 3j + 6k)$
- Nor $\iint_S \ F.\acute{n}ds = \iint_R \ F.\acute{n}\frac{dxdy}{|n.k|}$ where R is the projection of the surface S on xy plane . the region R is bounded by x axis , y axis and straight line 2x+3y=12, z=0.
- $= \iint_R \ (18zi + 12j + 3yk).\frac{1}{7}(2i + 3j + 6k) = \frac{1}{6}\iint_R \ (6z + 6 + 3y)dxdy$
- $= \int_0^6 \int_0^{y-\frac{2}{3}x}(6z + 6 + 3y)dxdy = 168.$

Green’s theorem:

- Green’s Theorem gives the relationship between a line integral around a simple closed curve C and a double integral over the plane region D bounded by C.
- Let C be a positively oriented, piecewise-smooth, simple closed curve in the plane and let D be the region bounded by C.
- If P and Q have continuous partial derivatives on an open region that contains D, then
- $$\oint_C (Pdx + Qdy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy$$

Q: Verify Green's theorem in plane for

$\oint_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$, where c, is the boundary of the region defined by $x = 0, y = 0$ and $x + y = 1.$

- Solution: By Green's theorem in plane , we have
- $\oint_c (Pdx + Qdy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy$
- The region R is bounded by the curve C and the curve C consist of straight line OP,PQ and QO where coordinate of P and Q are (1,0) and (0,1).
- Now $\oint_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$
- $= \iint_R \left(\frac{\partial}{\partial x}(4y - 6xy) - \frac{\partial}{\partial y}(3x^2 - 8y^2)\right) dxdy$
- $= 10 \int_0^1 \int_0^{1-x} ydydx$
- $= 5 \int_0^1 (1 - x^2)dx = \frac{5}{3}$
- Now for L.H.S of above

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- Along OP, $y = 0$ then $dy = 0$ and $x = 0$ to $x = 1$.
- Along PQ, $x = 1 - y$ then $dx = -dy$ and $y = 0$ to $y = 1$.
- Along QO, $x = 0$ then $dx = 0$ and $y = 1$ to $y = 0$.

- $\int_{OP} (3x^2 - 8y^2)dx + (4y - 6xy)\,dy +$
  $\int_{PQ} \oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy +$
  $\int_{QO} (3x^2 - 8y^2)dx + (4y - 6xy)dy$
- $= \int_0^1 3x^2 dx + \int_0^1 (11y^2 + 4y - 3)dy + \int_1^0 4y dy$
- $= \frac{5}{3}$
- Here L.H.S = R.H.S
- So Green's theorem is verified.

Stoke’s theorem:

- if F is any continuous differentiable vector function and S is a surface enclosed by a curve C, then
- $\oint_C F.dr = \iint_S (curlF).\acute{n}ds$ where $\acute{n}$ is the outward drawn unit normal vector at any point of the surface S and is drawn in the sense in which a right handed screw would move when rotated in the sense of description.
- Its gives the relation between line integral and surface integral.

Q: Verify Stoke's theorem for $F = yi + zj + xk$, where S is the upper surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

- Solution: let the boundary C of surface S is a circle in the xy- plane of radius unity and center origin. The equation o the curve C are $x^2 + y^2 = 1$, $z = 0$. Let us suppose $x = cost$, $y = sint$, $z = 0$, $0 \leq t \leq 2\pi$ are parametric equation of C . then we have
- $\oint_C F.dr = \oint_C (yi + zj + xk).(dxi + dyj + dzk)$
- $= \oint_C ydx$ as $z = 0, \quad dz = 0$
- $= \int_0^{2\pi} sint \frac{dx}{dt} dt = \int_0^{2\pi} -sin^2 t \, dt = -1/2 \int_0^{2\pi} (1 - cos2t) dt = -\pi$

Unit 5

- Now curl $\vec{V} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial x} & \frac{\partial}{\partial x} \\ y & z & x \end{vmatrix} = -i - j - k$
- Here $\acute{n} = k$ because surface S is the xy-plane.
- Now $\iint_S (curlF).\acute{n}ds = -\iint_S ds$
- $= -(area\ of\ the\ circle\ of\ radius\ 1) = -\pi$
- So Stoke's theorem is verified.

Gauss's Divergence theorem:

- if F be a continuously differentiable vector function in a region V and S is closed surface inclosing the region V then
- $\iint_S F.\acute{n} ds = \iiint_V div\, F. dv$
- where $\acute{n}$ is the outward drawn unit normal vector at any point of the surface S.
- Its gives the relation between surface integral and volume integral.

Q: using divergence theorem to evaluate

$\iint_S F.\acute{n}ds,\ whrer\ F = 4xi - 2y^2j + z^2k$ and S is the surface bounding the region

$x^2 + y^2 = 4, z = 0\ to\ z = 3.$

- Solution: By Divergence theorem , we have
- $\iint_S F.\acute{n}ds = \iiint_V div\,F.dv$
- $\iiint_V \left(\mathrm{i}\frac{\partial}{\partial \mathrm{x}} + j\frac{\partial}{\partial \mathrm{y}} + k\frac{\partial}{\partial \mathrm{z}}\right).(4xi - 2y^2j + z^2k)dv$
- $= \iiint_V (4 - 4y + 2z)dxdydz$
- $= \iint dxdy \int_0^3 (4 - 4y + 2z)dz$
- $= \iint (21 - 12y)dxdy$
- Putting $x = rcos\,\theta, y = rsin\,\theta$ we have
- $\int_0^{2\pi}\int_0^2 (21 - 12rsin\,\theta)rdrd\theta$
- On solving we get,
- $= 84\pi$

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Thank you!

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